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Minimal length phenomenology and the black body radiation

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Abstract

The generalized uncertainty principle (GUP) modifies the uncertainty relation between momentum and position giving room for a minimal length, as predicted by candidates theories of quantum gravity. Inspired by GUP, Planck's distribution is derived by considering a new quantization of the electromagnetic field. We elaborate on the thermodynamics of the black body radiation obtaining Wien's law and the Stefan–Boltzmann law. We show that such thermodynamics laws are modified at Planck-scale.

Keywords: quantum gravity phenomenology, minimal length, black body, generalized uncertainty principle

(Some figures may appear in colour only in the online journal)

1. Introduction

Candidate theories of quantum gravity, such as string theory, loop quantum gravity, as well as *gedanken* experiments in black hole physics predict the existence of a minimal uncertainty in the position [1–10]. Such a minimal length is in direct contradiction with the Heisenberg uncertainty principle. Thus, a modification to the Heisenberg principle has to be introduced that is expected to be relevant at the Planck scale. Such a modification is considered in phenomenological models of quantum gravity [11]. In particular, the generalized uncertainty principle (GUP) consists in modifying the standard position-momentum commutation relation by including a function of the momentum operator [12–25]. A typical model involves a quadratic modification of the form

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$$[q, p] = i\hbar[1 + \gamma^2 p^2], \tag{1}$$

where

$$\gamma = \frac{\gamma_0}{M_{\text{Pl}} c}, \tag{2}$$

with $M_{\text{Pl}} = \sqrt{\frac{\hbar c}{G}}$ the Planck mass and γ_0 a dimensionless parameter that determines the energy scale at which such a modification takes place. Such a parameter is determined experimentally. For example, in the case of a macroscopic harmonic oscillator, it has been possible to find an upper bound for such a parameter of the order of 10^4 [26]. If it is assumed to be of order unity, the modification will be relevant at the Planck energy.

This model has been applied to several low-energy systems searching for indirect quantum gravity effects. Examples of such indirect tests concern the quantum harmonic oscillator, condensed matter, and atomic experiments [21, 27–31]. Further studies have been pursued in statistical mechanics [32–35], where the GUP affects thermodynamic variables.

The black body radiation represents one of the most interesting problems in the history of physics. The solution to the black body problem with the introduction by Planck of the hypothesis of the quantized energy exchanges revealed the issues of classical mechanics and laid the foundation for the development of modern quantum mechanics. Specifically, Planck assumed that the energy exchange between the modes of radiation enclosed in a cavity is proportional to the frequency of the mode, $\Delta E = h\nu$. That means that the energy is not absorbed continuously, but discretely. Such a discretization is related to the discrete energy spectrum of a quantum harmonic oscillator; the energy difference between two neighbouring energy levels is proportional to the frequency of the oscillator. However, the introduction of a minimal length changes such energy differences [14, 21, 36]. Thus, equation (1) implies a change in Planck's postulate. In this paper, we elaborate on the modification of black body thermodynamics. By introducing a new modification on the radiation field inspired by the GUP, we intend to study Planck's distribution and the corresponding GUP modification. Studying the resulting expression, we obtain further features of the black body distribution, such as Wien's law and the Stefan–Boltzmann law. Similar considerations have been elaborated in [37–40] following different techniques. However, the novelty of the present approach consists in following Bose's statistical method [41] when studying the statistical properties of a photon gas.

The paper is organized as follows. In section 2, we introduce the quantization relation for the radiation field inspired by GUP. In section 3, we elaborate a statistical analysis using the modified spectrum to obtain Planck's distribution. In sections 4 and 5, we obtain the modified Wien's law and Stefan–Boltzmann law to complete the study of the black body radiation. In section 6, we conclude by presenting future perspectives.

