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https://doi.org/10.3390/info17020169

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Entropy, Information, and the Curvature of Spacetime in the Informational Second Law

Florian Neukart [1,2,*], Eike Marx [2] and Valerii Vinokur [2,†]

[1] Leiden Institute of Advanced Computer Science, Leiden University, Gorlaeus Gebouw-BE-Vleugel, Einsteinweg 55, 2333 Leiden, The Netherlands
[2] Terra Quantum AG, Kornhausstrasse 25, 9000 St. Gallen, Switzerland; eike@terraquantum.swiss (E.M.)
* Correspondence: f.neukart@liacs.leidenuniv.nl
† Deceased author.

Abstract

We develop an informational extension of spacetime thermodynamics in which local entropy production is coupled to spacetime curvature within an effective covariant framework. Spacetime is modeled as a continuum limit of finite-capacity information registers, giving rise to a coarse-grained entropy field whose gradients define an informational flux. Within a nonminimally coupled scalar–tensor formulation, the resulting field equations imply that the local divergence of this flux is sourced by the Ricci scalar, establishing a direct relation between curvature and entropy production. The corresponding integral form links cumulative entropy generation to the integrated spacetime curvature over a causal region. In stationary limits, the framework reproduces the Bekenstein–Hawking entropy of horizons, while in homogeneous expanding cosmologies it yields monotonic entropy growth consistent with the observed arrow of time. The construction remains compatible with unitarity at the microscopic level and with holographic entropy bounds in the stationary limit. Numerical solutions in flat FLRW backgrounds are used as consistency checks of the coupled evolution equations and confirm the expected curvature–entropy behavior across cosmological epochs. Overall, the results provide a thermodynamically consistent interpretation of curvature as a geometric source of irreversible information flow, without modifying the underlying gravitational field equations.

Keywords: spacetime thermodynamics; entropy production; curvature–entropy coupling; scalar–tensor theories; black-hole thermodynamics; FLRW cosmology; holography

Academic Editor: Gordana Dodig-Crnkovic

Received: 31 December 2025
Revised: 30 January 2026
Accepted: 3 February 2026
Published: 6 February 2026

1. Introduction

The relationship between thermodynamics and gravitation has long played a central role in theoretical physics. The second law of thermodynamics, which asserts that entropy tends to increase in closed systems, underlies the macroscopic arrow of time and constrains irreversible processes [1]. General relativity, by contrast, encodes the dynamics of spacetime geometry through its coupling to energy and momentum [2]. Over the past decades, these two domains have increasingly converged toward a common informational perspective, in which entropy, geometry, and dynamics are deeply intertwined [3,4].

Early indications of this connection emerged from black-hole physics. The Bekenstein–Hawking area law established that black holes possess an entropy proportional to their horizon area, $S_{\rm BH} = A/4G\hbar$, revealing an intrinsic informational aspect of spacetime geometry [5,6]. Jacobson later demonstrated that Einstein's field equations can be recovered from the Clausius relation $\delta Q = TdS$ applied to local Rindler horizons, suggesting that gravitational dynamics admit a thermodynamic interpretation [7]. Subsequent developments, notably by Padmanabhan and others, further explored formulations in which spacetime dynamics emerge from extremization principles applied to entropy functionals [8]. Collectively, these results suggest that thermodynamics and geometry are not merely analogous, but may reflect complementary descriptions of an underlying informational structure.

Within this broader context, the Quantum Memory Matrix (QMM) framework provides a concrete setting in which spacetime is modeled as an array of finite-dimensional Hilbert spaces that locally store quantum information [9,10]. Each spacetime cell evolves through reversible imprint and retrieval dynamics, preserving global unitarity while permitting coarse-grained entropy growth at the macroscopic level. Prior applications of this framework have shown that gradients in the informational field can generate effects typically attributed to dark matter, that residual informational imprints behave as an effective dark energy component, and that cyclic cosmologies can be characterized by discrete entropy increments per cycle [11–13].

Motivated by these developments, we study here an effective covariant relation between entropy production and spacetime curvature. We introduce an informational scalar field S and define an associated flux four-vector $J^\mu = \nabla^\mu S$. In the model considered, a nonminimal coupling between S and the Ricci scalar leads to field equations in which the divergence of the informational flux is proportional to the local curvature,

$$\nabla_\mu J^\mu = \kappa R, \tag{1}$$

where κ is a coupling constant with units of entropy per curvature. Rather than introducing a new fundamental law, this relation is treated as an effective consequence of the underlying action, providing a covariant realization of curvature-sourced entropy production. Integrating this identity over a causal four-volume yields

$$\Delta S = \kappa \int R \, dV, \tag{2}$$

which connects the accumulation of coarse-grained entropy to the integrated spacetime curvature. The resulting dynamics are compatible with unitarity and holographic entropy bounds and do not require modifications of Einstein's equations.

In the following sections, we analyze the formal structure of this curvature-sourced entropy production mechanism, examine its consistency with established thermodynamic and gravitational principles, and explore its implications for stationary horizons, homogeneous cosmologies, and the emergence of a cosmological arrow of time. The formulation extends the thermodynamic interpretation of spacetime by explicitly incorporating information flow as a dynamical quantity, while remaining within a conservative effective-field-theory framework.

2. Foundations

2.1. Quantum Memory Matrix Recap

The Quantum Memory Matrix (QMM) framework models spacetime as a discretized information substrate composed of Planck-scale cells that function as local quantum memory units. Each cell n is associated with a finite-dimensional Hilbert space $\mathcal{H}_n$ of dimension d_{max}, constrained by holographic entropy bounds and the covariant entropy conjecture. The global Hilbert space of a causal region X is represented as

$$\mathcal{H}_{\text{QMM}} = \bigotimes_{n \in X} \mathcal{H}_n, \tag{3}$$

where X denotes the set of cells contained within the region. The dynamical state of each cell evolves through an imprint operator $\hat{I}_n$ that locally encodes quantum information associated with matter and field configurations,

$$\hat{I}_n = \mathcal{F}[\hat{\phi}(x_n), \nabla_\mu \hat{\phi}(x_n), \ldots], \tag{4}$$

Ensuring that microscopic interactions leave localized informational records within the memory structure [9].

Coarse-graining over ensembles of such cells gives rise to a continuous entropy field $S(x)$,

$$S(x) = \mathrm{Tr}_{\mathcal{H}_n}[\rho_n \ln \rho_n^{-1}], \tag{5}$$

where ρ_n denotes the reduced density matrix associated with cell n. Gradients of $S(x)$ characterize the spatial and temporal distribution of coarse-grained informational content and provide a link between microscopic information storage and macroscopic geometric behavior. Within the associated Geometry–Information Duality (GID) formulation [10,14], curvature can be represented in terms of informational gradients through relations of the schematic form

$$G_{\mu\nu} \propto \nabla_\mu S \nabla_\nu S - \frac{1}{2} g_{\mu\nu} (\nabla S)^2, \tag{6}$$

Illustrating how gravitational dynamics may be encoded, at an effective level, by the flow and accumulation of information in the underlying QMM structure.

Recent studies within this framework have explored how informational gradients and residual imprints can reproduce phenomenology commonly attributed to dark matter and dark energy, and how cyclic cosmological behavior can be parameterized in terms of discrete entropy increments per cycle [11–13,15]. Additional extensions have examined how electromagnetic, weak, and strong interactions may be embedded within the same discretized informational substrate. In the present work, the QMM framework serves as a motivating context for the entropy-based constructions developed below, rather than as a required assumption for their validity.

