



Light bending in $F[g(\square)R]$ extended gravity theories

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ARTICLE INFO

Article history:

Received 2 February 2018

Accepted 20 February 2018

Available online 26 February 2018

Editor: M. Cvetič

Keywords:

Higher derivative gravity

Light bending

Brans–Dicke theory

Non-local gravity

Scalar–tensor theories

ABSTRACT

We show that in the weak field limit the light deflection alone cannot distinguish between different $R + F[g(\square)R]$ models of gravity, where F and g are arbitrary functions. This does not imply, however, that in all these theories an observer will see the same deflection angle. Owing to the need to calibrate the Newton constant, the deflection angle may be model-dependent after all necessary types of measurements are taken into account.

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1. Introduction

Modified gravity theories attract a lot of attention nowadays, this concerns in particular the models constructed with the scalar curvature and its covariant derivatives. In this Letter we discuss the deflection of light in the linear regime in a generic model of this type. This issue has already been discussed in the literature using a variety of approaches applied to specific models [1–10]. In the various references one can meet different conclusions. In some works the statement is that the bending of light occurs exactly in the same way as in general relativity (GR), while in other works it is claimed that the light bending effect can be used to distinguish between some of such theories and GR. The aim of the present work is to present a general discussion on the subject and clarify to which extent the bending of light, taken alone or together with other observables, can distinguish among the theories which depend on an arbitrary function $F(Y)$, where $Y = g(\square)R$, and g is a function of the d'Alembert operator.

The action of our interest is given by the expression

$$S_{\text{gen}} = \int d^4x \sqrt{-g} \left\{ -\frac{2}{\kappa^2} R + F[g(\square)R] \right\}, \quad (1)$$

where $\kappa^2 = 32\pi G$ and G is Newton's constant. The integrand of the gravitational action should be supplemented by the Lagrange matter density \mathcal{L}_M .

Our interest will be concentrated on the light bending by a weak gravitational field, which means that the background is considered to be a small deviation from the flat Minkowski space. This condition immediately provides significant simplification, as the non-linear in curvature terms are expected to have small effect on the bending of light. Then the influence of a non-flat background can be safely regarded as at least a second-order effect. Therefore, without losing generality, one can restrict consideration by the actions which are at most quadratic in scalar curvature, but may be non-polynomial in derivatives. In this way one arrives at the simplified action of the form

$$S = \int d^4x \sqrt{-g} \left[-\frac{2}{\kappa^2} R + f(\square)R + \mathcal{L}_M \right], \quad (2)$$

where $f(\square)$ is a function of d'Alembert operator, which is not necessarily analytic.

The theory (2) includes many interesting particular cases. Let us give a short list of the models of this type which have been discussed in the literature.

1) Taking $f(x) = \text{const.}$ gives the well-known $R + R^2$ gravity, which is the basis of the successful inflationary model of Starobinsky [11,12]. In this case the bending of light is exactly like in GR,

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as has been proved in the paper by Accioli et al. [2]. Our present consideration can be seen as a generalization of this work to the general model (1). Let us note that this statement concerns only the prediction of light deflection *without* taking other observables into account, as will be discussed in what follows.

2) $f(x)$ is a polynomial function of order N . The bending of light for the particular case of a linear function $f(\square)$ has been explored in the recent work [10]. The Ricci-squared term has also been included into consideration in [10], so the theory (2) represents only the part of the model which is related to scalar curvature. Here we generalize the corresponding results of [10] (see also [13]) to arbitrary polynomial or non-polynomial functions.

3) The logarithmic form factors $\ln(\square/\mu^2)$ are typical for the quantum corrections coming from the loops of massless fields, and correspond to the Minimal Subtraction scheme running of effective constants. At higher loops there may be higher powers of the same logarithms. In case of massive fields the expressions for the gravitational form factors are more complicated [14], but qualitatively are like $\ln((\square + m^2)/\mu^2)$ for the quantum field of mass m . At low energy this expression becomes a constant, which corresponds to the gravitational version of Appelquist and Carazzone decoupling theorem.

4) The functions $f_{-1}(\square) = \mu\square^{-1}$ and $f_{-2}(\square) = \mu\square^{-2}$ have been introduced in [14] in order to explain how the renormalization group running of the Einstein–Hilbert and cosmological terms can look like. The main source of importance of these terms for cosmological applications is that under the global scaling the $Rf_{-1}(\square)R$ -term behaves exactly like the Einstein–Hilbert term, and the $Rf_{-2}(\square)R$ -term behaves like the cosmological term. At the same time, since both terms are non-local, they do not reproduce the Einstein–Hilbert and cosmological terms exactly, and this leads to their fruitful use in cosmology, as suggested in [15] for the $R\varphi(\square^{-1}R)$ -type actions and in [16] for the $R\square^{-2}R$ -term (see further references therein). In view of this phenomenological success, it would be interesting to know what is the effect of these terms for the light bending.

In what follows we show that for the modified actions (2) in the weak field limit the deflection of light is not directly affected by the terms quadratic in scalar curvature. This does not mean, however, that the predictions for the light bending is the same in all the models (2), since the experimentally measured Keplerian mass of astronomical bodies can differ from theory to theory.

The paper is organized as follows. In Sec. 2 the bending of light is obtained on the basis of tree-level amplitudes, for the cases of GR and the models (2). The purely classical, geometric optics approach is used in Sec. 3, where we also discuss the correspondence with the Brans–Dicke models and the role of conformal transformations for the light bending and other observables. Finally, in Sec. 4 we draw our conclusions.

