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Magnetic brane solutions in Gauss–Bonnet–Maxwell massive gravity

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ABSTRACT

Magnetic branes of Gauss–Bonnet–Maxwell theory in the context of massive gravity is studied in detail. Exact solutions are obtained and their interesting geometrical properties are investigated. It is argued that although these horizonless solutions are free of curvature singularity, they enjoy a cone-like geometry with a conic singularity. In order to investigate the effects of various parameters on the geometry of conic singularity, its corresponding deficit angle is studied. It will be shown that despite the effects of Gauss–Bonnet gravity on the solutions, deficit angle is free of Gauss–Bonnet parameter. On the other hand, the effects of massive gravity, cosmological constant and electrical charge on the deficit angle will be explored. Also, a brief discussion related to possible geometrical phase transition of these topological objects is given.

1. Introduction

General relativity (GR) is one of the most successful theories in physics. Nonetheless, this theory could not predict precisely the fact that our universe has an accelerated expansion [1,2]. In order to interpret this expansion, some various candidates have been proposed, such as the cosmological constant idea [3], dark energy models [4,5] and modified gravities including Lovelock gravity [6], F(R) gravity models [7–9], scalar–tensor theories [10,11] and brane world cosmology [12,13]. The theory of Lovelock gravity is special between others, since this theory is ghost-free and also enjoys the principles of general relativity in higher dimensions.

On the other hand, GR includes the graviton as a massless particle, but the results of LIGO experiment, and also from theoretical point of view, it was shown that the graviton might be a massive particle with an upper limit on its mass [14–17]. Therefore, one may regard a generalization of Einstein's theory of gravity in the context of massive gravity. In addition, considering the massive gravitons improves our viewpoint about the cosmological constant problem [18]. Furthermore, the observational evidences suggest that about 95% energy of our universe is dark energy and dark matter [19] which is based on assumption that GR is equally valid at all length scales. So, the modifications of GR by massive gravitons could possibly change this scenario over large distances.

The first attempt regarding introduction of linear massive gravity was done by Fierz and Pauli [20,21]. Later, Boulware and Deser (BD) showed that this theory of massive gravity suffers the BD ghost instability at the nonlinear level [22]. The existence of ghost indicates that the theory under consideration is unstable. In order to avoid such instability, some other models of massive gravity were introduced. One of the ghost-free massive theories was introduced by Bergshoeff, Hohm and Townsend in which such theory was a three dimensional massive theory and is known as new massive gravity (see [23], for more details). However, this theory has some problems in four and higher dimensions. Recently, de Rham, Gabadadze and Tolley (dRGT) introduced another class of massive gravity [24] which is ghost-free

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in arbitrary dimensions. It is notable that, in this theory, the mass terms are produced by considering a reference metric. This reference metric plays a crucial role for constructing the massive theory of gravity [25]. Study of this theory showed that dRGT theory is stable [26,27], and it is free of BD ghost [26,27]. Black hole solutions and their thermodynamical properties, stability of various black holes and cosmological solutions of dRGT theory have been investigated in literature [28–34]. From astrophysical point of view, Katsuragawa et al., investigated the neutron stars in this gravity and found that the massive gravity leads to small deviation from the GR results [35].

Another massive gravity model with different reference metric was proposed by Vegh which is motivated by the applications of gauge/gravity duality [36]. Vegh showed that graviton may behave like a lattice and exhibits a Drude peak in this theory of massive gravity [36]. It is notable that, this theory is ghost-free and stable for arbitrary singular metric [37]. Charged black hole solutions and the existence of van der Waals like behavior in extended phase space have been studied in the context of this gravity [38,39]. Besides, the generalizations of such theory to include Born–Infeld electrodynamics [40], higher derivative gravity [41], and also gravity's rainbow [42] have been investigated. In addition, BTZ black hole solutions in massive gravity with linear and nonlinear electrodynamics have been studied in Ref. [43]. The hydrostatic equilibrium equation of neutron stars in the context of this massive gravity was extracted and it was found that the maximum mass of neutron stars can be more than $3.2M_{\odot}$ [44].

In summary, massive gravity have some interesting properties, such as (i) From cosmological point of view, massive gravity can be used to explain the cosmological constant problem [45], and also provides an interesting basis for self-acceleration of our universe without introducing the cosmological constant [46]. In other words, some of the massive terms of cosmological solutions can be regarded as an effective cosmological constant [47,48]. (ii) Graviton is one of the best candidates for dark matter [49]. (iii) The existence of massive gravitons provides extra polarization for the gravitational waves and also affects their propagation's speed [50]. Massive gravitons had considerable effect on the production of gravitational waves during inflation [51,52]. (iv) From astrophysical point of view, the existence of maximum mass of neutron stars more than $3.2M_{\odot}$ is possible in the massive gravity context [44]. (v) Considering the massive gravitons results into the existence of remnant for the temperature of black holes after evaporation which could explain the information paradox [43]. (vi) The existence of van der Waals behavior and critical phenomena for topological black holes is another interesting property of massive gravity [53].

On the other hand, one of the interesting higher derivative gravity models is the Lovelock theory which is the most generalization of Einstein gravity that includes properties of Einstein's tensor in the higher dimensions. This gravity enjoys only first and second-order derivatives of the metric function in the field equations, and also it is a ghost-free theory of gravity. It is notable that, in 4-dimensions, the Lovelock gravity reduces to the Einstein theory without any additional term. In other words, by considering the Lovelock gravity in higher dimensions, the additional terms will appear. The first three terms of Lovelock gravity is including the Einstein and Gauss–Bonnet (GB) gravities in the presence of cosmological constant. The GB gravity has interesting properties such as: (i) It is free of ghost particles [54,55]. (ii) The natural next-to-leading order term of the heterotic superstring effective action which plays a fundamental role in Chern–Simons gravitational theories is GB term [56]. (iii) The presence of GB gravity in addition to Einstein gravity may lead to the modified Renyi entropy [57]. This entropy violates specific inequality which must be hold for Renyi entropy. (iv) Regarding AdS/CFT correspondence, it was shown that considering GB gravity will modify shear viscosity, entropy, thermal conductivity and electrical conductivity [58]. The black holes, wormholes, cosmological solutions, stability and structure of stars in the context of GB gravity have been investigated in some literature [59–65].

