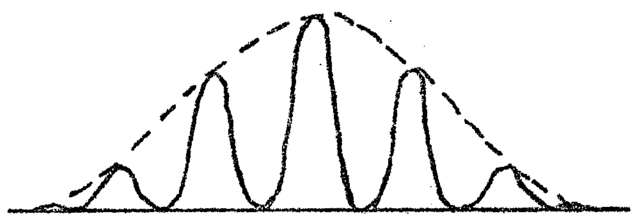
INTERFERENCE METHOD FOR OPTICAL ALIGNMENT

by Dr. Kenneth R. Trigger

Van Heel[1] suggests a straightforward method using an optical interference pattern to obtain an accuracy of 0.06 seconds of arc. This is achieved by observing the color transitions in the pattern formed by a white-light source. It is shown in Figure 1 that two square "slits" of dimensions w, separated a distance d, will inherently yield greater accuracy than one slit of width d.



d = 3w

FIGURE 1

The length of the path used in Van Heel's work was limited primarily because of the path in air and the intensity of the source. It appears possible to overcome both of these in a permanent installation at reasonable cost when compared to other alignment systems. The initial installation of a two-mile long, one-foot diameter pipe and necessary pumps to get a pressure between 10^{-1} and 1 torr will probably be 50% of the cost of the total system. Intense light sources are available. For example, the Osram long-arc, xenon-mercury lamp has a light output of one-kilowatt in the visible region with about six-kilowatts input.

Some improvement in the "coherent" light output can be made by enclosing the source in a spherical cavity with a small radiating hole. The power radiated, P_r, is:

$$P_r = \frac{Q_c}{Q_h} P_o$$

where P_o is the power input, Q_c is the total Q of the cavity, and Q_h is the contribution of the radiating hole. Since $Q_c \ll Q_h$

$$\frac{Q_c}{Q_h} = \frac{\text{area of radiation hole}}{\text{surface area of cavity} \times (1-r)} \tag{2}$$

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[1] Van Heel, A.C.S., PROGRESS IN OPTICS, I (1961), pp. 291-329, (Interscience)

with r being the reflectivity of the surface, neglecting other losses. The maximum radiation hole radius is,[2] for $d = 3w$,

$$\rho \lesssim \frac{f\lambda}{32w}$$

where f is the focal length of the condensing lens shown in Figure 2. The parameters ρ , f , and d are correlated to minimize the scattered light. Then

$$P_r \approx \frac{0.9}{4} \frac{\left(\frac{f\lambda}{32w}\right)^2}{R^2(1-r)} P_o$$

since about 90% of the power is transmitted within the first minima of the diffraction pattern, the incident intensity at the apertures is

$$I_i \approx \frac{P_r}{\pi(2w)^2}$$

The useful transmitted intensity at the telescope is, for $(L-z)\lambda/w = a$

$$I_t \approx 0.9 \times 4 I_i \left(\frac{w}{2a}\right)^2 = \frac{0.9 P_r}{4\pi}\left[\frac{w}{(L-z)\lambda}\right]^2$$

3) $$I_t \approx \frac{1.5 \times 10^{-5}}{1-r}\left[\frac{f}{(L-z)R}\right]^2 P_o$$

As may be noted in the above discussion, ρ is optimized for each value of $(L-z)$. With proper use of the telescope, ρ does not have to vary continuously. If $f = 200$ cm, $P_o = 1$ kw, R (radius of cavity) $= 7.5$ cm, and $r = 0.9$, then

$$I_t \approx \frac{10^2}{(L-z)^2}$$

Thus, at a distance of two miles, $I_t \approx 10^{-9}$ watts/cm.^{2}

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[2] Jenkins and White, FUNDAMENTALS OF PHYSICAL OPTICS (1937) p. 141 (McGraw - Hill)