2. Tree-level approach

The interaction between light and a weak gravitational field can be explored using the scattering approach by means of the tree-level Feynman diagrams [17,18]. Loop corrections may be included as well into this formalism [19–22]. In the present work our interest is concentrated on the classical behavior of the theories (2), and this includes the non-local form factors which can be attributed to the dressed propagator of gravitational modes. Hence in this framework we only need to deal with the tree-level diagrams. For the sake of consistency we shall start by briefly reviewing the use of the same method within GR, before going on in applying it to extended models (2).

2.1. Light bending in general relativity

The calculation for GR can be found in some books (see, e.g., [23,24]), but our derivation is carried out in a slightly different way, such that consequent generalization to the more complicated model (2) is straightforward.

The starting point is the metric written as a fluctuation around the Minkowski space, i.e., $g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$, with $\eta_{\mu\nu} = \text{diag}(+1, -1, -1, -1)$. Since gravity couples to matter via the energy–momentum tensor, in the first order in κ the interaction is described by

$$\mathcal{L}_{\text{int}} = -\frac{\kappa}{2} h^{\mu\nu} T_{\mu\nu}, \quad (3)$$

where $T^{\mu\nu}$ is the matter energy–momentum tensor in flat space–time, which defines the interaction vertex. For the electromagnetic field we have

$$T_{\text{em}}^{\mu\nu} = -\eta_{\alpha\beta} F^{\mu\alpha} F^{\nu\beta} + \frac{1}{4} \eta^{\mu\nu} F_{\alpha\beta} F^{\alpha\beta}, \quad (4)$$

with $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. We will also need the interaction with a minimal massive scalar,

$$T_{\text{scal}}^{\mu\nu} = \partial^\mu \phi \partial^\nu \phi - \frac{1}{2} \eta^{\mu\nu} (\partial_\alpha \phi \partial^\alpha \phi - M^2 \phi^2). \quad (5)$$

In the context of GR, gravitational interaction is related to the exchange of only one particle, the graviton, whose propagator can be written as

$$D_{\mu\nu,\alpha\beta}^{\text{GR}}(k) = \frac{P_{\mu\nu,\alpha\beta}^{(2)}}{k^2} - \frac{P_{\mu\nu,\alpha\beta}^{(0-s)}}{2k^2} \quad (6)$$

in the momentum–space representation. Here $P_{\mu\nu,\alpha\beta}^{(2)}$ and $P_{\mu\nu,\alpha\beta}^{(0-s)}$ are the spin-2 and spin-0 projectors (see, e.g., [25]) defined through

$$P_{\mu\nu,\alpha\beta}^{(2)} = \frac{1}{2} (\theta_{\mu\alpha} \theta_{\nu\beta} + \theta_{\mu\beta} \theta_{\nu\alpha}) - \frac{1}{3} \theta_{\mu\nu} \theta_{\alpha\beta}, \quad (7)$$

$$P_{\mu\nu,\alpha\beta}^{(0-s)} = \frac{1}{3} \theta_{\mu\nu} \theta_{\alpha\beta}, \quad (8)$$

where the longitudinal and transverse vector-space projectors are

$$\omega_{\mu\nu} = \frac{k_\mu k_\nu}{k^2}, \quad \text{and} \quad \theta_{\mu\nu} = \eta_{\mu\nu} - \omega_{\mu\nu}. \quad (9)$$

Let us note that the other parts of the gravitational propagator are irrelevant, because it is going to be directly contracted with the matter sources in the tree-level approximation. For the same reason we do not need to worry about the gauge-fixing dependence for the quantum gravitational field. After going on-shell this dependence will disappear anyway.

At the tree-level we can model the massive source which produces the gravitational field by a massive scalar field. Therefore we have to evaluate the exchange of graviton between such a scalar and a photon. The leading contribution to the amplitude \mathcal{M} associated to this process is of the order κ^2 and is related via the LSZ reduction formula to the function

$$G_4 = -\frac{\kappa^2}{4} \iint d^4 z_1 d^4 z_2 \langle 0 | T \phi(x) \phi(x') A_\mu(y) A_\nu(y') \times T_{\text{scal}}^{\alpha\beta}(z_1) T_{\text{em}}^{\rho\sigma}(z_2) h_{\alpha\beta}(z_1) h_{\rho\sigma}(z_2) | 0 \rangle_c. \quad (10)$$

Using Wick's theorem, the term $h_{\alpha\beta}(z_1) h_{\rho\sigma}(z_2)$ yields the graviton propagator (6). The energy–momentum tensor of the electromagnetic field satisfies the conservation law $\partial_\alpha T_{\text{em}}^{\alpha\beta} = 0$ and is

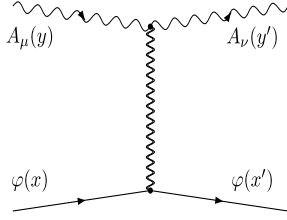


Fig. 1. Illustration of graviton exchange between a massive source and the photon.

traceless, $T_{\alpha\text{em}}^{\alpha} = 0$. Consequently, the terms proportional to $\omega_{\mu\nu}$ in the graviton propagator do not contribute to the expression (10), and hence to the scattering amplitude. We can thus replace the projectors $\theta_{\mu\nu}$ by the metric $\eta_{\mu\nu}$ in the graviton propagator. From the physical perspective the situation is such that photons interact only with the spin-2 sector of the propagator.

