Why is our Space-Time 4-Dimensional?

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In this 21st Century, we should claim that we live in the quantum 4-dimensional Minkowski space-time. We wish to "show" that the Standard Model is determined by this quantum 4-dimensional Minkowski space-time, through the dimensionlessness of all the couplings. Thus, we believe that the quantum 4-dimensional Minkowski space-time with the force-fields gauge-group structure $SU_c(3) \times SU_L(2) \times U(1) \times SU_f(3)$ built-in from the very beginning is the background for everything. The lepton world, of atomic sizes, is accepted by this background and offers us the eyes to see other things. The quark world, of the Fermi sizes, also accepted by the background, makes this entire world much more interesting. Unfortunately, we still don't know where the feeble gravitational force comes from.

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1 Prelude

The late Professor Henry Primakoff, my Ph.D. advisor, spoke at his 60th Birthday Symposium, beginning with the question, "Why is our space three-dimensional? Why our time one-dimensional?" In the late 1970's, I did not know the answer and even did not know this would be a meaningful question. I bet that Henry knew part of the answers except that he did not tell the other people.

Until these years (when I reached 60's and when there is no more pressure for the publications), I recognized that in fact these are deep meaningful questions. We attempt to describe the point-like particles by the complex scalar fields, the Dirac fields, and the force-fields gauge fields - there are a lot of mysteries (magics) happening with the 4-dimensional Minkowski space-time. In the 4-dimension, the complex scalar field is naturally born with the self-repulsive interaction, $\lambda(\phi^\dagger\phi)^2$ with positive λ determined by the 4-dimensional Minkowski space-time (*not* by the field ϕ itself). Thus, the complex scalar field, if alone, would be self-repulsive and **cannot exist**.

Once we assume that we live in the 4-dimensional Minkowski space-time, the Einstein relation, i.e., $E^2 = \vec{p}^{\,2} + m^2$, is the origin of everything. The Dirac fields, such as for the electrons, stems from the linearization of the Einstein relations, $E^2 = \vec{p}^{\,2} + m^2$. The resultant Dirac relation, i.e., $E = \beta m + \vec{\alpha}\cdot\vec{p}$ describes the electron so well. The Dirac algebra, satisfied by those sixteen 4×4 matrices, is the *first* place where we may ask why it is 4-dimensional.

One immediately realizes that if we make the space two-dimensional (and time one-dimensional), the linearization, similar to Dirac's, of the Einstein relation could not fly - it is simply not closed. In this way, we may rule out the dimension from three, from five, etc. The dimension of four, i.e., three spatial dimensions and one for time, is the one selected by the Einstein relation plus the Dirac linearization of the Einstein relation. The algebras, including the Dirac algebra, seem to be easily closed at four, but *not* at three or five. Period for the proof.

The Dirac linearization of the Einstein basic relation calls for the two components of the spin, in addition to the existence of the antiparticle. Again, the 4-dimensional Minkowski space-time is the origin of the spin, and also the origin of the antiparticle. Apparently the fundamental importance of the Dirac linearization of the Einstein basic relation was overlooked by physicists throughout the 20th Century. *All these should enter the textbooks in the 21st Century.*

In the 4-dimensional Minkowski space-time, a complex scalar field $\phi(x)$ cannot exist by itself, since the self-repulsive interaction $\lambda(\phi^\dagger\phi)^2$

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is born with it. This self-repulsive interaction exists because of the 4-dimensional Minkowski space-time, independent of the field $\phi(x)$ itself. This is another magic for the 4-dimensional Minkowski space-time. λ is dimensionless (and equals to $\frac{1}{8}$) in this 4-dimensional Minkowski space-time.

So, we live in the (quantum) 4-dimensional Minkowski space-time with the force-fields group $SU_c(3) \times SU_L(2) \times U(1) \times SU_f(3)$ gauge-group structure built-in from the very beginning [1]. The complex scalar (Higgs) fields now exist in pairs: $\Phi(1,2)$ (SM Higgs), $\Phi(3,2)$ (mixed family Higgs), and $\Phi(3,1)$ (purely family Higgs) jointly to make weak bosons $W^{\pm}$, Z^0 and the family bosons massive. This is the "background" of our world.

