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Research Article

Anisotropic Bulk Viscous String Cosmological Model in a Scalar-Tensor Theory of Gravitation

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Spatially homogeneous, anisotropic, and tilted Bianchi type-VI_0 model is investigated in a new scalar-tensor theory of gravitation proposed by Saez and Ballester (1986) when the source for energy momentum tensor is a bulk viscous fluid containing one-dimensional cosmic strings. Exact solution of the highly nonlinear field equations is obtained using the following plausible physical conditions: (i) scalar expansion of the space-time which is proportional to the shear scalar, (ii) the barotropic equations of state for pressure and energy density, and (iii) a special law of variation for Hubble's parameter proposed by Berman (1983). Some physical and kinematical properties of the model are also discussed.

1. Introduction

It is well known that Einstein's general theory of relativity has been successful in finding different models for the universe. Friedmann-Robertson-Walker (FRW) models describe spatially homogenous and isotropic universes. But, they have higher symmetries than the real universe and therefore they are probably poor approximations for very early universe. The measurements of the cosmic microwave background (CMB) anisotropy support the existence of anisotropies at the early universe [1–4]. Hence, in order to understand the early stages of evolution of the universe, spatially homogeneous anisotropic and tilted Bianchi type-Vl_0 cosmological models, are studied. In the tilted cosmological models the matter does not move orthogonally to the hyper surface of homogeneity. The general behavior of tilted cosmological models has been studied by King and Ellis [5], Collins and Ellis [6], and Bali and Sharma [7].

Sáez and Ballester [8] have developed a new scalar-tensor theory of gravitation in which the metric is coupled with a dimensionless scalar field in a simple manner. This coupling gives a satisfactory description of the weak fields. In spite of the dimensionless character of the scalar field, an antigravity regime appears in the theory. Also, this theory suggests a possible way to solve the "missing matter problem" in nonflat FRW cosmologies. The field equations given by Sáez and Ballester for the combined scalar and tensor fields are

$$R_{ij}-\frac{1}{2}g_{ij}R-\omega\emptyset^{n}\left(\emptyset_{,i}\emptyset_{,j}-\frac{1}{2}g_{ij}\emptyset_{,k}\emptyset^{,k}\right)=-8\pi T_{ij}, \quad (1)$$

and the scalar field $\emptyset$ satisfies the following equation:

$$2\emptyset^{n}\emptyset^{;i}_{;i}+n\emptyset^{n-1}\emptyset_{,k}\emptyset^{,k}=0. \quad (2)$$

Also, we have

$$T^{ij}_{;j}=0, \quad (3)$$

which is a consequence of the field equations (1) and (2). Here, ω and n are constants. T_{ij} is the energy tensor of the matter, R_{ij} is the Ricci tensor, R is the Ricci scalar, and comma and semicolon denote partial and covariant derivatives, respectively. Singh and Agrawal [9], Reddy and Venkateswara Rao [10], Reddy et al. [11], Mohanty and Sahu [12, 13], Adhav et al. [14], and Tripathy et al. [15] are some of the authors who have studied several aspects of the Sáez-Ballester scalar-tensor theory.

In recent years, there has been a considerable interest in the investigation of Bianchi-type cosmological models when the source for energy momentum tensor is a bulk viscous fluid containing one-dimensional cosmic strings. Bulk viscosity plays a significant role in the early evolution of the universe and contributes to the accelerated expansion phase of the universe popularly known as the inflationary phase. A review of the universe models with viscosity is given by Grøn [16]. Strings arise as a random network of stable line-like topological defects during the phase transition in the early universe. Massive closed loops of strings serve as seeds for the formation of large structures like galaxies and cluster of galaxies at the early stages of evolution of the universe. A good many authors have investigated about different aspects of string cosmological models either in the frame work of Einstein's theory or in the modified theories gravity [17–25].