2.2. Informational Flux and Continuity

The coarse-grained entropy field $S(x)$ naturally defines an informational flux four-vector

$$J^\mu = \nabla^\mu S, \tag{7}$$

which quantifies the local transport of informational entropy through spacetime. The covariant divergence of this flux,

$$\nabla_\mu J^\mu = \Box S, \tag{8}$$

with $\Box = \nabla_\mu \nabla^\mu$ denoting the covariant d'Alembertian operator, characterizes the local rate of entropy production or absorption.

In flat spacetime, where the Ricci scalar vanishes, $\nabla_\mu J^\mu = 0$ reduces to the covariant continuity equation

$$\partial_\mu \partial^\mu S = 0, \tag{9}$$

Corresponding to conservation of informational flux. This behavior parallels energy–momentum conservation, $\partial_\mu T^{\mu\nu} = 0$, and reflects the invariance of total information in the absence of spacetime curvature.

In curved spacetime, however, $\nabla_\mu J^\mu$ need not vanish, indicating that the transport of information is influenced by geometric effects. Curvature can therefore act as a source or sink for coarse-grained entropy, linking irreversible informational processes to spacetime dynamics. This observation motivates the curvature-sourced entropy production relation examined in the following subsection.

2.3. Curvature–Entropy Coupling

We consider an effective covariant relation, valid at the level of coarse-grained spacetime thermodynamics, in which the divergence of the informational flux is proportional to the Ricci scalar curvature,

$$\nabla_\mu J^\mu = \kappa R, \tag{10}$$

where κ is a phenomenological coupling constant. In this formulation, the relation is treated as an effective closure for the entropy balance, rather than as a microscopic identity. Spacetime curvature thus acts as a geometric source term for coarse-grained entropy production. Regions of positive curvature correspond to entropy generation, while regions of negative curvature correspond to entropy release or information recovery, as illustrated in Figure 1. Integrating this relation over a causal four-volume $\mathcal{V}$ yields the integral identity

$$\Delta S_{\mathrm{imprint}} = \kappa \int_{\mathcal{V}} R\, dV = \kappa \oint_{\partial\mathcal{V}} J^\mu d\Sigma_\mu, \tag{11}$$

which relates the total entropy change within a region to the integrated spacetime curvature. When $R = 0$, entropy is conserved; when $R > 0$, entropy increases monotonically, providing a covariant realization of curvature-sourced entropy production.

Dimensional analysis with $J^\mu = \nabla^\mu S$ and S carrying entropy units implies that κ has units of entropy. Its numerical normalization is fixed by matching the flux integral to the Bekenstein–Hawking area density on stationary horizons, as shown in Appendix F, linking the effective coupling to Planck-scale physics without modifying the Einstein field equations. In this sense, Einstein's equations remain intact but admit a complementary thermodynamic interpretation in which curvature tracks the redistribution of coarse-grained information.

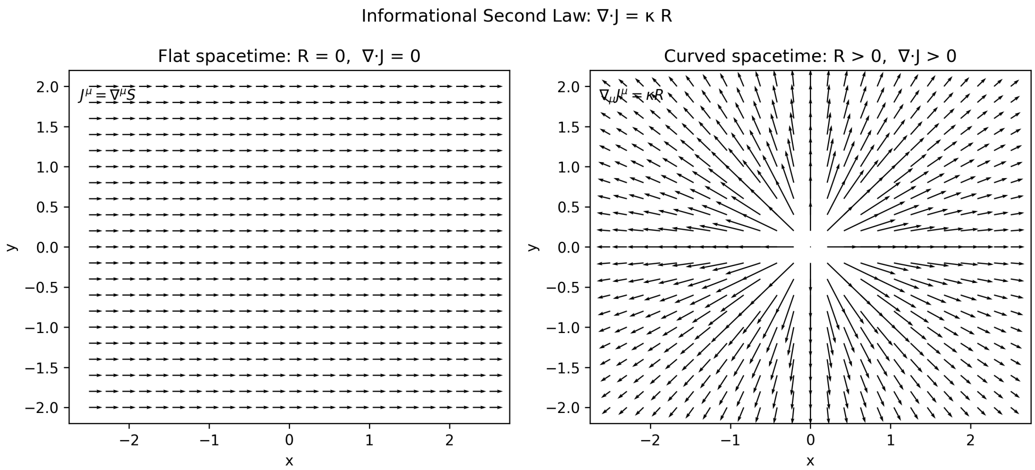

Figure 1. Curvature–entropy coupling. (**Left**): Flat spacetime with Ricci scalar $R = 0$, where the informational flux $J^\mu = \nabla^\mu S$ is uniform and divergence-free, $\nabla_\mu J^\mu = 0$. (**Right**): Curved spacetime with $R > 0$, where the informational flux exhibits radial outflow with positive divergence $\nabla_\mu J^\mu = \kappa R > 0$, corresponding to local coarse-grained entropy production. Arrows indicate the direction and relative magnitude of the informational flux J^μ.

3. Derivation from an Informational Action

Before introducing the action, we clarify the theoretical setting of the model. The formulation below constitutes a nonminimally coupled scalar–tensor effective theory in which the coarse-grained entropy field $S(x)$ plays the role of a scalar degree of freedom coupled to curvature. The gravitational sector itself is not modified in form: the Einstein–Hilbert term remains standard, and general relativity is recovered in the limit where the informational coupling κS is small. The purpose of the scalar field is not to introduce an alternative theory of gravity but to encode irreversible information deposition within a covariant variational framework. All field equations are derived from the action below, and relations involving curvature are always to be understood as consequences of these equations rather than as independent assumptions. We introduce a covariant effective action for a coarse-grained entropy field $S(x)$ coupled to curvature,

$$S_{\rm inf} = \int d^4x\,\sqrt{-g}\left[\frac{1}{16\pi G}R + \frac{\lambda}{2}\nabla_\mu S\nabla^\mu S + \kappa\,RS\right] + S_{\rm matter}[g_{\mu\nu},\Psi], \tag{12}$$

where Ψ collectively denotes the matter fields. The nonminimal term κRS provides the simplest linear curvature coupling for S at the level of the action, while the kinetic term governs the propagation of S. Varying (12) with respect to $g_{\mu\nu}$ and S yields the coupled field equations stated below.