The lepton world and, separately, the quark world are accepted by the "background" of our world [1]. They all come from Dirac's linearization of the Einstein relation $E^2 = \vec{p}^{\,2} + m^2$. The size of the quark world is typically $(1\,fermi)^3$, which is so much smaller than that of the lepton world of the atom size ($\sim (10^{-8}cm)^3$).

We may emphasize that the quark world is accepted by the "background" in view of its (123) symmetry (i.e., under $SU_c(3) \times SU_L(2) \times U(1)$). It is well-behaved when the energy is very large, or $Q^2 \to \infty$, or when the distance between two quarks is very small, or $r \to 0$. We propose that the lepton world is protected by another (123) symmetry (i.e., under $SU_L(2) \times U(1) \times SU_f(3)$) - so, well-behaved as $Q^2 \to \infty$, or $r \to 0$.

In fact, "the lepton world" or "the quark world" may be regarded not just as "a physical system" but also as "a mathematical system".

To be more specific, in the 4-dimensional Minkowski space-time, what is the behavior of a complex scalar field $\phi(x)$? A complex scalar field would have the following Lagrangian:

$$L = -(\partial_\mu \phi)^\dagger \partial_\mu \phi - \{m^2 \phi^\dagger \phi + \lambda(\phi^\dagger \phi)^2\}, \quad (1)$$

with the last two terms defining the minus of the potential, $-V(x)$.

λ has to be positive to stabilize the system. In only the 4-dimensional Minkowski space-time, λ is dimensionless - it must be a universal constant independent of the field itself, or, it is a characteristic of the 4-dimensional Minkowski space-time. We suspect that $\lambda = \frac{1}{8}$, although at present we do not know how to prove this basic aspect.

A positive λ means that this self-interaction is repulsive, and so this field cannot exist by itself. Note that, when the temperature is high enough (like in the early Universe), the mass term becomes irrelevant (i.e., very small by comparison). In fact, the Standard Model is a virtually dimensionless theory, without couplings with the dimensions except the "ignition" term. This dimensionless theory is also for the early Universe.

Thus, the lagrangian for the single complex scalar field ϕ is fixed (is given) except that the mass term could be adjusted, "fixed" in the 4-dimensional Minkowski space-time. The complex scalar field ϕ is self-repulsive and cannot exist.

That is why we must introduce the related complex scalar fields $\Phi(3,2)$ (mixed family Higgs) and $\Phi(3,1)$ (purely family Higgs) to lower the energy and to make the entire story. (In the notations, the first number refers to $SU_f(3)$ while the second for $SU_L(2)$.) This is the story on the origin of mass [2].

It is just right to have the three Higgs fields, and only the three, - $\Phi(1,2)$ (the Standard-Model Higgs), $\Phi(3,2)$ (mixed family Higgs), and $\Phi(3,1)$ (purely family Higgs) - to make the family gauge bosons all massive and to make the proper room for neutrino oscillations.

2 The Lepton World, the Quark World, and the "Background"

In the Standard Model [3], we live in the quantum 4-dimensional Minkowski space-time with the force-field gauge-group structure $SU_c(3) \times SU_L(2) \times U(1) \times SU_f(3)$ built-in from the outset. This is what we call "the background".

Then, what is the quark world? The quark world is a world of matter form, thus of the type of Dirac equations. It claims a rather small length scale, of about $10^{-13}cm$. The strong-interaction nature of $SU_c(3)$ explains such small size. The color $SU_c(3)$ gauge fields are already classified as part of "the background" - the quarks of three colors and of six flavors are building blocks of matter for the quark world. The quark world knows the gauge group $SU_c(3) \times SU_L(2) \times U(1)$, but not $SU_f(3)$ – the so-called (123) symmetry.

The quark world knows color $SU_c(3)$ well - the strong interaction that acts in the range of fermi's (i.e., $10^{-13}\,cm$). Everything larger than a few fermi would eventually cut off the influence of the strong interaction, unless some special arrangements are given (by the God).

The lepton world is very similar except that the scale is much bigger, at the atomic scale, or $10^{-8}\,cm$. But it does not know the color $SU_c(3)$, except indirectly.

