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Top-quark Yukawa coupling with a dimension-6 operator

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Abstract. We extend the standard top-quark Yukawa coupling with a dimension-6 operator in order to accommodate a CP violating complex phase with manifest gauge invariance. This leads to a new $ttHH$ contact interaction, along with many Goldstone boson couplings. We investigate the impact of the new interactions on a muon collider process $\mu^-\mu^+ \to \nu_\mu \bar{\nu}_\mu t\bar{t}H$ compared with the standard dimension-4 top-Yukawa coupling. The unitarity bounds on the coefficient of the new physics operator is obtained from $W_L^- W_L^+$ and HH initiated processes.

The observed baryon asymmetry of the universe (BAU) requires a new source of CP violation among elementary particles. We consider the top-quark Yukawa coupling with a CP-violating phase [1, 2]. This can be realized by introducing a complex Yukawa form:

$$\mathcal{L}^{ttH}_{\text{complex}} = -gH\bar{t}(\cos\xi + i\gamma_5 \sin\xi)t, \tag{1}$$

where the phase ξ $(-\pi \leq \xi \leq \pi)$ gives CP violation when $\sin\xi \neq 0$ [1]. We find that at high energies, we need a gauge invariant form of the interaction, by introducing a dimension-6 operator [3–6],

$$\mathcal{L}^{ttH}_{\text{SMEFT}} = \mathcal{L}^{ttH}_{\text{SM}} + \left\{ \frac{\lambda}{\Lambda^2} \left(Q^\dagger \phi t_R \right) \left(\phi^\dagger \phi - \frac{v^2}{2} \right) + \text{h.c.} \right\}, \tag{2}$$

with λ a complex number which measures deviation from the SM, Λ is the new physics scale, and

$$Q = \begin{pmatrix} t_L \\ b_L \end{pmatrix}, \qquad \phi = \begin{pmatrix} \frac{v+H+i\pi^0}{\sqrt{2}} \\ i\pi^- \end{pmatrix}. \tag{3}$$

A more generic higher dimensional operator case has been discussed e.g. in [6, 7]. We study consequences of the CP-violating top-Higgs coupling in the muon collider process $\mu^-\mu^+ \to \bar{\nu}_\mu \nu_\mu \bar{t}tH$ with the vector boson fusion production diagrams shown in Fig. 1. After expansion,

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[1]In the case where $\sin\xi = 1$, a CP-odd Htt interaction can still induce CP violation through amplitudes with a CP-even HVV interactions such as $VV \to t\bar{t}H, (V = W^\pm, Z)$. In contrast, in processes such as $gg \to t\bar{t}H$ and $\gamma\gamma \to t\bar{t}H$, no CP violating effects arise when $\sin\xi = 1$, in the tree-level, as the amplitudes with only purely CP-odd H couplings preserve the CP symmetry.

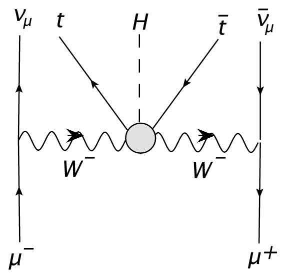


Figure 1. Weak boson fusion subdiagrams contributing to the process $\nu_\mu \bar{\nu}_\mu t\bar{t}H$.

the Yukawa Lagrangian becomes

$$\begin{aligned}
\mathcal{L}^{ttH}_{\mathrm{SMEFT}} &= -\sqrt{2}g_{\mathrm{SM}}\left(Q^\dagger \phi t_R\right) + \frac{\lambda}{\Lambda^2}\left(Q^\dagger \phi t_R\right)\left(\phi^\dagger\phi - \frac{v^2}{2}\right) + \text{h.c.}, \\
&= -m_t t_L^\dagger t_R - g_{\mathrm{SM}}\left[\left(H + i\pi^0\right)t_L^\dagger + i\sqrt{2}\pi^- b_L^\dagger\right]t_R \\
&\quad +(g_{\mathrm{SM}} - g e^{i\xi})\left\{H t_L^\dagger t_R + \frac{H}{v}\left[\left(H + i\pi^0\right)t_L^\dagger + i\sqrt{2}\pi^- b_L^\dagger\right]t_R\right\} \\
&\quad +(g_{\mathrm{SM}} - g e^{i\xi})\left\{\left(\frac{H^2 + (\pi^0)^2}{2v} + \frac{\pi^+\pi^-}{v}\right)t_L^\dagger t_R\right. \\
&\quad \left. + \frac{H^2 + (\pi^0)^2 + 2\pi^+\pi^-}{2v^2}\left[\left(H + i\pi^0\right)t_L^\dagger + i\sqrt{2}\pi^- b_L^\dagger\right]t_R\right\} + \text{h.c.},
\end{aligned} \tag{4}$$

