On $\pi_0 \to \gamma\gamma$ and the axial anomaly at $T \neq 0$ [†]

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Abstract

In vacuum, in the chiral limit the coupling of a pion to two on-shell photons is directly related to the coefficient of the axial anomaly in QED. This relationship is lost at any nonzero temperature. Explicit calculations show that the coupling decreases with temperature and vanishes at T_c, the temperature of chiral symmetry restoration.

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Consider QCD with two light flavors (*up* and *down*). The Lagrangian has an approximate $SU(2)_l \times SU(2)_r$ chiral symmetry, spontaneously broken to $SU(2)$ in vacuum. The resulting Goldstone bosons (the pions) have a direct coupling to the axial current,

$$\langle 0|J^a_{\mu 5}|\pi^b(P)\rangle = iP_\mu f_\pi \delta^{ab} \tag{1}$$

with $a, b = 1..3$ and $f_\pi \sim 93MeV$ is the pion decay constant. In the chiral limit the axial current is conserved and, consequently, the pions are massless.

Currents which are conserved classicaly may not remain so quantum mechanically. In particular, at one-loop order the third component of the axial current develops an anomalous divergence [1],

$$\partial_\mu J^3_{\mu 5} = -\frac{\alpha_{em}}{8\pi}\varepsilon_{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta} \tag{2}$$

where $F_{\mu\nu}$ is the QED field strength. A striking manifestation of the QED axial anomaly is to allow the π^0 to decay into two photons. Let us define the amplitude for $\pi^0 \to \gamma\gamma$ as

$$\mathcal{A} = g_{\pi\gamma\gamma}\,\varepsilon_{\alpha\beta\mu\nu}\,\epsilon^1_\alpha\epsilon^2_\beta\,k^1_\mu k^2_\nu \tag{3}$$

where k^1, k^2, and ϵ^1, ϵ^2 are the momenta and polarization vectors of the photons. In vacuum, the spontaneous breaking of chiral symmetry, (1), and the anomaly equation, (2), suffice to deduce the beautiful relation,

$$f_\pi\, g_{\pi\gamma\gamma} = \frac{1}{\pi}\,\alpha_{em}. \tag{4}$$

This formula is valid in the *chiral limit* and for *on-shell photons*. The *r.h.s.* of (4) comes from the anomaly of (2): without this term, $g_{\pi\gamma\gamma}$ would vanish, which is the content of the Sutherland-Veltman theorem [2]. Another remarkable theorem [1] states that the coefficient of the anomalous term is not affected by higher-order corrections, so that in the vacuum, (4) is exact.

What about finite temperature ? The non-renormalization of the axial anomaly holds at finite T or density [3, 4] but is (4) still valid ? Let us first assume so. We know [5] that, to leading order at low temperature and in the chiral limit, the pion decay constant changes as

$$f_\pi(T) = f_\pi\left(1 - \frac{T^2}{12 f_\pi^2}\right). \tag{5}$$

(for $N_f = 2$ light flavors). As the *r.h.s.* of (4) is T independent [1], one concludes that $g_{\pi\gamma\gamma}(T)$ *increases* at low T, $g_{\pi\gamma\gamma}(T) = g_{\pi\gamma\gamma}(1 + T^2/12f_\pi^2)$. At higher temperatures, $f_\pi(T)$ goes to zero at T_c, where chiral symmetry becomes manifest, which would imply that $g_{\pi\gamma\gamma}$ blows up at T_c. This is obviously absurd and (4) cannot hold as such at finite temperature. Actually, a closer look at the conditions under which (4) is derived, reveals that Lorentz invariance is crucial. In the presence of a thermal bath, Lorentz invariance is lost and (4) is not true at any non-zero temperature. Apparently, this has not been fully appreciated before (see [3] for related issues). A more detailled discussion of our claim based on Ward identities can be found in [6].

How is $g_{\pi\gamma\gamma}$ changing with temperature ? Because the direct connection with the coefficient of the anomaly is lost, this is a dynamical problem that can only be adressed by an explicit calculation. In this respect, finite T is similar to the problem of computing the amplitude for $\pi \to \gamma\gamma$ for *off-shell* photons (see *e.g* [7]): the Ward identities provide only limited information .

[1] We neglect $\delta\alpha_{em}(T) \sim \log(T)$, corrections. These are subleading with respect to the T^2 term of $f_\pi(T)$ in (5).

Consider first a linear sigma model with constituent quarks (see [8, 9] for details). This is a renormalizable theory and the pion has a pseudoscalar coupling to the quarks [2],

$$\mathcal{L}_{int} = g\phi\bar{q}_R q_L + h.c. = g\,\sigma\bar{q}q + ig\,\pi\bar{q}\gamma_5 q \tag{6}$$

The scalar potential is such that the chiral symmetry is spontaneously broken in vacuum, $\sigma \to \sigma_0 + \sigma$. Then, to leading order, $f_\pi = \sigma_0$ and $m_q = g\sigma_0$ and the π is a Goldstone boson. The π couples to two photons through the triangle diagram (fig. 1 below). Because the pion-quark coupling is pseudoscalar, $g\gamma_5$, and the photons vertices are vector-like, a mass insertion is necessary to restore chirality; thus Dirac trace brings in one power of the constituent quark mass. The remaining loop integral is ultraviolet finite,

$$g_{\pi\gamma\gamma} \propto \alpha_{em}\, g\, m_q \int d^4p\, \frac{1}{(p^2 + m_q^2)^3} \tag{7}$$

The constituent quark mass provides and infrared cut-off, so $I \propto 1/m_q^2$, and

$$g_{\pi\gamma\gamma} = \frac{\alpha_{em}}{\pi} g \frac{m_q}{m_q^2} = \frac{\alpha_{em}}{\pi} \frac{1}{f_\pi} \,. \tag{8}$$

Thus, as expected, the constituent quark model respects (4).

