

Are Real Mass Tachyons Responsible for Inflation?

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Abstract. The creation and annihilation of real mass tachyons is a possibility within the framework of parametrized Relativistic Quantum Theory (pRQT). This theory is used to consider the following question: are real mass tachyons responsible for cosmic inflation? In this paper we show that pRQT tachyon kinematics can provide a mechanism for achieving cosmic inflation.

1. Introduction

The Lambda CDM (Λ CDM) model of cosmology is the leading cosmological model [1-3]. One of the key features of the Λ CDM model is cosmic inflation [1, 4]. Cosmic expansion in the aftermath of the Big Bang is influenced by dark matter, which is believed to hinder expansion following inflation. In addition, dark energy is believed to accelerate expansion after approximately five billion years of expansion. The focus of this article is expansion during cosmic inflation.

Mechanisms for achieving cosmic inflation are typically associated with quantum fluctuations [5], but other mechanisms have been studied, such as MOND, or Modified Newtonian Dynamics, introduced by Milgrom [6-8]. An alternative inflation mechanism is considered here within the framework of parametrized Relativistic Quantum Theory (pRQT). The creation and annihilation of real mass tachyons is a feature of pRQT [9-11]. The associated tachyon kinematics in pRQT is used to show that tachyons could be responsible for achieving cosmic inflation within the pRQT framework.

2. What is Cosmic Inflation?

Cosmic inflation was suggested as a means of addressing issues associated with the Friedmann-Lemaître-Robertson-Walker (FLRW) metric. The FLRW metric was the standard Big Bang metric. It is sometimes referred to as the Robertson-Walker metric. The FLRW metric is a solution of Einstein's general relativistic equations. According to Wheeler, the relationship between spacetime and matter within the framework of general relativity can be summarized as follows: "Spacetime tells matter how to move; matter tells spacetime how to curve" [pg. 235 of 12].

The FLRW metric has issues associated with flatness, the horizon, and anisotropy. The flatness problem is associated with the observation that the present density of matter in the universe is approximately equal to critical density. This implies that the universe appears to be nearly flat or Euclidean. The flatness problem raises the question: why is the present density of matter in the universe approximately equal to critical density?

The Horizon problem is an issue that refers to the transfer of information between regions that are space-like separated. Regions that are space-like separated can be thought of as isolated systems because they cannot share information. An issue that is related to the horizon problem

arises when we recognize that the size of the universe is finite, that light has finite speed, and the cosmic background radiation (CBR) is in approximate thermal equilibrium. This raises the question: how was thermal equilibrium established throughout the universe if information can only be exchanged at a rate that does not exceed the speed of light?

CBR observations also suggest that differences present initially led to an anisotropic distribution of matter. The differences seem to be a relic of the Big Bang. What physical process can make space-like regions so similar and introduce differences that seed structure? This is the anisotropy issue.

One way to resolve these issues is to hypothesize that cosmic inflation occurs in the early universe as a period of accelerated expansion. Figure 1 shows the inflation era in the Guth inflationary model [4, 13]. The inflation model starts at a small radius and rapidly expands until it matches the FLRW metric of the standard model. The inflation model helped resolve the FLRW metric issues by magnifying local density fluctuations and equalizing the temperature of the universe that has been observed in the relatively smooth CBR.

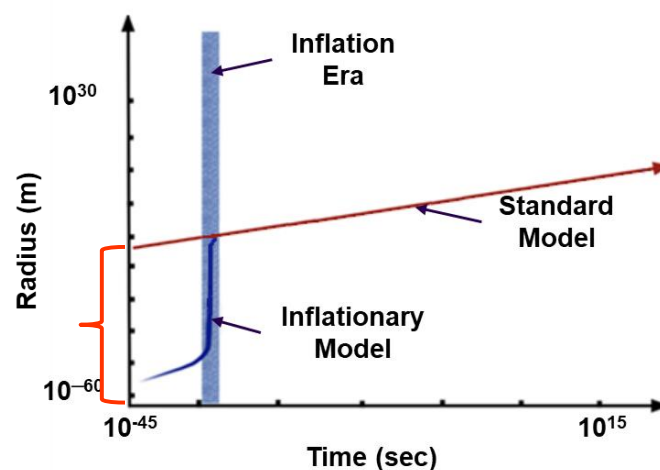


Figure 1. Cosmic inflation is a period of accelerated expansion.

