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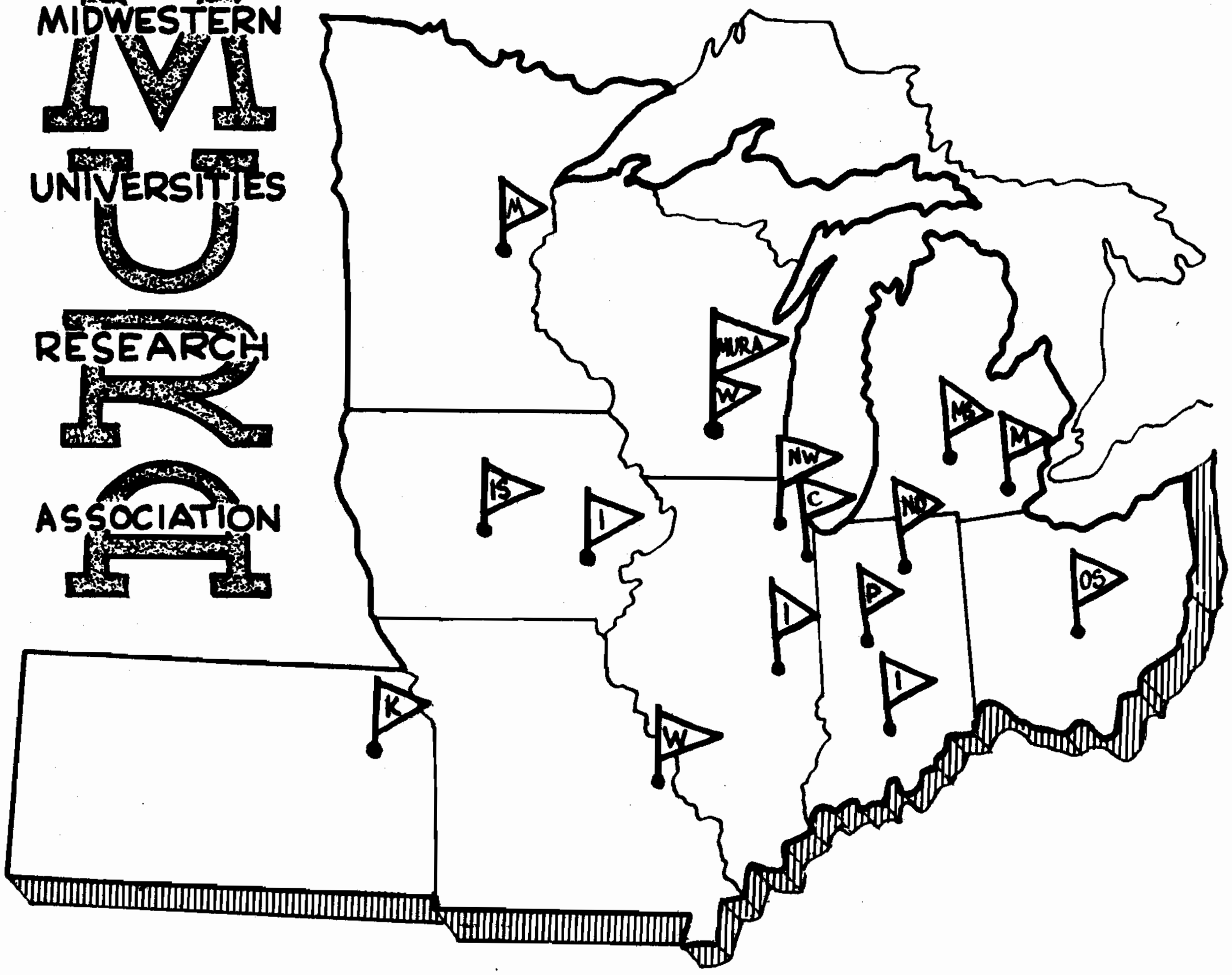

WINDING AND CONSTRUCTION ERRORS IN
RELUCTANCE POLE MAGNETS

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MIDWESTERN UNIVERSITIES RESEARCH ASSOCIATION*

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WINDING AND CONSTRUCTION ERRORS IN RELUCTANCE POLE MAGNETS.