The lowest order contribution to the scattering matrix comes from the tree diagram in Fig. 1, in which one photon with initial momentum p and one scalar with momentum q interchange one graviton, resulting in a final state with momenta p' and q' . The corresponding scattering amplitude is

$$\mathcal{M} = V_{\mu\nu}^{(\phi)}(q, q') D_{\text{GR}}^{\mu\nu, \alpha\beta}(k) V_{\alpha\beta\rho\sigma}^{(A)}(p, p') \epsilon^{\rho}(\mathbf{p}) \epsilon^{*\sigma}(\mathbf{p}'), \quad (11)$$

where we included the polarization vectors of the photons. The vertex functions are

$$V_{\mu\nu}^{(\phi)}(p, p') = -\frac{i\kappa}{2} [p_{\mu} p'_{\nu} + p'_{\mu} p_{\nu} + \eta_{\mu\nu}(M^2 - p \cdot p')] \quad (12)$$

for the scalar, and

$$\begin{aligned} V_{\alpha\beta\mu\nu}^{(A)}(p, p') &= -\frac{i\kappa}{2} [p \cdot p' (\eta_{\mu\alpha} \eta_{\nu\beta} + \eta_{\mu\beta} \eta_{\nu\alpha} - \eta_{\mu\nu} \eta_{\alpha\beta}) \\ &\quad + \eta_{\alpha\beta} p'_{\mu} p_{\nu} + \eta_{\mu\nu} (p_{\alpha} p'_{\beta} + p_{\beta} p'_{\alpha}) \\ &\quad - (\eta_{\beta\nu} p'_{\mu} p_{\alpha} + \eta_{\alpha\mu} p'_{\beta} p_{\nu} + \eta_{\alpha\nu} p'_{\mu} p_{\beta} + \eta_{\beta\mu} p'_{\alpha} p_{\nu})] \quad (13) \end{aligned}$$

for the photon.

As discussed above, in the amplitude (11) one can replace the propagator $D_{\text{GR}}^{\mu\nu, \alpha\beta}(k)$ by the expression

$$\frac{1}{2k^2} (\eta_{\mu\alpha} \eta_{\nu\beta} + \eta_{\mu\beta} \eta_{\nu\alpha}). \quad (14)$$

Further simplification is achieved by recalling that we are interested in the deflection of a light ray passing nearby a very massive object (such as a star or a galaxy), described by the scalar field. In this regime it is appropriate to work in the large-mass approximation for the scalar field, that means $\mathbf{q} = 0$, $\mathbf{q}' = -\mathbf{k}$, $M \gg |\mathbf{k}|$ and $M \gg |\mathbf{p}|$, and consider that the photon undergoes an elastic scattering with $|\mathbf{p}| = |\mathbf{p}'|$. For a vanishing momentum transfer the scalar-scalar-graviton vertex simplifies to $V_{\mu\nu}^{(\phi)} = -i\kappa M^2 \delta_{\mu}^0 \delta_{\nu}^0$. For the remaining vertex, it is useful to work in the Lorentz gauge for which $\epsilon^0 = 0$ and $\epsilon \cdot \mathbf{p} = 0$. It is then immediate to verify that Eq. (11) simplifies to

$$\mathcal{M} = -\frac{\kappa^2 M^2 E^2}{|\mathbf{k}|^2} \epsilon(\mathbf{p}) \cdot \epsilon^{*}(\mathbf{p}'). \quad (15)$$

Here $E = p_0 = p'_0$ is the energy of the photon. The polarization factor can be dropped in the limit of vanishing momentum transfer (see [26] for further discussion on helicity effects such as the flip in the case of more general, rotating sources). Also, we shall insert the normalization factors $\sqrt{2E_i}$ for each external particle. Recalling that $\kappa^2 = 32\pi G$, the scattering amplitude then reads

$$\mathcal{V}(\mathbf{k}) = -\frac{8\pi G M E}{|\mathbf{k}|^2}, \quad (16)$$

whose Fourier transform gives the interaction potential felt by the photon:

$$V(\mathbf{r}) = -\frac{2G M E}{|\mathbf{r}|}. \quad (17)$$

As expected this potential is twice the standard Newtonian potential, which is felt by non-relativistic particles [17]. This factor two, indeed, took an important role in the development of GR, since it was missing in the first light bending prediction made by Einstein based solely on the equivalence principle [27]. With the potential (17) above and using classical scattering theory it is possible to compute the deflection angle undergone by a light ray owed to the weak field produced by a massive object.¹

2.2. Light bending in extended theories

The approach described above can be easily adapted to extended gravity theories, in which the propagator of the gravitational interaction has a richer structure. For example, in the case of the action (2) with $f(x)$ being a polynomial function of degree N , there can be $N + 1$ scalar particles of mass $\mu_i \neq 0$ ($i = 0, 1, \dots, N$) along with the usual (massless) graviton. It is possible for the propagator to have degenerate poles, complex poles, or even to have solely the pole at $k^2 = 0$. The crucial point is that all those possibilities stem from modifications in the scalar sector of the propagator, while the spin-2 mode propagates in the same way as in GR. Namely, the propagator of the extended theory (2) can be written as

$$D_{\mu\nu, \alpha\beta}^{\text{ext.}}(k) = \frac{P_{\mu\nu, \alpha\beta}^{(2)}}{k^2} - \frac{P_{\mu\nu, \alpha\beta}^{(0-s)}}{2k^2 [1 - 3\kappa^2 k^2 f(-k^2)]}. \quad (18)$$

In order to gain extra degrees of freedom in the tensor sector one needs to include an $R_{\rho\sigma} \tilde{f}(\square) R^{\rho\sigma}$ -term into the action [28]. In the present work we do not consider this extension and deal only with the model (2).

