TN-66-6
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Revised
May 1966

MICROWAVE PHASE DEPENDENCE OF THE ACCELERATOR SECTIONS AND THE WAVEGUIDE NETWORK

I. INTRODUCTION

The electrical phase lengths of the waveguide feed networks and the accelerator sections must be correct for optimum acceleration of the beam. The phase length is dependent upon the mechanical length of the microwave structure and its phase constant, which are in turn dependent upon temperature, the external (atmospheric) pressure, the frequency, the dielectric constant of the medium inside the structure, and the initial dimensional tolerances of the components of the structure.

During "Stage I" operation a single, high-power, klystron amplifier will feed four, ten-foot, accelerator sections through a four-branched, rectangular waveguide network.[1] Since each section is an independent microwave structure, it is necessary to adjust the phase of the wave in each section so that the beam sees a continuous wave traveling the length of the accelerator with the proper phase velocity. Phasing of the machine is accomplished in several steps as follows:

1. The ten-foot sections are tuned[2] to the desired electrical phase length, and four of them are accurately spaced (with a periodicity of twenty-nine wavelengths per section) and aligned on a forty-foot girder assembly.
2. The waveguide feed network is installed, and the phase lengths of its four branches are adjusted[3] to differ by only integer numbers of wavelengths.
3. During accelerator turn-on, the forty-foot girders are phased relative to each other in sequence by an automatic phasing system[4] that adjusts a phase shifter in each girder station's closed-loop, klystron, phasing circuit.
4. The closed-loop phasing circuit continues to monitor any phase drift which can be compensated for manually while the machine is operating or automatically while the machine is in standby.

An expression is developed in Section II below for a change in phase length for rectangular waveguide and for the SLAC ten-foot, disc-loaded, circular waveguide, accelerator sections. Another technical note[5] discusses the detailed temperature dependence of both structures and only the pertinent results are presented herein. Section III gives some values for specific cases. A list of symbols is given at the end.

II. GENERAL EXPRESSIONS FOR PHASE LENGTH

The phase length of a microwave structure can be expressed by

$$\theta = \beta \ell, \tag{1}$$

where ℓ is the length and β is the phase constant, which is given by a dispersion relationship for the structure. For rectangular waveguide in the TE_{10} mode, the dispersion relationship is expressed analytically by

$$\beta = \frac{2\pi}{\lambda_g} = \sqrt{\epsilon'(\frac{\omega}{c})^2 - (\frac{\pi}{a})^2} \quad . \tag{2}$$

For the disc-loaded, circular waveguide that is used for the accelerator sections an exact expression for β is not available, and thus an experimentally determined ω-β diagram is used along with other experimentally determined coefficients. Since the β dependences are different, the rectangular waveguide will be treated separately from the circular waveguide.

A change in phase length can be expressed by the total derivative of θ ; therefore,

$$d\theta = \left(\frac{\partial \theta}{\partial \beta}\right) d\beta + \left(\frac{\partial \theta}{\partial \ell}\right) d\ell \quad . \tag{3}$$

For the rectangular waveguide, the parameters in equations (1) and (2) have the following functional dependencies: $\ell = \ell(T, \Delta\ell)$, $\beta = (\omega, a, \epsilon')$, $\epsilon' = \epsilon'(T, p, \Delta\epsilon')$ and $a = a(T, p, \Delta a)$. It should be noted that the effects due to ϵ' and p are included for completeness only and during accelerator operation $(\epsilon' - 1) \sim 0$ and $p \sim 0$. Figure 1 shows a piece of rectangular waveguide and identifies its dimensions. From the above functional expressions:

$$d\ell = \left(\frac{\partial \ell}{\partial T}\right) dT + \left(\frac{\partial \ell}{\partial \Delta\ell}\right) d\left(\Delta\ell\right) = \left(\frac{\partial \ell}{\partial T}\right) dT + d\left(\Delta\ell\right), \tag{4}$$

$$d\beta = \left(\frac{\partial \beta}{\partial \omega}\right) d\omega + \left(\frac{\partial \beta}{\partial a}\right) da + \left(\frac{\partial \beta}{\partial \epsilon'}\right) d\epsilon', \tag{5}$$

$$d\epsilon' = \left(\frac{\partial \epsilon'}{\partial T}\right) dT + \left(\frac{\partial \epsilon'}{\partial p}\right) dp + d(\Delta\epsilon'), \tag{6}$$

and

$$da = \left(\frac{\partial a}{\partial T}\right) dT + \left(\frac{\partial a}{\partial p}\right) dp + d(\Delta a) \quad . \tag{7}$$