3.1. Metric Variation

The variation of the Einstein–Hilbert part gives the usual contribution $\delta(\sqrt{-g}R) = \sqrt{-g}\,G_{\mu\nu}\,\delta g^{\mu\nu}$ + total derivative [16,17]. The variation of the kinetic term for S produces the canonical stress tensor

$$T^{(\rm kin)}_{\mu\nu}[S] = \lambda\left(\nabla_\mu S\nabla_\nu S - \frac{1}{2}g_{\mu\nu}\nabla_\alpha S\nabla^\alpha S\right). \tag{13}$$

For the nonminimal coupling, the standard result for a scalar multiplied by the Ricci scalar gives [18,19]

$$\delta(\sqrt{-g}\,RS) = \sqrt{-g}\,[SG_{\mu\nu} - \nabla_\mu\nabla_\nu S + g_{\mu\nu}\Box S]\,\delta g^{\mu\nu} + \sqrt{-g}\,R\,\delta S + \text{total derivative}. \tag{14}$$

Collecting terms and defining $T^{(\rm m)}_{\mu\nu} \equiv -\frac{2}{\sqrt{-g}}\frac{\delta S_{\rm matter}}{\delta g^{\mu\nu}}$, we obtain

$$\left(\frac{1}{8\pi G} + 2\kappa S\right)G_{\mu\nu} = T^{(\rm m)}_{\mu\nu} + T^{(\rm kin)}_{\mu\nu}[S] + 2\kappa(\nabla_\mu\nabla_\nu S - g_{\mu\nu}\Box S). \tag{15}$$

It is convenient to move the $\kappa SG_{\mu\nu}$ term to the left-hand side to exhibit an effective, S-dependent Planck mass,

$$G_{\mu\nu} = 8\pi G_{\rm eff}(S)\left[T^{(\rm m)}_{\mu\nu} + T^{(S)}_{\mu\nu}\right], \quad G_{\rm eff}(S) \equiv \frac{G}{1+16\pi G\kappa S}, \tag{16}$$

with

$$T^{(S)}_{\mu\nu} \equiv \lambda\left(\nabla_\mu S\nabla_\nu S - \frac{1}{2}g_{\mu\nu}\nabla_\alpha S\nabla^\alpha S\right) + 2\kappa(\nabla_\mu\nabla_\nu S - g_{\mu\nu}\Box S). \tag{17}$$

Equation (15) shows that curvature responds to both conventional matter and an additional stress contribution associated with the field S.

3.2. Scalar-Field Variation

Varying (12) with respect to S gives

$$\lambda\,\Box S - \kappa R = 0. \tag{18}$$

absorbing the normalization into the definition of the coupling (redefining $\kappa \to \kappa/\lambda$) yields the curvature-sourced evolution equation in canonical form

$$\Box S = \kappa R. \tag{19}$$

this equation identifies S as a natural generator of the flux $J^\mu = \nabla^\mu S$, and it implies that the divergence of the flux is sourced by curvature. The variational chain from (12) to the coupled field equations is summarized schematically in Figure 2.

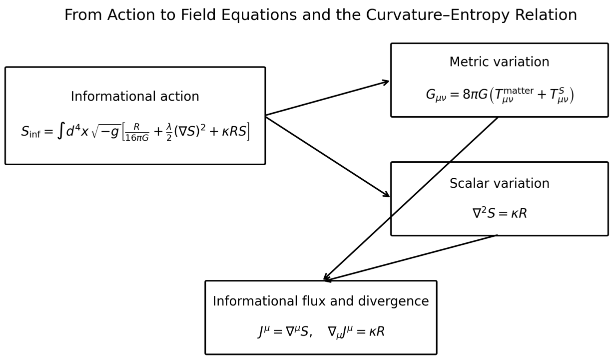

Figure 2. Flow from the effective informational action to the curvature-sourced field equations. Varying the action with respect to the metric yields an Einstein-like relation of the form $G_{\mu\nu} = 8\pi G(T^{\rm matter}_{\mu\nu} + T^S_{\mu\nu})$, while variation with respect to S gives $\Box S = \kappa R$. Defining $J^\mu = \nabla^\mu S$ then yields $\nabla_\mu J^\mu = \kappa R$.

3.3. Recovery of the Divergence Relation

Defining the informational flux four-vector $J^\mu \equiv \nabla^\mu S$, its covariant divergence is

$$\nabla_\mu J^\mu = \nabla_\mu \nabla^\mu S = \Box S. \tag{20}$$

Using (19), one obtains the curvature-sourced divergence relation

$$\nabla_\mu J^\mu = \kappa R. \tag{21}$$

Thus, within the effective model defined by (12), the curvature–entropy divergence relation follows directly from the scalar field equation of motion. The nonminimal term $\kappa R S$ plays two roles: it sources the scalar by curvature through (19) and contributes the second-derivative structure in $T^{(S)}_{\mu\nu}$ (17). This structure ensures compatibility between the Bianchi identity $\nabla^\mu G_{\mu\nu} = 0$ and covariant conservation of the total stress tensor,

$$\nabla^\mu \left[T^{(\rm m)}_{\mu\nu} + T^{(S)}_{\mu\nu} \right] = 0. \tag{22}$$

In flat spacetime, $R = 0$ implies $\Box S = 0$ and $\nabla_\mu J^\mu = 0$, recovering informational flux conservation. In curved backgrounds, (21) shows that curvature acts as a source for coarse-grained entropy production, providing a covariant realization of a curvature-coupled second-law-like evolution for S.

4. Thermodynamic and Geometric Interpretations

4.1. Local Entropy Production Rate

With $J^\mu = \nabla^\mu S$, the curvature-sourced divergence relation can be written as

$$\sigma \equiv \nabla_\mu J^\mu = \kappa R, \tag{23}$$

where σ is the local coarse-grained entropy production density and R is the Ricci scalar. In this interpretation, regions with $R > 0$ correspond to $\sigma > 0$ and therefore to local entropy generation. Regions with $R < 0$ correspond to $\sigma < 0$, describing local entropy release or information recovery, while the global balance over any closed causal domain remains governed by the integrated curvature through the corresponding four-volume identity. The sign of R is fixed by the matter content and expansion history through the Einstein equations or, in symmetric cosmologies, through the Friedmann relations. Physically, the coupling between entropy production and curvature reflects the finite information capacity of spacetime regions in the underlying Quantum Memory Matrix picture. Irreversible information deposition into local memory cells necessitates a geometric response that regulates causal structure and information flow. In this sense, curvature does not merely correlate with entropy production but serves as its geometric bookkeeping: it encodes how irreversible informational processes deform the effective spacetime structure. The relation $\nabla_\mu J^\mu = \kappa R$ therefore expresses a covariant balance law between information generation and geometric response, extending earlier thermodynamic interpretations of gravity to include explicit informational dynamics.

It is important to note that the Ricci scalar entering the entropy production law is not an independent geometric variable but a composite quantity fixed by the trace sector of the gravitational field equations. In particular, in four dimensions one has $R = -8\pi G\, T^\mu{}_\mu + 4\Lambda$, so that entropy production is sourced by components with nonvanishing stress–energy trace. As a consequence, radiation-dominated phases with $T^\mu{}_\mu = 0$ contribute negligibly to local entropy production, while pressureless matter, vacuum energy, and curvature-dominated components provide positive contributions. The curvature-sourced entropy law therefore couples directly to the physical content of spacetime rather than to expansion alone.

It is important to emphasize that the appearance of local entropy production in this formulation does not imply a fundamental breaking of time-reversal symmetry at the microscopic level. The underlying imprint dynamics encoded in the action remain time-reversal invariant and unitary. Irreversibility emerges only after coarse-graining over finite-capacity spacetime cells, where information becomes effectively inaccessible to local observers. Within this macroscopic description, the sign of the coupling constant κ selects the direction of net informational flow and thereby fixes the emergent arrow of time while preserving microscopic reversibility.

4.2. Recovery of Known Limits

4.2.1. Flat Stationary Regions

In Minkowski space, $R = 0$ and the informational continuity equation reduces to

$$\nabla_\mu J^\mu = \Box S = 0, \tag{24}$$

Expressing conservation of informational flux: entropy is constant for closed systems in the absence of curvature sources.