It is easy to see that, regardless of additional (compared to GR) degrees of freedom mediating gravitational interaction in the theory (2), the lowest order vertex associated to the interaction between photons and gravitational field is exactly the same as in GR. This happens because the extra modes are all in the scalar sector, which interacts through the trace of the energy-momentum tensor, as discussed in Sec. 2.1. In other words, photons interact only with the spin-2 (graviton) sector of the propagator, which is the same as GR, in spite of the $Rf(\square)R$ -terms in the action. This is the main reason of why the light bending alone cannot distinguish between original GR and this type of extensions. One can write for the Feynman amplitude

$$\begin{aligned} \mathcal{M} &= V_{\mu\nu}^{(\phi)}(q, q') D_{\text{ext.}}^{\mu\nu, \alpha\beta}(k) V_{\alpha\beta\rho\sigma}^{(A)}(p, p') \epsilon^{\rho}(\mathbf{p}) \epsilon^{*\sigma}(\mathbf{p}') \\ &= V_{\mu\nu}^{(\phi)}(q, q') \left(\frac{\eta^{\mu\alpha} \eta^{\nu\beta} + \eta^{\mu\beta} \eta^{\nu\alpha}}{2k^2} \right) \\ &\quad \times V_{\alpha\beta\rho\sigma}^{(A)}(p, p') \epsilon^{\rho}(\mathbf{p}) \epsilon^{*\sigma}(\mathbf{p}') \\ &= V_{\mu\nu}^{(\phi)}(q, q') D_{\text{GR}}^{\mu\nu, \alpha\beta}(k) V_{\alpha\beta\rho\sigma}^{(A)}(p, p') \epsilon^{\rho}(\mathbf{p}) \epsilon^{*\sigma}(\mathbf{p}'), \quad (19) \end{aligned}$$

¹ Let us note that one may also evaluate the unpolarized small-angle scattering cross section from (15) as in Refs. [2,8,10,17,18]. However, as discussed in [10], this quantum cross section cannot be generally applied to the deflection by astronomical bodies.

that is the same as in GR (16). All in all, light bending alone cannot distinguish among any two theories of the type (2). This general result is in agreement with the calculations of [5–9] for particular models by means of other methods, which will be also discussed in Sec. 3.

2.3. Light massive modes and rescaling of potential

Despite the equality of the interaction potential in the whole class of theories (2), the statement that all these theories predict the same deflection angle for light may be misleading. The possible effect is due to the difference between the product of the Newton's constant G and the mass M of the body which enters into the formulas for the deflection, and the one which comes from other measurements and/or observations. Indeed, G is measured in the experiments, e.g., the Cavendish-type ones. The measurement of the mass M can be achieved by detailed investigation of the orbits of astronomic bodies. As a result, in order to predict a deflection angle one has to take into account not only the interaction potential with relativistic particles, but also the non-relativistic potential resulting from the gravitational scattering of massive particles. This quantity can be computed, as in (17), as the Fourier transform of the non-relativistic scattering amplitude

$$\mathcal{M} = V_{\mu\nu}^{(\phi_1)} D^{\mu\nu,\alpha\beta} V_{\alpha\beta}^{(\phi_2)}, \quad (20)$$

where ϕ_1 and ϕ_2 are scalar fields with masses M and m . As the energy-momentum tensor for massive scalar fields has a non-vanishing trace, the non-relativistic potential depends on the form of the function $f(x)$ and hence on the massive parameters of the model.

For example, the non-relativistic potential for a polynomial $f(x)$ acquires Yukawa-like correction terms for each massive simple pole in the propagator [29–32],

$$V(\mathbf{r}) = -\frac{GMm}{|\mathbf{r}|} \left(1 + \frac{1}{3} \sum_{i=0}^N \prod_{\substack{j=0 \\ j \neq i}}^N \frac{\mu_j^2}{\mu_j^2 - \mu_i^2} e^{-|\mathbf{r}|\mu_i} \right). \quad (21)$$

If the quantities μ_i are real and are much larger than a typical scale r_{lab}^{-1} of experimental measurements of G , then the Yukawa terms are suppressed and the potential reduces to the Newtonian one. On the other hand, if the masses of the extra degrees of freedom satisfy $\mu_i \ll r_{\text{lab}}^{-1}$ for a given experimental/observational system, then the exponentials can be approximated by the unit, and it is possible to show that the potential is roughly 4/3 of Newton's one [30,31]. In this case our laboratory measurements would actually measure an effective $G_{\text{eff}} = \frac{4}{3}G$, thus the predicted bending angle for a light-ray passing close to the solar limb would be 3/4 of the general relativity's prediction.

This simple example shows that the function $f(x)$ can have an indirect influence on the prediction of deflection angles. The effect is indirect because it does not concern the bending of light itself, but the measured value of the product GM , which appears in the expression for the bending angle. In the realistic astrophysical situations all this concerns only the models with very light, albeit massive, gravitational degrees of freedom.

3. Geometric optics and null-geodesic structure

Solving the wave equation may be viewed as a cleaner way of evaluating the bending of light, based on the assumption that the source of the gravitational field is classical, not quantum. As

discussed in [10] this procedure avoids possible subtleties which appear when using the calculations via Feynman diagrams. For this reason, in this section we describe how the higher-derivative and non-local extensions (2) manifest in the bending of light using classical, geometric arguments.