Combining equations (3) through (7) yields,

$$(d\theta)_{w/g} = \left[\left(\frac{\partial \theta}{\partial \beta}\right) \left(\frac{\partial \beta}{\partial \omega}\right)\right] d\omega + \left[\left(\frac{\partial \theta}{\partial \beta}\right)\left\{\left(\frac{\partial \beta}{\partial a}\right)\left(\frac{\partial a}{\partial T}\right) + \left(\frac{\partial \beta}{\partial \epsilon'}\right)\left(\frac{\partial \epsilon'}{\partial T}\right)\right\} + \left(\frac{\partial \theta}{\partial \ell}\right)\left(\frac{\partial \ell}{\partial T}\right)\right] dT$$

$$+ \left[\left(\frac{\partial \theta}{\partial \beta}\right)\left\{\left(\frac{\partial \beta}{\partial a}\right)\left(\frac{\partial a}{\partial p}\right) + \left(\frac{\partial \beta}{\partial \epsilon'}\right)\left(\frac{\partial \epsilon'}{\partial p}\right)\right\}\right] dp + \left[\left(\frac{\partial \theta}{\partial \beta}\right)\left(\frac{\partial \beta}{\partial a}\right)\right] d(\Delta a) + \left[\left(\frac{\partial \theta}{\partial \beta}\right)\left(\frac{\partial \beta}{\partial \epsilon'}\right)\right] d(\Delta \epsilon') + \left[\frac{\partial \theta}{\partial \ell}\right] d(\Delta \ell). \tag{8}$$

The equations for a, ℓ, and ϵ' are:

$$\ell = \ell_o (1+\alpha T) + \Delta \ell , \tag{9}$$

$$a = a_o + a_o \alpha T + a_o \kappa_p(p) + \Delta a, \tag{10}$$

and

$$\epsilon' = 1 + (\epsilon'_o - 1) \frac{pT_o}{p_o T} + \Delta \epsilon' . \tag{11}$$

From equations (1), (2), (9), (10), (11) and experimental data:

$$\left(\frac{\partial \theta}{\partial \beta}\right) = \ell, \quad \left(\frac{\partial \theta}{\partial \ell}\right) = \beta , \quad \left(\frac{\partial \beta}{\partial \omega}\right) = \frac{\epsilon' \omega}{c^2 \beta} = \frac{\epsilon' \lambda g}{\lambda_o^2 f} , \quad \left(\frac{\partial \beta}{\partial \epsilon'}\right) = \frac{1}{2\beta} \left(\frac{\omega}{c}\right)^2 = \frac{\pi \lambda g}{\lambda_o^2}$$

$$\left(\frac{\partial \beta}{\partial a}\right) = \frac{\pi \lambda_g}{2a^3} , \quad \left(\frac{\partial a}{\partial T}\right) = a_o \alpha , \quad \left(\frac{\partial a}{\partial p}\right) = a_o \frac{\partial \kappa_p(p)}{\partial p} , \quad \frac{\partial \kappa_p(p)}{\partial p} = 0.85 \times 10^{-7} (\mathrm{Torr})^{-1} ,$$

$$\left(\frac{\partial \epsilon'}{\partial T}\right) = - \frac{(\epsilon' - 1)\, pT_o}{p_o T^2} , \text{ and } \left(\frac{\partial \epsilon'}{\partial p}\right) = \frac{(\epsilon' - 1)\, T_o}{p_o T} .$$

Also, $(\epsilon_o' - 1) \sim 576 \times 10^{-6}$, $T_o = 273^{\circ}K$ and $p_o = 760$ Torr. By substituting the above values into equation (8) and by expressing the change in phase length in electrical degrees per unit length, the following result is obtained:

$$\left(\frac{d\theta}{\ell}\right)_{w/g} = \frac{4320}{\lambda_g}\Bigg[\underbrace{\left(\frac{\lambda_g}{\lambda_o}\right)^2 \frac{\epsilon' df}{f}}_{\text{frequency}} + \Bigg\{\underbrace{\left[\left(\frac{\lambda_g}{\lambda_c}\right)^2 + 1\right]\alpha}_{\text{width and length thermal expansion}} + \underbrace{\left(\frac{\lambda_g}{\lambda_o}\right)^2 \frac{(\epsilon'-1)\,T_o}{2p_o}\left(\frac{p}{T^2}\right)}_{\text{dielectric temperature}}\Bigg\} dT$$

$$+ \Bigg\{\underbrace{\left(\frac{\lambda_g}{\lambda_c}\right)^2 \frac{\partial\kappa_p(p)}{\partial p}}_{\text{pressure deformation}} + \underbrace{\left(\frac{\lambda_g}{\lambda_o}\right)^2 \frac{(\epsilon'-1)\,T_o}{2p_o T}}_{\text{dielectric pressure}}\Bigg\} dp + \underbrace{\left(\frac{\lambda_g}{\lambda_o}\right)^2 \frac{d(\Delta\epsilon')}{2}}_{\text{dielectric composition}} + \underbrace{\left(\frac{\lambda_g}{\lambda_c}\right)^2 \frac{d\Delta a}{a}}_{\text{width tolerances}} + \underbrace{\frac{d(\Delta\ell)}{\ell}}_{\text{length tolerances}}\Bigg] ({}^{o}\phi)\cdot(\text{ft})^{-1} . \quad (12)$$