2. GUP modification to the radiation field

In this section, we review the GUP modification to the electromagnetic field quantization. According to [42], such a modification is introduced by modifying the generalized coordinates and momenta of the electromagnetic field $\mathbf{q_k}$ and $\mathbf{p_k}$, respectively, as follows

$$[\mathbf{q_k}, \mathbf{p_{k'}}] = i\hbar\delta_{\mathbf{k},\mathbf{k'}}[1 + \gamma_{\text{EM}}^2 \mathbf{p_k}^2], \tag{3}$$

where γ_{EM} corresponds to the deformation parameter for the electromagnetic field with units of inverse (energy)$^{1/2}$

$$\gamma_{\mathrm{EM}} = \frac{\gamma_0}{\sqrt{E_{\mathrm{Pl}}}}. \tag{4}$$

Here, $E_{\mathrm{Pl}} = M_{\mathrm{Pl}}c^2$ is the Planck energy. The transverse electric field $\vec{E}_{\mathrm{T}}$ is written in terms of $\mathbf{q_k}$ and $\mathbf{p_k}$ as [43]

$$\vec{E}_{\mathrm{T}} = \sum_{\vec{k}} \sqrt{\frac{1}{\varepsilon_0 V}} \vec{\epsilon}_k \left[\omega^k \mathbf{q_k} \sin\theta - \mathbf{p_k} \cos\theta\right], \tag{5}$$

where k is the magnitude of the wavenumber used to label the different modes and ω^k is the corresponding frequency. Furthermore, θ is the phase angle, $\vec{\epsilon}_k$ the polarization vector, and V the cavity volume. Using equation (5), the units for the generalized momenta of the radiation field are (energy)$^{1/2}$. With the deformed quantization rule in equation (3), the energy spectrum with GUP for a mode with wave vector $\mathbf{k}$ is [21]

$$E_n^k = \hbar\omega^k \left\{ \left(n + \frac{1}{2}\right) + \frac{\hbar\omega^k}{4}\gamma_{\mathrm{EM}}^2 (1 + 2n + 2n^2) \right\}. \tag{6}$$

Here, we see that, differently from the standard theory, the energy difference between the ground state and a state n is

$$\Delta E_n^k = \hbar\omega^k n + \zeta^k n(n+1), \tag{7}$$

where $\zeta^k = \frac{1}{2}(\hbar\omega^k\gamma_{\mathrm{EM}})^2$. This term carries the modification to the energy due to GUP. It is worth observing that in the limit $\gamma_{\mathrm{EM}} \to 0$, we recover the usual expression for the energy difference.

3. Modified Planck's law

Here, we follow the argument introduced by Bose to derive Planck's distribution [41]. Considering the statistics of indistinguishable photons, we construct a distribution Z_n^k for the number of photons in the state n with an infinitesimal range of frequencies $\mathrm{d}\omega^k$ centered around ω^k. By introducing the energy equation (7), the total energy is then

$$E = \sum_k (\Delta E_1^k Z_1^k + \Delta E_2^k Z_2^k + \Delta E_3^k Z_3^k + \cdots) = \sum_k \left[\hbar\omega^k N^k + \zeta^k (N^k + M^k)\right], \tag{8}$$

where

$$N^k = \sum_n n Z_n^k, M^k = \sum_n n^2 Z_n^k. \tag{9}$$

For any distribution Z_n^k, the occupation number for the system in a range $\mathrm{d}\omega^k$ around ω^k is A^k, expressed by

$$A^k = \sum_n Z_n^k. \tag{10}$$

The probability of observing a particular distribution Z_n^k is related to the number of different ways that particular distribution can be formed, that is

$$W = \prod_k \frac{A^k!}{\prod_n Z_n^k!}. \tag{11}$$

For a large number of photons, we adopt the following expression using Stirling's approximation

$$\log W = \sum_k A^k \log A^k - \sum_k \sum_n Z_n^k \log Z_n^k. \tag{12}$$

The quantity $\log W$ can be understood as a probability distribution of the microstates. Thus, maximizing such a quantity corresponds to look for the equilibrium condition for the system, as its maximum value is related to the most probable distribution [44]. Therefore, we determine the distribution $\bar{Z}_n^k$ that maximizes W. Furthermore, we note that the system is subject to two constraints i.e. equations (8) and (10). By applying Lagrange's method of undetermined multipliers

$$\delta \left\{ \log W + \sum_k \tau^k \sum_n Z_n^k + \alpha \sum_k \left[\hbar\omega^k N^k + \zeta^k (N^k + M^k) \right] \right\} = 0, \tag{13}$$

where τ^k and α are Lagrange multipliers. The variation with respect to Z_n^k gives

$$\sum_k \sum_n \delta Z_n^k \left\{ 1 + \log Z_n^k + \tau^k + \alpha \left[n\hbar\omega^k + \zeta^k n(1+n) \right] \right\} = 0. \tag{14}$$