Very recently, Reddy et al. [26] have investigated Kaluza-Klein bulk viscous cosmic string universe in Sáez-Ballester theory while Naidu et al. [27] have discussed the same universe in Brans-Dicke [28] scalar-tensor theory of gravitation. Reddy et al. [29] presented LRS Bianchi type-II universe with cosmic strings and bulk viscosity in the $f(R,T)$ theory of gravity proposed by Harko et al. [30] while Reddy et al. [31] have studied Kaluza-Klein bulk viscous cosmic string model in $f(R,T)$ gravity. Naidu et al. [32] have obtained a Bianchi type-V bulk viscous string model in $f(R,T)$ gravity. Reddy et al. [33] have discussed LRS Bianchi type-II bulk viscous cosmic string cosmological model in the scale covariant theory of gravitation formulated by Canuto et al. [34]. Also, Kiran and Reddy [35] have established the nonexistence of Bianchi type-III bulk viscous string cosmological models in $f(R,T)$ gravity.

Motivated by the above investigations of bulk viscous cosmic string Bianchi type models in modified theories of gravitation, we, in this paper, investigate spatially homogeneous, anisotropic, and tilted Bianchi type-VI_0 cosmological model in the presence of bulk viscous fluid with one-dimensional cosmic strings. The paper is organized as follows. Section 2 deals with the derivation of the field equations in Sáez-Ballester theory in Bianchi type-VI_0 space-time when the source for energy momentum tensor is bulk viscous fluid with one dimensional cosmic strings. Section 3 is devoted to the solutions of the nonlinear field equations under some specific physical conditions. In Section 4, we discuss some physical and kinematical properties of the cosmological model. Section 5 contains some conclusions.

2. Metric and Field Equations

We consider the spatially homogenous, anisotropic, and tilted Bianchi type-VI_0 space-time described by the following metric:

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 e^{-2\alpha x} dy^2 + C^2 e^{2\alpha x} dz^2, \quad (4)$$

where A, B, and C are functions of cosmic time t and α is a constant.

The energy momentum tensor for a bulk viscous fluid containing one-dimensional cosmic strings is given by

$$T_{ij} = (\rho + p) u_i u_j + p g_{ij} - \lambda x_i x_j,$$
$$\overline{p} = p - 3\zeta H, \quad (5)$$

where ρ is the rest energy density of the system, $\zeta(t)$ is the coefficient of bulk vissocity, $3\zeta H$ is usually known as bulk viscous pressure, H is Hubble's parameter, and λ is string tension density.

Also, $u^i = \delta^i_4$ is a four-velocity vector which satisfies

$$g_{ij} u^i u_j = -x^i x_j = -1, \quad u^i x_i = 0. \quad (6)$$

Here, we, also consider ρ, p, and λ as functions of time t only.

Using comoving coordinates and (5)-(6), Sáez-Ballester field equations (1)–(3) for the metric (4) take the following form:

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4}{B}\frac{C_4}{C} + \frac{\alpha^2}{A^2} - \frac{\omega}{2}\emptyset^n \emptyset_4^2 = -8\pi(\overline{p} - \lambda), \quad (7)$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4}{A}\frac{C_4}{C} - \frac{\alpha^2}{A^2} - \frac{\omega}{2}\emptyset^n \emptyset_4^2 = -8\pi \overline{p}, \quad (8)$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4}{A}\frac{B_4}{B} - \frac{\alpha^2}{A^2} - \frac{\omega}{2}\emptyset^n \emptyset_4^2 = -8\pi \overline{p}, \quad (9)$$

$$\frac{A_4}{A}\frac{B_4}{B} + \frac{B_4}{B}\frac{C_4}{C} + \frac{A_4}{A}\frac{C_4}{C} - \frac{\alpha^2}{A^2} + \frac{\omega}{2}\emptyset^n \emptyset_4^2 = 8\pi\rho, \quad (10)$$

$$\frac{B_4}{B} - \frac{C_4}{C} = 0, \quad (11)$$

$$\rho_4 + (\rho + \overline{p})\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) - \lambda\frac{A_4}{A} = 0, \quad (12)$$

$$\emptyset_{44} + \emptyset_4\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) + \frac{n}{2}\frac{\emptyset_4^2}{\emptyset} = 0. \quad (13)$$

Here, and in what follows, a subscript 4 after an unknown function indicates differentiation with respect to t.

The spatial volume is given by

$$V = ABC = a^3, \quad (14)$$

where $a(t)$ is the scale factor of the universe.