During operation, as mentioned above, $p \sim 0$ and $(\epsilon' - 1) \sim 0$. Therefore, $dp \sim d\Delta\epsilon' \sim 0$ and equation (12) becomes

$$\left(\frac{d\theta}{\ell}\right)_{w/g} = \frac{4320}{\lambda_g}\left[\left(\frac{\lambda_g}{\lambda_o}\right)^2 \frac{df}{f} + \left\{\left(\frac{\lambda_g}{\lambda_c}\right)^2 + 1\right\}\alpha dT + \left(\frac{\lambda_g}{\lambda_c}\right)^2 \frac{d(\Delta a)}{a} + \frac{d(\Delta\ell)}{\ell}\right] ({}^{o}\phi)\cdot(\text{ft})^{-1} . \quad (13)$$

Equation (13) is the desired expression for the change in phase length due to frequency changes, temperature changes, and dimensional tolerance, and it is evaluated in Section III and Table 1.

For the disc-loaded circular waveguide, the parameters in equations (1) and (2) have the following functional dependencies: $\ell = \ell(T, \Delta\ell)$, $\beta = \beta(\epsilon', \omega, 2a, 2b)$, $\epsilon' = \epsilon'(T, p, \Delta\epsilon')$, $2a = a(T, \Delta(2a))$, and $2b = b(T, p, \Delta(2b))$. The variation of β with ℓ and t is assumed second order and is neglected here. This can be shown by deriving the temperature dependence of the θ by frequency scaling and comparing the result with the result of the following derivation. Figure 2 shows a typical cavity of the accelerator section. The phase length of each cavity is given by equation (1) and the total phase length of a section is

$$(\theta)_{acc} = \sum_{n=1}^{86} \theta = \sum_{n=1}^{86} \beta_n \ell_n , \quad (14)$$

where a standard ten-foot (119.8465"@113.0° F) section has 86 cavities.

From equation (14) the equation similar to equation (3) is

$$(d\theta)_{acc} = \sum_{n=1}^{86} d\theta_n = \sum_{n=1}^{86} \left\{ \left(\frac{\partial\theta_n}{\partial\beta_n}\right) d\beta_n + \left(\frac{\partial\theta_n}{\partial\ell_n}\right) d\ell_n \right\} . \tag{15}$$

Equations similar to equations (4) through (7) can be written from the above functional dependencies.

$$d\ell = \left(\frac{\partial\ell}{\partial T}\right) dT + d(\Delta\ell) \tag{16}$$

$$d\beta = \left(\frac{\partial\beta}{\partial\epsilon'}\right) d\epsilon' + \left(\frac{\partial\beta}{\partial\omega}\right) d\omega + \left(\frac{\partial\beta}{\partial(2a)}\right) d(2a) + \left(\frac{\partial\beta}{\partial(2b)}\right) d(2b) \tag{17}$$

$$d\epsilon' = \left(\frac{\partial\epsilon'}{\partial T}\right) dT + \left(\frac{\partial\epsilon'}{\partial p}\right) dp + d(\Delta\epsilon') \tag{18}$$

$$d(2a) = \left(\frac{\partial(2a)}{\partial T}\right) dT + \left(\frac{\partial(2a)}{\partial\Delta(2a)}\right) d\Delta(2a) \tag{19}$$

$$d(2b) = \left(\frac{\partial(2b)}{\partial T}\right) dT + \left(\frac{\partial(2b)}{\partial p}\right) dp + \left(\frac{\partial(2b)}{\partial\Delta(2b)}\right) d\Delta(2b) \tag{20}$$

Putting equations (16) through (20) into equation (15) yields:

$$\begin{aligned}(d\theta)_{acc} = \sum_{n=1}^{86} \Bigg\{ & \left[\left(\frac{\partial\beta_n}{\partial\beta_n}\right)\left(\frac{\partial\beta_n}{\partial\epsilon'}\right)\left(\frac{\partial\epsilon'}{\partial T}\right) + \left(\frac{\partial\theta_n}{\partial\beta_n}\right)\left(\frac{\partial\beta_n}{\partial(2a)}\right)\left(\frac{\partial(2a)}{\partial T}\right) + \left(\frac{\partial\theta_n}{\partial\beta_n}\right)\left(\frac{\partial\beta_n}{\partial(2b)}\right)\left(\frac{\partial(2b)}{\partial T}\right) + \left(\frac{\partial\theta_n}{\partial\ell_n}\right)\left(\frac{\partial\ell_n}{\partial T}\right)\right] dT \\ & + \left[\left(\frac{\partial\theta_n}{\partial\beta_n}\right)\left(\frac{\partial\beta_n}{\partial(2a)}\right)\right] d\Delta(2a) + \left[\left(\frac{\partial\theta_n}{\partial\beta_n}\right)\left(\frac{\partial\beta_n}{\partial(2b)}\right)\right] d\Delta(2b) + \left[\left(\frac{\partial\theta_n}{\partial\ell_n}\right)\right] d\Delta\ell_n + \left[\left(\frac{\partial\theta_n}{\partial\beta_n}\right)\left(\frac{\partial\beta_n}{\partial\omega}\right)\right] d\omega \\ & + \left[\left(\frac{\partial\theta_n}{\partial\beta_n}\right)\left(\frac{\partial\beta_n}{\partial\epsilon'}\right)\right] d\Delta\epsilon' + \left[\left(\frac{\partial\theta_n}{\partial\beta_n}\right)\left(\frac{\partial\beta_n}{\partial\epsilon'}\right)\left(\frac{\partial\epsilon'}{\partial p}\right) + \left(\frac{\partial\theta_n}{\partial\beta_n}\right)\left(\frac{\partial\beta_n}{\partial(2b)}\right)\left(\frac{\partial(2b)}{\partial p}\right)\right] dp \Bigg\} . \end{aligned} \tag{21}$$