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ABSTRACT: The effects of errors in the placement of return windings and errors of construction in reluctance pole magnets are discussed. It is found that the perturbation of the field by a misplaced winding in a reluctance pole magnet is smaller than the perturbation occurring in a conventional magnet with pole-face windings largely because the windings are moved back from the median plane by the thickness of the reluctance pole. The perturbation of the field due to an error in construction of the reluctance pole can, however, be quite serious.

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I. INTRODUCTION

Nielsen[1] has recently proposed a magnet structure involving the introduction of a suitable reluctance path for the return of the gap flux. The proposal seems to have two advantages. Firstly, the reluctance pole magnet can be so designed that some or all of the return winding cam be lumped so that a thick coil of distributed return winding is avoided. Secondly, the reluctance pole will partially shield the gap from errors in the winding of the distributed return coil. How much shielding is provided by the reluctance pole is one point to be investigated in this report.

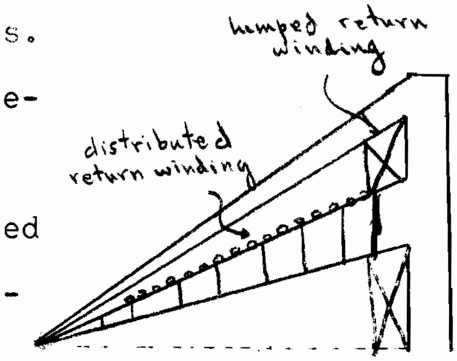


Fig. 1

The reluctance pole envisioned by Nielsen would be made up of iron with the spacing and thickness of the laminae adjusted so as to have a suitable permeability to flux in the radial direction. The construction of such a reluctance pole would seem to be a rather delicate matter and one would like to know how serious is the perturbation of the field produced by an error in the construction of the reluctance pole - an error such as a displacement of a lamina or a deviation of the spacing of the laminae from the design value. This point will also be discussed in this report.

Winding Error with Iron Shield:

As Kerst[2] has pointed out the displacement of a wire carrying a current I from a radius R to R + ΔR is equivalent to the introduction of a current-I at R and a current +I at R + ΔR. The problem now is to what extent does the reluctance pole shield the gap from the perturbing field due to this current pair? This problem arises in another correction. Kerst has suggested that in order to reduce the effect of winding errors in a conventional magnet a thin sheet of iron, fully saturated so that its permeability is low, can be introduced below the face coils.

To determine the extent to which the reluctance pole or the saturated iron sheet will shield gap from a perturbing field due to winding errors let us first consider an infinite slab of uniform thickness t and permeability μ . A wire carrying a current q is located in air a distance s above the slab. Denote the region above the slab by I, the slab itself by II and the region below the slab by III.

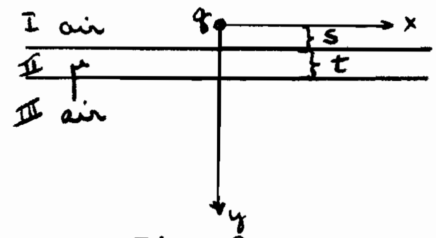


Fig. 2

The vector potential $\vec{A}$, where $\vec{B} = \vec{\nabla} \times \vec{A}$, can be chosen to have only one non-vanishing component, the z-component, which will be denoted by A^{I}, A^{II} and A^{III} in the three regions. A^{II} and A^{III} can have no singularities in their respective regions while the only singularity that A^{I} can have within region I is at $x = y = 0$. Furthermore A^{I}, A^{II}, A^{III} satisfy the boundary conditions

$$A^{I} = A^{II}, \quad \mu \frac{\partial A^{I}}{\partial y} = \frac{\partial A^{II}}{\partial y}, \quad \text{at} \quad y = s,$$
$$A^{III} = A^{II}, \quad \mu \frac{\partial A^{III}}{\partial y} = \frac{\partial A^{II}}{\partial y}, \quad \text{at} \quad y = s + t. \tag{1}$$

These conditions define A uniquely.