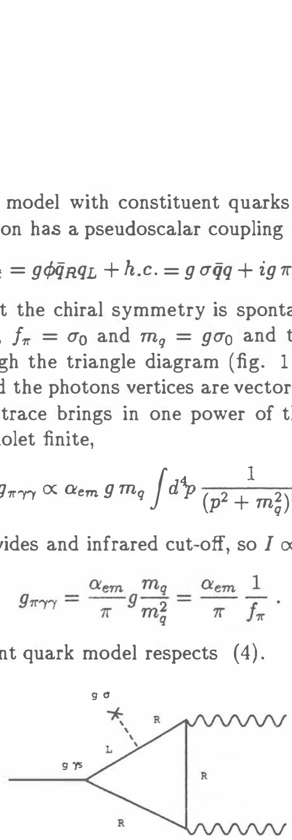

Figure 1: π_0 couples to $\gamma\gamma$ through the triangular diagram. L and R refer to quark chiralities.

It is easy to extend this calculation to finite T. As there is no dependence in the external momenta, we compute the integral I in the imaginary time formalism. Then

$$I \to I_T = T \sum_{n=-\infty}^{+\infty} \int d^3p \frac{1}{(p^2 + m_q^2)^3} \tag{9}$$

with $q_0 = \pi(2n+1)T$. As T gets close to T_c, $m_q \ll T$, and T provides the infrared cut-off, $I_T \propto 1/T^2$. Consequently, near and below T_c,

$$g_{\pi\gamma\gamma}(T) \propto \alpha_{em}\, g^2 \frac{f_\pi(T)}{T^2} \to 0 \tag{10}$$

This calculation suggests that the coupling goes to zero at T_c as a consequence of chiral symmetry restoration. For further arguments, see [8].

Now, let us consider the low temperature limit. At low T, the dynamics is dominated by the massless pions and we use a gauged non-linear sigma model with a Wess-Zumino term [10],

$$\begin{aligned} \mathcal{L} &= \frac{1}{2}(D_\mu \pi^a)^2 + \frac{1}{6f_\pi^2}(\pi^a \partial_\mu \pi^a)^2 + \ldots \\ &+ \left(\frac{e^2 N_c}{96\pi^2}\right) \frac{1}{f_\pi} \pi^0 \varepsilon_{\alpha\beta\mu\nu} F_{\alpha\beta} F_{\mu\nu} + \ldots \end{aligned} \tag{11}$$

[2]We consider a simple $U(1) \times U(1)$ model. The extension to the non-abelian case is trivial.

where the dots stands for operators with more pions or higher derivatives. At tree level, the effective Lagrangian (11) is normalized so that (4) holds [10]. Pion one-loop corrections (fig. 2) to the anomalous vertex are easily computed. Consider first vacuum corrections [11]. In the chiral limit and for on-shell photons, these simply amount to replace f_π in (11) by its renormalized, physical value. This is how the Adler-Bardeen theorem, which states that the anomaly is not renormalized beyond one loop order, works at the level of the effective action. In the chiral limit $m_\pi \to 0$ and for on-shell photons, the $\mathcal{O}(P^4)$ anomalous operator of (11) is the unique operator which contributes to the amplitude for $\pi_0 \to \gamma\gamma$; all other operators are $\mathcal{O}(P^6)$ or higher, and vanish on the photon mass shell. Hence, all the divergences have to be absorbed by (11).

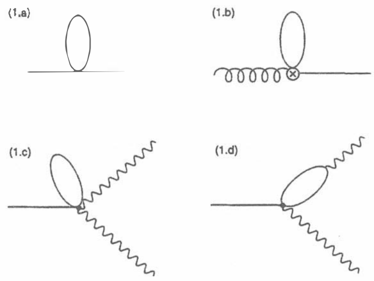

Figure 2: pion one-loop corrections to $\pi_0 \to \gamma\gamma$ in the gauged WZW model.

At finite temperature, $T \ll f_\pi$, a careful calculation [6] reveals [3] that

$$g_{\pi\gamma\gamma}(T) = g_{\pi\gamma\gamma}\left(1 - \frac{T^2}{12 f_\pi^2}\right) \tag{12}$$

Hence, $g_{\pi\gamma\gamma}$ decreases like $f_\pi(T)$ at low temperature (5), consistent with (10) close to T_c. That the temperature dependence of $g_{\pi\gamma\gamma}$ can be non-trivial is due to the fact that at $T \neq 0$, unlike in the vacuum, it is possible to add new $\mathcal{O}(P^4 T^2/f_\pi^2)$ terms to the Lagrangian (11) [6, 13]. These terms are non-local, similar to the hard thermal loops of hot QCD [14].

The result (10) shows that the amplitude for $\pi \to \gamma\gamma$ vanishes at the critical temperature. With non-zero quark current quark masses, this translates to a decrease of the effective coupling near T_c, with $f_\pi(T_c) \sim 1/3 f_\pi$. Also, in the linear sigma model, if $\pi \to \gamma\gamma$ is suppressed, $\pi\sigma \to \gamma\gamma$ is not [8]. This is because the mass insertion, $\propto \sigma_0$, is equivalent to the insertion of a σ particle.

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