As each δZ_n^k is arbitrary, we can impose the curly brackets to vanish obtaining

$$\bar{Z}_n^k = B^k \, \mathrm{e}^{-\alpha\hbar\omega^k n} \, \mathrm{e}^{-\alpha\zeta^k n(n+1)}, \tag{15}$$

where

$$B^k = \mathrm{e}^{-1-\tau^k}. \tag{16}$$

The second exponential in equation (15) carries the contribution due to GUP. Assuming for the moment that $\alpha\zeta^k \ll 1$, a condition that will be clarified and verified once a physical meaning will be associated with α, we can approximate equation (15) writing

$$\bar{Z}_n^k = B^k \, \mathrm{e}^{-\alpha\hbar\omega^k n} \left[1 - \alpha\zeta^k n(n+1) \right]. \tag{17}$$

It is worth noticing that, by definition, $\bar{Z}_n^k$ is a non-negative quantity. However, the minus sign implies that, for some values of n larger than some value $n_{\max}$, we have $\bar{Z}_n^k < 0$. Such values of n are therefore not physical within the approximation and cannot be considered. The maximum value for n for each mode k is then found as

$$n_{\max}^k = \left\lfloor \left(\frac{1}{\alpha\zeta^k} + \frac{1}{4} \right)^{1/2} - \frac{1}{2} \right\rfloor. \tag{18}$$

In turn, such a maximum value for n^k corresponds to a maximum value for the energy

$$E_{\max}^k = \hbar\omega^k + \zeta^k \left\{ \left(\frac{1}{\alpha\zeta^k} + \frac{1}{4} \right)^{1/2} + \frac{1}{2} \right\}. \tag{19}$$

Having the most probable distribution equation (17), we can proceed following Bose's approach [41]. We calculate equations (9) and (10) using the geometric series and its derivatives. With the further condition $\gamma_{\mathrm{EM}}/\sqrt{\alpha} \ll 1$, we can express the sums in equation (9) as ranging from $n = 0$ up to infinity instead of up to n^k_{max}. Specifically, the difference between the two sums, that ranging up to infinity and that ranging up to n^k_{max}, is negligible under the assumption $\gamma_{\mathrm{EM}}/\sqrt{\alpha} \ll 1$, to be clarified once a physical meaning will be assigned to α. Thus, for the mean occupation number in the mode k, we obtain the following expression

$$\begin{aligned} A^k &= \sum_n \bar{Z}^k_n \\ &= \frac{B^k}{(1-\mathrm{e}^{-\alpha\hbar\omega^k})^3}\left[(1-\mathrm{e}^{-\alpha\hbar\omega^k})^2 - 2\alpha\zeta^k\,\mathrm{e}^{-\alpha\hbar\omega^k}\right], \end{aligned} \tag{20}$$

which can be solved for B^k. Then, substituting in equation (9), we find

$$\begin{aligned} N^k &= \sum_n n\bar{Z}^k_n \\ &= \frac{A^k}{\mathrm{e}^{\alpha\hbar\omega^k}-1}\left\{\frac{(1-\mathrm{e}^{-\alpha\hbar\omega^k})^2 - 2\alpha\zeta^k(1+2\,\mathrm{e}^{-\alpha\hbar\omega^k})}{(1-\mathrm{e}^{-\alpha\hbar\omega^k})^2 - 2\alpha\zeta^k\,\mathrm{e}^{-\alpha\hbar\omega^k}}\right\}, \end{aligned} \tag{21}$$

$$\begin{aligned} M^k &= \sum_n n^2\bar{Z}^k_n \\ &= \frac{A^k\,\mathrm{e}^{-\alpha\hbar\omega^k}}{(1-\mathrm{e}^{-\alpha\hbar\omega^k})^2} \\ &\quad\times\left\{\frac{(1-\mathrm{e}^{-\alpha\hbar\omega^k})^2(1+\mathrm{e}^{-\alpha\hbar\omega^k}) - 2\alpha\zeta^k(1+7\,\mathrm{e}^{-\alpha\hbar\omega^k}+4\,\mathrm{e}^{-2\alpha\hbar\omega^k})}{(1-\mathrm{e}^{-\alpha\hbar\omega^k})^2 - 2\alpha\zeta^k\,\mathrm{e}^{-\alpha\hbar\omega^k}}\right\}. \end{aligned} \tag{22}$$