The inflation model is known as the Λ CDM model. CDM refers to cold, dark matter in addition to ordinary matter. The term Λ refers to the cosmological constant. It represents the energy density of empty space and is associated with dark energy. The duration of expansion in the figure is between 10^{-35} second to 10^{-32} second. The radius of expansion is approximately 10^{-50} m to 10^{-10} m. The approximate rate of expansion is given by the ratio of the radius of expansion to the duration of expansion. If the radius of expansion is 10^{-50} m in a duration of 10^{-35} second, the rate of expansion is 10^{-15} m/s, which is bradyonic. If the radius of expansion is 10^{-10} m in a duration of 10^{-32} second, the rate of expansion is 10^{+22} m/s, which is tachyonic. The estimated tachyonic rate of expansion is discussed further in a subsequent section.

3. Prelude to Real Mass Tachyons

Most theories of tachyonic physics use tachyons with imaginary mass because the square of particle mass is negative (many references are presented in [11]). In this section, we introduce

pRQT, the theoretical framework for real mass tachyons. Real mass tachyons and a model of tachyon kinematics are discussed in the next section.

3.1 Parametrizing Spacetime Observations

We can interpolate between two events by introducing a parameter s that lets us parametrize spacetime observations. Figure 2 displays a parametrized particle world-line [14]. The system is deterministic if we only allow a single world-line. It is probabilistic if we allow many possible world-lines. The variables x , t , and s are independent variables, while the means $\langle x \rangle$ and $\langle t \rangle$ may be correlated using the parameter s . To retain the manifestly covariant character of the theory, the parameter s needs to be an invariant parameter.

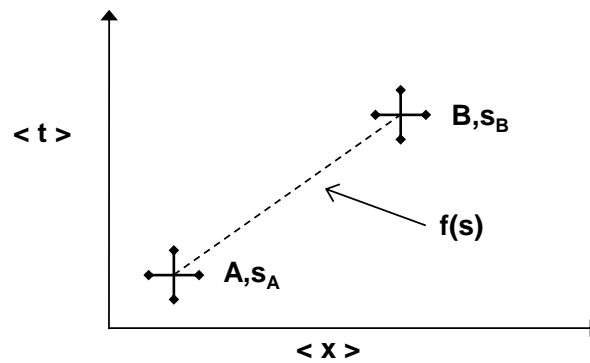


Figure 2. Parametrizing a particle world-line.

Feynman [15, 16] used path integrals to derive a relativistic quantum mechanical equation with invariant evolution parameter. He suggested a path integral formulation where probability amplitudes were defined in terms of an action S . The action S is expressed in terms of the Lagrangian L by

$$S(x_i, t_i) = \int L(\dot{x}_i, \dot{t}_i) ds \quad (3.1)$$

where $\dot{x} = \frac{dx}{ds}$, $\dot{t} = \frac{dt}{ds}$. The evolution of the probability amplitude was found by integrating over all possible paths from spacetime point A to spacetime point B:

$$\Phi(x_{i+1}, t_{i+1}, s + \varepsilon) = \frac{1}{N} \int e^{iS(x_i, t_i)/\hbar} \Phi(x_i, t_i, s) dx_i d(ct_i) \quad (3.2)$$

As an example, assume the Lagrangian for a free particle with mass m is

$$L(\dot{x}, \dot{t}) = \frac{m}{2} \left[\left(c \frac{dt}{ds} \right)^2 - \left(\frac{dx}{ds} \right)^2 \right] \quad (3.3)$$