From cosmological point of view, it was proposed that the early universe was plugged with number of phase transitions. During these phase transitions, different regions were collectively regarded different minima in the set of possible states to fall in. This resulted into formation of different regions with specific boundaries. Alongside of these phase transitions, specific symmetries were broken which resulted into formation of different topological defects. These topological defects were located on the boundary of different regions and in the essence, they are due to disagreement between two different regions regarding their choices for the minima. The final structure of the topological defects and their geometrical and physical properties depend on the broken symmetry during the phase transition. Among different topological defects, one can name: (i) Domain walls which are originated from broken discrete symmetry and divide universe into blocks. (ii) Cosmic strings which are arisen from the breaking of the axial or cylindrical symmetry with applications in grand unified particle physics in the electroweak scale. (iii) Monopoles which carry magnetic charge and are formed due to a broken spherical symmetry. (iv) Textures which are due to breaking of several symmetries. The topological defects contain information regarding the early universe and its phase transitions [66,67]. Furthermore, it was argued that they have important role in the large-scale structure of universe [66,67]. The effects of these astrophysical objects on the Cosmic Microwave Background (CMB) have been explored in Refs. [68,69]. In addition, it was proposed that dark matter may be originated from these topological defects [70,71]. Also, it was shown that these topological defects have gravitational lensing property [72] which is due to the modification in trajectory of the photon on these topological defects depending on deficit angle. So far, wide range of studies regarding the topological defects were done which among them one can point out: (i) The cosmic strings in the presence of Maxwell field [73,74]. (ii) The superconducting property of these topological defects in Einstein [75], dilaton [76] and Brans–Dicke [77] theories. (iii) The QCD applications of the magnetic strings [78,79] and their roles in quantum theories [80,81]. (iv) The stability of the cosmic strings through quantum fluctuations [82]. (v) The existence of the limits on the cosmic string tension by extracting signals of cosmic strings from CMB temperature anisotropy maps [83]. (vi) The spectrum of gravitational wave background produced by cosmic strings [84]. (vii) The evolution of domain walls in de Sitter universe [85]. (viii) The production of gravitational waves from decaying domain walls [86]. For further studies regarding the cosmological topological defects, we refer the reader to Refs. [87–92]. Considering the wide applications of topological defects, GB theory and massive gravity, it is interesting to study the effects of GB massive gravity on the properties of conic geometry.

In this paper, we are interested in topological defects which are known as horizonless magnetic solutions (see refs. [90,93,94], for more details). These solutions are not black hole but they contain conic singularity. The main motivation is to understand the effects of the massive gravity alongside of GB gravity on geometrical and physical properties of the magnetic solutions. We will emphasize on the role and effects of different parameters on deficit angle of the solutions and show that depending on choices of different parameters, there might be discontinuity and change of signature for deficit angle which mark the existence of geometrical phase transition for these objects.

2. Basic field equations and exact solutions

In order to study horizonless magnetic branes, we consider the following metric for d-dimensions

$$ds^2 = -\frac{\rho^2}{l^2}dt^2 + \frac{d\rho^2}{g(\rho)} + l^2 g(\rho)d\varphi^2 + \frac{\rho^2}{l^2}h_{ij}dx_i dx_j, \quad i, j = 1, 2, 3, ..., d-3 \tag{1}$$

where $g(\rho)$ is an arbitrary function of radial coordinate ρ which should be determined. In addition, the scale length factor l is related to the cosmological constant Λ, and $h_{ij}dx_i dx_j$ is the Euclidean metric on the $(d-3)$-dimensional submanifold. Moreover, the angular coordinate φ is dimensionless and range in $0 \le \varphi \le 2\pi$ while the range of x_i's is $(-\infty, +\infty)$.

Due to the fact that we are interested in studying the Maxwell–GB-massive gravity, we consider the d-dimensional action of GB-massive gravity coupled to Maxwell electrodynamics

$$\mathcal{I}_G = -\frac{1}{16\pi}\int_{\mathcal{M}} d^d x\sqrt{-g}\left[\mathcal{L}_E + \alpha\mathcal{L}_{GB} - 2\Lambda - \mathcal{F} + m^2\sum_{i=1}^{4} c_i \mathcal{U}_i(g, f)\right], \tag{2}$$

where the Lagrangian of Einstein gravity is the Ricci scalar, $\mathcal{L}_E = \mathcal{R}$, α is the GB coefficient with dimension $(length)^2$, and $\mathcal{L}_{GB}$ is the Lagrangian of GB gravity

$$\mathcal{L}_{GB} = R_{\mu\nu\tau\sigma}R^{\mu\nu\tau\sigma} - R_{\mu\nu}R^{\mu\nu} + \mathcal{R}^2. \tag{3}$$