with $g_{\mathrm{SM}} = \frac{m_t}{v}$ and $g_{\mathrm{SM}} - g e^{i\xi} = \frac{\lambda v^2}{\sqrt{2}\Lambda^2}$. There are new couplings, $ttHH$ and $ttHHH$, proportional to $g_{\mathrm{SM}} - g e^{i\xi}$, which will lead to different CPV parameter ξ dependence compared with the complex Yukawa parametrization Eq.(1). In the phenomenological study, we will show how different the prediction can be at certain kinematical region, especially at high energies.

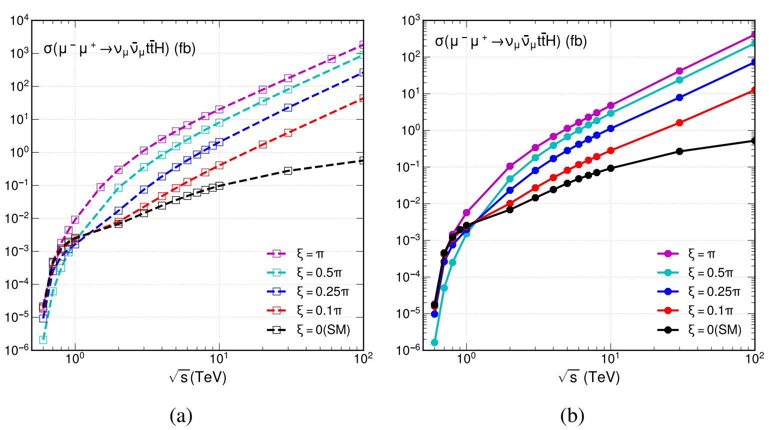


Figure 2. Total cross section of $\nu_\mu \bar{\nu}_\mu t\bar{t}H$ production at a muon collider with $\sqrt{s}$ dependence at $\xi = 0, 0.1\pi, 0.25\pi, 0.5\pi, \pi$ (a) in the complex Yukawa model, (b) in the SMEFT model.

Comparing the complex Yukawa model and the one with an extra dimension-6 operator, the $ttHH$ coupling Lagrangian

$$\mathcal{L}_{ttHH} = \frac{3(g_{\rm SM} - g e^{i\xi})}{v} \frac{HH}{2} t_L^\dagger t_R + \text{h.c.}, \tag{5}$$

contributes to the weak boson fusion process

$$W^-(q, 0) W^+(\bar{q}, 0) \to t\bar{t}H. \tag{6}$$

The amplitudes in the SMEFT framework should hence be

$$\mathcal{M}(W_L^- W_L^+ \to t\bar{t}H) = \mathcal{M}_{\rm complex\,Yukawa} + \mathcal{M}_{ttHH}. \tag{7}$$

With straightforward calculation, we obtain the analytical expression

$$\mathcal{M}_{ttHH} = \frac{3}{v^2} \left[\mp 2p_t (g_{\rm SM} - g\cos\xi) - i m_{tt} g \sin\xi \right] \frac{\hat{s} - 2m_W^2}{\hat{s} - m_H^2}, \tag{8}$$

when the t and $\bar{t}$ have the same helicity $\pm\frac{1}{2}$ in the $t\bar{t}$ rest frame. The Goldstone boson equivalence theorem (GBET) [8, 9] tells that the amplitudes for the process (6) should approach to those of the process

$$\pi^-\pi^+ \to t\bar{t}H. \tag{9}$$

From the Lagrangian (4), we can tell that the amplitudes are dominated by the contact $\pi^+\pi^- t\bar{t}H$ term, which gives

$$\mathcal{M}^{\pm\pm}_{\pi\pi ttH} = \frac{1}{v^2} \left[\mp 2p_t (g_{\rm SM} - g\cos\xi) - i m_{tt} g \sin\xi \right]. \tag{10}$$