This problem can readily be solved by the method of images. We first determine the location of the images. We shall have

$$A^{I} = -q \ln(x^2 + y^2) - \sum_i q_i^{I} \ln\left(x^2 + [y - d_i^{I}]^2\right)$$

where q_i^{I} is an image current located at $x = 0$, $y = d_i^{I}$.

Similarly
$$A^{I} = -\sum_i q_i^{II} \ln\left(x^2 + [y - d_i^{II}]^2\right) ,$$
$$A^{III} = -\sum_i q_i^{II} \ln\left(x^2 + [y - d_i^{II}]^2\right) .$$

The restriction on the locations of the singularities in these functions imply that
$$d_i^{I} > s \quad ; \quad d_i^{II} < s \quad \text{or} \quad d_i^{II} > s+t \quad ; \quad d_i^{III} < s + t \quad .$$

It is clear that in order to satisfy the boundary conditions at y = s we must include in A^{II}, corresponding to the term $\ln(x^2 + [y - d_i^{I}]^2)$ in A^{I} a term involving an image at d_i^{II} where
$$[s - d_i^{II}]^2 = [s - d_i^{I}]^2$$
so that

either $d_i^{II} = 2s - d_i^{I}$ or $d_i^{II} = d_i^{I}$. (2a,b)

In a similar fashion one finds

either $d_i^{II} = 2(s+t) - d_i^{III}$ or $d_i^{II} = d_i^{III}$. (3a,b)

The location of the images can now be determined. Corresponding to the current at y = 0 it follows from Eq. (2b) that for region II there will be an image at y = 0. From the occurrence of this image it follows from Eqs. (2a) and (3b) that for region I there can be an image at y = 2s and for region III an image at y = 0. Corresponding to this latter image the use of Eq. (3a) indicates an image for region II at y = 2(s + t). Now using successively Eqs. (2b), (2a), (3b), (3a) one obtains the following sets of images:

$$q_m^{I} \text{ at } y = 2s + 2nt \quad , \quad n = 0, 1, 2, \cdots ,$$
$$q_m^{II} \text{ at } y = 2s + 2nt \quad , \quad n = 1, 2, \cdots ,$$
$$\text{and } y = 2nt \quad , \quad n = 0, -1, -2, \cdots ,$$
$$q_m^{III} \text{ at } y = 2nt \quad , \quad n = 0, -1, -2, \cdots .$$

Thus we obtain

$$A^{I} = -q\ln(x^2+y^2) - \sum_{n=0}^{\infty} q_n^{I} \ln\left(x^2+[y-2s-2nt]^2\right),$$

$$A^{II} = -\sum_{n=1}^{\infty} q_n^{II} \ln\left(x^2+[y-2s-2nt]^2\right) - \sum_{n=0}^{\infty} q_{-n}^{II} \ln\left(x^2+[y+2nt]^2\right),$$

$$A^{III} = -\sum_{n=0}^{\infty} q_{-n}^{III} \ln\left(x^2+[y+2nt]^2\right) . \tag{4}$$

Application of the boundary condition yields the following equations:

$$\begin{aligned}
q + q_0^{I} &= q_0^{II} ,\\
\mu(q - q_0^{I}) &= q_0^{II} ,\\
&\left.\begin{aligned} q_n^{I} &= q_{-n}^{II} + q_n^{II} ,\\ \mu q_n^{I} &= -q_{-n}^{II} + q_n^{II} ,\end{aligned}\right\} n \neq 0\\
q_{-n}^{III} &= q_{-n}^{II} + q_{n+1}^{II} ,\\
\mu q_{-n}^{III} &= q_{-n}^{II} - q_{n+1}^{II} .
\end{aligned} \tag{5}$$

