DARK ENERGY AND STRUCTURE FORMATION

J.-M.SCHWINDT
Institut fuer Theoretische Physik, Philosophenweg 16, D-69120 Heidelberg, Germany

We discuss the influence of dark energy on structure formation, especially the effects on σ_8. Our interest is particularly focused on quintessence models with time-dependent equation of state and non-negligible quintessence component in the early universe. We obtain an analytic expression for σ_8 valid for a large class of dark energy models. We conclude that structure formation is a good indicator for the history of dark energy and use our results to set constraints on quintessence models.

1 Introduction

1.1 *Dark Energy*

There is evidence of dark energy contributing up to about 70% of the total energy of the universe [1,2]. The nature of dark energy is an open question, a cosmological constant or a dynamical scalar field[3] called quintessence[4] being two major options. Telling the difference between a cosmological constant and quintessence or between different quintessence models is complicated because of the non-genericness of quintessence.

If quintessence couples only gravitationally to matter, the only way of detection is possibly the exploration of its time dependent energy density and equation of state, as the relative energy density fluctuations $\delta\rho_d/\rho_d$ within the horizon are negligible [5]. In order to find this time dependence, measurements of different epochs are necessary. For that reason, the interplay between quintessence and nucleosynthesis [6], cosmic microwave background (CMB) [7], weak lensing[8] and Supernovae Ia data [9] has recently been explored.

Also, the theory of structure formation can in principle test the history of quintessence in the large range of redshift $z \in [0,\ 10^4]$. As has been noticed in [5,10,11], the presence of dark energy can influence the growth of structure in the universe from matter radiation equality onwards. In particular, σ_8, the rms density fluctuations averaged over $8h^{-1}$Mpc spheres, is a sensitive parameter.

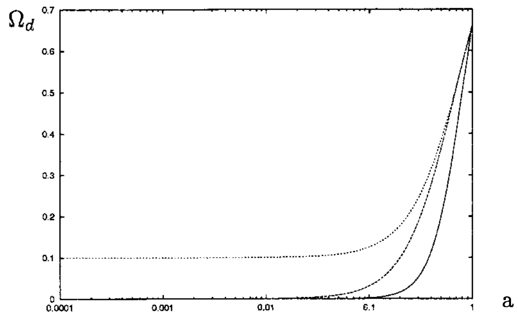

Figure 1: Three types of dark energy with $\Omega_{\rm d}^0 = 2/3$. Solid line: Cosmological Constant, $w = -1$. Dashed line: Quintessence with (almost) constant w, here $w = -0.6$, corresponding to an inverse power law potential. Dotted line: Quintessence model with early quintessence. Structure formation turns out to be very sensitive to the area between these curves.

Supernovae Ia observations [1] indicate that the equation of state $w_{\rm d} \equiv p_d/\rho_d$ of dark energy is negative today. This gives rise to the fine-tuning problem: We have $(\Omega_{\rm d}(a)/\Omega_m(a)) \propto a^{-3\bar{w}_d}$, where a is the scale factor and $\bar{w}_{\rm d}$ is an appropriate mean value for the equation of state. If $w_{\rm d}$ has always been negative, like in the case of the cosmological constant, $\Omega_{\rm d}(a)$ has been extremely small in the early universe, and its importance just today lacks a natural explanation. The problem can be surrounded if we assume that $w_{\rm d}$ became negative relatively recently and ρ_d has scaled in the past like radiation or matter [12,13]. In this case, a quintessence component in the early universe has to be taken into account. We will call such models 'models with early quintessence' and pay particular attention to them.

CMB measurements [2,14] suggest that the universe is flat, $\Omega \equiv \Omega_{\rm m} + \Omega_{\rm r} + \Omega_{\rm d} = 1$ and we assume this throughout the talk. (We use here conventions where 0 denotes today's value of a quantity and the subscript m, r and d denote matter, radiation and dark energy respectively.)

1.2 Structure Formation

The structure growth exponent f of the density contrast δ_k is defined as

$$f(a) \equiv \frac{d \ln \delta_k(a)}{d \ln a}, \tag{1}$$

and is roughly k-independent for a wide range of k. One can use linearized General Relativity in the synchronous gauge to compute $f(a)$. For sub-horizon modes in SCDM models, one obtains $f \to 0$ in the radiation and $f = 1$ in the matter eras. In general, the sub-horizon growth of density perturbations is governed by

$$\frac{df}{d \ln a} + f^2 + \left(2 - \frac{1}{2}\left[3 + 3w_{\rm d}(a)\Omega_d(a) + \Omega_r(a)\right]\right) f - \frac{3}{2}\Omega_m = 0. \tag{2}$$