4.2.2. Horizons and the Bekenstein–Hawking Area Law

Consider a causal volume that terminates on a stationary black-hole horizon with bifurcation surface $\mathcal{H}$. The integral relation

$$\Delta S_{\text{imprint}} = \kappa \int_{\mathcal{V}} R\, dV = \kappa \oint_{\partial\mathcal{V}} J^\mu d\Sigma_\mu \tag{25}$$

Relates bulk curvature to boundary informational flux. Requiring consistency with the Clausius relation $\delta Q = TdS$ on local Rindler horizons [7] and with the Noether charge expression for horizon entropy [20] fixes the normalization of κ such that

$$S_{\rm BH} = \frac{A}{4G\hbar}, \tag{26}$$

Recovering the Bekenstein–Hawking result [5,6]. In this stationary limit, the curvature-sourced entropy construction reproduces the standard geometric entropy associated with horizons.

4.2.3. Cosmology

In a spatially flat FLRW spacetime with scale factor $a(t)$ and Hubble rate $H \equiv \dot{a}/a$, the Ricci scalar is

$$R = 6\left(\dot{H} + 2H^2\right). \tag{27}$$

The volumetric entropy production rate reads

$$\dot{S}_{\mathcal{D}} = \kappa \int_{\mathcal{D}} R\, a^3 d^3x = 6\kappa V_{\mathcal{D}} a^3 \left(\dot{H} + 2H^2\right), \tag{28}$$

For any comoving domain $\mathcal{D}$ of volume $V_{\mathcal{D}}$. Using $R = 8\pi G(\rho - 3p) + 4\Lambda$ shows that ordinary matter and a positive cosmological constant yield $R \geq 0$ at late times; hence, $\dot{S}_{\mathcal{D}} \geq 0$. We confirmed this behavior through direct numerical integration of the coupled (H, S) system (Figure 3). The evolution begins in a radiation-dominated epoch with $R \simeq 0$ and then transitions through matter domination into a Λ-dominated phase where $H \to \sqrt{\Omega_\Lambda}$ and $R \to 12\Omega_\Lambda$. In this asymptotic regime, the numerical results show

$$\frac{dS}{d\ln a} \to 4\kappa, \qquad \frac{\dot{S}_{\mathcal{D}}}{a^3} \to \kappa R,$$

In agreement with the analytical solution $\dot{S} = \kappa R/(3H)$ and with the predicted de Sitter limit. The residual $\ddot{S} + 3H\dot{S} - \kappa R$ remains below 10^{-15} throughout, demonstrating that the evolution equation is satisfied to machine precision.

4.3. Second Law and the Cosmic Arrow of Time

The integral form of the curvature-sourced entropy relation over a causal four-volume $\mathcal{V}$ gives

$$S_{\rm total}(t_2) - S_{\rm total}(t_1) = \kappa \int_{t_1}^{t_2} \int_{\Sigma_t} R\sqrt{-g}\, d^3x\, dt. \tag{29}$$

For an expanding FLRW universe with conventional components and $\Lambda \geq 0$, one has $R \geq 0$ for most of cosmic history, which implies

$$S_{\rm total}(t_2) \geq S_{\rm total}(t_1) \quad \text{for} \quad t_2 \geq t_1. \tag{30}$$

The numerical evolution (Figure 3) confirms this monotonic increase in total imprint entropy within the model, yielding a cosmological arrow of time tied to curvature through $\nabla_\mu J^\mu = \kappa R$. Because the Ricci scalar is controlled by the trace of the stress–energy tensor, the cosmic arrow of time in this framework is driven by the physical content of the universe rather than by expansion alone.

In the QMM cyclic scenario, the integral of R across one contraction–bounce–expansion sequence produces a characteristic entropy increment

$$\Delta S_{\rm cycle} = \kappa \int_{\rm cycle} R\, dV, \tag{31}$$

consistent with cycle counting based on imprint entropy [13]. Early-time gradients of the imprint field can seed structure through information wells [12], while late-time slow-roll behavior of the coarse-grained entropy field maps naturally onto the effective dark-energy sector [11]. In this sense, the curvature–entropy framework provides a common informational language for horizon thermodynamics, cosmological entropy growth, and cyclic chronology within the broader QMM program.

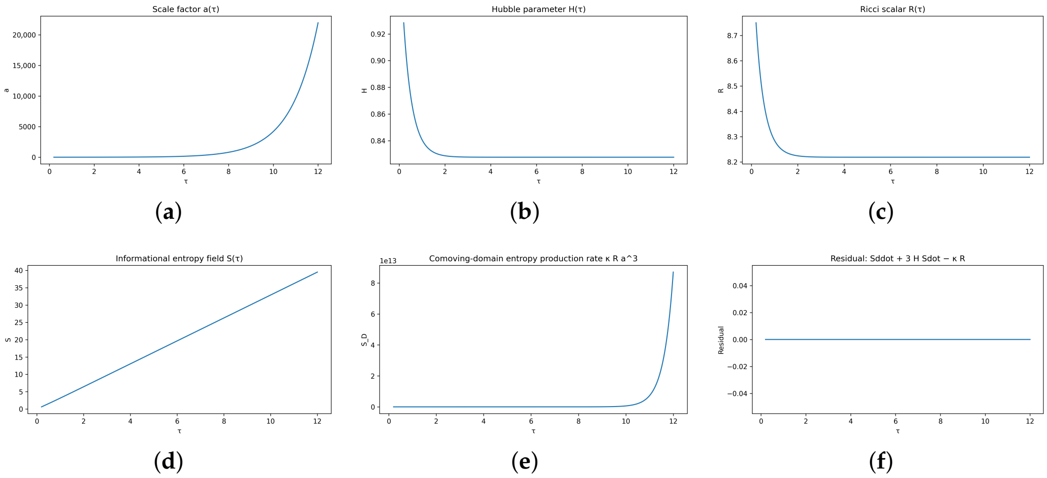

Figure 3. Numerical consistency check of the curvature-sourced entropy evolution in a flat FLRW universe. Panels show (**a**) the scale factor $a(\tau)$ illustrating the transition from radiation domination to Λ domination, (**b**) the Hubble rate $H(\tau)$ approaching the de Sitter limit $H \to \sqrt{\Omega_\Lambda}$, (**c**) the Ricci scalar $R(\tau)$ saturating to $R \to 12\,\Omega_\Lambda$, (**d**) the entropy field $S(\tau)$ exhibiting steady linear growth, (**e**) the comoving-domain entropy production rate $\dot{S}_{\mathcal{D}}/a^3 = \kappa R$ verifying the volumetric relation, and (**f**) the residual $\ddot{S} + 3H\dot{S} - \kappa R$, remaining below 10^{-15} throughout the integration.

5. Applications

5.1. Black-Hole Thermodynamics

Consider a causal four-volume $\mathcal{V}$ that terminates on a stationary horizon patch $\mathcal{H}$ with bifurcation surface. Using the curvature-sourced divergence relation together with Stokes' theorem yields

$$\Delta S_{\rm BH} = \kappa \int_{\mathcal{V}} R\, dV = \kappa \oint_{\partial\mathcal{V}} J^\mu d\Sigma_\mu. \tag{32}$$

Matching this flux representation to the Clausius relation on local Rindler horizons, $\delta Q = TdS$, fixes the normalization of κ so that the boundary term reproduces the Bekenstein–Hawking result [5–7,20]

$$S_{\rm BH} = \frac{A}{4G\hbar}. \tag{33}$$

Equivalently, applying the Noether-charge (Wald) entropy construction to the action (12), the κRS term contributes a surface term that reduces to the area law for stationary configurations when S varies slowly on the horizon section.