The expressions for scalar of expansion θ and shear scalar σ^2 are (kinematical parameters)

$$\theta = u^i_{;j} = \frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}. \quad (15)$$

Hubble parameter H and the mean anisotropy parameter are defined as

$$3H = \theta = 3\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right),$$
$$3A_h = \sum_{I=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2, \quad \Delta H_i = H_i - H, \ i = 1, 2, 3, \quad (16)$$
$$2\sigma^2 = \sigma^{ij}\sigma_{ij} = 3A_h^2 - H^2.$$

3. Solutions and the Model

Equation (12) gives, on integration,

$$B = kC, \tag{17}$$

where k is a constant of integration which can be chosen as unity without any loss of generality, so that we have

$$B = C. \tag{18}$$

Using (18), the field equations (7)–(13) reduce to the following system of independent equations:

$$2\frac{B_{44}}{B} + \left(\frac{B_4}{B}\right)^2 + \frac{\alpha^2}{A^2} - \frac{\omega}{2}\emptyset^n\emptyset_4^2 = -8\pi\left(\overline{p} - \lambda\right), \tag{19}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4}{A}\frac{B_4}{B} - \frac{\alpha^2}{A^2} - \frac{\omega}{2}\emptyset^n\emptyset_4^2 = -8\pi\overline{p}, \tag{20}$$

$$2\frac{A_4}{A}\frac{B_4}{B} + \left(\frac{B_4}{B}\right)^2 - \frac{\alpha^2}{A^2} + \frac{\omega}{2}\emptyset^n\emptyset_4^2 = 8\pi\rho, \tag{21}$$

$$\emptyset_{44} + \emptyset_4\left(\frac{A_4}{A} + 2\frac{B_4}{B}\right) + \frac{n}{2}\frac{\emptyset_4^2}{\emptyset} = 0. \tag{22}$$

Now, (19)–(22) are a system of four independent equations in six unknowns A, B, p, $\emptyset$, ρ, and λ. Also, the equations are highly nonlinear. Hence, to find a determinate solution, we use the following plausible physical conditions.

(i) Variation of Hubble's parameter proposed by Berman [36] that yields constant deceleration parameter models of the universe is defined by

$$q = -\frac{aa_{44}}{a_4^2} = \text{constant}. \tag{23}$$

(ii) The shear scalar σ^2 is proportional to scalar expansion θ of the space-time (4) so that we can take

$$A = B^m, \tag{24}$$

where $m \neq 0$ is a constant (Collins et al. [37]).

(iii) For a barotropic fluid, the combined effects of the proper pressure and the bulk viscous pressure can be expressed as

$$\begin{aligned} \overline{p} &= p - 3\zeta H = \varepsilon\rho, \\ p &= \varepsilon_o\rho, \end{aligned} \tag{25}$$

where $\varepsilon = \varepsilon_o - \beta$ $(0 \le \varepsilon_o \le 1)$ and $\varepsilon, \varepsilon_o$, and β are constants

Now, (23) admits the following solution:

$$a(t) = (ct + d)^{1/(1+q)}, \tag{26}$$

where $c \neq 0$ and d are constants of integration. This equation implies that the condition for expansion of the universe is $1 + q > 0$.

Now from (14), (18), (24), and (26), we obtain metric potentials as

$$\begin{aligned} A &= (ct + d)^{3m/(2m+1)(1+q)}, \\ B = C &= (ct + d)^{3/(2m+1)(1+q)}. \end{aligned} \tag{27}$$

Using (27) and by a suitable choice of coordinates and constants (i.e., taking $d = 0$ and $c = 1$), the metric (4) can be written as

$$\begin{aligned} ds^2 = &-dt^2 + t^{6m/(2m+1)(1+q)}dx^2 \\ &+ t^{6/(2m+1)(1+q)}\left[e^{-\alpha x}dy^2 + e^{2\alpha x}dz^2\right]. \end{aligned} \tag{28}$$

4. Physical Discussion of the Model

Equation (28) represents the anisotropic Bianchi type-VI_0 bulk viscous string cosmological model in Sáez-Ballester scalar-tensor theory of gravitation with the following expressions for physical and kinematical parameters which are significant in the physical discussion of the cosmological model.