The equations for 2a and 2b are:

$$(2a) = 2a_o (1 + \alpha T) + \Delta(2a) \tag{22}$$

$$(2b) = 2b_o (1 + \alpha T) + \tau_p (2b_o) + \Delta(2b) . \tag{23}$$

Since the structure operates in a $2\pi/3$ mode,

$$\ell_o \sim \frac{\lambda_o}{3} \quad \text{and} \quad \beta_o \sim \frac{2\pi}{\lambda_o} \sim \frac{2\pi}{3\ell_o} \quad .$$

The partial derivatives of equation (21) can be evaluated from equations (1), (11), (22) and (23) and from experimental coefficients.[6] They are as follows:

$$\left(\frac{\partial\theta_n}{\partial\beta_n}\right) = \ell_n \sim \ell_o = \frac{\lambda_o}{3} \,, \quad \left(\frac{\partial\theta_n}{\partial\ell_n}\right) = \beta_n \sim \beta_o = \frac{2\pi}{\lambda_o} = \frac{2\pi c}{f_o} \,, \quad \left(\frac{\partial\beta_n}{\partial\omega}\right) = \frac{1}{\left(\frac{\partial\omega}{\partial\beta_n}\right)} = \left(\frac{1}{v_g}\right)_n \quad ,$$

$$\left(\frac{\partial f}{\partial(2a)}\right) = k_a, \quad \left(\frac{\partial f}{\partial(2b)}\right) = k_b, \quad \left(\frac{\partial\epsilon'}{\partial T}\right) = -\left(\epsilon' - 1\right)\frac{pT_o}{p_o T^2} \,, \quad \frac{\partial\epsilon'}{\partial p} = \left(\epsilon' - 1\right)\frac{T_o}{p_o T} \quad ,$$

$$\frac{\partial(2a)}{\partial T} \sim \alpha(2a) \,, \quad \frac{\partial(2b)}{\partial T} \sim \alpha(2b), \quad \frac{\partial\ell_n}{\partial T} \sim \alpha\ell_o = \frac{\alpha\lambda_o}{3} \,, \quad \frac{\partial(2b)}{\partial p} \sim (2b)\frac{\partial\tau_p}{\partial p}, \frac{\partial\beta_n}{\partial\epsilon'} = \,?$$

The terms $\left(\frac{\partial\beta_n}{\partial(2b)}\right)$ and $\left(\frac{\partial\beta_n}{\partial(2a)}\right)$ may be obtained by considering $d\omega = \left(\frac{\partial\omega}{\partial\beta}\right) d\beta + \left(\frac{\partial\omega}{\partial(2b)}\right) d(2b)$ when $d\omega = 0$, which means for $\omega = \omega(\beta, b)$ that $\frac{d\beta}{d(2b)} = \frac{\partial\beta}{\partial(2b)}$, and therefore,

$$\left(\frac{\partial\beta_n}{\partial(2b)}\right) = -\left(\frac{\partial\omega}{\partial(2b)}\right) \Big/ \left(\frac{\partial\omega}{\partial\beta_n}\right) = -\left(\frac{2\pi}{v_g}\right)\left(\frac{\partial f}{\partial(2b)}\right)$$

and similarly for (2a).