The action for the free particle is

$$S(x_i, t_i) = \varepsilon \frac{m}{2} \left[c^2 \left(\frac{t_{i+1} - t_i}{\varepsilon} \right)^2 - \left(\frac{x_{i+1} - x_i}{\varepsilon} \right)^2 \right] \quad (3.4)$$

The path integral approach yields a Stueckelberg-like equation for a free particle

$$\frac{\partial \Phi}{\partial s} = \frac{i\hbar}{2m} \left[\frac{1}{c^2} \frac{\partial^2 \Phi}{\partial t^2} - \frac{\partial^2 \Phi}{\partial x^2} \right] \quad (3.5)$$

If we write the parametrized wave function Φ as the stationary state solution

$$\Phi(x,t,s) = \Phi_{KG}(x,t) \exp \left[-i \frac{M^2 c^2 s}{2m\hbar} \right] \quad (3.6)$$

we obtain the Klein-Gordon equation

$$M^2 c^2 \Phi_{KG} = -\hbar^2 \partial^\mu \partial_\mu \Phi_{KG} \quad (3.7)$$

3.2 Dynamical Evolution of a Relativistic System

Figure 3 illustrates two different spacetime diagrams evolving with respect to an invariant parameter s . A single spacetime diagram is a static system relative to the invariant parameter. A dynamic system consists of multiple spacetime diagrams that are linked by an invariant parameter.

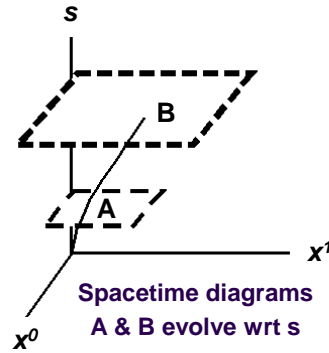


Figure 3. The evolution of spacetime diagrams with respect to an invariant evolution parameter.

The dynamical evolution of a relativistic system can be expressed using a unitary operator, that is, a function of a scalar evolution parameter. Several possible generators of invariant parameter translations are possible for a parametrized field equation. Examples of distinct generators were presented by Fock [17], Stueckelberg [18-20], Nambu [21], and Johnson [22]. The Stueckelberg approach led to the parametrized field equation

$$i\hbar \frac{\partial \psi(y,s)}{\partial s} = \left[\frac{\pi^\mu \pi_\mu}{2m} + V_I \right] \psi(y,s) \quad (3.8)$$

where V_I is the interaction potential. The operator π^μ is

$$\pi^\mu = \frac{\hbar}{i} \frac{\partial}{\partial y_\mu} - \frac{e}{c} A^\mu \quad (3.9)$$

with four-vector potential A^μ .

3.3 Probability-Based Relativistic Dynamics – pRQT

Equation (3.8), referred to here as the Stueckelberg equation, can be derived within the framework of pRQT, a probability-based approach. We begin by outlining the probability concepts that are needed to develop the kinematics of real mass tachyons.

A positive definite conditional probability density $\rho(y|s)$ that depends on spacetime y and invariant parameter s can be described in terms of a wave function $\psi(y,s)$ as

$$\rho(y|s) = \psi^*(y,s) \psi(y,s) \quad (3.10)$$

The probability density is normalized over spacetime

$$\int_D \rho(y|s) dy = 1 \quad (3.11)$$

The continuity equation that includes an invariant parameter is

$$\frac{\partial \rho}{\partial s} + \frac{\partial \rho V^\mu}{\partial y^\mu} = 0 \quad (3.12)$$

where Einstein's summation convention for Greek indices is assumed and the term ρV^μ represents the μ^{th} component of probability flux of a particle.