The solution of the first two equations is

$$q_0^{I} = -\left(\frac{1-\mu}{1+\mu}\right) q ,$$

$$q_0^{II} = \frac{2\mu}{1+\mu} q .$$

The remaining equations can now be solved for successively larger values of n with the following result:

$$q^{I}_{n} = \left(\frac{1-\mu}{1+\mu}\right)^{2n-1} \frac{4\mu}{(1+\mu)^2}\, q \quad , \quad n \neq 0 ,$$

$$q^{II}_{n} = \left(\frac{1-\mu}{1+\mu}\right)^{2n-1} \frac{2\mu}{1+\mu}\, q \quad , \quad n > 0 ,$$

$$q^{II}_{n} = \left(\frac{1-\mu}{1+\mu}\right)^{-2n} \frac{2\mu}{1+\mu}\, q \quad , \quad n \leq 0 ,$$

$$q^{III}_{n} = \left(\frac{1-\mu}{1+\mu}\right)^{-2n} \frac{4\mu}{(1+\mu)^2}\, q \quad , \quad n \leq 0 .$$

Region III (the gap) is the region of interest. There one obtains

$$A^{III} = -\frac{4\mu}{(1+\mu)^2}\, q \sum_{n=0}^{\infty} \left(\frac{1-\mu}{1+\mu}\right)^{2n} \ln\left(x^2+[y+2nt]^2\right),$$

and

$$H^{III}_{y} = -\frac{\partial A^{III}}{\partial x} = \frac{8\mu}{(1+\mu)^2}\, q x \sum_{n=0}^{\infty} \frac{\left(\frac{1-\mu}{1+\mu}\right)^{2n}}{x^2+[y+2nt]^2} .$$

If $\left(\frac{1-\mu}{1+\mu}\right)^2$ is small this is a rapidly converging sum. If, for example, $\mu = 10$ then $\left(\frac{1-\mu}{1+\mu}\right)^2 \approx 2/3$ and the leading term in H^{III}_{y} is approximately $1/3\left(\frac{2qx}{x^2+y^2}\right)$ where $\frac{2qx}{x^2+y^2}$ is the y component of the field due to an unshielded current of magnitude q. Note that this leading term is independent of t so that no appreciable improvement in the effectiveness of the slab as a shield can be obtained merely by thickening it. In the case of a reluctance pole, however, there is an advantage in having it thick due to the fact that the windings are moved farther away from the gap.

Kerst's suggestion shield poses a slightly different problem from the one that has been solved in the foregoing in tha t the displaced wire is backed by iron as shown in the sketch.

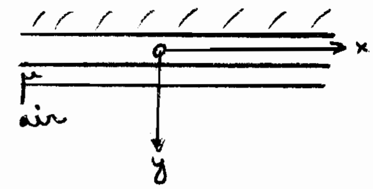


Fig. 3

In this case one obtains, for the vertical component of the magnetic field below the slab,

$$H_y = \frac{8\mu}{\mu+1} g x \sum_{n=0}^{\infty} \frac{\left(\frac{1-\mu}{1+\mu}\right)^n}{x^2+[y+2nt]^2} .$$

The terms of the sum alternate in sign. If they are taken in pairs one obtains a rapidly converging sum of positive terms. By considering only the leading pair of terms we obtain

$$H_y > \frac{8\mu}{\mu+1} g x \left\{ \frac{1}{x^2+y^2} - \left(\frac{\mu-1}{\mu+1}\right) \frac{1}{x^2+(y+2t)^2} \right\}.$$

If $\mu = 10$ and $t \ll y$ this becomes

$$H_y > \frac{1}{3}\left(\frac{2gx}{x^2+y^2}\right),$$

the quantity in parentheses being the field due to the wire backed by iron in the absence of the shield. Curiously, if $x \ll y$ and $y \lesssim 6t$ the field is enhanced by the presence of the shield.

Winding Errors in Reluctance Pole Magnets

The reluctance pole made up of laminae of iron may be expected to differ from the preceding problem partly because of the presence of an iron shield above the reluctance pole (which would have only a small effect if the shield is as far as a gap width away from the reluctance pole) but largely because of the tensor character of the permeability of the reluctance pole.