Equation (21) can be evaluated now in terms of known parameters and is given below without the ϵ' and p dependent terms, since final testing and operation are at $p \sim 0$ and $(\epsilon' - 1) \sim 0$. Care has been taken to use the appropriate average value when the quantity varies along the length of the section.

$$(d\theta)_{acc} = \left\{\left[\frac{86 \times 120}{f_o} \overline{\left(\frac{c}{v_g}\right)}\right] df - \left[86 \times 120\,\alpha\left\{\overline{\left(\frac{c}{v_g}\right)}\frac{\left(\overline{2a}\right)\left(\overline{k_a}\right)}{f_o} + \overline{\left(\frac{c}{v_g}\right)}\frac{\left(\overline{2b}\right)\left(\overline{k_b}\right)}{f_o} - 1\right\}\right] dT \right.$$
$$\left. - \left[\frac{86 \times 120}{f_o}\overline{\left(\frac{c}{v_g}\right)}\left(\overline{k_a}\right)\right] d\Delta(2a) - \left[\frac{86 \times 120}{f_o}\overline{\left(\frac{c}{v_g}\right)}\left(\overline{k_b}\right)\right] d\Delta(2b) + \left[\frac{360}{\lambda_o}\right] d\Delta\ell_{acc}\right\}\left({}^o\phi\right) \quad . \qquad (24)$$

Equation (24) is the phase length dependency equation for an accelerator section and is comparable to equation (13) for waveguide. If $(d\theta)_{acc}$ is derived using a frequency scaling argument $[a \propto f^{-1}$, etc., and $a = a_o(1 + \alpha T)]$, which includes effects of ℓ and t, the result is the same as equation (24), if $(\overline{2a})(\overline{k_a}) + (\overline{2b})(\overline{k_b}) = f_o$ in equation (24). In Section III this is shown to be a good approximation in practice; i.e., terms due to ℓ and t are much smaller than those due to a and b. Thus the above derivation, giving specific dependence on (2a) and (2b), is approximately correct.

III. SPECIFIC VALUES OF PHASE LENGTH DEPENDENCY

Equations (13) and (24) may be expressed in terms of phase length coefficients as follows:

$$\left(\frac{d\theta}{\ell}\right)_{w/g} = \theta'_f df + (\theta'_a + \theta'_\ell)\, dT + \theta'_{\Delta a}\, d\Delta a + \theta'_{\Delta\ell}\, d\Delta\ell \tag{25}$$

and

$$\left(d\theta\right)_{acc} = \theta_f df + (\theta_{2a} + \theta_{2b} + \theta_\ell)\, dT + \theta_{\Delta(2a)}\, d\Delta(2a)$$
$$+ \theta_{\Delta(2b)}\, d\Delta(2b) + \theta_{\Delta\ell}\, d\Delta\ell_{acc} \quad . \tag{26}$$

These coefficients are given in Tables 1 and 2 for: $f_o = 2856(MH_z)$, $\lambda_g = 6.0239''$, $\lambda_o = 4.133''$, $\lambda_c = 5.680''$, $\alpha = 9.3 \times 10^{-6}\, (^\circ F)^{-1}$, $a = 2.840''$, $(\overline{k_a}) = 0.22\times 10^9\, (Hz)\cdot(in)^{-1}$, $(\overline{k_b}) = -0.93\times 10^9\, (Hz)\cdot(in)^{-1}$, $\overline{\left(\frac{c}{v_g}\right)} = 82.6$, $(\overline{2a}) = 0.91$ (in), and $(\overline{2b}) = 3.25$ (in).

The values of θ_f and θ'_f have been measured and agree with those given in Tables 1 and 2. The values $\theta_T = +7.9\, (^\circ\phi)\cdot(^\circ F)^{-1}$ and $\theta'_T = 0.0144\, (^\circ\phi)\cdot(^\circ F\cdot ft)^{-1}$ (or $\theta_T = 0.85\, (^\circ\phi)\cdot(^\circ F)^{-1}$ and $1.0\, (^\circ\phi)\cdot(^\circ F)^{-1}$ for the network's 59-foot and 70-foot branches, respectively) are comparable to measured values, also. It should be noted that the only phase coefficients that vary during accelerator operation are θ_f and θ_T, since the effects of dimensional tolerances are adjusted to be minimum during accelerator fabrication and installation.

Variations in $d(\Delta a)$ and $d(\Delta \ell)$ in equation (25) are compensated for within $\pm 2.5^{o}\phi$ for each network branch after installation by squeezing the waveguide walls.[3] Variations in $d\Delta(2a)$ and $d\Delta(2b)$ in equations (26) are compensated for within $\pm 2.5^{o}\phi$ after fabrication by indenting each accelerator cavity wall.[2] The error due to $d\Delta\ell$ in equation (26) is less than $\pm$ 0.020 inches for a ten-foot section, which means $\pm 1.8^{o}\phi$.

In another technical note[5] the change in temperature of the network and the accelerator section is shown to be dependent upon the ambient temperature, the water flow rate, the pressure drops in the water lines, the thermal resistance of the structures, the microwave power level and the stability of the water temperature regulator. During accelerator operation the temperature variables that may change or be changed are the ambient temperature, the water flow rate, and the microwave power level. The effect of the microwave power level can be divided into two parts. One is the simple variation of the operating level of a klystron, $d\eta$, and the other is the variation in the power actually absorbed, $d\Delta P$, due to unequal power division and attenuation. The latter term is more or less constant during operation, as are the thermal resistance, water pressure change and the fluctuation in the water temperature out of the regulator, (dT_r); therefore, these constants will all be lumped into a single maximum temperature difference term, dT_x.