In terms of the size consideration, $10^{-13}\,cm$ versus $10^{-8}\,cm$ so much different, the quark world really has nothing to do with the lepton world. Thus, it is an independent question to ask whether the gauge group $SU_f(3)$ would see the lepton world, or whether to see the quark world. On this point, we also are curious why the strong-interaction group $SU_c(3)$ sees only the quark world, but not the lepton world.

"Our world" is the combination of the background, the quark world, and the lepton world - so, it is quite complicated but in fact all of them are (quantum) point-like particles. Amazingly, each of them could be represented as a branch of mathematics - or, relativistic quantum mechanics and quantum field theory [4].

The decomposition of the Standard Model could make our thoughts much clearer, eventually to adopt a language which is more precise [5]. Such as: we live in the (quantum) 4-dimensional Minkowsi space-time with the force-fields gauge-group structure $SU_c(3)\times SU_L(2)\times U(1)\times SU_f(3)$ built-in from the outset - as the "background" of our world. This background supports the quark world. For some reason, it also supports the lepton world.

To treat the family concept in a force-fields gauge-group framework, we regard [6] $((\nu_\tau,\,\tau)_L,\,(\nu_\mu,\,\mu)_L,\,(\nu_e,\,e)_L)$ $(columns)$ $(\equiv\,\Psi(3,2))$ as the $SU_f(3)$ triplet and $SU_L(2)$ doublet. It is essential to complete the (extended) Standard Model [3] by working out the Higgs dynamics in detail [2]. It is also essential to realize the role of neutrino oscillations - it is the change of a neutrino in one generation (flavor) into that in another generation; or, we need to have the coupling $ih\bar\Psi_L(3,2)\times\Psi_R(3,1)\cdot\Phi(3,2)$, exactly the coupling introduced by Hwang and Yan [6]. Then, it is clear [3] that the mixed family Higgs $\Phi(3,2)$ must be there. The remaining purely family Higgs $\Phi(3,1)$ helps to complete the picture, so that the eight gauge bosons are massive in the $SU_f(3)$ family gauge theory [7].

Remember that the story is pretty much fixed if the so-called "gauge-invariant derivative", i.e. D_μ in the kinetic-energy term $-\bar\Psi\gamma_\mu D_\mu\Psi$, is given for a given basic unit [4]. It seems that this aspect is as fundamental as the Einstein relation, $E^2=\vec p^2+m^2$.

Thus, we have, for the up-type right-handed quarks u_R, c_R, and t_R,

$$D_\mu=\partial_\mu-ig_c\frac{\lambda^a}{2}G^a_\mu-i\frac{2}{3}g'B_\mu, \qquad (2)$$

and, for the rotated down-type right-handed quarks d'_R, s'_R, and b'_R,

$$D_\mu=\partial_\mu-ig_c\frac{\lambda^a}{2}G^a_\mu-i(-\frac{1}{3})g'B_\mu. \qquad (3)$$

On the other hand, we have, for the $SU_L(2)$ quark doublets such as $(u_L,\,d'_L)$,

$$D_\mu=\partial_\mu-ig_c\frac{\lambda^a}{2}G^a_\mu-ig\frac{\vec\tau}{2}\cdot\vec A_\mu-i\frac{1}{6}g'B_\mu. \qquad (4)$$

That is, we are using d'_R, d'_L, etc., consistently. In the quark world, the down-type quarks are always rotated - that means that the so-called GIM are always there.

The mass term from the old Standard-Model way is given by

$$\begin{aligned} L_m \;=\; & -G_1\{\bar d'_R\Phi^\dagger(1,2)Q_{1,L}+h.c.\}\\ & -G'_1\{\bar s'_R\tilde\Phi^\dagger(1,2)Q_{1,L}+h.c.\}\\ & -G_2\{\bar s'_R\Phi^\dagger(1,2)Q_{2,L}+h.c.\}\\ & -G'_2\{\bar c_R\tilde\Phi^\dagger(1,2)Q_{2,L}+h.c.\}\\ & -G_3\{\bar b'_R\Phi^\dagger(1,2)Q_{3,L}+h.c.\}\\ & -G'_3\{\bar t_R\tilde\Phi^\dagger(1,2)Q_{3,L}+h.c.\}. \qquad (5)\end{aligned}$$

Note that the six couplings $G_{1,2,3}$ and $G'_{1,2,3}$ in principle can be adjusted. The unitary mixings, the so-called GIM mechanism, for the down-type quarks helps to forbid the weak neutral current, at the expense of introducing peculiar cross-mass terms. How to detect these "peculiar" interactions through the SM Higgs studies would be something of importance and urgency.