Also, $\Lambda = -(d-1)(d-2)/2l^2$ is the negative cosmological constant, $\mathcal{F} = F_{\mu\nu}F^{\mu\nu}$ is the Maxwell invariant where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the Faraday tensor and A_μ is the gauge potential. In addition, f is a fixed symmetric tensor, c_i's are some constants, and $\mathcal{U}_i$'s are symmetric polynomials of the eigenvalues of matrix $\mathcal{K}^\mu_\nu = \sqrt{g^{\mu\alpha}f_{\alpha\nu}}$

$$\mathcal{U}_1 = [\mathcal{K}], \quad \mathcal{U}_2 = [\mathcal{K}]^2 - \left[\mathcal{K}^2\right], \quad \mathcal{U}_3 = [\mathcal{K}]^3 - 3[\mathcal{K}]\left[\mathcal{K}^2\right] + 2\left[\mathcal{K}^3\right],$$
$$\mathcal{U}_4 = [\mathcal{K}]^4 - 6\left[\mathcal{K}^2\right][\mathcal{K}]^2 + 8\left[\mathcal{K}^3\right][\mathcal{K}] + 3\left[\mathcal{K}^2\right]^2 - 6\left[\mathcal{K}^4\right].$$

Varying the action (2) with respect to $g_{\mu\nu}$ and A_μ, one can obtain the field equations as

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}(\mathcal{R} - 2\Lambda) + \alpha G^{GB}_{\mu\nu}\mathcal{L}_{GB} + m^2\chi_{\mu\nu} = T_{\mu\nu}, \tag{4}$$

$$\partial_\mu\left(\sqrt{-g}F^{\mu\nu}\right) = 0, \tag{5}$$

where $G^{GB}_{\mu\nu}$, $\chi_{\mu\nu}$ and the energy–momentum tensor $(T_{\mu\nu})$ are, respectively

$$G^{GB}_{\mu\nu} = 2(R_{\mu\tau\sigma\lambda}R_\nu^{\tau\sigma\lambda} - 2R_{\mu\tau\nu\sigma}R^{\tau\sigma} - 2R_{\mu\lambda}R^\lambda_\nu + \mathcal{R}R_{\mu\nu}) - \frac{1}{2}\mathcal{L}_{GB}g_{\mu\nu}, \tag{6}$$

$$\chi_{\mu\nu} = -\frac{c_1}{2}(\mathcal{U}_1 g_{\mu\nu} - \mathcal{K}_{\mu\nu}) - \frac{c_2}{2}\left(\mathcal{U}_2 g_{\mu\nu} - 2\mathcal{U}_1\mathcal{K}_{\mu\nu} + 2\mathcal{K}^2_{\mu\nu}\right) - \frac{c_3}{2}(\mathcal{U}_3 g_{\mu\nu} - 3\mathcal{U}_2\mathcal{K}_{\mu\nu} +$$
$$6\mathcal{U}_1\mathcal{K}^2_{\mu\nu} - 6\mathcal{K}^3_{\mu\nu}) - \frac{c_4}{2}(\mathcal{U}_4 g_{\mu\nu} - 4\mathcal{U}_3\mathcal{K}_{\mu\nu} + 12\mathcal{U}_2\mathcal{K}^2_{\mu\nu} - 24\mathcal{U}_1\mathcal{K}^3_{\mu\nu} + 24\mathcal{K}^4_{\mu\nu}), \tag{7}$$

$$T_{\mu\nu} = 2F_{\mu\lambda}F^\lambda_\nu - \frac{1}{2}\mathcal{F}g_{\mu\nu}. \tag{8}$$

Now, we are going to obtain the d-dimensional magnetic solutions in GB gravity coupled to Maxwell electromagnetic field. In order to obtain exact solutions, we should make a choice for the reference metric. We consider the following ansatz metric [95]

$$f_{\mu\nu} = diag(-\frac{c^2}{l^2}, 0, 0, \frac{c^2}{l^2}h_{ij}), \tag{9}$$

where c is a constant and its values must be positive.

Using the metric ansatz (9), we can obtain the following explicit forms for $\mathcal{U}_i$'s [95]

$$\mathcal{U}_1 = \frac{d_2 c}{\rho}, \ \mathcal{U}_2 = \frac{d_2 d_3 c^2}{\rho^2}, \ \mathcal{U}_3 = \frac{d_2 d_3 d_4 c^3}{\rho^3}, \ \mathcal{U}_4 = \frac{d_2 d_3 d_4 d_5 c^4}{\rho^4}, \tag{10}$$

which $d_i = d - i$. Due to our interest to investigate the magnetic solutions, we assume the vector potential as

$$A_\mu = h(\rho)\delta^\varphi_\mu. \tag{11}$$

Using the Maxwell equation (5) and the metric (1), one finds the following differential equation for the only nonzero component of Faraday tensor

$$d_2 F_{\varphi\rho} + \rho F'_{\varphi\rho} = 0, \tag{12}$$

where the "prime" denotes differentiation with respect to ρ. Equation (12) has the following solution

$$F_{\varphi\rho} = \frac{q}{\rho^{d_3}}, \tag{13}$$

where q is an integration constant which is related to electrical charge. Regarding $F_{\varphi\rho}$ as the nonzero component of $F_{\mu\nu}$ and inserting Eq. (1) in Eq. (4), one can obtain

$$\begin{aligned} d_2 l^2 \rho \left[\rho^2 - 2\alpha d_3 d_4 g(\rho)\right] g'(\rho) + 2\rho^4 F_{\varphi\rho}^2 - d_2 d_3 d_4 l^2 \left\{ m^2 \left[\frac{c c_1 \rho^3}{d_3 d_4} + \frac{c^2 c_2 \rho^2}{d_4} + c^3 c_3 \rho + d_5 c^4 c_4\right] \right. \\ \left. - \frac{\rho^2}{d_4}\left(g(\rho) + \frac{2\Lambda\rho^2}{d_2 d_3}\right) + \alpha d_5 g^2(\rho) \right\} = 0, \end{aligned} \tag{14}$$

$$\begin{aligned} l^2 \rho^{d_3} \left[\rho^2 - 2\alpha d_3 d_4 g(\rho)\right] g''(\rho) + 2 d_3 l^2 \rho^{d_3} \left[\rho - \alpha d_4 g(\rho)\right] g'(\rho) - 2\rho^{d_1} F_{\varphi\rho}^2 - 4\alpha d_3 d_4 d_5 l^2 \rho^{d_4} g(\rho) g'(\rho) \\ - d_3 l^2 \left\{ m^2 \left[c c_1 \rho^3 + d_4 c^2 c_2 \rho^2 + d_4 d_5 c^3 c_3 \rho + d_4 d_5 d_6 c^4 c_4\right] + \alpha d_4 d_5 d_6 g^2(\rho) - d_4 \rho^2 \left(g(\rho) + \frac{2\Lambda\rho^2}{d_3 d_4}\right) \right\} = 0 \end{aligned} \tag{15}$$