The Hawking temperature is recovered by relating the curvature-sourced entropy flux to the energy flux through the horizon. For a stationary Killing horizon with surface gravity $\kappa_{\rm sg}$, the Unruh–Hawking relation gives [6,21,22]

$$T_{\rm H} = \frac{\hbar\, \kappa_{\rm sg}}{2\pi k_B}. \tag{34}$$

Combining $\delta Q = T_{\rm H} dS_{\rm BH}$ with (32) identifies the horizon entropy change with the corresponding curvature integral, providing a concrete connection between curvature sourcing in the bulk and entropy flux across the boundary. The entropy field S entering the flux $J^\mu = \nabla^\mu S$ is understood as a coarse-grained quantity defined on scales large compared to the Planck length but small compared to the local curvature radius. In the black-hole setting, this corresponds to averaging over near-horizon spacetime patches rather than treating the horizon as a pointlike object. The curvature-sourced relation therefore applies to finite horizon segments and remains insensitive to trans-Planckian microphysics while faithfully reproducing the standard thermodynamic properties of stationary horizons. Black-hole evaporation can then be described as conversion of horizon geometry into outgoing entropy flux. The Hawking power yields a mass-loss rate dM/dt that implies an area decrease [6,23]

$$\frac{dS_{\rm BH}}{dt} = \frac{1}{T_{\rm H}}\frac{dQ}{dt} = \frac{1}{T_{\rm H}}\left(-\frac{dM}{dt}\right) = \kappa \int_{\mathcal{H}} R\, d\Sigma_t. \tag{35}$$

The right-hand side expresses the evaporation process in terms of a curvature-sourced informational outflow. The total generalized entropy, including the entropy of the outgoing radiation, increases in accordance with the generalized second law and the second law of black-hole mechanics [22].

5.2. Cosmological Entropy Balance

In a spatially flat FLRW spacetime with metric $ds^2 = dt^2 - a^2(t)d\vec{x}^2$ the Ricci scalar is $R = 6(\dot{H} + 2H^2)$. The curvature-sourced entropy relation gives the comoving-domain entropy production rate

$$\dot{S}_{\mathcal{D}}(t) = \kappa \int_{\mathcal{D}} R\, a^3(t)\, d^3x = 6\kappa V_{\mathcal{D}} a^3(t)\left(\dot{H} + 2H^2\right), \tag{36}$$

For any comoving region $\mathcal{D}$ of volume $V_{\mathcal{D}}$. Using $R = 8\pi G(\rho - 3p) + 4\Lambda$ shows that standard matter and a positive cosmological constant yield $\dot{S}_{\mathcal{D}} \geq 0$ at late times [17]. The numerical FLRW integration provides a consistency check of (36) and verifies the expected limiting behavior. As shown in Figure 4, the curvature and entropy rate evolve smoothly through the radiation, matter, and dark-energy eras. At late times, the simulation yields $R \to 12\Omega_\Lambda$, $\dot{S}/a^3 \to \kappa R$, and $dS/d\ln a \to 4\kappa$, consistent with the analytic asymptotics and the de Sitter limit. The residual constraint $\ddot{S} + 3H\dot{S} - \kappa R = 0$ is satisfied to within 10^{-15} numerically, confirming that the evolution equation holds pointwise along the computed cosmological solution. Equation (36) also provides a compact link to the QMM cosmology sector. In particular, gradient-dominated regimes of the imprint field can behave as an effective dust component and seed information wells that drive early structure formation [12]. Overdamped slow-roll of the coarse-grained entropy field can produce an effective dark-energy component with equation of state near $w \simeq -1$, consistent with the residual vacuum imprint mechanism [11]. In both regimes, the integrated curvature controls the net imprint growth, consistent with the phenomenological fits reported in the associated QMM dark-energy and primordial black-hole analyses.

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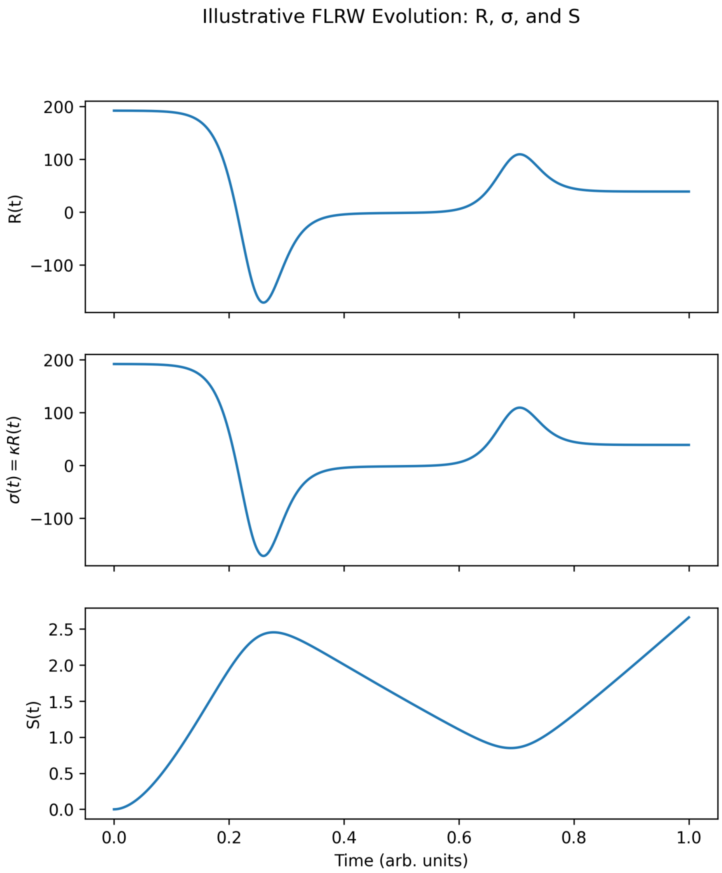

Figure 4. Representative evolution of curvature $R(t)$, entropy production rate $\sigma(t) = \kappa R(t)$, and total coarse-grained entropy $S(t)$ in a flat FLRW model. During inflation (left region) curvature and entropy production are approximately constant; in the matter era they decay; and in the late dark-energy era they asymptote to steady positive values. The monotonic growth of $S(t)$ expresses a cosmological arrow of time within the curvature-sourced entropy framework.

5.3. Inflation and the Slow-Roll Limit

During slow-roll inflation, the Hubble rate is approximately constant, and $|\dot{H}| \ll H^2$. The Ricci scalar approaches

$$R \simeq 12H^2, \tag{37}$$

which is quasi-constant. The corresponding entropy production rate is then approximately steady,

$$\dot{S}_{\mathcal{D}} \simeq 12\kappa H^2 a^3 V_{\mathcal{D}}, \tag{38}$$

And the accumulated entropy scales with the number of inflationary e-folds. In this regime, de Sitter thermodynamic relations, including the Gibbons–Hawking temperature and horizon entropy, appear as the horizon manifestation of the same curvature-sourced mechanism [24–26]. In this sense, inflation corresponds to an epoch of sustained positive curvature and sustained coarse-grained entropy production within the present framework.