The above two equations tell

$$\mathcal{M}_{ttHH} \approx 3\mathcal{M}_{\pi\pi ttH}, \tag{11}$$

whereas the GBET tells,

$$\mathcal{M}_{\rm complex\,Yukawa} + \mathcal{M}_{ttHH} \approx \mathcal{M}_{\pi\pi ttH}, \tag{12}$$

at high energies. Therefore it is straightforward to obtain the relation with high energy approximation.

$$\mathcal{M}_{\rm complex\,Yukawa} \approx -2\mathcal{M}_{\pi\pi ttH}. \tag{13}$$

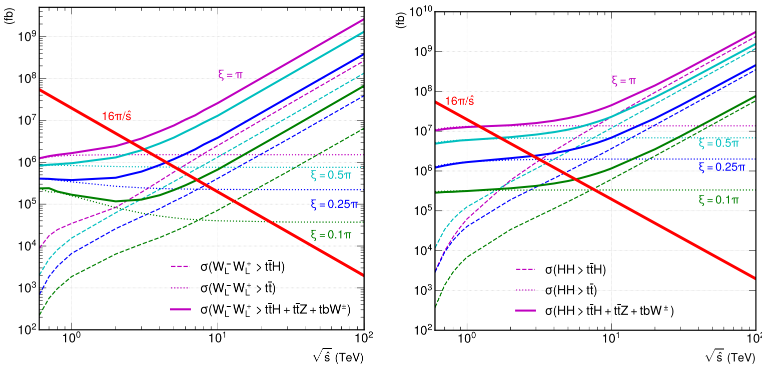
The above relation for the scattering amplitudes $\mathcal{M}(W_L^- W_L^+ \to ttH)$ explains the factor of 4 numerical difference of the total cross section in the complex Yuakwa model and the SMEFT model as shown in Figure 2(a) and 2(b).

The perturbative unitarity constraints of the SMEFT model can be obtained by the $J = 0$ amplitudes of the weak boson fusion process. In the optical theorem, for the $W_L^- W_L^+$ process, the final state is summed over all $J = 0$ final states,

$$2\text{Im}\,\langle i| T |i\rangle = \sum_f |\langle f| T |i\rangle|^2, \tag{14}$$

with initial state $|i\rangle = \left|W_L^- W_L^+(J=0)\right\rangle$. The unitarity bound

$$|\text{Im}\,\langle i| T |i\rangle| < |\langle i| T |i\rangle| < 16\pi \tag{15}$$



(a) (b)

Figure 3. Unitarity bound from $W_L^- W_L^+$ and HH channel summing over $2 \to 2$ and $2 \to 3$ processes.

becomes

$$\sum_f \sigma_{\text{tot}}(W_L^- W_L^+ \to f; J = 0) < \frac{16\pi}{\hat{s}}. \tag{16}$$

However, from the SMEFT Lagrangian, the large $ttHH$ and $ttHHH$ couplings make the HH channel grows faster than the weak boson fusion channel. We show in Figure 3 for both cases. We note that the cases above the red line $\frac{16\pi}{\hat{s}}$ violating unitarity bound are non physical. The bounds are obtained from amplitudes of Λ^{-2} order, since the $J = 0$ amplitudes at high energies grow with energies by the dimension-5 and 6 vertices.

In summary, we investigate the strong energy dependence of the cross section for process $\mu^-\mu^+ \to \nu_\mu \bar{\nu}_\mu t\bar{t}H$ in the complex top Yukawa model as well as the SMEFT framework. By comparing the two frameworks, we find that the highest-energy cross-section is reduced to one-quarter of the result predicted by the complex top Yukawa model, while maintaining the same energy dependence in the SMEFT with one dimension-6 operator (4). This is shown explicitly by the dominant subamplitude for $W_L^- W_L^+ \to t\bar{t}H$ by using the GBET. The perturbative unitarity constraints are also obtained by summing up all $2 \to 2$ and $2 \to 3$ processes for $W_L^- W_L^+$ and HH scattering ampiltudes.

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