The analogy between the propagation of massless particles in curved space-time and geometrical optics in a medium can be regarded as one of the most straightforward methods for describing the gravitational deflection of light. One can consult, e.g., Refs. [33–36] for the detailed description of this method. Let us give a brief survey of the main results. For a static and spherically symmetric source the metric is assumed to have an isotropic form,

$$ds^2 = (1 + 2\Phi)dt^2 - (1 - 2\Psi)[dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)], \quad (22)$$

where $\Phi = \Phi(r)$ and $\Psi = \Psi(r)$ only. It is possible to demonstrate that in this case light propagation in the geometric optics limit is equivalent to the propagation in the flat space in a medium with the effective local refractive index

$$n(r) = \sqrt{\frac{1 - 2\Psi(r)}{1 + 2\Phi(r)}}. \quad (23)$$

The deflection angle can be evaluated with Snell–Descartes law (see [34] for an explicit calculation). This method is also equivalent to the use of Maxwell equations in a space-time which represents a small fluctuation around Minkowski one [33,35].

In the case of GR, to the first order in G , the metric (22) associated to a point source of mass M is given by the potentials

$$\Phi(r) = \Psi(r) = -\frac{GM}{r}. \quad (24)$$

Due to the smallness of these quantities, Eq. (23) yields the effective refractive index in GR,

$$n_{\text{GR}}(r) = 1 + \frac{2GM}{r} + O(G^2). \quad (25)$$

Let us derive the effective refractive index for extended models of gravity (2) with an arbitrary function $f(x)$. To this end we rewrite the potentials Φ and Ψ in terms of the new functions Φ_* and Ψ_* ,

$$\begin{aligned} \Phi(r) &= V(r) + \Phi_*(r), & \Psi(r) &= V(r) + \Psi_*(r), \\ \text{with } \Delta V &= \frac{\kappa^2 \rho}{8}. \end{aligned} \quad (26)$$

In the last expression ρ is the static density of matter sourcing the field. It is clear that for GR the equations of motion must yield $\Phi_* = \Psi_* \equiv 0$. The solution in the form (22) for the field generated by a static mass distribution can be obtained from the trace and the (0, 0)-component of the equations of motion, which read

$$4[1 - 3h(-\Delta)]\Delta(\Phi - 2\Psi) = -\kappa^2 \rho, \quad (27)$$

$$4[h(-\Delta) - 1]\Delta\Phi - 8h(-\Delta)\Delta\Psi = -\kappa^2 \rho, \quad (28)$$

where

$$h(\square) = \left[1 + 2\kappa^2 f(\square)\square \right]. \quad (29)$$

Let us remember that in the static case \square boils down to $-\Delta$. Using the Ansatz (26) it is straightforward to show that the functions Φ_* and Ψ_* satisfy the equations

$$\frac{1-3h(-\Delta)}{1-h(-\Delta)} \Delta\Phi_* = \frac{\kappa^2\rho}{8}, \quad \frac{1-3h(-\Delta)}{1-h(-\Delta)} \Delta\Psi_* = -\frac{\kappa^2\rho}{8}, \quad (30)$$

from which we assume $\Phi_*(\mathbf{r}) = -\Psi_*(\mathbf{r})$.

Taking the point-like mass source $\rho(\mathbf{r}) = M\delta^{(3)}(\mathbf{r})$, the effective refractive index is

$$n_{\text{ext}}(r) = \sqrt{\frac{1-2V-2\Psi_*}{1+2V+2\Phi_*}} \\ = 1 - (V - \Phi_*) - (V + \Phi_*) + O(G^2) \quad (31)$$

$$= 1 + \frac{2GM}{r} + O(G^2). \quad (32)$$

The above expression is the same one which stems from GR (25). Once again, we see that the light deflection alone cannot be used to distinguish between the two theories. This matches and generalizes the previous results of [2,5–9] in the context of the $R + R^2$ gravity and of [10] for the sixth-order gravity. A quick inspection of (31) reveals that this happens because of the cancellation of the R -extended contribution, which appears with exactly the same value, but with an opposite sign, in the h_{00} and h_{11} components. However, as it was discussed in the previous section, in order to define the light bending one has to establish the quantity GM from a qualitatively different measurement.

To better explain this conclusion one may write the metric (22) in the PPN-inspired form [37], with

$$\Phi = -\frac{G_{\text{eff}}M}{r}, \quad \Psi = -\frac{\gamma G_{\text{eff}}M}{r}, \quad (33)$$

being G_{eff} the quantity measured in a given experiment. Let us stress that it is not generally possible to cast theories of the form (2) into the PPN formulation [37]. This happens because the non-relativistic limit in these theories may be non-Newtonian, since the potential is not proportional to r^{-1} only but may have an additional non-trivial dependence on r , as discussed in [4,38–40]. Indeed, the quantities G_{eff} and γ are actually functions of r . In the phenomenologically relevant models of this sort one can thus expect a scale for which it is possible to observe deviations from the Newtonian mechanics, such as anomalous precession of elliptical orbits [41,42]. The force law can be investigated in such a regime and the free parameters of the model determined, including the gravitational constant G . Nevertheless, it may happen that these effects are not observable at a certain scale \bar{r} for which $G_{\text{eff}}(\bar{r})$ and $\gamma(\bar{r})$ are roughly constant and the laws of mechanics are close to the ones of Newton. In this scenario one can work with the metric in the PPN form above (see, e.g., [4,39,40]). Then, from the relations

$$\Phi = V + \Phi_* = V \left(1 + \frac{\Phi_*}{V}\right) \quad \text{and}$$

$$\Psi = V - \Phi_* = \gamma V \left(1 + \frac{\Phi_*}{V}\right)$$

in the vicinity of \bar{r} one reads off

$$G_{\text{eff}} = \left(1 + \frac{\Phi_*}{V}\right) G \quad \text{and} \quad \gamma = \frac{V - \Phi_*}{V + \Phi_*}. \quad (34)$$

Assuming that $\Phi_*(\bar{r}) \ll V(\bar{r})$ in a certain region, it follows that

$$\gamma(\bar{r}) \approx 1 - \frac{2\Phi_*(\bar{r})}{V(\bar{r})}.$$

On the other hand, if the condition $\Phi_*(\bar{r}) \ll V(\bar{r})$ does not hold, the parameter γ may be very different from the value $\gamma_{\text{GR}} = 1$

of GR. Then the bending angle can vary from theory to theory. For instance,² from (32) the deflection θ undergone by a light ray with impact parameter b is $\theta = 4GMb^{-1}$. In terms of G_{eff} this result has the form

$$\theta = \frac{4G_{\text{eff}}M}{(1 + \Phi_*/V)b} = 2(1 + \gamma) \frac{G_{\text{eff}}M}{b}, \quad (35)$$

with γ defined in (34).