Following standard arguments, the density of modes in a cavity around a wave number of magnitude k and per unit of volume can be found by counting the wavelengths admissible in the cavity [44]. Such a procedure is not affected by a minimal length and the density of modes per unit volume is then given by the ordinary Rayleigh–Jeans expression

$$\mathcal{Q}(k)\mathrm{d}k = \frac{k^2}{\pi^2}\,\mathrm{d}k. \tag{23}$$

To find the same density but as a function of the frequency ω^k, we need to employ a dispersion relation. In general, quantum gravity theories predict modifications of dispersion relations. However, studies have showed that such deformations may be relevant only at scales comparable or higher than the Planck scale [45, 46]. Since the framework of the present analysis is based on quantities with values much below the Planck scale, as it will be further clarified below, we can safely neglect the effects of a modification of the dispersion relation for electromagnetic waves, assuming $\omega^k = ck$. Therefore, the number of photons in a frequency range $[\omega^k, \omega^k + \mathrm{d}\omega^k]$ can be written as follows

$$\mathcal{R}(\omega)\mathrm{d}\omega^k = \frac{(\omega^k)^2}{\pi^2 c^3}\,\mathrm{d}\omega^k. \tag{24}$$

Following the approach in [41], the total number of fundamental cells in phase space is equal to the number of possible ways of placing a photon in the relevant volume. Thus, integrating equation (24) over the volume of the cavity V, we find that the number of cells allowed in an infinitesimal interval of frequencies around ω^k is

$$A^k = \frac{V(\omega^k)^2}{\pi^2 c^3}\,\mathrm{d}\omega^k. \tag{25}$$

Let us now define entropy S using the probability function W from equation (12), according to Boltzmann entropy definition. We then find

$$\begin{aligned} S &= k_\mathrm{B}\ \log\ W \\ &= k_\mathrm{B}\left\{\alpha E - \sum_k 3A^k\ \log(1-\mathrm{e}^{-\alpha\hbar\omega^k}) - A^k\ \log\left[(1-\mathrm{e}^{-\alpha\hbar\omega^k})^2 - 2\alpha\zeta^k\,\mathrm{e}^{-\alpha\hbar\omega^k}\right]\right\}, \end{aligned} \tag{26}$$

where k_B is the Boltzmann constant. To find equation (26), we have used equations (8) and (10). Thus, considering the relation between entropy and temperature, $\frac{\partial S}{\partial E} = 1/T$, we get that the Lagrange multiplier α acquires the value $1/k_\mathrm{B}T$. Now, we can indeed see that the conditions $\alpha\zeta^k \ll 1$, that is $\hbar\omega^k/k_\mathrm{B}T \ll E_\mathrm{Pl}/\hbar\omega^k$, and $\gamma/\sqrt{\alpha} \ll 1$, that is $k_\mathrm{B}T/E_\mathrm{Pl} \ll 1$, are valid for values of frequency and temperature away from the Planck scale. Therefore, the results obtained via approximations that relied on such constraints are valid under the same conditions for the frequency and temperature.

From equation equation (8), and using equations (21) and (22), we can obtain the total energy

$$E = \sum_k\left[\hbar\omega^k N^k + \zeta^k(N^k + M^k)\right] = V\int_0^\infty \rho_\mathrm{T}(\omega)\mathrm{d}\omega, \tag{27}$$

where $\rho_\mathrm{T}(\omega)$ is the energy density. Since we want to study the effects of a minimal length in the black body radiation, the following transformation is introduced $\rho_\mathrm{T}(\lambda) = -\rho_\mathrm{T}(\omega)\frac{\mathrm{d}\omega}{\mathrm{d}\lambda}$. Thus, the energy density as a function of the wavelength λ is