Using Eqs. (14) and (15), and after some calculations, we can obtain the metric function $g(\rho)$ as

$$g(\rho) = \frac{\rho^2}{2 d_3 d_4 \alpha}\left(1 \pm \sqrt{1 + \frac{8 d_3 d_4 \alpha}{d_1 d_2}\left[\Lambda - \frac{d_1 m_0}{2\rho^{d_1}} - \frac{d_1 d_3 q^2}{\rho^{2d_2}} - \mathcal{M}(m)\right]}\right), \tag{16}$$

in which m_0 is an integration constant which is related to the mass parameter and $\mathcal{M}(m)$ is the massive term

$$\mathcal{M}(m) = \frac{d_1 m^2}{2\rho^4}\left(c c_1 \rho^3 + d_2 c_2 c^2 \rho^2 + d_2 d_3 c_3 c^3 \rho + d_2 d_3 d_4 c_4 c^4\right).$$

As one can see from Eq. (16), the solution has two branches with "−" and "+" signs. The suitable sign should be chosen such that the obtained solutions reduce to the Einstein-massive-Maxwell solutions introduced in [95] as α goes to zero. In order to have desired metric function, we should choose the minus sign branch. Considering Eq. (16), one finds that Λ-term is the dominant term for large values of ρ, and therefore, the asymptotical behavior of the solution (16) is anti de Sitter (AdS) or de Sitter (dS) provided $\Lambda < 0$ or $\Lambda > 0$. It is worthwhile to mention that in the absence of massive parameter ($m = 0$), the metric function (16) reduces to the GB–Maxwell solution [93].

3. Geometric properties

In order to study the properties of spacetime (1), one can calculate the Kretschmann scalar as

$$R_{\mu\nu\lambda\kappa} R^{\mu\nu\lambda\kappa} = \left(\frac{d^2 g(\rho)}{d\rho^2}\right)^2 + 2 d_2 \left(\frac{1}{\rho}\frac{dg(\rho)}{d\rho}\right)^2 + 2 d_2 d_3 \left(\frac{g(\rho)}{\rho^2}\right)^2. \tag{17}$$

Obviously, considering the metric function (16), the Kretschmann scalar (17) diverges at $\rho = 0$ and one may think that there is a curvature singularity located at $\rho = 0$, but this conclusion is not valid since the spacetime will never achieve $\rho = 0$. Assuming that r_+ is the largest root of the metric function $g(\rho)$, and therefore, the metric function $g(\rho)$ is negative for $\rho < r_+$ and positive for $\rho > r_+$. The function $g_{\rho\rho} = g(\rho)$ cannot be negative (which occurs for $\rho < r_+$) because of the changing in the metric signature. Therefore, we cannot extend the spacetime to $\rho < r_+$, and the valid region is $r_+ \leqslant \rho < \infty$. To get rid of this incorrect extension, we introduce a new radial coordinate r as

$$r^2 = \rho^2 - r_+^2 \Longrightarrow d\rho^2 = \frac{r^2}{r^2 + r_+^2} dr^2, \tag{18}$$

in which $\rho \geq r_+$ leads to $0 \leq r < \infty$. Now, by applying this coordinate transformation, it is possible to obtain the metric (1) in the following form

$$ds^2 = -\frac{r^2 + r_+^2}{l^2} dt^2 + \frac{r^2}{\left(r^2 + r_+^2\right) g(r)} dr^2 + l^2 g(r) d\varphi^2 + \frac{r^2 + r_+^2}{l^2} dX^2, \tag{19}$$

where the coordinates φ and r are in the range $0 \leq \varphi < 2\pi$ and $0 \leq r < \infty$, as usual. The metric function $g(r)$ (Eq. (16)) is now given as

$$g(r) = \frac{r^2 + r_+^2}{2 d_3 d_4 \alpha}\left(1 - \sqrt{1 + \frac{8 d_3 d_4 \alpha}{d_1 d_2}\left[\Lambda - \frac{d_1 m_0}{2\left(\sqrt{r^2 + r_+^2}\right)^{d_1}} - \frac{d_1 d_3 q^2}{\left(r^2 + r_+^2\right)^{d_2}} - \mathcal{M}_+(m)\right]}\right), \tag{20}$$