As long as $\Omega_d(a)$ is small, this equation is approximately solved (in the matter era) by

$$f(a) \approx 1 - \frac{3}{5}\Omega_d(a) + O((\Omega_d(a))^2). \tag{3}$$

The COBE normalization [15] of the CMB power spectrum determines σ_8 for any given model by essentially fixing the fluctuations at decoupling. This prediction is to be compared to values of σ_8 infered from other experiments, such as cluster abundance constraints which yield [16]

$$\sigma_8 = (0.5 \pm 0.1)\Omega_m^{-\gamma}, \tag{4}$$

where γ is slightly model dependent and usually $\gamma \approx 0.5$. A model where these two σ_8 values do not agree can be ruled out. SCDM without dark energy and with standard values $\Omega_{\rm d}^0 = 0.65$, $h = 0.65$, $n = 1$, $\Omega_b h^2 = 0.02$ with $-1 < \bar{w} < -0.5$ for instance gives $\sigma_8^{\rm cmb} \approx 1.5$, $\sigma_8^{\rm clus.} \approx 0.5 \pm 0.1$ and is hence incapable of meeting both constraints. ΛCDM gives $\sigma_8^{\rm cmb} \approx 0.9$, $\sigma_8^{\rm clus.} \approx 0.9 \pm 0.2$ and is consistent with the data.

2 The Influence of Dark Energy on σ_8

There are several differences in the computation of the CMB-normalized σ_8-value between a universe with dark energy and SCDM:

Equality shift: The scale factor at matter radiation equality is $a_{\rm eq} = \Omega_{\rm r}^0/\Omega_{\rm m}^0 = 4.3 \times 10^{-5} h^{-2} (1 - \Omega_{\rm d}^0)^{-1}$. If $\Omega_{\rm d}^0 \approx 0.6$, then $a_{\rm eq}$ is larger than in SCDM by a factor of 2.5. Therefore f starts growing much later for the σ_8-relevant modes, leading to a substantially lower σ_8-value. This effect is the strongest for many dark energy models. We circumvent the difficulty of computing this effect analytically by comparing models with the *same* dark energy content today and therefore identical values of $a_{\rm eq}$. It is then sufficient to determine σ_8 numerically (by CMBFAST [17] or CMBEASY [18]) for one model of this class, e.g. ΛCDM.

Matter depletion: From Equation (2) we see that a decrease of Ω_m leads to a decrease of f.

Accelerated expansion: Also from Equation (2) we see that an accelerated expansion, i.e. a lower value of $w_{\rm d}(a)\Omega_d(a)$, leads to a decrease of f.

Normalization shift: The different age of universes with dark energy leads to a shift in the normalization of δ_k with respect to the CMB power spectrum [10].

Our main result is an estimate of the CMB-normalized σ_8-value for a very general class of Quintessence models Q just from the knowledge of their "background solution" $[\Omega_{\rm d}(a),\ w_{\rm d}(a)]$ and the σ_8-value of the ΛCDM model Λ with the same amount of dark energy today $\Omega_\Lambda^0 = \Omega_{\rm d}^0(\Lambda)$:

$$\frac{\sigma_8(Q)}{\sigma_8(\Lambda)} \approx (a_{\rm eq})^{3\bar{\Omega}_{\rm d}^{\rm sf}/5} \left(1 - \Omega_\Lambda^0\right)^{-(1+\bar{w}^{-1})/5} \sqrt{\frac{\tau_0(Q)}{\tau_0(\Lambda)}}, \tag{5}$$

where

$$\bar{\Omega}_{\rm d}^{\rm sf} \equiv [\ln a_{\rm tr} - \ln a_{\rm eq}]^{-1} \int_{\ln a_{\rm eq}}^{\ln a_{\rm tr}} \Omega_{\rm d}(a)\ {\rm d}\ln a, \quad \frac{1}{\bar{w}} = \frac{\int_{\ln a_{\rm tr}}^0 \Omega_{\rm d}(a)/w(a)\, d\ln a}{\int_{\ln a_{\rm tr}}^0 \Omega_{\rm d}(a)\, d\ln a}. \tag{6}$$

If Q is a model with 'early quintessence', $\bar{\Omega}_{\rm d}^{\rm sf}$ is an average value for the fraction of dark energy during the matter dominated era, before $\Omega_{\rm d}$ starts growing rapidly at scale factor $a_{\rm tr}$. If Q is a model without early quintessence, $\bar{\Omega}_{\rm d}^{\rm sf}$ is zero. The effective equation of state $\bar{w}$ is an average value for $w_{\rm d}$ during the time in which $\Omega_{\rm d}$ is growing rapidly. Finally, τ_0 is the conformal age of the universe. Equation (5) in combination with (4) can be used to make general statements about the consistency of quintessence models with σ_8-constraints.
The first factor comes from the matter depletion effect during the early matter era ($a < a_{tr}$). The second factor is due to the matter depletion and the accelerated expansion effects in the recent epoch ($a > a_{tr}$). The last factor is an approximation for the normalization shift. The exponents in the first two factors are proportional to the area between the curves in figure 1. Using CMBEASY simulations of several quintessence models we found equation (5) to be precise within 2-5%.