$$\begin{aligned} \rho_\mathrm{T}(\lambda) = \frac{8\pi hc}{\lambda^5}&\left\{\frac{1+\frac{hc}{2\lambda}\gamma_\mathrm{EM}^2}{\mathrm{e}^{\frac{hc}{\lambda k_\mathrm{B}T}}-1}\left[\frac{\left(\mathrm{e}^{\frac{hc}{\lambda k_\mathrm{B}T}}-1\right)^2 - \frac{(hc\gamma_\mathrm{EM})^2}{\lambda^2 k_\mathrm{B}T}\,\mathrm{e}^{\frac{hc}{\lambda k_\mathrm{B}T}}\left(2+\mathrm{e}^{\frac{hc}{\lambda k_\mathrm{B}T}}\right)}{\left(\mathrm{e}^{\frac{hc}{\lambda k_\mathrm{B}T}}-1\right)^2 - \frac{(hc\gamma_\mathrm{EM})^2}{\lambda^2 k_\mathrm{B}T}\,\mathrm{e}^{\frac{hc}{\lambda k_\mathrm{B}T}}}\right]\right. \\ &\left. + \frac{\frac{hc}{2\lambda}\gamma_\mathrm{EM}^2\,\mathrm{e}^{\frac{hc}{\lambda k_\mathrm{B}T}}}{\left(\mathrm{e}^{\frac{hc}{\lambda k_\mathrm{B}T}}-1\right)^2}\left[\frac{\left(\mathrm{e}^{\frac{hc}{\lambda k_\mathrm{B}T}}-1\right)^2\left(\mathrm{e}^{\frac{hc}{\lambda k_\mathrm{B}T}}+1\right) - \frac{(hc\gamma_\mathrm{EM})^2}{\lambda^2 k_\mathrm{B}T}\,\mathrm{e}^{\frac{hc}{\lambda k_\mathrm{B}T}}\left(4+7\,\mathrm{e}^{\frac{hc}{\lambda k_\mathrm{B}T}}+\mathrm{e}^{\frac{2hc}{\lambda k_\mathrm{B}T}}\right)}{\left(\mathrm{e}^{\frac{hc}{\lambda k_\mathrm{B}T}}-1\right)^2 - \frac{(hc\gamma_\mathrm{EM})^2}{\lambda^2 k_\mathrm{B}T}\,\mathrm{e}^{\frac{hc}{\lambda k_\mathrm{B}T}}}\right]\right\}. \end{aligned} \tag{28}$$

It is worth noting that in the limit $\gamma_\mathrm{EM} \to 0$, the extra terms that carry the modification on the energy density vanish. In such a limit, $\rho_\mathrm{T}(\lambda)$ is reduced consistently to the usual expression

$$\rho_{0\mathrm{T}}(\lambda) = \frac{8\pi\hbar c}{\lambda^5}\frac{1}{e^{\frac{hc}{\lambda k_\mathrm{B}T}}-1}. \tag{29}$$

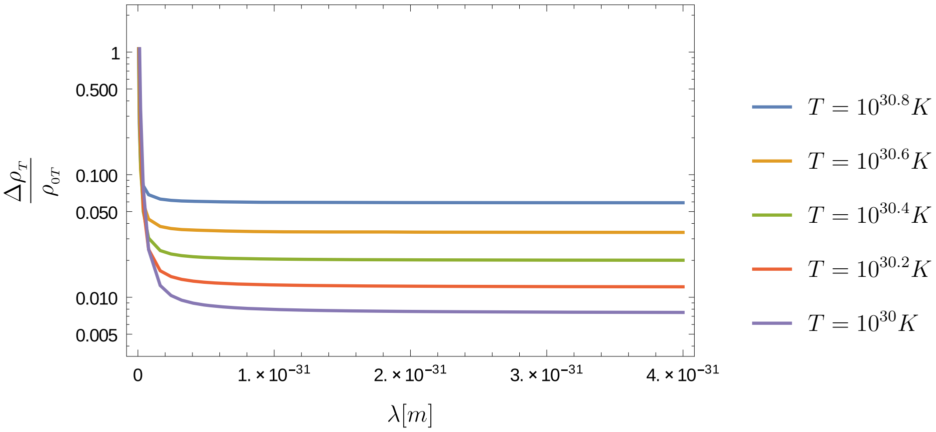

Figure 1. Relative modification of the energy density $\frac{\Delta\rho_\mathrm{T}}{\rho_{0\mathrm{T}}}$ for different temperatures. For higher temperatures, the difference between the standard energy density and the modified one with GUP is larger.