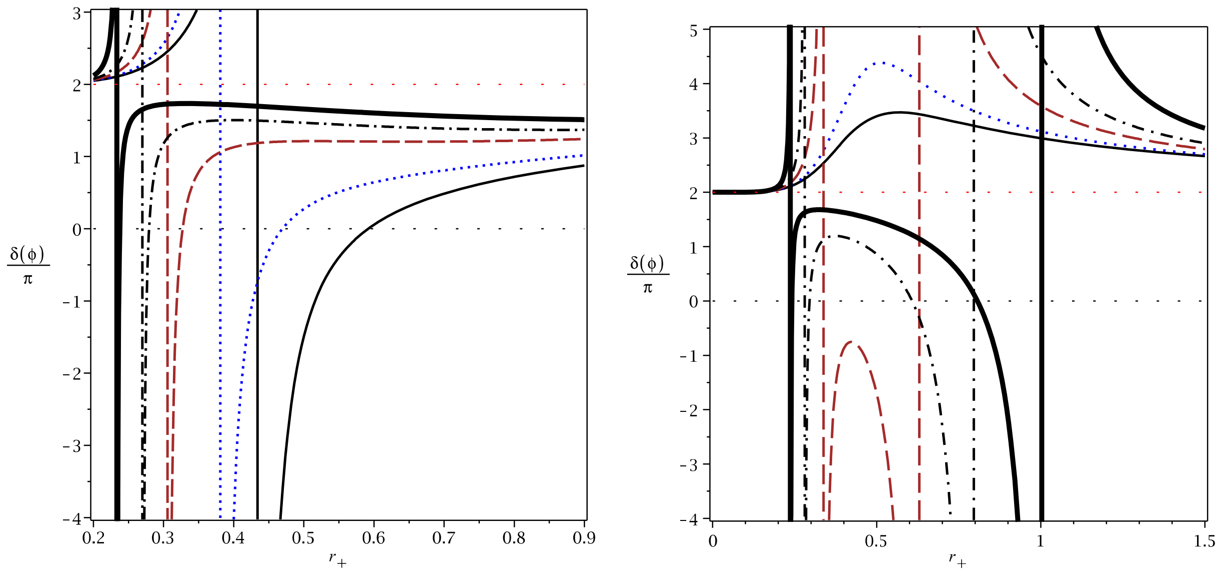

Fig. 1. $\delta(\phi)$ versus r_+ for $l=c=c_1=c_2=c_3=c_4=\alpha=1$, $q=0.1$ and $d=5$; $m=0$ (continuous line), $m=0.3$ (dotted line), $m=0.55$ (dashed line), $m=0.7$ (dashed–dotted line) and $m=0.9$ (bold continuous line). Left panel: AdS solutions and right panel: dS solutions.

where

$$\mathcal{M}_+(m)=\frac{d_1 m^2}{2\left(r^2+r_+^2\right)^2}\left(cc_1^3\left(\sqrt{r^2+r_+^2}\right)^3+d_2c_2c^2\left(r^2+r_+^2\right)+d_2d_3c_3c^3\sqrt{r^2+r_+^2}+d_2d_3d_4c_4c^4\right).$$

Now, considering the new metric function, it is a matter of calculation to show that all curvature invariants are finite (divergence free) at the origin and also other values of $0\leq r<\infty$ (contrary to black hole solutions). Therefore, this spacetime has no curvature singularity and no horizon. Using the Taylor expansion, in the vicinity of $r=0$, we have

$$g(r)=g(r)\left|_{r=0}+\left(\frac{dg(r)}{dr}\right|_{r=0}\right)r+\frac{1}{2}\left(\frac{d^2g(r)}{dr^2}\right|_{r=0}\right)r^2+\mathcal{O}\left(r^3\right)+\ldots\ , \tag{21}$$

in which

$$g(r)\left|_{r=0}=\frac{dg(r)}{dr}\right|_{r=0}=0. \tag{22}$$

On the other hand, the spacetime (19) has a conic geometry with a conical singularity at $r=0$, because the limit of the ratio "*circumference/radius*" is not 2π ($\lim_{r\longrightarrow 0}\frac{1}{r}\sqrt{\frac{g_{\varphi\varphi}}{g_{rr}}}\neq 1$). It is notable that, this singularity can be removed if one exchanges the coordinate φ with the following period

$$Period_{\varphi}=2\pi\left(\lim_{r\longrightarrow 0}\frac{1}{r}\sqrt{\frac{g_{\varphi\varphi}}{g_{rr}}}\right)^{-1}=2\pi\left(1-4\mu\right), \tag{23}$$

in which μ is

$$\mu=\frac{1}{4}\left[1-\frac{2}{lr_+}\left(\frac{d^2g(r)}{dr^2}\right|_{r=0}\right)^{-1}\right]=\frac{1}{4}-\frac{d_2r_+^{2d_{1/2}}}{4l\Omega}, \tag{24}$$

and Ω is given as

$$\Omega=\left[2\Lambda r_+^4-cd_2\Upsilon m^2\right]r_+^{2d_2}+2d_3^2q^2r_+^4, \tag{25}$$

where

$$\Upsilon=d_3d_4d_5c_4c^3+d_3d_4c_3c^2r_++d_3c_2cr_+^2+c_1r_+^3. \tag{26}$$

This result shows that the metric (19) describes a spacetime which is locally flat near the origin but has a conical singularity at $r=0$ with a deficit angle $\delta(\phi)=8\pi\mu$. In order to investigate the effects of different parameters on deficit angle, we plot the deficit angle versus r_+ in various figures (see Figs. 1–4).

First of all, the upper parts of thin dotted line in figures is forbidden area. In order to have a better picture regarding the behavior of solutions, we have presented this part in figures. There is no restriction on negative values of the deficit angle. In addition, in order to study the effects of cosmological constant, we have plotted two set of diagrams for variation of different parameters.