One important observation: The couplings g_c, g, g', and six G's all are dimensionless. Or, they are determined by the 4-dimensional Minkowski space-time. They may be the property of the quantum 4-dimensional Minkowski space-time, *even though we don't know how to prove it as a theorem for the quark world.*

Now, we turn our attention to the lepton world, which includes the electrons as one well-known species.

Since we put all six objects as a representation in the group, we agree that all these objects are point-like Dirac particles - *not* the mixture of Dirac particles and Majorana particles. Moreover, particles of the second or third generation must be of the same characteristics as the particles of the first generation. All these are the group theory in mathematics - we physicists sometime forget the mathematics ABC.

Of course, we are too far in proving experimentally that these neutrinos are also point-like Dirac particles. The regularities, or the symmetries, sort of give us the confidence in all this regarding the Standard Model.

In the lepton world, we introduce the family triplet, $(\nu_\tau^R,\, \nu_\mu^R,\, ,\nu_e^R)$ (column), under $SU_f(3)$. Since the minimal Standard Model does not see the right-handed neutrinos, it would be a natural way to make an extension of the minimal Standard Model. Or, we have, for $(\nu_\tau^R,\, \nu_\mu^R,\, \nu_e^R)$,

$$D_\mu = \partial_\mu - i\kappa \frac{\bar{\lambda}^a}{2} F_\mu^a. \tag{6}$$

and, for the left-handed $SU_f(3)$-triplet and $SU_L(2)$-doublet $((\nu_\tau^L,\, \tau^L),\, (\nu_\mu^L,\, \mu^L),\, (\nu_e^L,\, e^L))$ (all columns),

$$D_\mu = \partial_\mu - i\kappa \frac{\bar{\lambda}^a}{2} F_\mu^a - ig\frac{\vec{\tau}}{2}\cdot \vec{A}_\mu + i\frac{1}{2} g' B_\mu. \tag{7}$$

The right-handed charged leptons form the triplet $\Psi_R^C(3,1)$ under $SU_f(3)$, since it were singlets their common factor $\bar{\Psi}_L(\bar{3},2)\Psi_R(1,1)\Phi(3,2)$ for the mass terms would involve the cross terms such as $\mu \to e$.

The neutrino mass term assumes the *unique* form:

$$i\frac{h}{2}\bar{\Psi}_L(3,2)\times \Psi_R(3,1)\cdot \Phi(3,2) + h.c., \tag{8}$$

Here the Higgs field $\Phi(3,2)$ is the mixed family Higgs, because it carries some nontrivial $SU_L(2)$ charge. In fact, the charged part of $\Phi(3,2)$ does not experience the spontaneous symmetry breaking (SSB), as worked out explicitly in [2].

We wish to note that, in view of that if (ϕ_1,ϕ_2) is an $SU(2)$ doublet then $(\phi_2^\dagger, -\phi_1^\dagger)$ is another doublet, we could form $\tilde{\Phi}^\dagger(3,2)$ from the doublet-triplet $\Phi(3,2)$. Thus, we also have

$$i\frac{h^C}{2}\bar{\Psi}_L(3,2)\times \Psi_R^C(3,1)\cdot \tilde{\Phi}^\dagger(3,2) + h.c., \tag{9}$$

which gives rise to the imaginary off-diagonal (hermitian) elements in the 3×3 mass matrix, so removing the equal masses of the charged leptons.

Now, everything is dimensionless. It has to do with the 4-dimensional Minkowski space-time; it has nothing to do with the individual fields. Hence, this story is absolutely beautiful - the act of the Einstein relation and of its Dirac's linearization in the quantum 4-dimensional Minkowski space-time. Either the lepton world or the quark world is the necessary attachment of the 4-dimensional Minkowski space-time. A beautiful result!!

To understand "The Origin of Mass" [2], we realize that, before the spontaneous symmetry breaking (SSB), the Standard Model does not contain any parameter that is pertaining to "mass", but, after the SSB, all particles in the Standard Model acquire the mass terms as it should - a way to explain "the origin of mass". In this way, we sort of tie "the origin of mass" to the effects of the SSB, or the generalized Higgs mechanism.