Let us consider the same geometrical arrangement shown in Fig. 2 assuming, however, that the permeability of the slab to flux parallel to the x-axis is μ_x and the permeability to flux parallel to the y-axis is μ_y. A situation approximating the reluctance pole arises if μ_y is allowed to approach infinity while μ_x is of order 10.

The distinction between this case and the isotropic case arises out of the fact that within the slab the curl of $\vec{B}$ is not zero, but rather

$$\nabla \times \vec{H} = 0 = \nabla \times \left(\frac{1}{\mu_x} B_x \vec{i} + \frac{1}{\mu_y} B_y \vec{j} \right) .$$

Therefore the differential equation satisfied by A within the slab is

$$\frac{1}{\mu_y} \frac{\partial^2 A}{\partial x^2} + \frac{1}{\mu_x} \frac{\partial^2 A}{\partial y^2} = 0 .$$

The boundary conditions that A must satisfy are the same as the conditions stated in Eqs. (1) save for the replacement of μ by μ_x.

This problem can also be solved by the method of images. In place of $\ln(x^2 + (y - d)^2)$ in the expression for the vector potential within the slab one must use $\ln(\mu_y x^2 + \mu_x (y-d)^2)$ or, more conveniently for the consideration of large values of μ_y, $\ln\left(x^2 + \frac{\mu_x}{\mu_y}[y-d]^2\right)$.

The analysis now parallels that for the isotropic case. One finds that the images are located as follows:

Region I: q_n^{I} at $y = 2s + 2n\kappa t$, $n = 0, 1, 2, \cdots$,

Region II: q_n^{II} at $y = \frac{1}{\kappa}\left[-s(1-\kappa) + 2n\kappa t\right]$, $n = 0, -1, -2, \cdots$,
and $y = \frac{1}{\kappa}\left[s(1+\kappa) + 2n\kappa t\right]$, $n = 1, 2, \cdots$,

Region III: q_n^{III} at $y = t(1-\kappa) + 2n\kappa t$, $n = 0, -1, -2, \cdots$,

where

$$\kappa^2 = \frac{\mu_x}{\mu_y} .$$

The vector potential in the three regions is then

$$A^{I} = -q \ln(x^2 + y^2) - \sum_{n=0}^{\infty} q_n^{I} \ln\left(x^2 + [y - 2s - 2n\kappa t]^2\right),$$

$$A^{II} = -\sum_{n=0}^{\infty} q_{-n}^{II} \ln\left(x^2 + [\kappa y + s(1-\kappa) + 2n\kappa t]^2\right) - \sum_{n=1}^{\infty} q_n^{II} \ln\left(x^2 + [\kappa y - s(1+\kappa) - 2n\kappa t]^2\right),$$

$$A^{III} = -\sum_{n=0}^{\infty} q_{-n}^{III} \ln\left(x^2 + [y - t(1-\kappa) + 2n\kappa t]^2\right) .$$

If the boundary conditions are now applied the resulting equations are exactly like Eq. (5) except that μ is replaced by

$$\bar{\mu} = \frac{\mu_x}{\kappa} = \sqrt{\mu_x \mu_y}$$

and the solutions will of course be the same but for this substitution. Thus

$$q_{-n}^{III} = \frac{4\bar{\mu}}{(1+\bar{\mu})^2}\left(\frac{1-\bar{\mu}}{1+\bar{\mu}}\right)^{2n} q = \frac{4\mu_x}{(\kappa+\mu_x)^2}\left(\frac{\kappa-\mu_x}{\kappa+\mu_x}\right)^{2n} q\kappa$$

and we obtain

$$H_y^{III} = \frac{8\mu_x}{(\kappa+\mu_x)^2}\, qx\kappa \sum_{n=0}^{\infty} \frac{\left(\frac{\kappa-\mu_x}{\kappa+\mu_x}\right)^{2n}}{x^2+[y-t(1-\kappa)+2n\kappa t]^2} \,.$$