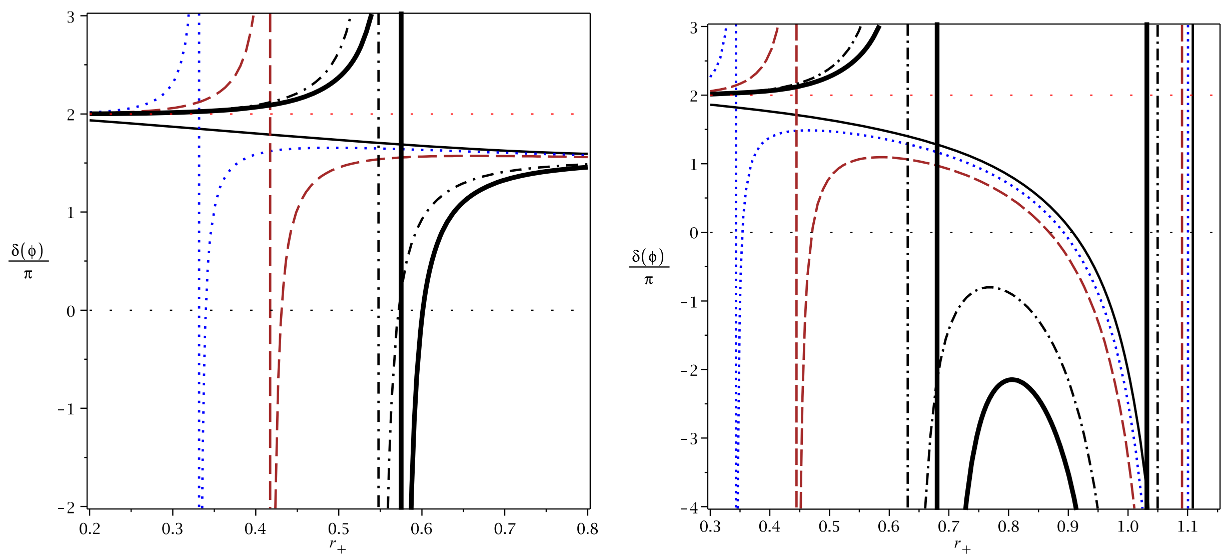

Fig. 2. $\delta(\phi)$ versus r_+ for $l = c = c_1 = c_2 = c_3 = c_4 = \alpha = m = 1$ and $d = 5$; $q = 0$ (continuous line), $q = 0.2$ (dotted line), $q = 0.3$ (dashed line), $q = 0.5$ (dashed–dotted line) and $q = 0.55$ (bold continuous line). Left panel: AdS solutions and right panel: dS solutions.

For AdS spacetime, depending on choices of different parameters, deficit angle could have different number of the roots, regions of positivity/negativity and divergency. Plotted diagrams for variations of the massive parameter (left panel of Fig. 1), electric charge (left panel of Fig. 2) and dimensions (up panels of Fig. 4), show that there exists a divergency and a root (located after divergency) for deficit angle. Before the divergency, deficit angle is positive valued but it is located at prohibited area. Therefore, in this region, there is no value available for deficit angle. Between the divergency and the root, deficit angle is negative, whereas after the root, it is positive valued within permitted region. The divergency and the root are decreasing functions of the massive parameter (left panel of Fig. 1), while they are increasing functions of the electric charge (left panel of Fig. 2) and dimensions (up panels of Fig. 4). Interestingly, in the absence of the charge, deficit angle is positive valued without any root and divergency (continuous line in left panel of Fig. 2). This shows that existences of negative valued deficit angle, the root and the divergency are due to contributions of the electric charge (in case of $c_i > 0$). Here, we have considered the variation of c_1 as a case example to study the effects of massive coefficients on deficit angle (up panels of Fig. 3). Evidently, for very small values of the c_1, deficit angle could have three divergencies with three roots. The number of roots and divergencies are decreasing function of this parameter. Meaning, by increasing c_1, it is possible to have, two divergencies with three roots, one divergency with three roots, one divergency with two roots in which one of them is extreme and one divergency with one root (for more details see up panels of Fig. 3).

For dS spacetime, the situation is different. Here, it is possible to have one of the following cases: I) Deficit angle is always positive but is not in permitted area (right panel of Fig. 1 for small values of m). II) Two divergencies without any root. In this case, between the divergencies, the deficit angle is negative valued. Otherwise, although the deficit angle is positive, it is not located at the permitted area. III) Two divergencies with one extreme root with similar behavior to previous case. IV) Two divergencies with two roots. Between the roots, the deficit angle is positive and within permitted area. While between the smaller (larger) root and the smaller (larger) divergency, the deficit angle is negative. V) One root and one divergency with root being before divergency. This case is only observed for absence of the electric charge (right panel of Fig. 2). VI) One divergency in which for both sides of the divergency, the deficit angle is positive but not in permitted area. Evidently, the number of roots is an increasing function of the massive parameter (right panel of Fig. 1), c_1 (down panels of Fig. 3) and dimensions (down panels of Fig. 4) while it is a decreasing function of the electric charge (right panel of Fig. 2). Here, we see that contrary to AdS case, in the absence of electric charge, root, negative values and divergency exist for deficit angle which are originated from contribution of the cosmological constant.

The magnetic solutions presented here contain a conical singularity. The geometrical structure of solutions is determined by deficit angle. In other words, positivity/negativity, root and divergencies of the deficit angle determine the total structure of solutions. For the positive values of deficit angle, the geometrical shape of solutions is cone-like one. On the contrary, the negative values of it show an extra angle known as surplus angle. This extra angle results into a saddle-like cone as the structure of solutions. The absence of deficit angle shows that our solutions are brane without any missing segment. In other words, in this case, no topological defect is observable. The presence of divergency indicates a type of phase transition for our solutions. The same could also apply for root of the deficit angle (except extreme ones), since the total structure of magnetic solutions changes in this case.

4. Conclusions

In this paper, we have considered magnetic solutions of Einstein gravity with three generalizations: massive gravity, adding cosmological constant and Gauss–Bonnet gravity. We have obtained the analytical solutions and investigated their geometrical structure. Among properties of the solutions, the deficit angle was studied thoroughly since it plays a crucial role in geometrical structure of the solutions.