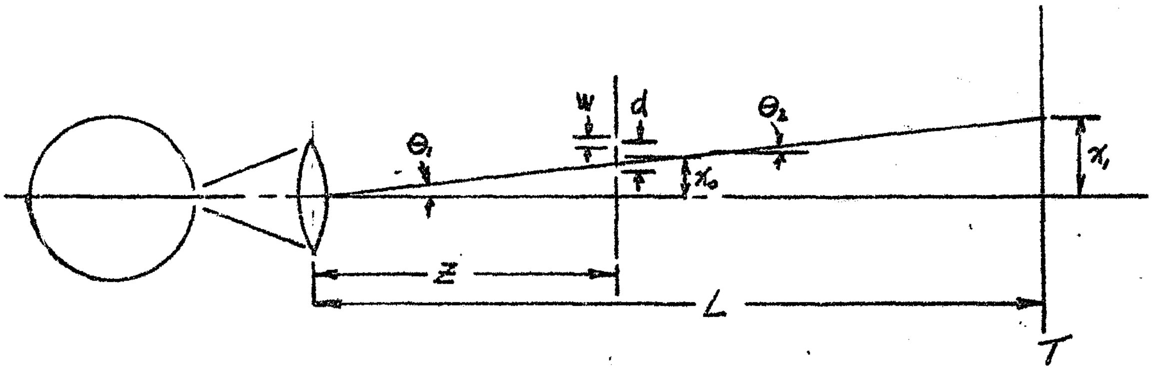

FIGURE 2

The sensitivity of the normal human eye is adequate to detect considerably less. For example, α Centauri, one of the brightest stars as viewed by an observer on earth, provides an intensity of 3×10^{-13} watts/cm^2. From the above discussion it is seen that in the worst case a factor of more than 3,000 can be obtained for accelerator alignment. Increasing the available intensity by an order of magnitude should not be difficult. With a source and detector at each end the maximum value of $(L - z)$ can be halved gaining a factor of four. The limit on the reflectivity is 0.97, a factor of three more. Of course, a good telescope will also help.

Since it appears possible to construct an adequate system, some further geometrical factors will be considered. The intensity distribution at the telescope, T, in Figure 2 is given by:

$$4)\quad I = I_0 \frac{\sin^2 \alpha}{\alpha^2} \cos^2 \beta$$

$$\alpha = \frac{\pi w}{\lambda}\left(\sin\theta_1 - \sin\theta_2\right)$$

$$\beta = \frac{\pi d}{\lambda}\left(\sin\theta_1 - \sin\theta_2\right)$$

$$\sin\theta_1 = \frac{x_0}{z}$$

$$\sin\theta_2 = \frac{x_1 - x_0}{L - z}$$

Applying this to accelerator conditions, for the position of maximum intensity at T,

$$5)\quad x_0 = \frac{z}{L} x_{10}$$

and the distance between monochromatic minima positions is:

$$\Delta x_{11} = (L - z)\frac{\lambda}{d}$$

The accuracy in determining x_{10} is

$$\delta x_{10} = e(L - z)\frac{\lambda}{d}$$

where e is the ratio of the accuracy with which one can obtain the distance between fringes divided by the distance between fringes. The corresponding accuracy in x_0 is:

$$6)\quad \delta x_0 = e\,\frac{z(L-z)}{L}\,\frac{\lambda}{d}$$

Since Δx_{11} should not be too large for telescopic viewing, let Δx_{11} = 1 cm., then:

$$\delta x_{10} = e\ cm. \quad \text{and} \quad \delta x_0 = \frac{z}{L} e\ cm.$$

Thus, the error in δx_0 is e cm. so that it appears reasonable to align the accelerator to greater accuracy than ±0.1 cm by optical means leaving adequate margin for fabrication and measurement inaccuracies. It is to be noted that Van Heel has been able to achieve e ≈ 1/1200 by using the color contrast.

There is one advantage in the optical technique that should not be overlooked. This is the comparative simplicity in processing the data to provide information for realignment. With the use of Equation 5), direct conversion to horizontal and vertical position is obtained. Although this data could be put into a computer to find the center of the distribution, a presentation such as Figure 3 will generally suffice to determine the center by inspection. Placing an interference diaphragm into position in the light tube, locating the position, recording and plotting the data, and removing the diaphragm should be a reasonably fast operation. With a diaphragm every forty feet, the total time required to accumulate the data would be less than five or six hours. With the information in the form of Figure 3, realignment could begin immediately.

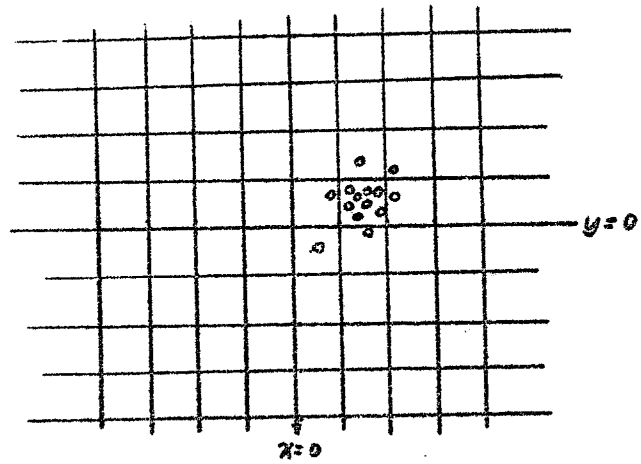

FIGURE 3

Another potential advantage lies in the possibility of incorporating the vacuum pipe into the accelerator support structure. It appears that a 12" aluminum pipe has sufficient torsional rigidity in itself to satisfy the total torsional requirement.[3] The stiffness in bending would have to be increased by a factor of 8 by a suitable truss.

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[3] J. Beverly Whitehouse, Private Conversation.

[4] I would like to thank R. Helm for many helpful discussions.