We can study the modification in the energy density by plotting the relative modification

$$\frac{\Delta\rho_\mathrm{T}}{\rho_{0\mathrm{T}}} = \frac{\rho_{0\mathrm{T}} - \rho_\mathrm{T}}{\rho_{0\mathrm{T}}}. \tag{30}$$

In figure 1, we show the ratio for different temperatures. As it can be observed, the larger difference takes place at larger temperatures.

4. Wien's law

In the black body distribution, Wien's law establishes a relation between a given temperature and the wavelength of the maximum of the distribution. In particular, the higher the temperature of the black body, the lower the wavelength of the maximum. As we are including a minimal length we expect a modification of this law at high temperatures. Wien's Law can be deduced by finding the maximum of the distribution in equation (28). In the standard theory, that is, in the limit $\gamma_\mathrm{EM} \to 0$, this procedure leads to a constant quantity $x = \frac{hc}{\lambda_0 k_\mathrm{B} T} = 5 + W(0, -5\,\mathrm{e}^{-5})$, where $W(z)$ is the Lambert W function. In the present case, we consider the approximated expression

$$x = \frac{hc}{\lambda_0 k_\mathrm{B} T}\left(1 - \frac{\delta\lambda}{\lambda_0}\right), \tag{31}$$

where λ_0 is the wavelength that satisfies Wien's law in the standard cases and $\delta\lambda$ is the shift on the wavelength of the maximum due to GUP.

In order to simplify the expression, we consider an expansion up to the first order in $k_\mathrm{B} T \gamma_\mathrm{EM}^2$. Such approximation is justified for temperatures much smaller than the Planck temperature. By differentiating equation (28) with respect to λ and imposing the maximum condition, we get

$$k_\mathrm{B} T \gamma_\mathrm{EM}^2 x\,\mathrm{e}^x \left[x^2(\mathrm{e}^{2x} + 4\,\mathrm{e}^x + 1) - 8x(\mathrm{e}^{2x} - 1) + 6(\mathrm{e}^x - 1)^2\right] - x\,\mathrm{e}^x(\mathrm{e}^x - 1)^2 + 5(\mathrm{e}^x - 1)^3 = 0. \tag{32}$$

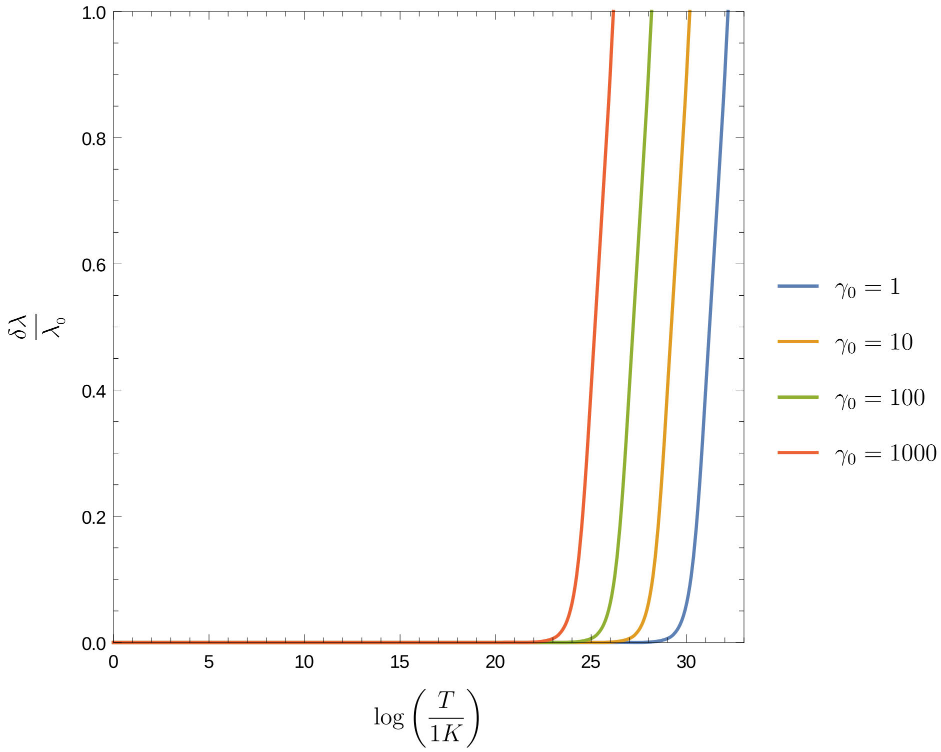

Figure 2. Relative shift $\frac{\delta\lambda}{\lambda_0}$ as a function of the temperature for different values of the parameter γ_0.