In fact, that sets the unique stage for the dimensionless interaction $\lambda(\phi^\dagger\phi)^2$ in the 4-dimensional Minkowski space-time. We realized that, to begin with, we have the three Higgs fields and, then, the (elusive) purely family Higgs $\Phi(3,1)$ could work the best as the "ignition" channel, even though this deviates from the standard wisdom of using $\Phi(1,2)$ as the "ignition" channel.

Thus, we have to have the various Higgs at our disposal, but not too many in view of the repulsive nature of these scalar fields. In the model [3], we have the Standard-Model Higgs $\Phi(1,2)$, the purely family Higgs $\Phi(3,1)$, and the mixed family Higgs $\Phi(3,2)$, with the first label for $SU_f(3)$ and the second for $SU_L(2)$. We need another triplet $\Phi(3,1)$ since all eight family gauge bosons are massive [7].

In another arXiv paper [5], we try to introduce the joint-group space, "$SU_c(3)\times SU_L(2)\times U(1)\times SU_f(3)$ Minkowski space-time", in the effort of trying to find out what would be the constraints on the complex scalar fields. First of all, we have to recognize the special importance of the dimensionless interaction $\lambda(\phi^\dagger\phi)^2$, the only pure number λ for the 4-dimensional low-spin fields. We find $\lambda = \frac{1}{8}$, without knowing the underlying reason. Secondly, those unrelated complex fields could be described by $\lambda(\phi_a^\dagger\phi_a + \phi_b^\dagger\phi_b)^2$ (with $a \neq b$), through a repulsive interaction. Thus, we can write an "attractive" interaction, $(\phi_a^\dagger \cdot \phi_b) \cdot (\phi_b^\dagger \cdot \phi_a)$, for only those related complex fields. We use the "maximal" attraction in understanding the origin of mass [2].

In other words, we believe that we live in the quantum 4-dimensional Minkowski space-time with the force-fields gauge-group structure $SU_c(3) \times SU_L(2) \times U(1) \times SU_f(3)$ built-in from the very beginning. Only in this world (i.e., our world), the complex scalar fields have those "peculiar" characteristics - these characteristics are determined by the 4-dimensional Minkowski space-time, *not* by the fields themselves.

We may write down all the terms for potentials among the three Higgs fields, subject to (1) that they are renormalizable, and (2) that symmetries are only broken spontaneously (i.e., the generalized Higgs mechanism). This is what we do in constructing the Standard Model [3].

In the U-gauge, we choose to have

$$\begin{aligned} \Phi(1,2) &= (0, \frac{1}{\sqrt{2}}(v+\eta)), \\ \Phi^0(3,2) &= \frac{1}{\sqrt{2}}(u_1+\eta_1', u_2+\eta_2', u_3+\eta_3'), \\ \Phi(3,1) &= \frac{1}{\sqrt{2}}(w+\eta', 0, 0), \end{aligned} \quad (10)$$

all in columns. The five components of the complex triplet $\Phi(3,1)$ get absorbed by the $SU_f(3)$ family gauge bosons and the neutral part of $\Phi(3,2)$ has three real parts left - together making all eight family gauge bosons massive.

Basically, we may write down a general renormalizable lagrangian, like in our work in constructing the Standard Model [3]. But, because of three "cooperative" complex scalar Higgs fields, because of a universal λ, and because of only one "ignition" point, the real lagrangian becomes rather simple [2]. It seems that λ $(= \frac{1}{8})$ should obey some sort of non-renormalization theorem, owing to that it is determined completely by the 4-dimensional Minkowski space-time, *not* by the complex scalar fields themselves.

It is an interesting question to ask why there is only one "ignition" channel, and whether we could do away with this "ignition" channel. When we examine such questions much further, we realize that, without the dimensional "ignition" term, our theory becomes completely dimensionless, all couplings in the background, in the quark world, and in the lepton world — they are determined completely by the 4-dimensional Minkowski space-time, *not* by its contents. That is why we have the faith in this theory, the so-called "Standard Model".