Let us consider a simple example. In the case of a constant function $f(\square) \equiv (6\alpha)^{-2}$ (that is, $R + R^2$ gravity) with a point-like mass as a source one has ($m \equiv \alpha\kappa^{-1}$)

$$\Phi_*(r) = -\frac{GMe^{-mr}}{3r}, \quad \gamma = \frac{3 - e^{-mr}}{3 + e^{-mr}}. \quad (36)$$

If m is sufficiently large, the Yukawa term has a range shorter than the scale of the recent torsion-balance experiments. Then for all practical purposes the quantity G_{eff} measured in laboratory is equal to G and $\gamma = 1$. On the other hand, if the scalar is very light, with a range much larger than the Solar System scales, then the measured solar mass in terms of Newtonian mechanics is such that

$$(GM)_{\text{eff}} = \left(1 + \frac{1}{3}e^{-mr}\right)GM \approx \frac{4GM}{3},$$

and $\gamma \approx 1/2$. This example shows that one cannot assume $\Phi_* \ll V$ in general. Inserting $\gamma = 1/2$ into (35) one finds the bending angle $\theta = \frac{3}{4}\theta_{\text{GR}}$, in agreement with [1] (see also the example briefly discussed in Sec. 2.3). The PPN parameter γ in (36) was derived in [6], where it was also argued that its correct value should be, instead, $\gamma = 1$ in view of the fact that $\Phi + \Psi = 2V$ as in (32). As we have shown above there is no conflict between Eqs. (32) and (36) after one takes into account the observable quantities in (32). The result $\gamma = 1$ was found in [4] under the assumption that the free parameter m is such that the theory does not violate known experimental results. This is in agreement with our discussion, i.e., considering that m is such that it may affect the Newtonian force law only at sub-millimeter scale. Finally, the deflection of light in this model was investigated also in Refs. [2,5,7,9] with the conclusion that it cannot be used to distinguish between GR and $R + R^2$ at the leading order in G (see [9] for calculations at the next order). According to the discussion in the present work this assertion is partially correct, since any prediction is based on the quantity GM and the measured $(GM)_{\text{eff}}$ can differ from GR and quadratic gravity.

3.1. Relation to conformal transformation of the background

The description of the bending of light in terms of the effective refractive index is closely related to the fact that light rays follow null-geodesics associated to the metric $g_{\mu\nu}$. In fact, the statement that the refractive index for the extended model (2) equals general relativity's one to the first order in G can be translated to the geodesic structure of the manifolds in both theories in terms of conformal transformations. This association has been carried out, for example, in [43,44] for the model $R + R^2$, and in [29,30,45] for the more general model which is a polynomial function of d'Alembertian. Using the metric (22) parametrized as (26) and with the result $\Psi_* = -\Phi_*$ obtained above it is cursory to verify that

$$g_{\mu\nu}^{\text{ext}} = (1 + 2\Phi_*)g_{\mu\nu}^{\text{GR}}, \quad (37)$$

² The derivation of this result can be found in textbooks on GR, for the terms of Snell-Descartes law see [34].

were only the terms to order G were kept. Here

$$g_{\mu\nu}^{\text{GR}}(r) = \eta_{\mu\nu} + 2V\delta_{\mu\nu}$$

is GR's weak-field metric associated to the same matter configuration in the de Donder gauge.

Since null-geodesics are conformal invariant, up to $O(G)$ the null-geodesics of GR are also null-geodesics in the extended model (2). As a result, light rays undergo the same gravitational deflection in both theories. On the other hand, time- and space-like geodesics are not conformally mapped to geodesics, and massive particles ought to deflect in a different manner. As mentioned before, the reasoning presented in this section parallels the description in terms of Maxwell equations on a curved background. The conformal transformation $g_{\mu\nu}^{\text{GR}} \mapsto g_{\mu\nu}^{\text{ext}}$ does not affect the equations of motion of the electromagnetic field, as we discussed above (see also [46]).

The result (37) turns out to be the linearized version of the theorem proved in [29] on the relation of the model (2), scalar-tensor theories and GR with extra scalar fields.³ By evoking the same arguments in terms of conformal transformations (see, e.g., [47]), it follows that in the generalized Brans–Dicke (BD) theory [48] in the form

$$S_{\text{BD}} = \int d^4x \sqrt{-g} [\phi R + \omega g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - U(\phi) + \mathcal{L}_M] \quad (38)$$

the non-trivial contribution to the bending of light also comes from the observable value $(GM)_{\text{eff}}$ rather than to a new force acting on light rays.