Equation (32) can be solved numerically for $\frac{\delta\lambda}{\lambda_0}$ with the condition $\frac{\delta\lambda}{\lambda_0} < 1$. In figure 2, we show the temperature dependence of relative shift for the wavelength of the maximum for different values of γ_0. We observe that the modification grows with the temperature reaching the value $\frac{\delta\lambda}{\lambda_0} = 1$. For such a value and beyond, the approximation in equation (31) cannot be considered valid. Specifically, for $\gamma_0 = 1$, the approximation breaks close to the Planck temperature. Consistently with the approximation, the modification $\delta\lambda$ goes to 0 for much smaller temperatures.

5. Stefan–Boltzmann law

The Stefan–Boltzmann law describes the total power radiated by a cavity with volume *V* at an absolute temperature *T*. More precisely, the law establishes that the radiance of a black body, that is the amount of energy radiated per unit of surface is proportional to the fourth power of the absolute temperature

$$R_\mathrm{T} = \sigma T^4, \tag{33}$$

where the proportionality constant, called the Stefan–Boltzmann constant, is $\sigma = \frac{2\pi^5 k_\mathrm{B}^4}{15h^3c^2}$. This law can be derived from the total energy emitted by the black body by integrating equation (28) and using the relation between spectral radiance and the energy density $R_\mathrm{T}(\lambda)\mathrm{d}\lambda = \frac{c}{4}\rho_\mathrm{T}(\lambda)\mathrm{d}\lambda$. For simplicity, we expand equation (28) up to the first order in $k_\mathrm{B}T\gamma_\mathrm{EM}^2$. Such approximation

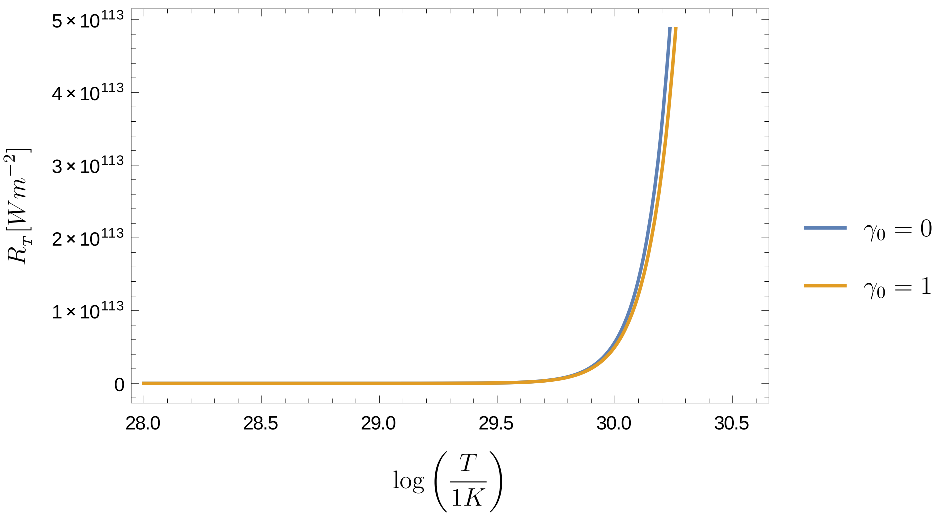

Figure 3. Plot of the radiance of a black body R_{T}. The solid blue line represents the Stefan–Boltzmann law in the ordinary theory. The solid orange line corresponds to the modified law in equation (35).