In short, we begin with [2]

$$\begin{aligned} V_{Higgs} = \; & \mu_2^2\Phi^\dagger(3,1)\Phi(3,1) \\ & +\lambda(\Phi^\dagger(1,2)\Phi(1,2) \\ & +cos\theta_P\Phi^\dagger(3,2)\Phi(3,2))^2 \\ & +\lambda(-4cos\theta_P)(\Phi^\dagger(\bar{3},2)\Phi(1,2)) \\ & \quad (\Phi^\dagger(1,2)\Phi(3,2)) \\ & +\lambda(\Phi^\dagger(3,1)\Phi(3,1) \\ & +sin\theta_P\Phi^\dagger(3,2)\Phi(3,2))^2 \\ & +\lambda(-4sin\theta_P)(\Phi^\dagger(\bar{3},2) \\ & \quad \Phi(3,1))(\Phi^\dagger(3,1)\Phi(3,2)) \\ & +\lambda_2'\Phi^\dagger(\bar{3},1)\Phi(3,1)\Phi^\dagger(1,2)\Phi(1,2) \\ & +(terms\ in\ i\delta's \\ & \quad and\ in\ decay). \end{aligned} \quad (11)$$

These are two prefect squares minus the other extremes, to guarantee the positive definiteness, when the minus μ_2^2 was left out. (θ_P may be referred to as "Pauchy's angle".)

We obtain [2, 3]

$$v^2(3cos^2\theta_P - 1) = sin\theta_P cos\theta_P w^2. \quad (12)$$

And the SSB-driven η' yields

$$w^2(1-2sin^2\theta_P) = -\frac{\mu_2^2}{\lambda} + (sin2\theta_P - tan\theta_P)v^2. \quad (13)$$

These two equations show that it is necessary to have the driving term, since $\mu_2^2 = 0$ implies that everything is zero. Also, $\theta = 45°$ is the (lower) limit.

The mass squared of the SM Higgs η is $2\lambda cos\theta_P u_i u_i$ (noting the factor of two), as known to be $(125\ GeV)^2$. The famous v^2 is the number divided by 2λ, or $(125\ GeV)^2/(2\lambda)$. Using PDG's for e, $sin^2\theta_W$, and the W-mass [8], we find $v^2 = 255\ GeV$. So, we set $\lambda = \frac{1}{8}$, a simple model indeed.

The mass squared of η' is $-2(\mu_2^2 - sin\theta_P u_1^2 + sin\theta_P(u_2^2+u_3^2))$. The other condensates are $u_1^2 = cos\theta_P v^2+sin\theta_P w^2$ and $u_{2,3}^2 = cos\theta_P v^2 - sin\theta_P w^2$ while the mass squared of η'_1 is $u_1^2\lambda$, those of $\eta'_{2,3}$ be $u_{2,3}^2\lambda$. The mixings among η'_i themselves are neglected in the paper.

There is no SSB for the charged Higgs $\Phi^+(3,2)$. The mass squared of ϕ_1 is $\lambda(cos\theta_P v^2 - sin\theta_P w^2) + \frac{\lambda}{2}u_i u_i$ while $\phi_{2,3}$ be $\lambda(cos\theta_P v^2 + sin\theta_P w^2) + \frac{\lambda}{2}u_i u_i$.

A further look of these equations tells that $3cos^2\theta_P - 1 > 0$ and $2sin^2\theta_P - 1 > 0$. A narrow range of θ_P is allowed (greater than 45° while less than 57.4°, which is determined by the group structure). For illustration, let us choose $cos\theta_0 = 0.6$ and work out the numbers as follows: (Note that $\lambda = \frac{1}{8}$ is used.)

$$\begin{aligned}
& 6w^2 = v^2, \\
& -\mu_2^2/\lambda = 0.32v^2; \\
\eta: \quad & m^2(\eta) = (125\,GeV)^2, \\
& v^2 = (250\,GeV)^2; \\
\eta': \quad & m^2(\eta') = (51.03\,GeV)^2, \\
& w^2 = v^2/6; \\
\eta'_1: \quad & m^2(\eta'_1) = (107\,GeV)^2, \\
& u_1^2 = 0.7333v^2; \\
\eta'_{2,3}: \quad & m^2(\eta'_{2,3}) = (85.4\,GeV)^2, \\
& u_{2,3} = 0.4667v^2; \\
\phi_1: \quad & mass = 100.8\,GeV; \\
\phi_{2,3}: \quad & mass = 110.6\,GeV. \qquad (14)
\end{aligned}$$

All numbers appear to be reasonable. Since the new objects need to be accessed in the lepton world, it would be a challenge for our experimental colleagues.