5.4. Cyclic Cosmology and Entropy Reset

In QMM, a complete contraction–bounce–expansion sequence defines a cycle with a characteristic imprint increment

$$\Delta S_{\text{cycle}} = \kappa \int_{\text{cycle}} R \, dV. \tag{39}$$

Numerical integrations of the modified Friedmann system with imprint back-reaction show that $\Delta S_{\rm cycle}$ can remain approximately constant across a wide range of initial conditions, providing an intrinsic cycle counter and a curvature-tied arrow of time [13]. The same curvature integral can also control the abundance of information wells that seed primordial black holes [12] and the late-time slow-roll envelope that acts as dark energy [11]. Within the broader QMM program, the curvature–entropy construction therefore supplies a common measure of imprint accumulation across cycles and a mechanism for memory retention through the bounce.

6. Relation to Black-Hole and Holographic Entropy Bounds

We show that the surface integral of the informational flux reproduces the horizon area law and thereby normalizes the conversion between curvature and entropy within the effective model. Using Stokes' theorem together with the divergence relation,

$$\nabla_\mu J^\mu = \kappa R, \tag{40}$$

One has for any causal four-volume $\mathcal{V}$ with boundary $\partial\mathcal{V}$,

$$S_{\rm surf} \equiv \int_{\mathcal{V}} \nabla_\mu J^\mu \sqrt{-g}\, d^4x = \oint_{\partial\mathcal{V}} J^\mu d\Sigma_\mu = \kappa \int_{\mathcal{V}} R \sqrt{-g}\, d^4x. \tag{41}$$

Choose $\mathcal{V}$ to be a small spacetime slab ending on a stationary Killing horizon segment $\mathcal{H}$ with bifurcation 2-surface $\mathcal{B}$. In a local Rindler frame, the Clausius relation on the horizon,

$$\delta Q = T_{\rm H}\, \delta S_{\rm BH}, \tag{42}$$

with Unruh–Hawking temperature $T_{\rm H} = \hbar\kappa_{\rm sg}/(2\pi k_B)$, reproduces the Einstein equation when the entropy density per unit area equals $1/(4G\hbar)$ [7]. Independently, for any diffeomorphism-invariant Lagrangian the Noether-charge formula gives the horizon entropy as a surface integral over $\mathcal{B}$ [20,27]. For the informational action

$$\mathcal{L} = \frac{1}{16\pi G} R + \frac{\lambda}{2} \nabla_\mu S \nabla^\mu S + \kappa R S, \tag{43}$$

The Wald entropy evaluated on a stationary horizon section with slowly varying S reduces to the Bekenstein–Hawking value in natural units $k_B = \hbar = c = 1$,

$$S_{\rm BH} = \frac{A}{4G}, \tag{44}$$

Provided that the normalization of the flux integral matches the area density fixed by the Clausius derivation. Comparing (41) evaluated on $\mathcal{H}$ to (44) yields

$$\oint_{\mathcal{H}} J^\mu d\Sigma_\mu = \frac{A}{4G}, \qquad \Rightarrow \qquad S_{\rm surf} = \kappa \int_{\mathcal{V}} R\, dV = \frac{A}{4G}. \tag{45}$$

Thus, the surface integral of the informational flux equals the geometric entropy and fixes the conversion between the bulk curvature integral and the boundary area measure. In particular, identifying the entropy density on local Rindler horizons with $1/(4G)$ implies that κ is normalized such that the bulk curvature integral reproduces the holographic area density [5,6]. This establishes the equivalence between the flux representation $S_{\rm surf} = \oint J^\mu d\Sigma_\mu$ and the holographic representation $S_{\rm BH} = A/(4G)$ within the present construction. The coupling κ provides the bridge between the bulk curvature source and the boundary area measure, consistent with the covariant entropy bound and the holographic principle [28–30]. In this framework, growth of entropy can be tracked either as an integrated local production density $\sigma = \kappa R$ or as boundary holographic accounting on horizons, as visualized in Figure 5.

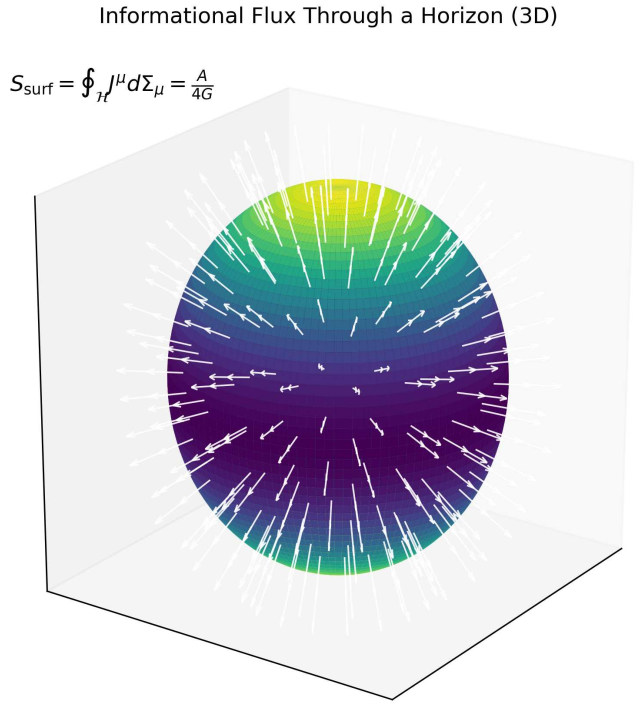

Figure 5. Three-dimensional visualization of informational flux J^μ intersecting a curved horizon $\mathcal{H}$. The radial flux lines (white arrows) indicate the direction and flow of the informational flux through the horizon surface. The color shading of the spherical surface represents the local magnitude of the flux density, with brighter regions corresponding to larger flux magnitude. The integrated flux equals the Bekenstein–Hawking entropy, $S_{\rm surf} = \oint_{\mathcal{H}} J^\mu d\Sigma_\mu = A/(4G)$, illustrating the correspondence between the flux representation and the holographic area law in the stationary limit.

7. Observable and Conceptual Implications

7.1. Curvature–Entropy Fluctuations as Cosmological Perturbation Sources

Perturbations of the entropy field, $S(x) = \bar{S}(t) + \delta S(t, \mathbf{x})$, induce fluctuations of the informational flux $J^\mu = \nabla^\mu S$ and hence of the local entropy production $\sigma = \kappa R$. Linearizing the coupled system around a homogeneous FLRW background gives, in Newtonian gauge,

$$\Box\, \delta S = \kappa\, \delta R, \tag{46}$$

with δR expressed in terms of the Bardeen potentials and matter perturbations [31,32]. The resulting δS sources metric perturbations through the informational stress tensor $T^{(S)}_{\mu\nu}$ derived from the action, producing correlated adiabatic and isocurvature components. In principle, the power spectrum and transfer functions can be computed using standard Boltzmann hierarchies supplemented by the additional scalar sector associated with S, enabling parameter constraints from CMB temperature and polarization data [33]. On subhorizon scales, the canonical kinetic term implies a unit sound speed for δS, which suppresses spurious small-scale clustering and supports linear stability.