The simplified expressions for the change in temperature as given by Table 6 of reference 5 are:

$$(dT)_{w/g} \sim \left[dT_r + 0.012\, dT_a - \begin{Bmatrix} .47 \\ .78 \end{Bmatrix} \frac{dw}{w} + 1.860\, d\eta + dT_x \right] {}^{o}F \qquad (27)$$

for a rectangular waveguide network branch and

$$(dT)_{acc} \sim \left[dT_r + 0.006\, dT_a - \begin{Bmatrix} 1.90 \\ 2.05 \end{Bmatrix} \frac{dw}{w} + 3.500\, d\eta + dT_x \right] {}^{o}F \qquad (28)$$

for an accelerator section. The two values in the brackets are for different ambient temperature conditions (see Tables 5 and 6 of reference 5). Equations (25), (26), (27) and (28) may be combined to give

$$\left(\frac{d\theta}{\ell}\right)_{w/g} = \theta'_f df + \theta'_T dT_r + \theta'_{T_a} dT_a + \theta'_w \frac{dw}{w} + \theta'_\eta d\eta + \theta'_T dT_x + \theta'_{\Delta a} d\Delta a + \theta'_{\Delta\ell} d\Delta\ell \qquad (29)$$

and

$$(d\theta)_{acc} = \theta_f df + \theta_T dT_r + \theta_{T_a} dT_a + \theta_w \frac{dw}{w} + \theta\eta\, d\eta + \theta_T dT_x$$

$$+ \theta_{\Delta(2a)} d\Delta(2a) + \theta_{\Delta(2b)} d\Delta(2b) + \theta_{\Delta\ell} d\Delta\ell_{acc} \quad . \qquad (30)$$

The additional phase coefficients θ_{Ta}, θ_w and θ_η , which pertain to the ambient air temperature, the water flow rate and the operating level, respectively, are given in Tables 1 and 2, also.

As in Section III of reference 5, reasonable estimates can be made of the maximum $d\theta$ under various conditions. It will be assumed for the following estimates that $df = \pm 0.5$ (kHz), $\overline{T}_a = 63^\circ F$, and $\eta = 1$ (P = 21.6 kW). Ambient temperature variations from branch to branch or section to section are indicated by dT_a. For accelerator sections on the same girder, equation (30) and Table 2 yield:

$$(d\theta)_{acc} = \pm .15 \pm 0 \pm .14 \pm 1.6 \pm 0 \pm 2.7 \pm 2.5 \pm 0 \pm 1.8 = \underline{\pm 8.9^\circ\phi} ,$$

where $\quad dT_r = 0,\ dT_a = \pm 3^\circ F,\ \frac{dw}{w} = \pm\frac{1}{10},\ d\eta = 0,\ dT_x = \pm .34^\circ F,$

the section tuning error is $\pm 2.5^\circ\phi$ and the section length is $\pm 1.8^\circ\phi$. For waveguide branches on the same network, equation (29), Table 1, and ℓ = 70 feet yield:

$$(d\theta)_{w/g} = \pm .02 \pm 0 \pm .06 \pm .15 \pm 0 \pm .5 \pm 2.5 \pm 0 = \underline{\pm 3.2^\circ\phi},$$

where $\quad dT_r = 0,\ dT_a = \pm 5^\circ F,\ \frac{dw}{w} = \pm\frac{1}{6},\ d\eta = 0,\ dT_x = \pm .51^\circ F,$

and the branch is tuned to $\pm 2.5^\circ\phi$. For sections on different girders and branches on different networks, the maximum phase length differences are:

$$(d\theta)_{acc} = \pm .15 \pm 0 \pm .28 \pm 1.6 \pm 2.8 \pm 2.5 \pm 0 \pm 1.8 = \underline{\pm 11.8^\circ\phi},$$

where the conditions are the same as above except $dT_a = \pm 6^\circ F$ and $d\eta = \pm 1/10$, and

$$(d\theta)_{w/g} = \pm .02 \pm 0 \pm .12 \pm .3 \pm .18 \pm .6 \pm 2.5 \pm 0 = \pm \underline{3.7^\circ\phi},$$

where the conditions are the same as above except $dT_a = \pm 10^\circ F$, $\frac{dw}{w} = \pm \frac{1}{3}$,

$d\eta = \pm \frac{1}{10}$, and $dT_x = \pm .61^\circ F$.

For sections and branches in different sectors, $dT_r = \pm 0.1^\circ F$, so $\pm 0.1^\circ \phi$ and $\pm .8^\circ \phi$ should be added to the branch and section errors, respectively. A $10^\circ F$ change in the average ambient air temperature produces a phase shift of $0.12^\circ \phi$ and $0.47^\circ \phi$ in the branches and sections, respectively. Furthermore, it should be remembered that the effective misphasing of the wave with the beam is the sum of the branch error and half the section error. In the above analysis the total errors are the maximum possible, since the individual errors were assumed to be additive and maximum. The actual cumulative error for the whole machine should be considerably less, if a more careful weighting of the errors or the rms error is considered.