Our major interest is in the value of H_y^{III} in the limit of large μ_y or small κ. Note that as κ approaches zero the sum tends to infinity but is multiplied by κ so that the result is indeterminate. To get an estimate of H_y^{III} replace $\left(\frac{\kappa-\mu_x}{\kappa+\mu_x}\right)^{2n}$ by $e^{-\frac{4n\kappa}{\mu_x}}$ (the smaller the value of κ, the smaller is the error made in this substitution). Then, neglecting κ as compared with 1 and μ_x we obtain

$$H_y^{III} \approx \frac{8}{\mu_x}\, qx\kappa \sum_{n=0}^{\infty} \frac{e^{-\frac{4n\kappa}{\mu_x}}}{x^2+[y-t+2n\kappa t]^2}$$

$$\approx \frac{8}{\mu_x}\, qx \int_0^{\infty} \frac{e^{-\frac{4}{\mu_x}v}}{x^2+[y-t+2tv]^2}\, dv$$

where we have put $v = n\kappa$. If we set

$$ux = (y-t) + 2tv$$

this becomes

$$H_y^{III} \approx \frac{8}{\mu_x}\, q\, \frac{e^{\frac{2(y-t)}{\mu_x t}}}{2t} \int_{\frac{y-t}{x}}^{\infty} \frac{e^{-\frac{2x}{\mu_x t}u}}{1+u^2}\, du \,.$$

For small x, such that $\frac{y-t}{x} \gg 1$, the 1 in the denominator of the integrand can be ignored and by putting $\frac{2x}{\mu_x t}$ u = z we obtain

$$H_y^{III} \approx \frac{8qx}{(\mu_x t)^2}\, e^{\frac{2(y-t)}{\mu_x t}} \int_{\frac{2(y-t)}{\mu_x t}}^{\infty} \frac{e^{-z}}{z^2}\, dz$$

$$\approx \frac{8qx}{(\mu_x t)^2}\, e^{\frac{2(y-t)}{\mu_x t}} \left\{ \frac{\mu_x t}{2(y-t)}\, e^{-\frac{2(y-t)}{\mu_x t}} + E_i\left(-\frac{2(y-t)}{\mu_x t}\right) \right\} .$$

For small values of $\frac{2(y-t)}{\mu_x t}$ the first term is dominant, the exponential integral going to infinity only logarithmically as y-t approaches zero. (The value of $\mu_x t$ for the reluctance pole is of the order of 50.) Thus, for small x,

$$H_y^{III} \sim \frac{4qx}{\mu_x t(y-t)} \quad .$$

This result can be compared with the field due to an unshielded wire, $\frac{2qx}{y^2}$ (for $x << y$). A fairer comparison, however, is with the field due to a wire backed by iron at a distance y-t from the wire, for this is the situation when a winding is misplaced in a conventional magnet. This field, when x << y, is approximately

$$H_y \approx \frac{4qx}{(y-t)^2}$$

so that

$$\frac{H_y^{III}}{H_y} \sim \frac{y-t}{\mu_x t} \quad .$$

Thus if $\mu_x t = 50$ and y-t = 5 the perturbation of the field by the winding error in the magnet with the reluctance pole is about one-tenth as great as the perturbation in the conventional magnet. It should be emphasized, however, that this benefit is conferred not by the ability of the reluctance pole to shield out perturbing fields but rather by the fact that the return windings are moved back from the median plane by the thickness of the reluctance pole as compared with a conventional magnet with the same gap.

Reluctance Error in Reluctance Pole Magnets

Consider a scaling structure for which t, the thickness of the reluctance pole is proportional to the radius, $t = t_0 r$, and the permeability is a constant, μ_0. The permeability is given by

$$\frac{1}{\mu_0} = \frac{w_2}{w_1 + w_2}$$

where w_2 = distance between iron laminae,

w_1 = width of a lamina.