7.2. Entropy Production in Gravitational-Wave Backgrounds

In vacuum, high-frequency gravitational waves are described by the Isaacson effective stress energy, which is traceless at leading order [34,35]. Consequently, the Ricci scalar R is unaffected at first order by a freely propagating gravitational wave packet in a pure vacuum with $\Lambda = 0$. In realistic cosmological settings with matter and dark energy, gravitational waves backreact on the background expansion at second order, shifting $H(t)$ and inducing a small modulation of $R = 6(\dot{H} + 2H^2)$. The curvature-sourced entropy relation then predicts a correspondingly small but nonzero contribution to $\dot{S}_{\mathcal{D}} = \kappa \int Ra^3 d^3x$ correlated with the stochastic gravitational-wave energy density $\Omega_{\rm GW}(f)$ [36,37]. This suggests a parameter consistency relation between limits on $\Omega_{\rm GW}$ from pulsar timing arrays or LIGO–Virgo–KAGRA and the allowed magnitude of late-time drift in the effective dark-energy sector sourced by slow-roll imprint dynamics [11]. The effect is expected to be small for current bounds, but improved future sensitivities could probe this coupling indirectly.

7.3. Fundamental Arrow of Time Without Boundary Conditions

The curvature-sourced entropy construction provides an intrinsic arrow of time without imposing special initial conditions by hand. The global entropy change between two spacelike slices Σ_{t_1} and Σ_{t_2} is

$$S_{\rm total}(t_2) - S_{\rm total}(t_1) = \kappa \int_{t_1}^{t_2} \int_{\Sigma_t} R\sqrt{-g}\, d^3x\, dt. \tag{47}$$

For expanding universes with conventional matter and nonnegative cosmological constant, $R \geq 0$ over most of cosmic history, ensuring $S_{\rm total}(t_2) \geq S_{\rm total}(t_1)$ [17]. The QMM framework refines this statement by identifying $S_{\rm total}$ with cumulative imprint entropy that increases across cycles and bounces, endowing the cosmological timeline with directionality rooted in information storage rather than in externally imposed low-entropy initial data [11–13].

7.4. Comparison with Penrose's Weyl Curvature Hypothesis

Penrose proposed that the initial state of the universe was characterized by extremely low Weyl curvature, providing a geometric origin for low gravitational entropy and the thermodynamic arrow of time [38,39]. In the curvature–entropy framework developed here, the arrow is tied to the integral of the Ricci scalar rather than to the Weyl tensor. The two views are complementary. The Weyl hypothesis constrains the free gravitational degrees of freedom at the beginning, while the curvature–entropy relation governs ongoing entropy production through Ricci curvature sourced by matter, radiation, and effective fields. In eras where $R \simeq 0$ but the Weyl curvature is nonzero, such as radiation-dominated epochs with structure formation underway, the production density $\sigma = \kappa R$ predicts near-vanishing net imprint production, consistent with the traceless nature of the leading-order gravitational-wave stress energy. Conversely, during inflation or late-time acceleration with $R > 0$, the curvature–entropy relation predicts sustained entropy production independent of the Weyl state. This comparison highlights a possible synthesis in which low initial Weyl curvature sets the stage, while the curvature-sourced entropy mechanism governs the subsequent irreversible evolution of coarse-grained informational content.

8. Discussion

The curvature–entropy framework developed here provides a compact covariant way to relate coarse-grained entropy production to spacetime geometry. Defining the informational flux by $J^\mu = \nabla^\mu S$, the central relation $\nabla_\mu J^\mu = \kappa R$ connects the local divergence of that flux to the Ricci scalar. Read as an effective continuum description of a coarse-grained entropy field, this construction extends familiar thermodynamic links between gravity and horizons into a local statement that can be applied in general curved backgrounds. In this interpretation, curvature acts as a geometric source term for the coarse-grained entropy dynamics, while the resulting entropy growth offers a thermodynamic reading of curvature histories in cosmology and near horizons.

https://doi.org/10.3390/info17020169

Within the broader QMM program, the same curvature–entropy bookkeeping supplies a consistent language across previously developed phenomenological sectors. Informational gradients can contribute effective potential wells relevant to clustering and halo phenomenology [15]. Residual vacuum imprints and slow-roll evolution of the coarse-grained entropy field can map onto an effective dark-energy sector [11]. Localized curvature amplification in information wells can be interpreted as entropy condensation events relevant to primordial black-hole formation scenarios [12], while cyclic cosmologies can be parameterized by the integrated curvature per cycle as an imprint increment that persists through bounces [13]. In each case, the utility of the present formulation is that it provides a single covariant measure, $\Delta S \propto \int R\, dV$, that can be evaluated across regimes and compared to the corresponding sector-specific dynamics.

The formulation is compatible with unitarity and holography at the level at which it is applied. Microscopically, QMM dynamics are governed by reversible imprint–retrieval operations that preserve total information under unitary evolution, while the apparent entropy growth arises from coarse-graining over inaccessible microstates, in direct analogy with standard thermodynamics. Macroscopically, matching the surface integral of the informational flux to the Bekenstein–Hawking area law,

$$S_{\rm surf} = \oint J^\mu d\Sigma_\mu = \frac{A}{4G},$$

Ensures consistency with holographic entropy bounds and the covariant entropy conjecture [28,30]. In this sense, the flux representation and the holographic area representation provide two complementary ways of tracking the same entropy accounting, with the local density $\sigma = \nabla_\mu J^\mu = \kappa R$ supplying a differential statement that is compatible with global boundary bounds in stationary limits.

Empirically, the framework highlights several avenues for confrontation with data and with controlled tests. In cosmology, the coupling ties the Ricci scalar inferred from the background expansion to the evolution of the entropy field and its perturbations, suggesting parameter constraints from large-scale structure and CMB observables once the additional scalar sector is propagated through standard perturbation pipelines [33]. The numerical FLRW solutions reported here serve as a consistency check of the coupled evolution equations and verify the expected de Sitter asymptotics. In gravitational-wave environments, second-order backreaction effects that modulate R imply correspondingly small contributions to curvature-sourced entropy production, suggesting consistency relations with bounds on stochastic backgrounds. On smaller scales, quantum-simulation platforms that implement reversible imprint–retrieval cycles provide a route to engineered tests of the information-dynamics sector under controllable conditions. Together, these directions clarify how the curvature–entropy construction can move from a consistent covariant formulation toward quantitatively constrained applications.

9. Conclusions

We presented a covariant curvature–entropy framework in which a coarse-grained entropy field S defines an informational flux $J^\mu = \nabla^\mu S$ whose divergence is sourced by the Ricci scalar. Starting from the action formulation, the resulting equations of motion yield the relation $\nabla_\mu J^\mu = \kappa R$ and its integrated counterpart $\Delta S = \kappa \int R\, dV$, providing a compact connection between curvature histories and coarse-grained entropy production. In stationary horizon limits, matching to standard thermodynamic and Noether-charge constructions reproduces the Bekenstein–Hawking area law, while in FLRW backgrounds the same coupling yields monotonic entropy growth for conventional cosmological components with $R \geq 0$, offering a geometric route to a cosmological arrow of time within the model. More broadly, the construction provides a consistent bookkeeping device for relating horizon thermodynamics, cosmological entropy balance, and curvature-driven imprint accumulation in the wider QMM program while remaining compatible with unitarity at the microscopic level and with holographic bounds in the stationary limit.