The PPN parameter γ for massive Brans–Dicke (mBD) theories, i.e., with $\omega \neq 0$ and $U''(\phi) \neq 0$, was computed in [39,40,49,50], with the result

$$\gamma_{\text{mBD}} = \frac{2\omega + 3 - e^{-mr}}{2\omega + 3 + e^{-mr}}, \quad (39)$$

where m depends on ω and on the form of U . For a suitable potential U and in the limit $\omega \rightarrow 0$, it is possible to show that γ_{mBD} above reduces to the expression in (36), evaluated for the $R + R^2$ gravity. This is expected, since this theory is dynamically equivalent to a BD theory with $\omega = 0$ but with a non-trivial potential U [51]. Light deflection (or the PPN parameter γ) has been computed in the scalar-tensor formulation in [52,49,50] with the results consistent to those presented in the previous sections, as expected. We note that if $m \ll r^{-1}$ (i.e., the scalar field has a range much larger than the experimental scale) then $\gamma_{\text{mBD}} \approx (2\omega + 2)/(2\omega + 4)$, which gives $\gamma_{\text{mBD}} \approx 1/2$ for the quadratic equivalent model with $\omega = 0$, in agreement to the example discussed in the last section. It is important to notice, however, that this only occurs for a very light massive field; in the case $m \gg r^{-1}$ and $\omega = 0$ one has $\gamma_{\text{mBD}} \approx 1$. This situation is different from the massless BD theory, for which $\omega = 0$ implies in $\gamma_{\text{BD}} = 1/2$ independently of the scale.

4. Conclusions

The gravitational deflection of light is one of the main predictions of GR, which has a growing precision of measurement. It is quite reasonable to use this effect to test the models of modified gravity, including higher-derivative and non-local ones. In the literature one may find different conclusions on the bending of light

within the simplest extension of GR, the $R + R^2$ gravity. A recurrent statement is that light bending cannot distinguish between this theory and GR in the linear regime [2,5–9]. This result directly follows from the fact that in quadratic gravity light is not subjected to an extra potential other than the Newtonian one. However, the situation may change if taking into account what are the true observable quantities in the theory. In this context the predictions for the light bending can differ, as the Keplerian mass of the object responsible for the deflection is theory-dependent [1,4,39,40,49,50]. Similar results have been obtained for the bending of light in the sixth-order gravity [10,13].

In the present work we extended such considerations to the whole family of gravity theories based on the Einstein–Hilbert action with an extra term $Rf(\square)R$. Namely, we presented different arguments to show that in the weak field limit the deflection of photons is not *directly* affected by the terms quadratic in scalar curvature, regardless of the form of the function $f(x)$. It turns out that this fact is closely related to the conformal invariance of Maxwell equations. It is worth noticing that in the weak-field approximation the result applies also to the general models of the form (1).

The form of $f(x)$ can produce an indirect effect on the light bending through the redefinition of the (measured) Keplerian mass M_{eff} . In particular, the effective PPN parameter γ was calculated as a quantity dependent on the function $f(x)$, assuming that its spatial dependence can be neglected in some regimes. This derivation confirms that the predictions of light bending can differ from theory to theory among the class (2), even though they have the same interaction potential with light (written in terms of G).

Finally, it is worth to point out that the introduction of the terms of the second order in Ricci and/or Riemann tensors—such as $R_{\mu\nu}g(\square)R^{\mu\nu}$, for a local or non-local function $g(x)$ —changes the propagator of the spin-2 sector, which yields a *direct* effect in the light bending [8,10,13]. More general results in this direction will be presented elsewhere.

Acknowledgements

B.L.G. is thankful to CNPq for supporting his Ph.D. project. I.Sh. is grateful to CNPq, FAPEMIG and ICTP for partial support of his work.

References

- [1] E. Pechlaner, R. Sexl, *Commun. Math. Phys.* 2 (1966) 165.
- [2] A. Accioly, A.D. Azeredo, E.C. de Rey Neto, H. Mukai, *Braz. J. Phys.* 28 (1998) 2.
- [3] S. Capozziello, V.F. Cardone, A. Troisi, *Phys. Rev. D* 73 (2006) 104019, arXiv:astro-ph/0604435.
- [4] T. Clifton, *Phys. Rev. D* 77 (2008) 024041.
- [5] M. Lubini, C. Tortora, J. Näf, Ph. Jetzer, S. Capozziello, *Eur. Phys. J. C* 71 (2011) 1834, arXiv:1104.2851.
- [6] Ch.P.L. Berry, J.R. Gair, *Phys. Rev. D* 83 (2011) 104022, arXiv:1104.0819; Ch.P.L. Berry, J.R. Gair, *Phys. Rev. D* 85 (2012) 089906 (Erratum).
- [7] A. Stabile, An. Stabile, *Phys. Rev. D* 85 (2012) 044014, arXiv:1108.4721.
- [8] A. Accioly, J. Helayël-Neto, B. Giacchini, W. Herdy, *Phys. Rev. D* 91 (2015) 125009, arXiv:1506.00270.
- [9] H. Liu, X. Wang, H. Li, Y. Ma, *Eur. Phys. J. C* 77 (2017) 723, arXiv:1508.02647.
- [10] A. Accioly, B.L. Giacchini, I.L. Shapiro, *Phys. Rev. D* 96 (2017) 104004, arXiv:1610.05260.
- [11] A.A. Starobinsky, *Phys. Lett. B* 91 (1980) 99.
- [12] A.A. Starobinsky, *Sov. Astron. Lett.* 9 (1983) 302.
- [13] A. Accioly, B.L. Giacchini, I.L. Shapiro, *Eur. Phys. J. C* 77 (2017) 540, arXiv:1604.07348.
- [14] E.V. Gorbar, I.L. Shapiro, *J. High Energy Phys.* 02 (2003) 021; E.V. Gorbar, I.L. Shapiro, *J. High Energy Phys.* 06 (2003) 004.
- [15] S. Deser, R.P. Woodard, *Phys. Rev. Lett.* 99 (2007) 111301, arXiv:0706.2151.
- [16] M. Maggiore, *Fundam. Theor. Phys.* 187 (2017) 221, arXiv:1606.08784; E. Belgacem, Y. Dirian, S. Foffa, M. Maggiore, *Nonlocal gravity. Conceptual aspects and cosmological predictions*, arXiv:1712.07066.