If we write the wavefunction as

$$\psi(y, s) = [\rho(y|s)]^{1/2} \exp[i\xi(y, s)] \quad (3.13)$$

and a velocity four-vector

$$V^\mu(y, s) = \frac{1}{m} \left[\hbar \frac{\partial \xi(y, s)}{\partial y_\mu} - \frac{e}{c} A^\mu(y, s) \right] \quad (3.14)$$

we obtain the probability flux

$$\rho V^\mu = \frac{i\hbar}{2m} \left[\psi^* \frac{\partial \psi}{\partial y_\mu} - \psi \frac{\partial \psi^*}{\partial y_\mu} \right] - \frac{eA^\mu}{mc} \psi^* \psi \quad (3.15)$$

Substituting these equations into the continuity equation and rearranging lets us derive a single particle Stueckelberg equation

$$i\hbar \frac{\partial \psi(y, s)}{\partial s} = \left[\frac{\pi^\mu \pi_\mu}{2m} + V_I \right] \psi(y, s) \quad (3.16)$$

with interaction potential V_I and four-momentum operator with minimal coupling

$$\pi^\mu = \frac{\hbar}{i} \frac{\partial}{\partial y_\mu} - \frac{e}{c} A^\mu \quad (3.17)$$

The expectation value of operator Ω is

$$\langle \Omega \rangle \equiv \int_D \psi^* \Omega \psi dy \quad (3.18)$$

The above procedure can be used to construct an N-body Stueckelberg equation [23]. The N-body formulation has been used to design an evolution parameter clock [24, 25]. A model of tachyon kinematics constructed from the N-body formulation is reviewed in the next section.

4. Real Mass Tachyons

A distinction is made here between real mass tachyons and imaginary mass tachyons commonly associated with more traditional, non-parametrized theories. The pRQT framework can be used to show that particles can exist as tachyons with real mass. In addition, the pRQT framework can be used to explain how a particle can transition between bradyon and tachyon regions of spacetime. These results are then used to determine if cosmic inflation can be explained as a tachyon phenomenon. Real mass tachyons and a model of tachyon kinematics [9-11] are discussed in this section.

4.1 Free Particle Mass in pRQT

The free particle Stueckelberg equation is

$$i\hbar \frac{\partial \psi_f}{\partial s} = - \frac{\hbar^2}{2m} \frac{\partial^2 \psi_f}{\partial y^\mu \partial y_\mu} \quad (4.1)$$

with the solution

$$\psi_f(y, s) = \eta^{1/2} \exp \left[-\frac{i\hbar}{2m} (k^\mu k_\mu) s + i k_\mu y^\mu \right] \quad (4.2)$$

The constant η is calculated by imposing probability normalization, and the four-vector k^μ is the wavenumber. The observable world-line of the free particle is expressed in terms of expectation values as

$$\delta \langle y^\mu \rangle \delta \langle y_\mu \rangle = \frac{\hbar^2 \langle k^\mu \rangle \langle k_\mu \rangle}{m^2} \delta s^2 \quad (4.3)$$

Equation (4.3) can be written in the classical limit of negligible dispersion as

$$\delta \langle y^\mu \rangle \delta \langle y_\mu \rangle = \frac{\hbar^2 \langle k^\mu k_\mu \rangle}{m^2} \delta s^2 \quad (4.4)$$

The observable on-shell mass of the free particle is

$$\langle p^\mu p_\mu \rangle = \hbar^2 \langle k^\mu k_\mu \rangle \equiv m^2 c^2 \quad (4.5)$$

Timelike and spacelike motion can be displayed by rewriting Equation (4.3) as

$$\frac{m^2}{\delta s^2} = \frac{\hbar^2 \langle k^\mu \rangle \langle k_\mu \rangle}{\delta \langle y^\mu \rangle \delta \langle y_\mu \rangle} \quad (4.6)$$