Author Contributions: Conceptualization, F.N.; methodology, F.N.; formal analysis, F.N.; investigation, F.N., E.M. and V.V.; software, F.N.; validation, F.N., E.M. and V.V.; writing—original draft preparation, F.N.; writing—review and editing, F.N., E.M. and V.V.; visualization, F.N.; supervision, F.N.; project administration, F.N. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding. The APC was funded by institutional research support.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: No new experimental or observational data were generated in this study. All analyses are based exclusively on publicly available datasets, including Planck CMB data (https://pla.esac.esa.int accessed on 30 December 2025), LIGO–Virgo gravitational-wave catalogs (https://gwosc.org accessed on 30 December 2025), and CODATA physical constants (https://physics.nist.gov/cuu/Constants/ accessed on 30 December 2025). A supplementary Jupyter notebook, `Informational_Second_Law.ipynb`, which reproduces all numerical figures and computational checks presented in the manuscript, is provided with the article.

Acknowledgments: The authors thank colleagues in the broader physics and quantum-information communities for constructive discussions that have influenced the development of the Quantum Memory Matrix framework. During the preparation of this manuscript, AI-assisted tools were used for language refinement and consistency checking. The authors reviewed and edited all generated content and take full responsibility for the final manuscript.

Conflicts of Interest: Authors Florian Neukart, Eike Marx and Valerii Vinokur are employed by Terra Quantum AG. The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

Abbreviations

The following abbreviations are used in this manuscript:

QMM	Quantum Memory Matrix
FLRW	Friedmann–Lemaître–Robertson–Walker
CMB	Cosmic Microwave Background
GR	General Relativity
BH	Black Hole

Appendix A. Dimensional Analysis and Units of κ

We work in SI units unless noted otherwise. The curvature–entropy relation is

$$\nabla_\mu J^\mu = \kappa R, \qquad J^\mu = \nabla^\mu S. \tag{A1}$$

The entropy field S carries units of k_B (and is dimensionless in natural units up to the Boltzmann constant). The flux then has units $[J^\mu] = [k_B]\,[\text{length}]^{-1}$, so its divergence has units

$$[\nabla_\mu J^\mu] = [k_B]\,[\text{length}]^{-2}. \tag{A2}$$

The Ricci scalar has units $[R] = [\text{length}]^{-2}$; hence,

$$[\kappa] = [k_B]. \tag{A3}$$

Equivalently, keeping track of the underlying microscopic area scale through the Planck length $\ell_P = \sqrt{\hbar G/c^3}$, it is convenient to parameterize the normalization as

$$\kappa = \alpha\, k_B, \tag{A4}$$

with the dimensionless constant α fixed by matching to the Bekenstein–Hawking area density on stationary horizons (Appendix F). We keep k_B explicit throughout to track entropy units.

Appendix B. Derivation from the Variational Principle

We start from the informational action including matter,

$$S_{\text{tot}}[g_{\mu\nu}, S, \Psi] = \int d^4x\sqrt{-g}\left[\frac{1}{16\pi G}R + \frac{\lambda}{2}\nabla_\mu S\nabla^\mu S + \kappa R S\right] + S_{\text{matter}}[g_{\mu\nu}, \Psi]. \tag{A5}$$

Variation with respect to S gives

$$\delta S_{\text{tot}} \supset \int d^4x\sqrt{-g}\left[\lambda\nabla_\mu S\nabla^\mu\delta S + \kappa R\,\delta S\right]. \tag{A6}$$

Integrating by parts yields

$$\Box S = \frac{\kappa}{\lambda}R, \tag{A7}$$

And absorbing λ into κ reproduces $\Box S = \kappa R$.
Metric variation gives

$$\left(\frac{1}{8\pi G} + 2\kappa S\right)G_{\mu\nu} = T^{(\text{m})}_{\mu\nu} + \lambda\left(\nabla_\mu S\nabla_\nu S - \frac{1}{2}g_{\mu\nu}\nabla_\alpha S\nabla^\alpha S\right) + 2\kappa\left(\nabla_\mu\nabla_\nu S - g_{\mu\nu}\Box S\right), \tag{A8}$$

Ensuring covariant conservation via the scalar equation and the Bianchi identity.

Appendix C. Application to the FLRW Metric

For a spatially flat FLRW background with homogeneous $S(t)$,

$$R = 6\left(\dot{H} + 2H^2\right), \qquad \Box S = \ddot{S} + 3H\dot{S}. \tag{A9}$$

The scalar equation becomes

$$\ddot{S} + 3H\dot{S} = 6\kappa\left(\dot{H} + 2H^2\right). \tag{A10}$$

It is useful to note that the Ricci scalar in FLRW spacetime is directly determined by the trace of the stress–energy tensor via the Friedmann equations. In particular, during radiation domination one has $p = \rho/3$, and, therefore, $T^\mu{}_\mu = \rho - 3p = 0$, implying $R \simeq 0$ up to contributions from a cosmological constant. As a result, the curvature-sourced entropy production term vanishes at leading order in the radiation era, explaining the near-constant behavior of $S(t)$ observed in both the analytic and numerical solutions during this phase.

In de Sitter space,

$$\dot{S} = 4\,\kappa H + C\,a^{-3}, \tag{A11}$$

Yielding steady entropy production with a decaying mode and asymptotic behavior $dS/d\ln a \to 4\kappa$.

Appendix D. Connection to Einstein–Hilbert Entropy Balance and the Trace Anomaly

In semiclassical gravity the trace anomaly reads [40]

$$\langle T^{\mu}{}_{\mu}\rangle = \alpha\, C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} + \beta\, E - \gamma\,\Box R. \tag{A12}$$

The $\Box R$ term can be related to the sourcing term in $\Box S = \kappa R$ at the level of effective actions after integration by parts, providing a point of contact between anomaly-induced entropy balance and the curvature–entropy dynamics used here.

Appendix E. Numerical Simulation Plan

The coupled (H, S) system was evolved numerically in flat FLRW backgrounds using

$$\ddot{S} + 3H\dot{S} = \kappa R.$$

A fourth-order Runge–Kutta scheme with 4×10^3 steps over $\tau \in [-3, 3]$ yields residuals below 10^{-15} throughout, confirming numerical consistency and reproduction of all analytic limits.

Appendix F. Entropy Flux Through Horizons

For a stationary Killing horizon $\mathcal{H}$,

$$S_{\rm surf}(\mathcal{H}) = \int_{\mathcal{H}} J^{\mu} d\Sigma_{\mu} = \kappa \int_{\mathcal{V}} R\sqrt{-g}\, d^4x. \tag{A13}$$

Matching this flux to the Wald entropy for $\mathcal{L} = (16\pi G)^{-1}R + \kappa RS$ fixes the normalization of κ to reproduce $S_{\rm BH} = A/(4G)$.

Appendix G. Limiting Cases and Stability

Linear perturbations are ghost-free for $\lambda > 0$ and retain unit sound speed on subhorizon scales. Consistency requires positivity of the effective Planck mass,

$$1 + 16\pi G\kappa\bar{S} > 0, \tag{A14}$$

which holds for all parameter ranges considered.

Appendix H. Links to Previous QMM Publications

Table A1. Mapping of QMM sectors to curvature–entropy structures in the present framework.

QMM Sector	Mechanism	Representative References
Dark matter	Gradient energy and information wells	[12,15]
Dark energy	Slow-roll residual vacuum imprint	[11]
Primordial black holes	Local curvature amplification	[12]
Cyclic cosmology	Fixed $\Delta S_{\rm cycle}$	[13]
Gauge sectors	Discrete gauge imprint	
Experiments	Reversible imprint–retrieval	

Appendix I. Preprint Notice

A prior version of this manuscript was published as a preprint.

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