³ Indeed, it can be shown that the higher-derivative theories whose Lagrangian is formed by functions of $\square^n R$ (for all $n \in \mathbb{N}$) are dynamically equivalent to a (multi)scalar-tensor theory, which is then conformal equivalent to GR with scalar fields [45].

- [17] B.M. Barker, S.N. Gupta, R.D. Haracz, *Phys. Rev.* 149 (1966) 1027.
- [18] D. Boccaletti, V. de Sabbata, C. Gualdi, P. Fortini, *Nuovo Cimento A* 48 (1967) 58.
- [19] N.E.J. Bjerrum-Bohr, J.F. Donoghue, B.R. Holstein, L. Planté, P. Vanhove, *Phys. Rev. Lett.* 114 (2015) 061301, arXiv:1410.7590.
- [20] N.E.J. Bjerrum-Bohr, B.R. Holstein, L. Planté, P. Vanhove, *Phys. Rev. D* 91 (2015) 064008, arXiv:1410.4148.
- [21] N.E.J. Bjerrum-Bohr, J.F. Donoghue, B.R. Holstein, L. Planté, P. Vanhove, *J. High Energy Phys.* 2016 (2016) 117, arXiv:1609.07477.
- [22] D. Bai, Y. Huang, *Phys. Rev. D* 95 (2017) 064045, arXiv:1612.07629.
- [23] M.D. Scadron, *Advanced Quantum Theory*, third edition, World Scientific Publishing, Singapore, 2007.
- [24] A. Zee, *Quantum Field Theory in a Nutshell*, Princeton University Press, Princeton, 2003.
- [25] I.L. Buchbinder, S.D. Odintsov, I.L. Shapiro, *Effective Action in Quantum Gravity*, IOP Publishing, Bristol, 1992.
- [26] E. Guadagnini, *Phys. Lett. B* 548 (2002) 19, arXiv:gr-qc/0207036.
- [27] A. Einstein, *Ann. Phys. (Berl.)* 35 (1911) 898.
- [28] M. Asorey, J.L. López, I.L. Shapiro, *Int. J. Mod. Phys. A* 12 (1997) 5711, arXiv:hep-th/9610006.
- [29] H.-J. Schmidt, *Class. Quantum Gravity* 7 (1990) 1023.
- [30] I. Quandt, H.-J. Schmidt, *Astron. Nachr.* 312 (1991) 97, arXiv:gr-qc/0109005.
- [31] B.L. Giacchini, *Phys. Lett. B* 766 (2017) 306, arXiv:1609.05432.
- [32] L. Modesto, T. de Paula Netto, I.L. Shapiro, *J. High Energy Phys.* 1504 (2015) 098, arXiv:1412.0740.
- [33] A.P. Lightman, D.L. Lee, *Phys. Rev. D* 8 (1973) 364.
- [34] E. Fischbach, B. Freeman, *Phys. Rev. D* 22 (1980) 2950.
- [35] T. Padmanabhan, *Gravitation: Foundations and Frontiers*, Cambridge University Press, Cambridge, 2010.
- [36] J.L. Anderson, *Principles of Relativity Physics*, Academic Press, New York, 1967.
- [37] C.M. Will, *Theory and Experiment in Gravitational Physics*, second edition, Cambridge University Press, Cambridge, UK, 1993.
- [38] S. Capozziello, A. Stabile, A. Troisi, *Phys. Lett. B* 686 (2010) 79, arXiv:1002.1364.
- [39] L. Perivolaropoulos, *Phys. Rev. D* 81 (2010) 047501.
- [40] J. Alsing, E. Berti, C.M. Will, H. Zaglauer, *Phys. Rev. D* 85 (2012) 064041.
- [41] D. Borka, P. Jovanović, V. Borka Jovanović, A.F. Zakharov, *J. Cosmol. Astropart. Phys.* 11 (2013) 050, arXiv:1311.1404.
- [42] S. Capozziello, D. Borka, P. Jovanović, V. Borka Jovanović, *Phys. Rev. D* 90 (2014) 044052, arXiv:1408.1169.
- [43] B. Whitt, *Phys. Lett. B* 145 (1984) 176.
- [44] P. Teyssandier, *Class. Quantum Gravity* 6 (1989) 219.
- [45] D. Wands, *Class. Quantum Gravity* 11 (1994) 269, arXiv:gr-qc/9307034.
- [46] H.A. Buchdahl, *J. Aust. Math. Soc.* 1 (1959) 99.
- [47] A. Barros, C. Romero, *Phys. Lett. A* 245 (1998) 31, arXiv:gr-qc/9712080.
- [48] C. Brans, R.H. Dicke, *Phys. Rev.* 124 (1961) 925.
- [49] G.J. Olmo, *Phys. Rev. Lett.* 95 (2005) 261102.
- [50] G.J. Olmo, *Phys. Rev. D* 72 (2005) 083505.
- [51] P. Teyssandier, Ph. Tourrenc, *J. Math. Phys.* 24 (1983) 2793.
- [52] J. O'Hanlon, *Phys. Rev. Lett.* 29 (1972) 137.