Despite the effects of Gauss–Bonnet gravity on the magnetic solutions, it was shown that deficit angle is not affected by this gravity. Whereas the generalization to massive gravity and adding the cosmological constant resulted into diverse modifications in the deficit angle. It was pointed out that positivity or negativity of the deficit angle, its roots and divergencies were both massive and Λ parameters

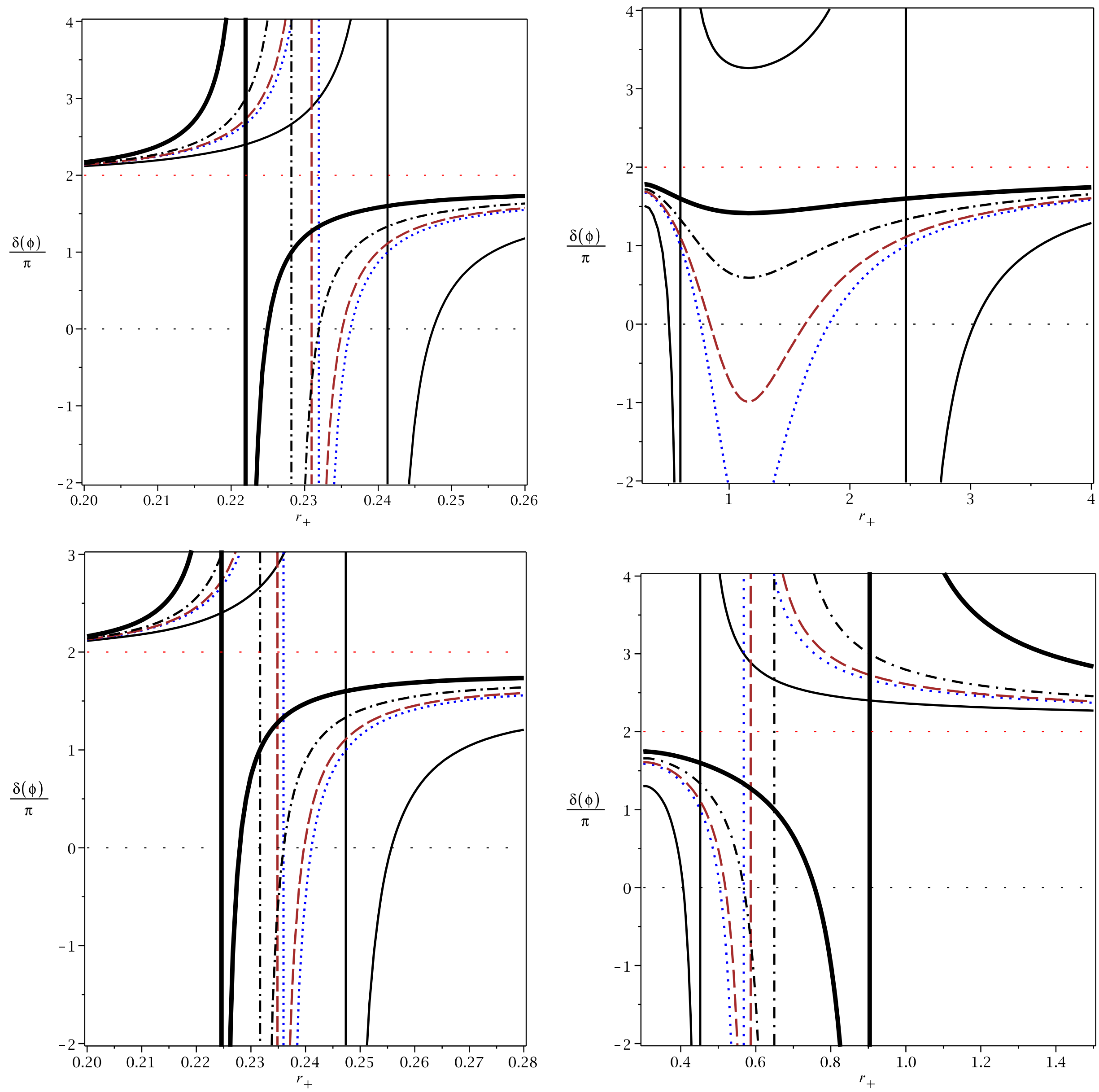

Fig. 3. $\delta(\phi)$ versus r_+ for $l = c = c_2 = c_3 = c_4 = \alpha = m = 1$, $q = 0.1$ and $d = 5$; $c_1 = -11$ (continuous line), $c_1 = -7$ (dotted line), $c_1 = -6.5$ (dashed line), $c_1 = -5$ (dashed–dotted line) and $c_1 = -1$ (bold continuous line). Up panels: AdS solutions (different scales) and down panels: dS solutions (different scales).

dependent. In other words, existence of the massive gravitons modified the structure of solutions on a significant level, and the nature of background of spacetime has also a significant role on total structure of the solutions. One of the interesting results of this paper was related to the existence of divergencies in deficit angle. In the case of AdS spacetime, this was due to the presence of the electrical charge. On the contrary, for dS case, even in the absence of the electrical charge, we encountered with the divergencies. In this case, existence of divergency was due to the contributions of the cosmological constant and massive parameters.

Also, it is worth mentioning that, despite the positive deficit angle (deleted segment) which is bounded by the value of 2π, the negative deficit angle (added segment or surplus angle) is unbounded, and therefore, one can conclude that the range of real deficit angles is from $-\infty$ to 2π. On the other hand, considering this fact that the metric function could be interpreted as a potential (see for example chapter 9 of Ref. [96], for more details), it is arguable that the singular points could be interpreted as phase transitions. Likewise, the geometrical structure of the obtained solutions in this gravity in the case of positive and negative values of the deficit angle is different. For positive deficit angle, the geometrical structure of the object is cone-like with a deficit angle whereas for the negative deficit angle, the structure will be saddle-like with a surplus angle. So, one may state that due to these differences in the structure of the obtained solutions, the roots of the deficit angle represent another type of phase transition point.