The effects of atmospheric pressure on the structures and the nature of the dielectric inside the structures were brushed over in Section II because good experimental results are not available. However, it is interesting to note that a ten-foot accelerator section increases in phase length by about $280^\circ \phi$ in going from vacuum ($< 25 \times 10^{-3}$ Torr) to one atmosphere of dry nitrogen[7]. The waveguide increases in length by about $0.8\ (^\circ \phi) \cdot (\mathrm{ft})^{-1}$ in going from vacuum to atmospheric pressure or about $1.05 \times 10^{-3}\ (^\circ \phi) \cdot (\mathrm{ft} \cdot \mathrm{Torr})^{-1}$.

Changes in the barometric pressure can affect the relative phase lengths from branch to branch, as well as the absolute phase length of a branch, because of the differences in branch lengths and metal elasticity.[8] If the barometric pressure changes $\pm 5\%$, the phase length can change by as much as $\pm 0.014\ (^\circ \phi) \cdot (\mathrm{ft})^{-1}$. That corresponds to $\pm 1.0\ (^\circ \phi)$ for a 70 foot branch or $\pm 0.15\ (^\circ \phi)$ for the difference between a 70 foot and a 59 foot branch. Since $\frac{\partial \tau_p}{\partial p}$ is not known for the accelerator sections, nothing can be said about the effect of barometric pressure on them other than it is the same, to first order, for all ten-foot sections. Therefore, the waveguide networks generally have less than $\pm 0.15\ (^\circ \phi)$ error due to normal barometric pressure variations, if all the waveguide branches are in the same state of anneal.

IV. CONCLUSION

The phase length of an accelerator section is eight to ten times more sensitive to temperature and frequency changes than a waveguide network branch. A frequency increase of 10 kHz lengthens a waveguide branch by about $0.3^{\circ}\phi$ and an accelerator section by about $3.0(^{\circ}\phi)$; so the average phase shift the beam sees is: $0.3 + 1.5 = 1.8\ (^{\circ}\phi)\cdot(10\ \mathrm{kHz})^{-1}$. The accelerator operation level has the greatest effect on the phase length thru the temperature. As the accelerator goes from zero microwave power to full Stage I power ($\eta = 1$), the branches increase in length by about $1.8\ ^{\circ}\phi$ and the sections by about $28\ ^{\circ}\phi$, which means an average phase shift of about $16\ ^{\circ}\phi$. Variations in phase length due to the accuracy of the water temperature controller unit are less that $\pm 1\ ^{\circ}\phi$.

REFERENCES

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2. SLAC-PUB-71 and SLAC Report No. 32, pp. 44-50.
3. SLAC-PUB-89, SLAC Report No. 34, pp. 52-59, and Internal Memorandum, "Instruction Manual for the Rectangular Waveguide Network Phasing Machine," J. Weaver, 1966.
4. SLAC-PUB-104.
5. SLAC-TN-66-5.
6. SLAC-TN-63-60, p. 7.
7. Private communication with D. Rogers.
8. Private communication with R. Alvarez.

LIST OF SYMBOLS

	a	-	inside width of rectangular waveguide - (in.)
	$(2a)$	-	hole diameter of accelerator section discs - (in.)
	$(2b)$	-	inside diameter of accelerator section cylinders - (in.)
	f	-	frequency - (Hz)
	k_a	-	frequency-hole-diameter coefficient for accelerator discs - $(Hz)\cdot(mil)^{-1}$
	k_b	-	frequency-cylinder-diameter coefficient for accelerator cylinder - $(Hz)\cdot(mil)^{-1}$
	ℓ	-	length - (ft.)
	p	-	air pressure - (Torr)
	T	-	temperature - (^{o}F or ^{o}K)
	(v_g/c)	-	group velocity - (normalized with respect to the velocity of light)
	w	-	water flow rate - $(lb.)\cdot(hr.)^{-1}$
	α	-	thermal expansion coefficient $(^{o}F)^{-1}$
	β	-	phase constant - $(in.)^{-1}$
	Δ	-	used as a prefix to indicate dimensional errors or tolerance - (mils.)
	ϵ'	-	relative dialectric constant
	η	-	fraction of Stage I full power (21.6 kw)
	θ	-	electrical phase length - (radian or $^{o}\phi$)
	$\kappa_p(p)$	-	elastic pressure compression factor for waveguide
	λ_c	-	cutoff wavelength - (in.)
	λ_g	-	guide wavelength - (in.)
	λ_o	-	free space wavelength - (in.)
	τ_p	-	elastic pressure compression factor for accelerator section
	$^{o}\phi$	-	used to indicate degrees of electrical phase length
	ω	-	frequency - (radian)
Subscript	a	-	relates to waveguide width temperature expansion
	Δa	-	relates to changes in waveguide width
	$2a$	-	relates to disc hole diameter temperature expansion
	$\Delta(2a)$	-	relates to changes in disc hole diameter
	$2b$	-	relates to cylinder inside diameter temperature expansion
	$\Delta(2b)$	-	relates to changes in cylinder inside diameter
	acc	-	relates to a ten-foot accelerator section
	f	-	relates to frequency