3 Conclusions

We have analyzed the effects of dark energy on structure formation. We found that σ_8 is a very promising indicator for constraining dark energy models, especially the amount of early quintessence $\bar{\Omega}_{\rm d}^{\rm sf}$. The CMB-normalized value of σ_8 depends on all cosmological parameters. As a rough guide for the strength of these dependencies around *standard values*(see above) we get

–Increasing h by 0.1 $\Rightarrow$ Increase of σ_8 by 20 %
–Increasing $\Omega_{\rm d}^0$ by 0.1 $\Rightarrow$ Decrease of σ_8 by 20%
–Increasing n by 0.1 $\Rightarrow$ Increase of σ_8 by 25%
–Increasing $\bar{w}$ by 0.1 $\Rightarrow$ Decrease of σ_8 by 5-10%
–Increasing $\Omega_b h^2$ by 0.01 $\Rightarrow$ Decrease of σ_8 by 10%
–Increasing $\bar{\Omega}_{\rm d}^{\rm sf}$ by 0.1 $\Rightarrow$ Decrease of σ_8 by 50%

An overall bound for early quintessence can be obtained from Equation (4) if we assume $h < 0.75$, $\Omega_{\rm d}^0 > 0.5$ and $n < 1.2$. One finds

$$\bar{\Omega}_{\rm d}^{\rm sf} \lessapprox 0.2. \tag{7}$$

This is of the same order as the bound from big bang nucleosynthesis, $\Omega_{\rm d}^{\rm bbn} < 0.2$ [19,20]. This bound can be substantially improved by more precise determinations of h, $\Omega_{\rm d}^0$ and n. For standard values (see above) we obtain $\bar{\Omega}_{\rm d}^{\rm sf} \lessapprox 0.04$.

Acknowledgements

This talk is based on work in collaboration with Christof Wetterich and Michael Doran. I would like to thank them for their support. I would also like to thank the organizers of the conference.

References

1. A. G. Riess *et al.*, Astron. J. **116** (1998) 1009; S. Perlmutter *et al.*, Astrophys. J. **517** (1999) 565.
2. C. B. Netterfield *et al.*, astro-ph/0104460.
3. C. Wetterich, Nucl. Phys. B **302** (1988) 668; P. J. Peebles and B. Ratra, Astrophys. J. **325** (1988) L17; Phys. Rev. D **37** (1988) 3406.
4. R. R. Caldwell, R. Dave and P. J. Steinhardt, Phys. Rev. Lett. **80** (1998) 1582.
5. P. G. Ferreira and M. Joyce, Phys. Rev. D **58**, 023503 (1998).
6. R. Bean, S. H. Hansen and A. Melchiorri, astro-ph/0104162.
7. L. Amendola, Phys. Rev. Lett. **86** (2001) 196; M. Doran, M. Lilley and C. Wetterich, astro-ph/0105457; P. S. Corasaniti and E. J. Copeland, astro-ph/0107378.
8. D. Huterer, astro-ph/0106399.
9. D. Huterer and M. S. Turner, astro-ph/0012510; J. Weller and A. Albrecht, astro-ph/0106079.
10. M. Doran, J. Schwindt and C. Wetterich, Phys. Rev. D **64**, 123520 (2001).
11. C. Skordis and A. Albrecht, astro-ph/0012195; K. Benabed and F. Bernardeau, astro-ph/0104371.
12. A. Hebecker and C. Wetterich, Phys. Lett. B **497** (2001) 281.
13. A. Albrecht and C. Skordis, Phys. Rev. Lett. **84** (2000) 2076.
14. A. T. Lee *et al.*, astro-ph/0104459.
15. E. F. Bunn and M. J. White, Astrophys. J. **480** (1997) 6.
16. L. Wang and P. J. Steinhardt, Astrophys. J. **508** (1998) 483.
17. U. Seljak and M. Zaldarriaga, Astrophys. J. **469**, 437 (1996).
18. M. Doran, Contribution to the XXXVIIth Rencontres de Moriond, The Cosmological Model.
19. C. Wetterich, Astron. Astrophys. **301** (1995) 321.
20. M. Birkel and S. Sarkar, Astropart. Phys. **6** (1997) 197.