The importance of topological defects lies within the fact that they were formed during the early universe phase transition. This indicates that these topological defects carry in formation regarding the early universe which could help us draw a better picture regarding the evolution of universe. That being said, it is very important to see what type of geometrical structure these solutions could have. This point could be understood through studies that are conducted in the context of deficit angle. The roots, divergencies and signature of the

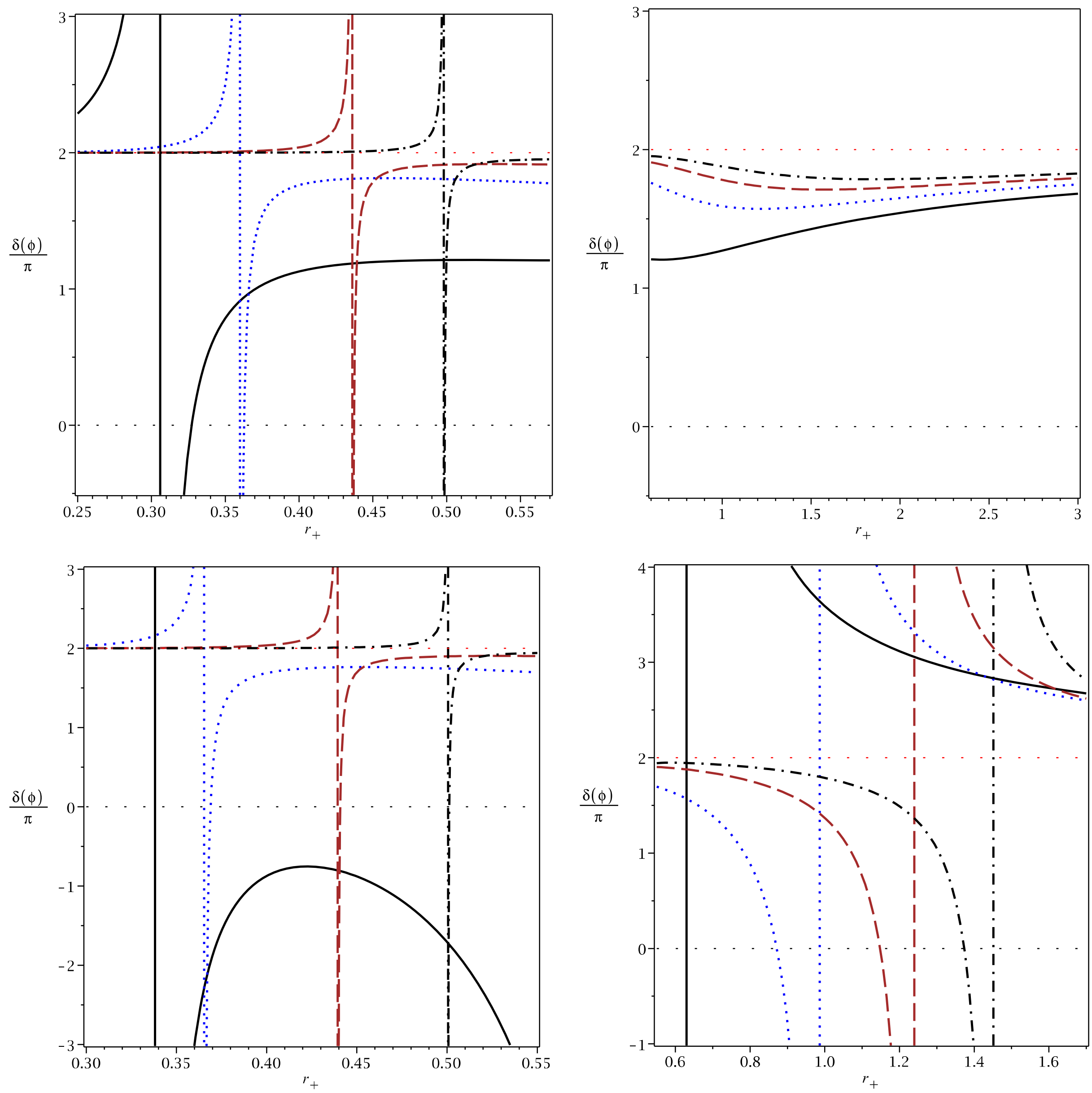

Fig. 4. $\delta(\phi)$ versus r_+ for $l = c = c_1 = c_2 = c_3 = c_4 = \alpha = 1$, $m = 0.55$ and $q = 0.1$; $d = 5$ (continuous line), $d = 6$ (dotted line), $d = 7$ (dashed line) and $d = 8$ (dashed–dotted line). Up panels: AdS solutions (different scales) and down panels: dS solutions (different scales).

deficit angle play the key roles in describing the geometrical, hence physical properties of topological defects. The horizonless magnetic solutions belong to one of the classes of topological defects. In addition, these topological defects have lensing property which enables them to be detected if instrumental advances reached certain level. The lensing property is governed by the factor which is known as deficit angle [72,97–99]. Therefore, we are expecting to have different lensing property in the context of negative deficit angle versus positive ones. This highlights the importance of roots and divergencies of deficit angle, hence phase transition like behaviors that were observed for the solutions.

Taking into account the conceptions of deficit and surplus angles, one may discuss a typical geometrical phase transition. In other words, it will be interesting to analyze the effects of different generalizations on such transition. In addition, one may regard a nonlinear gauge field and investigate its effects on the geometrical properties of the magnetic solutions. Moreover, it will be fascinating to use the cut-and-paste method for gluing two copies of the mentioned spacetime by a traversable bridge with a typical wormhole interpretation [100–102]. We will address these subjects in the future works.

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