If we recognize that the change δs of the invariant evolution parameter is positive, then $m^2 > 0$ when $\frac{\hbar^2 \langle k^\mu \rangle \langle k_\mu \rangle}{\delta \langle y^\mu \rangle \delta \langle y_\mu \rangle} > 0$ for timelike and spacelike motion. This can be illustrated by rearranging and plotting $\frac{m^2}{\delta s^2} = \frac{\hbar^2 \langle k^\mu \rangle \langle k_\mu \rangle}{\delta \langle y^\mu \rangle \delta \langle y_\mu \rangle}$ as $z = \frac{x}{y}$ for independent variables x, y . Figure 4 shows regions of real mass exist for both timelike and spacelike motion when $z = \frac{x}{y} = \frac{m^2}{(\delta s)^2} > 0$.

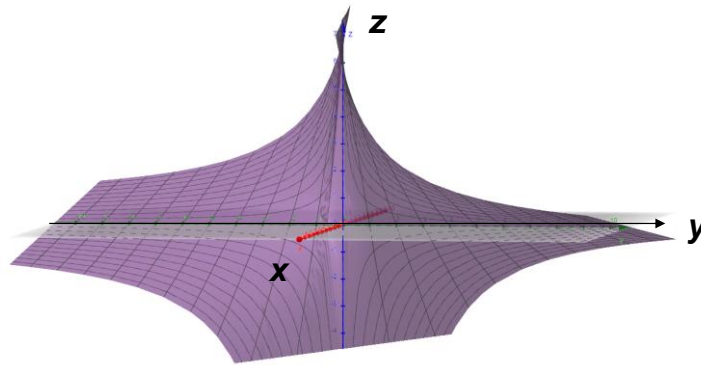


Figure 4. Regions of free particle real mass.

4.2 Particle Transitions Across the Light Cone

Section 4.1 shows that real mass tachyons can exist within the pRQT framework. The question becomes: how do massive particles cross the light cone? A mechanism for achieving the transition has been discussed previously [9-11] and is outlined here.

The mechanism of interest has a non-relativistic analog: the transition between energy states. In the non-relativistic quantum case, time-dependent interaction potentials enable transitions between energy states such as those observed in lasers. The pRQT mechanism is based on a mass state transition at a scattering vertex. In this case, an invariant evolution parameter-dependent

interaction can be represented as the interaction of a projectile with a target to yield a product particle, i.e. Projectile (Ψ) + Target (Φ_T) \rightarrow Product. A physical model is a particle beam scattering off a thin foil.

A field equation for the interaction of a projectile with a target to yield a product particle is

$$i\hbar \frac{\partial \Psi}{\partial s} = -\frac{\hbar^2}{2m_\Psi} \frac{\partial^2 \Psi}{\partial x^\mu \partial x_\mu} + g(\Phi_T + \Phi_T^*)\Psi \quad (4.7)$$

It can be written in the s -dependent perturbation form

$$i\hbar \frac{\partial \Psi}{\partial s} = K_0 \Psi + K_1 \quad (4.8)$$

where K_0 refers to the unperturbed operator and K_1 refers to the s -dependent interaction term $g(\Phi_T + \Phi_T^*)\Psi$. The perturbation is Hermitian if it satisfies the constraint

$$\int [K_1^* \Psi - \Psi^* K_1] d^4x = 0 \quad (4.9)$$

The eigenfunction expansion of the wave function $\Psi(x, s)$ is

$$\Psi(x, s) = \int a_\xi(s) \psi_\xi(x, s) d\xi \quad (4.10)$$

where $a_\xi(s)$ are expansion coefficients and ψ_ξ are solutions for the unperturbed system K_0 . The transition probability amplitude to state ψ_α is

$$a_\xi = a_\xi^0 - \frac{i}{\hbar} \int_0^s [\int \psi_\xi^* K_1 d^4x] ds' \quad (4.11)$$

and the transition probability density is

$$P_\xi = a_\xi^* a_\xi \quad (4.12)$$

for a mass state transition.

