

Dark Matter in the Standard Model Extension in Non-Commutative Geometry (NCG)

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Abstract. It is for the most part expected that dark matter is important to clarify the rotation of the galaxy, It has effectively been seen that the non-commutative geometry background can achieve this objective similarly. The objective of this study is to investigate a relationship between non-commutative geometry and certain aspect of dark matter. We are relying on a basic mathematical expression argument that indicates that the appearance of dark matter in galaxies and galaxy clusters with regard to flat rotation curves is similarly a result of non commutative geometry.

1. Introduction

Non-commutative geometry may be described informally as the study of spaces with non-commutative function algebra[1]. Throughout this century, both mathematicians and physicists have been interested in this topic. Non-commutative geometry, on the other hand, was underdeveloped in comparison to classical geometry (the geometry of spaces whose algebra of functions is commuting)[2].

This has just begun to alter with the introduction of (independent) generalized by Rham differential algebras on the "non-commutative manifold" by Connes [3] and Dubois-Violette [4] [5]. Since then, non-commutative geometry has been improved and refined to the point where it now encompasses many of the techniques used in classical geometry [6] [7].

Modern astrophysical and cosmological models confront two major theoretical challenges, which can be summed up as the dark energy and dark matter issues[8]. The purpose of this research is to investigate several characteristics of dark matter using non-commutative geometry, which predicts that The mass within the radius r sphere increases in line with r in the external radial direction.

As a result, dark matter isn't required to compute flat galaxy rotation curves. A similar statement can be made about the spin curves of galaxy clusters, and the most important conclusion is that non-reciprocal geometry can explain the background of certain aspects of dark matter.

2. Non commutative geometry

We commence by calculating the generic spherically symmetrical static metric in units where $c = G = 1$:



$$ds^2 = -e^{2\phi(r)} dt^2 + \frac{dr^2}{1 - \frac{2m(r)}{r}} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \quad (1)$$

The efficient mass inside a sphere of radius r with $m(r) = 0$ is $m(r)$. We additionally require that $\lim_{r \rightarrow \infty} m(r)/r = 0$.

The only nonzero additions of the stress-energy tensor are $T_t^t = -\rho(r)$ the energy density, which is due to the spherical symmetry. $T_r^r = \rho(r)$ the radial pressure, $T_\theta^\theta = T_\phi^\phi = \rho_t(r)$ the lateral pressure.

Die Einsteinian field equations may be expressed in the following way:

$$\rho(r) = \frac{2m'}{8\pi r^2} \quad (2)$$

$$\rho_r(r) = \frac{1}{8\pi} \left[-\frac{2m}{r^3} + \frac{2\phi'}{r} \left(1 - \frac{2m}{r} \right) \right] \quad (3)$$

$$\rho_r(r) = \frac{1}{8\pi} \left(1 - \frac{2m}{r} \right) \left[\phi'' - \frac{2mr' - 2m}{2r(r-2m)} \phi' + (\phi')^2 + \frac{\phi'}{r} - \frac{2mr' - 2m}{2r^2(r-2m)} \right] \quad (4)$$

The conservation law $T_{\alpha\beta}^\alpha = 0$ means that :

$$\rho_r' + \phi' \rho + \phi' \rho_r + \frac{2\rho_r}{r} - \frac{2\rho_t}{r} = 0 \quad (5)$$

Only equations (2) and (3) are truly necessary, which is a perspective that will be exploited later. Next, we introduce the concept of non-commutative geometry, which is an area that is defined by the following string hypothesis result: On D-brane, directions can become noncommuting administrators [9].

$[x^\mu, x^\nu] = i\theta^{\mu\nu}$ is the commutator and $\theta^{\mu\nu}$: is an antisymmetric matrix.

The fundamental concept, as described in Refs. [10, 11], is that non-commutativity results in the substitution of spread items for point like structures. (The objective is to remove the divergences inherent in general relativity.) Instead of the Dirac delta work, a Gaussian conveyance of minimal length $\sqrt{\beta}$ is often used to produce the spreading effect [12].

A same, however less difficult, The route is to anticipate that the energy thickness of the static and circularly symmetric gravitational source has the structure of a molecule [15].

$$\rho(r) = \frac{M\sqrt{\beta}}{\pi^2(r^2 + \beta)^2} \quad (6)$$

To use Eq. (6), one can keep the conventional Einstein field conditions because the Einstein tensor retains its distinctive structure, but the pressure energy tensor is altered [13]. As a result, the length scale does not have to be limited to the Planck scale.

It is additionally noted in Ref. [12] that non commutative math is a characteristic property of spacetime and doesn't rely upon a specific component like bend.

The gravitational source in Eq. (6) brings about a spread mass. As in Refs. [11, 12], the Schwarzschild arrangement of the Einstein field conditions related with the spread source.

Where :

$$ds^2 = -\left(1 - \frac{2M_\beta(r)}{r}\right) dt^2 + \left(1 - \frac{2M_\beta(r)}{r}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \quad (7)$$

Here the spread mass is discovered to be

$$M_{\beta}(r) = \int_0^r 4\pi(r')^2 \rho(r') dr' = \frac{2M}{\pi} \left(\tan^{-1} \frac{r}{\sqrt{\beta}} - \frac{r\sqrt{\beta}}{r^2 + \beta} \right) \quad (8)$$

where M is the source's hard and fast mass at the moment. Since $\lim_{r \rightarrow 0} M_{\beta}(r)/r = 0$, there is no peculiarity at $r = 0$.