If one of the laminae had a thickness $\varpi_1+\delta$ the effect would be like having, in a continuous medium, the permeability changed from μ_0 to μ over a width $\varpi_1+\varpi_2$ where

$$\frac{1}{\mu} = \frac{\varpi_2}{\varpi_1+\varpi_2+\delta} \approx \frac{\varpi_2}{\varpi_1+\varpi_2} - \frac{\varpi_2\delta}{\varpi_1+\varpi_2} = \frac{1}{\mu_0} - \frac{1}{\mu_0}\frac{\delta}{\varpi_1+\varpi_2} .$$

If one of the non-magnetic spaces had a thickness $\varpi_2+\delta$, the equivalent permeability over a width $\varpi_1+\varpi_2$ would be given by

$$\frac{1}{\mu} = \frac{\varpi_2+\delta}{\varpi_1+\varpi_2+\delta} \approx \frac{\varpi_2}{\varpi_1+\varpi_2} + \frac{\delta}{\varpi_1+\varpi_2}\left(1-\frac{\varpi_2}{\varpi_1+\varpi_2}\right) = \frac{1}{\mu_0} + \frac{\delta}{\varpi_1+\varpi_2}\left(1-\frac{1}{\mu_0}\right) .$$

Since $\mu_0 \sim 10$, the latter is clearly the more serious perturbation.

One error that would seem likely to arise is the displacement of a block of laminations d cm. wide. This leads to an increase in the space between a pair of laminae at one point and a decrease in the spacing at a point d cm. away.

If, between R and R + $(\varpi_1 + \varpi_2)$ the permeability of the reluctance pole differs from its design value, μ_0 , then the potential drop occurring in the distance $\varpi_1 + \varpi_2$ differs from its design value, v_d , by the amount

$$v - v_d = \int_R^{R+(\varpi_1+\varpi_2)} B_i \left(\frac{1}{\mu} - \frac{1}{\mu_0}\right) dr$$

where B_i is the flux density within the reluctance pole. For B_i we can use the flux density in the unperturbed structure. If $\varpi_1 + \varpi_2$ is sufficiently small so that B_i does not change appreciably in the distance then

$$v - v_d = B_i \left(\frac{1}{\mu} - \frac{1}{\mu_0}\right)(\varpi_1+\varpi_2) .$$

B_i is given by

$$B_i(R) = \frac{1}{Rt}\int_0^R (H+L)r\, dr$$

where H is the field in the gap and L is the leakage field, that is, the field above the reluctance pole. For a scaling structure L is proportional to H. If we put

$$L = \alpha H \;, \quad H = H_0 r^k \;, \quad t = t_0 r$$

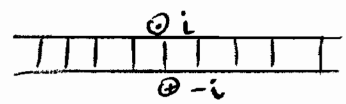
then

$$B_1(R) = \frac{(1+\alpha)}{t_0 (k+2)} H_0 R^k$$

and consequently

$$\nu - \nu_\lambda = \left(\frac{1}{\mu} - \frac{1}{\mu_0}\right) \frac{(1+\alpha)(w_1 + w_2)}{t_0 (k+2)} H_0 R^k \;.$$

The effect of such a perturbation of the permeability is roughly equivalent to that of a pair of currents straddling the reluctance pole as shown in Fig. 4. The magnitude of the current i (in emu) is given by



$$4\pi i = \nu - \nu_\lambda = \left(\frac{1}{\mu} - \frac{1}{\mu_0}\right) \frac{(1+\alpha)(w_1 + w_2)}{t_0 (k+2)} H_0 R^k \;.$$

Fig. 4.

If the error in the permeability is due to the spacing of two laminae differing by δ from the design value, then, inserting the appropriate value of $(\frac{1}{\mu} - \frac{1}{\mu_0})$, we obtain

$$4\pi i = \delta \frac{(\mu_0 - 1)(1+\alpha)}{\mu_0 t_0 (k+2)} H_0 R^k \;.$$

If a block of laminae of width d is shifted a distance δ then, in effect, two pairs of currents appear as shown in Fig. 5, one pair at the position of the increased spacing and one pair at the position of the decreased spacing. The current i has approximately the same magnitude at both positions if H does not change appreciably in the distance d.

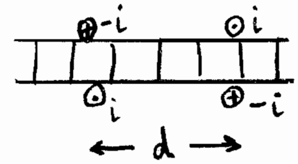


Fig. 5.