is justified for small values of the temperature compared to the Planck temperature. The total energy per unit volume is then

$$\begin{aligned} R_{\mathrm{T}} &= \frac{2\pi^5 k_{\mathrm{B}}^4}{15h^3c^2}T^4 - \frac{2\pi k_{\mathrm{B}}^4}{h^3c^2}(k_{\mathrm{B}}T\gamma_{\mathrm{EM}}^2)T^4\int_0^\infty \frac{x^4\,\mathrm{e}^x(1+x-\mathrm{e}^x+x\,\mathrm{e}^x)}{(\mathrm{e}^x-1)^3}\,\mathrm{d}x \\ &= \frac{2\pi^5 k_{\mathrm{B}}^4}{15h^3c^2}T^4(1-16k_{\mathrm{B}}T\gamma_{\mathrm{EM}}^2). \end{aligned} \tag{34}$$

By substituting the value of the Stefan–Boltzmann constant, we obtain the following equation

$$R_{\mathrm{T}} = \sigma T^4(1-16k_{\mathrm{B}}T\gamma_{\mathrm{EM}}^2). \tag{35}$$

This is the first-order modification of the Stefan–Boltzmann law by including a minimal measurable length. Due to the existence of a minimal length, and therefore a minimal wavelength, modes corresponding to shorter wavelengths are expected to not contribute to the spectrum. Thus, the energy radiated by a blackbody considering GUP is smaller than the standard value, in agreement with equation (35). We notice that in the limit $\gamma_{\mathrm{EM}} \to 0$, we recover the usual Stefan–Boltzmann law for the black body equation (33).

In figure 3, we plot the radiance for both the GUP modification and the ordinary case.

6. Conclusions

Statistical mechanics, as well as thermodynamics, may offer indirect evidence of quantum gravity effects related to a minimal measurable length. The effects of such a length modifies the potentials as well as the laws established in both disciplines. However, the energy at which such effects become relevant is still outside the range offered by current experiments.

In this paper, we analyzed the effects of a minimal measurable length on the black body spectrum. To do so, we have considered a quantization procedure for the electromagnetic field inspired by the GUP. One of the effects of such a procedure is that of modifying the dependence of the quanta of energy on the frequency. Using Bose's approach [41], we obtained the Planck distribution that matches with the standard expression in the limit $\gamma_{\mathrm{EM}} \to 0$. The modified energy density at any given temperature results to be smaller than the energy density in the standard theory for the same temperature excluding values close to Planck length. This is consistent with known features of models of quantum mechanics with a minimal length. Specifically, first we notice that GUP changes the de Broglie relation between the (physical) momentum and the wave number of a system [25]. In particular, the model in equation (1) implies that larger wave numbers, and therefore shorter wavelengths, are associated with a value of momentum, and therefore of energy, larger than in the ordinary case. Therefore, since short wavelengths are characterized by higher values of energy than in the standard case, they also result harder to excite, contributing less to the energy distribution. Elaborating on Wien's law using the modified energy density, we found that GUP effects shift the maximum of the distribution. We observed that such a modification depends on the temperature. For much smaller temperatures compared to the Planck temperature, the modification goes to zero, consistently with the approximation. The modified Stefan–Boltzmann law was obtained by integrating the spectral radiance related to the modified energy density. The results suggest that the total energy radiated is lower than in the ordinary case at high temperatures. Such an effect is compatible with results obtained in DSR for a photon gas [47]. For both modifications, the effects of a minimal measurable length are temperature-dependent.

The importance of the black body radiation lies in its applications, in thermodynamics as well as other contexts. For example, the adsorption and emission of black holes make them similar to a black body [48]. However, black holes cannot absorb wavelengths longer than their size [49]. Furthermore, *gedanken* experiments in black hole thermodynamics consider black holes with the size of the order of Planck length. For such systems, the Hawking temperature is of the order of Planck temperature. At such a temperature, as we have seen in the paper, the Planck distribution is expected to be modified by quantum gravitational effects. Thus, the modification considered here may have a role in the thermodynamics of Planckian black holes. Furthermore, the study of black body radiation applies to cosmology as well [50]. The microwave background radiation is observed to be an almost perfect black body with a temperature of 2.7 K [51]. Thus, a modification in the Planck distribution due to the GUP can play a role in obtaining new information in the early stage of the universe by considering quantum gravity corrections.

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Data availability statement

No new data were created or analysed in this study.

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