The formalism of s -dependent perturbation theory applied to the interaction $g(\Phi_T + \Phi_T^*)\Psi$ in Equation (4.7) yields two four-momentum constraints

$$\begin{aligned} \text{Four - Momentum} \\ k_\alpha &= k_a + K_b \\ k_\alpha &= k_a - K_b \end{aligned} \quad (4.13)$$

where subscripts a, b, α denote the projectile, target, and product particles. We also have the associated set of mass constraints

$$\begin{aligned} \text{Mass} \\ q_\alpha &= q_a + Q_b \\ q_\alpha &= q_a - Q_b \end{aligned} \quad (4.14)$$

The masses in this model refer to free particle masses given by

$$q_n = \frac{\hbar^2 (k_n)_\sigma (k_n)^\sigma}{2m_n \hbar} \quad (4.15)$$

where the Einstein summation convention applies to index σ and subscript n denotes the type of particle.

Possible mass state transitions are outlined in Table 1. Letter B denotes a bradyon and letter T denotes a tachyon. If different targets and projectiles are considered, different products are possible. As an example, the top line shows that a projectile bradyon interacting with a target bradyon can produce a bradyon for either mass constraint. A tachyon can be produced when the mass constraint is $q_\alpha = q_a - Q_b$. The product that results from the interaction depends on all kinematic constraints. More discussion and numerical examples are provided by Fanchi [9-11].

Table 1. Possible Mass State Transitions

Target Q_b	Projectile q_a	Product-1 $q_\alpha = q_a + Q_b$	Product-2 $q_\alpha = q_a - Q_b$
B	B	B	B
			T
B	T	B	T
		T	
T	B	B	B
		T	
T	T	T	B
			T

5. Real Mass Tachyons and Cosmic Inflation

The ideas discussed above for producing and detecting real mass tachyons are used here to see if real mass tachyons can provide a mechanism for cosmic inflation. Consider a simple hypothetical model that is based on the assumption that real mass tachyons were produced during the Big Bang and its aftermath. The tachyons were subject to the kinematics derived within the pRQT framework. Cosmic inflation occurs as the fastest tachyons move away from the center of mass-energy of the system. Cosmic inflation stops when the fastest tachyon reaches the aphelion of its orbit. The hypothesis that the inflationary period matches the FLRW metric implies that tachyons do not achieve escape velocity but are bound by the curvature of spacetime. If we use Figure 1, the Guth figure, depicting cosmic inflation, the tachyon rate of expansion is estimated by dividing the radius of expansion 10^{-10} m by the duration of expansion 10^{-32} sec to find the approximate rate of expansion to be 10^{22} m/s $\approx 3.34 \times 10^{13}$ c. We assume the tachyon speed is equal to the rate of expansion:

$$v_0 \approx 10^{22} \text{ m/s} \approx 3.34 \times 10^{13} \text{ c.} \quad (5.1)$$

In a previous publication [26], a formulation of pRQT was developed for a particle moving in curved spacetime assuming the metric did not depend on the invariant evolution parameter. Given this assumption, the pRQT framework in curved spacetime was applied to a free particle in flat spacetime and a particle in the Schwarzschild metric. The wavefunction of the particle had the expected $1/r$ dependence on the radius of curvature. The equivalent mass of the system can be estimated from the tachyon speed v_0 associated with the aphelion of the orbit. Since the shape of the orbit is unknown, the simplest orbit to assume is a circular orbit with radius r_0 . In this case and at this level of approximation, the equivalent mass of the system M is given by

$$M \approx \frac{r_0 v_0^2}{G} \approx 1.5 \times 10^{45} \text{ kg} \quad (5.2)$$

where G is the gravitational constant.

6. Conclusions

The real mass tachyon model presented here appears to be a possible mechanism for cosmic inflation. Questions remain. For example, what were the tachyon particles, how were the tachyon particles produced, and what was the metric associated with the trajectory of the bound tachyon. Answers to questions like these can improve the model and increase its suitability for testing.

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