To get some notion of how bad the effect of such construction errors might be let us use some representative numbers. Put $\mu_0 t_0 = .5$ (if $\mu_0 = 10$ then with $t_0 = .05$ the reluctance pole is 5 cm. thick at 1 m. radius, which is reasonable), $\mu_0 = 10$, $\alpha = 1$, $H_0 R^k = 10^3$ oersteds, k = 8. Then

$$4\pi i = 3600\,\delta$$

so that an effective current of about 3 emu appears for each .1 mm. deviation of the spacing from its correct value.

The perturbing field due to such an array of currents can be estimated using the results found earlier for the field due to a current in the presence of a slab with permeability μ_x in the direction perpendicular to the laminations and permeability μ_y (approximately infinite) in the direction parallel to the laminations. It turns out that one is not far off if one considers only the lower pair of currents (the currents that effectively appear on the gap side of the reluctance pole) and treats them as though they were backed by iron of infinite permeability. One can then make an immediate comparison between the effects of such a construction error in a reluctance pole and a winding error, such as Kerst has considered, in a conventional magnet. In the latter case, if a band of windings of width d is shifted by δ then a current pair appears on the surface of the pole, the magnitude of each current being given by

$$4\pi i \approx \frac{k\,HG}{R}\,\delta$$

where R is the radius at which the winding error occurs and H and G are, respectively, the vertical component of the field and the width of the gap at the radius R. In this case if $\frac{G}{R} = .1$, k = 8, and $H = 10^3$ oersteds then

$$4\pi i = 800\,\delta$$

so that an effective current of about .65 emu appears for each .1 mm.

displacement of the wires. Thus the tolerances for the spacing of the laminae of the reluctance pole must be about five times as stringent as the tolerances for the spacing of the windings of a conventional magnet.

Superimposed on, and partially compensating, the effect of the perturbation of the permeability is the change in the apparent density of the backwindings on the reluctance pole due to the variation of the spacing of the laminae. This effect, which is nearly equivalent to a displacement of the backwindings, is, in any case, no more serious than the displacement of backwindings in a conventional magnet and is, in fact, as we have seen, less serious because of the greater distance of the windings from the median plane. The compensation of the large effect of the permeability perturbation will therefore be slight.

Erratum for MURA Report 249

In MURA-249 Kerst has pointed out the relation between a class of reluctance pole structures and eigenpoles. In his expression for the potential V (where $\vec{B} = -\nabla V$) within the reluctance pole, Eq. (5), the quantity V_0 appears as a coefficient. This is not the same as the V_0 appearing in his Eq. (1), the expression for the potential in air. If we replace the former V_0 by a V_1 then the correct expression for the permeability of the reluctance pole contains as a factor V_1/V_0 . This factor should appear in Kerst's Eq. (7).

One can determine V_1/V_0 from the condition of continuity of the normal component of $\vec{B}$. Thus

$$\frac{1}{r}\frac{\partial V_{air}}{\partial \theta}\bigg|_{\theta=\frac{\pi}{2}-\varsigma} = \frac{1}{r}\frac{\partial V_{rel.\ pole}}{\partial \theta}\bigg|_{\theta=\frac{\pi}{2}-\varsigma},$$

and one obtains

$$\frac{V_1}{V_0} = \left[\frac{P_{k+1}(\cos\alpha)}{\frac{d}{d\theta}P_{k+1}(\cos\theta)} \cdot \frac{\frac{d}{d\theta}P_{k+1}(\cos\theta) - \frac{P_{k+1}(\cos\beta)}{Q_{k+1}(\cos\beta)}\frac{d}{d\theta}Q_{k+1}(\cos\theta)}{P_{k+1}(\cos\alpha') - \frac{P_{k+1}(\cos\beta)}{Q_{k+1}(\cos\beta)}Q_{k+1}(\cos\alpha')} \right]_{\theta=\frac{\pi}{2}-\varsigma} .$$

References:

1. MURA Minutes No. 24, March 18, 1957

2. D. W. Kerst, MURA Report-249