As for the range of validity, $\frac{1}{3} \le cos^2\theta_P \le \frac{1}{2}$. The first limit refers to $w^2 = 0$ while the second for $\mu_2^2 = 0$.

Before the family Higgs are discovered, we may try to "guess" the various couplings, using our common senses. κ, the basic coupling of the family gauge bosons would be the most important in this "guess". The electroweak coupling g is 0.6300 while the strong QCD coupling $g_s = 3.545$ (order of unity); my first guess for κ would be about 0.1 (which is rather small). The masses of the family gauge bosons would be estimated by using $\frac{1}{2}\kappa \cdot w$, so slightly less than $10\,GeV$. (In the numerical example with $cos\theta_P = 0.6$, we have $6w^2 = v^2$ or $w = 102\ GeV$. This gives $m = 5\ GeV$ as the estimate.) So, the range of the family forces, existing in the lepton world, would be 0.04 *fermi*.

In [2], the term that ignites the SSB is chosen to be with η', the purely family Higgs. This in turn ignites EW SSB and others. It explains the origin of all the masses, in terms of the spontaneous symmetry breaking (SSB). SSB in $\Phi(3,2)$ is driven by $\Phi(3,1)$, while SSB in $\Phi(1,2)$ from the driven SSB by $\Phi(3,2)$, as well. The different, but related and each self-repulsive, complex scalar fields can accomplish so much, to our surprise. And these Higgs are exactly those the gauge fields (i.e., the force-fields) demand.

In [2], we have the prediction (or, post-prediction) that $m_{SM}(\eta) = v/2$, a relation satisfied well. Our criterion is that the "ignition" should cover the "entire" Higgs sector.

The "uniqueness" in the determination of the λ means that the choice of the potential is unique [2] and so the Standard Model is unique. The angle θ_P is the only unknown.

3 Episode

We are astonished to discover that the 4-dimensional Minkowski space-time is the origin of the Standard Model. The Einstein basic relation, $E^2 = \vec{p}^{\,2} + m^2$, and its Dirac's linearization play the role of generating the "background", the lepton world, and the quark world, leading to the Standard Model [3].

So far our knowledge concerning the Standard Model does not give us any clue regarding the origin of the extremely feeble gravitational force. Einstein might be surprised that his basic relation and Dirac's linearization is the origin of the Standard Model, while leaving the question of the feeble gravitational force completely open.

References

[1] W-Y. Pauchy Hwang, The Universe, **3-1**, 3 (2015); "The Origin of Fields (point-like Particles)".

[2] W-Y. Pauchy Hwang, The Universe, **2-2**, 34 (2014); "The Origin of Mass".

[3] W-Y. Pauchy Hwang, arXiv:1340.4705v2 [hep-ph] 25 Aug (2015); "The Standard Model".

[4] Ta-You Wu and W-Y. Pauchy Hwang, "Relativistic Quantum Mechanics and Quantum Fields" (World Scientific, 1991), currently being updated into the second edition.

[5] W-Y. Pauchy Hwang, arXiv:1409.6077v1 [hep-ph] 22 Sep 2014.

[6] W-Y. Pauchy Hwang and Tung-Mow Yan, The Universe, **1-1**, 5 (2013); arXiv:1212.4944v1 [hep-ph] 20 Dec 2012.

[7] W-Y. Pauchy Hwang, Nucl. Phys. **A844**, 40c (2010); W-Y. Pauchy Hwang, Int. J. Mod. Phys. **A24**, 3366 (2009); the idea first appeared in hep-ph, arXiv: 0808.2091; talk presented at 2008 CosPA Symposium (Pohang, Korea, October 2008), Int. J. Mod. Phys. Conf. Ser. **1**, 5 (2011); plenary talk at the 3rd International Meeting on Frontiers of Physics, 12-16 January 2009, Kuala Lumpur, Malaysia, published in American Institute of Physics 978-0-7354-0687-2/09, pp. 25-30 (2009).

[8] Particle Data Group, "Review of Particle Physics", J. Phys. G: Nucl. Part. Phys. **37**, 1 (2010); and its biennual publications.