Because of the spreading, the mass of the molecule relies upon β , just as on r . As in the instance of the Gaussian model, the mass of the molecule is zero at the middle and quickly increments to M . Therefore, from a distance the spreading is not, at this point noticed, and we get a conventional molecule :

$$\lim_{\beta \rightarrow 0} M_{\beta}(r) = M$$

So the changed Schwarzschild arrangement turns into a normal Schwarzschild arrangement in the cutoff.

3. The dark-matter

The presence of dark matter was at that point speculated in the 1930s by Zwicky and others. The suggestions there of were not perceived until the 1970s the point at which it was seen that cosmic systems display level pivot bends (consistent speeds) adequately a long way from the galactic focus. This perception demonstrates that the matter in the cosmic system increments straightly the outward spiral way [13].

To understand why, consider that m_1 to be the mass of a star, v to be its constant velocity, and m_2 to be the mass of everything else. Now multiply m_1 by the centripetal acceleration to get the result.

$$m_1 \frac{v^2}{r} = m_1 m_2 \frac{G}{r^2} \quad (9)$$

Where G is consistent with Newton's gravity. We have the straight structure with geometrized units ($G = c = 1$):

$$m_2 = rv^2 \quad (10)$$

as attested. Eq. (10) basically describes the dark matter theory. Consider a small spherical shell with radius $r = r_0$. So rather than a spread item situated at the beginning, we currently have a spread round surface. We take into account spreading the outward spiral way just, since that is the simple form of the spread molecule at the beginning. The energy thickness in Eq. (6) should thusly be supplanted by :

$$\rho(r) = \frac{M_{r_0} \sqrt{\beta}}{\pi^2 [(r - r_0)^2 + \beta]^2} \quad (11)$$

$$m_{\beta}(r - r_0) = \frac{2M_{r_0}}{\pi} \left(\tan^{-1} \frac{r - r_0}{\sqrt{\beta}} - \frac{(r - r_0)\sqrt{\beta}}{(r - r_0)^2 + \beta} \right) \quad (12)$$

$$\lim_{\beta \rightarrow 0} m_{\beta}(r - r_0) = M_{r_0}$$

As a result, the shell's mass is zero at $r = r_0$ and quickly increases to M_{r_0} . As we move the spiral outwards, We can substitute "change in mass per unit length in the r -heading" for "mass of the shell," which will still be expressed by $m_{\beta}(r - r_0)$.

$$dM_T(r - r_0) = \frac{dM_T(r - r_0)}{dr} dr = m_\beta(r - r_0)dr$$

The total spread mass in this manner is calculated as follows:

$$\begin{aligned} M_T(r - r_0) &= \int_0^{r-r_0} m_\beta(r') dr' = \frac{2M_{r_0}}{\pi} \left[(r - r_0) \tan^{-1} \frac{(r - r_0)}{\sqrt{\beta}} - \sqrt{\beta} \ln[(r - r_0)^2 + \beta] \right] \\ &= \frac{2M_{r_0}}{\pi} (r - r_0) \left[\tan^{-1} \frac{(r - r_0)}{\sqrt{\beta}} - \sqrt{\beta} \frac{\ln[(r - r_0)^2 + \beta]}{r - r_0} \right] \\ \lim_{r \rightarrow \infty} \left[\tan^{-1} \frac{(r - r_0)}{\sqrt{\beta}} - \sqrt{\beta} \frac{\ln[(r - r_0)^2 + \beta]}{r - r_0} \right] &= \frac{\pi}{2} - 0 \end{aligned} \quad (13)$$

$$M_T(r - r_0) = M_{r_0}(r - r_0) \quad (14)$$

The geometric understanding of the gravitational draw because of dark matter is especially in the soul of Einstein's hypothesis, which takes the role of the idea of gravity power through the mathematical idea of the bend. To interface Eq. (14) to the extraneous speed, we get back to Eq. (13) and see that

$$= \frac{2M_{r_0}}{\pi} \left[(r - r_0) \tan^{-1} \frac{(r - r_0)}{\sqrt{\beta}} - \sqrt{\beta} \ln[(r - r_0)^2 + \beta] \rightarrow M_{r_0}(r - r_0) \right]$$

Because β is Powerfully tiny to begin with, if r is sufficiently large, we have

$$M_T(r) \approx M_{r_0}(r - r_0)$$

From the equation(10) we find :

$$\begin{aligned} v^2 r &\approx M_{r_0}(r - r_0) \\ v^2 &\approx M_{r_0} \left(1 - \frac{r_0}{r}\right) \end{aligned}$$

As a result, we deduce v^2 is around equivalent to the adjustment of the spread mass per unit [14] .

$$\rho(r) = \rho_0 \left(1 + \frac{r^2}{r_c^2}\right)^{-3\beta/2} \quad (15)$$

r_c is the core radius , β and ρ_0 are constants[15]

$$M(r) = \frac{3K_B \beta T}{\mu m_p G} \frac{r^3}{r_c^2 + r^2} \quad (16)$$

Boltzmann's constant, gas temperature, and proton mass are all represented by K_B , T and m_p , respectively, and $\mu \approx 0.61$ is the gas's mean atomic weight.

It is important to note that for large r , $M(r)$ has the same linear form as that found in the preceding section.

4. Conclusion

This paper talks about certain parts of non-commutative geometry. It's anything but a particular part of dark matter, computing level world pivot bends. The foundation math ends up being steady with the presumption of dark matter, whose essential sign is a straightly expanding mass the external spiral way. Associations with different parts of dark matter, like the arrangement of the construction, are left open. Notwithstanding, dark matter doesn't rise up out of the non-commutative calculation in the cosmology setting, as confirmed by our expansion of world groups.

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