ℓ	–	relates to waveguide length temperature expansion
$\Delta\ell$	–	relates to changes in length
o	–	relates to nominal or initial value
r	–	relates to temperature regulator
T	–	relates to combined temperature effect
T_a	–	relates to ambient air temperature
T_r	–	relates to temperature regulator temperature
w	–	relates to water flow rate
w/g	–	relates to waveguide
η	–	relates to fraction of Stage I full power
' prime	–	relates to waveguide phase coefficients
$\bar{}$ bar	–	indicates average value.

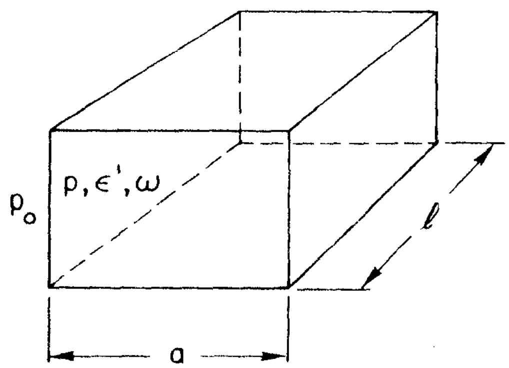

FIG. 1.

RECTANGULAR WAVEGUIDE

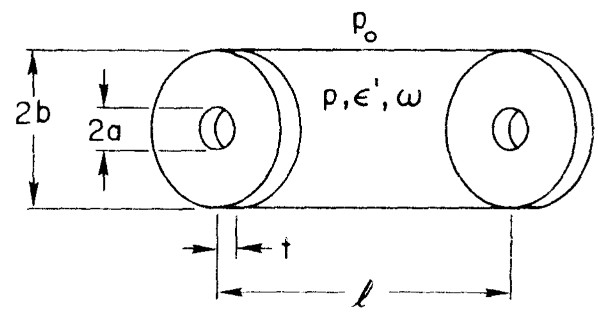

FIG. 2.

DISC-LOADED CIRCULAR WAVEGUIDE

TABLE 1

Phase Length Coefficients for a 70-Foot Waveguide Network Branch--See Equations (25) and (29)

$\theta'_f \frac{(^o\phi)}{(MHz)(ft)}$	$\theta'_T \frac{(^o\phi)}{(^oF)(ft)}$ $=\theta'_a + \theta'_\ell$	$\theta'_a \frac{(^o\phi)}{(^oF)(ft)}$	$\theta'_\ell \frac{(^o\phi)}{(^oF)(ft)}$	$\theta'_{Ta} \frac{(^o\phi)}{(^oF)(ft)}$	$\theta'_w \frac{(^o\phi)}{(ft)}$	$\theta'_\eta \frac{(^o\phi)}{(ft)}$	$\theta'_{\Delta a} \frac{(^o\phi)}{(mil)(ft)}$	$\theta'_{\Delta \ell} \frac{(^o\phi)}{(mil)}$
+ 0.532	+ 0.0144	+ 0.0076	+ 0.0068	+ 0.00017	$-\begin{Bmatrix}0.0078\\0.0127\end{Bmatrix}$	+ 0.025	+ 0.283	+ 0.0596

TABLE 2

Phase Length Coefficients For a Ten-Foot Accelerator Section--See Equations (26) and (30)

$\theta_f \frac{(^o\phi)}{(kHz)}$	$\theta_T \frac{(^o\phi)}{(^oF)}$ $=\theta_{2a}+\theta_{2b}+\theta_\ell$	$\theta_{2a} \frac{(^o\phi)}{(^oF)}$	$\theta_{2b} \frac{(^o\phi)}{(^oF)}$	$\theta_\ell \frac{(^o\phi)}{(^oF)}$	$\theta_{Ta} \frac{(^o\phi)}{(^oF)}$	$\theta_w\ (^o\phi)$	$\theta_\eta\ (^o\phi)$	$\theta_{\Delta(2a)} \frac{(^o\phi)}{(mil)}$	$\theta_{\Delta(2b)} \frac{(^o\phi)}{(mil)}$	$\theta_{\Delta\ell} \frac{(^o\phi)}{(mil)}$
+ 0.30	+ 7.9	- 0.55	+ 8.4	+ 0.096	+ 0.047	$-\begin{Bmatrix}15.0\\16.2\end{Bmatrix}$	+ 27.7	-66	+ 278	+ 0.088