Spatial volume is

$$V^3 = t^{3/(1+q)}. \tag{29}$$

Scalar expansion is

$$\theta = \frac{3}{(1+q)t}. \tag{30}$$

The mean Hubble parameter is

$$H = \frac{1}{(1+q)t}. \tag{31}$$

The mean anisotropy parameter is

$$A_h = 2\frac{(m-1)^2}{(m+2)^2}. \tag{32}$$

The shear scalar is

$$\sigma^2 = 3\frac{(m-1)^2}{(m+2)^2(1+q)^2t^2}. \tag{33}$$

Energy density is

$$\begin{aligned} 8\pi\rho = {} & \alpha^2 t^{-6m/(1+q)(m+2)} - \frac{\omega}{2}\emptyset_0^2 t^{-6/(1+q)} \\ & - \frac{9(2m+1)}{(1+q)^2(m+2)^2t^2}. \end{aligned} \tag{34}$$

Isotropic pressure is

$$\begin{aligned} 8\pi p = \varepsilon_0 \Bigg[& \alpha^2 t^{-6m/(1+q)(m+2)} - \frac{\omega}{2}\emptyset_0^2 t^{-6/(1+q)} \\ & - \frac{9(2m+1)}{(1+q)^2(m+2)^2t^2} \Bigg]. \end{aligned} \tag{35}$$

Coefficient of bulk viscosity is

$$8\pi\zeta = \frac{(\varepsilon_0 - \varepsilon)(1+q)}{9} \times t\left[\alpha^2 t^{-6m/(1+q)(m+2)} - \frac{\omega}{2}\emptyset_0^2 t^{-6/(1+q)} - \frac{9(2m+1)}{(1+q)^2(m+2)^2 t^2}\right]. \quad (36)$$

The scalar field in the model is

$$\emptyset = \left[\frac{\emptyset_0(n+2)}{2}\frac{(1+q)}{(q-2)}t^{(1+q)/(q-2)}\right]^{2/(n+2)}. \quad (37)$$

String tension density is

$$8\pi\lambda = \frac{3(m-1)(q-2)}{(1+q)^2(m+2)t^2} - 2\alpha^2 t^{-6m/(1+q)(m+2)}. \quad (38)$$

Using the above results, we now discuss the behavior of the cosmological model given by (30). The result (29) shows that the model is expanding with time since $1+q>0$. It can be observed that the space-time given by (28) has no initial singularity, that is, at $t=0$. It can also be observed that $\theta, H, \rho, p, \lambda$, and ζ decrease with time and approach zero as $t \to \infty$ and they all diverge at $t=0$. The scalar field increases with time and at $t=0$, it vanishes.

Also, since $A_h \neq 0$ and $\sigma^2/\theta^2 \neq 0$, the universe remains anisotropic throughout the evolution of the universe. It is also interesting to note from (32) and (33) that when $m=1$, $A_h = 0$ and $\sigma^2 = 0$ and hence the universe becomes isotropic and shear free. Also, when $\alpha = 0$ and $m=1$, we observe, from (38), that $\lambda = 0$ which shows that strings do not survive in this particular case. Bulk viscosity, in the model, decreases as t increases which is in accordance with the well-known fact that bulk viscosity decreases with time and leads to inflationary model [38].

5. Conclusions

Bulk viscosity, cosmic strings scalar fields, and Bianchi models play a significant role in the discussion of the early stages of the evolution of the universe and in inflationary cosmology. Hence, we have investigated here a spatially homogeneous, anisotropic, and tilted Bianchi type-VI_0 cosmological model in the framework of a scalar-tensor theory of gravitation proposed by Sáez and Ballester [8] in the presence of bulk viscous fluid containing one-dimensional cosmic strings. The model is obtained using the special law of variation for Hubble's parameter proposed by Berman [36], scalar expansion of the space-time which is proportional to shear scalar (Collins [37]), and the barotropic equation of state for pressure and energy density. It is observed that the model is expanding, nonsingular, and nonrotating. It is also observed that all the physical and kinematical parameters of the model diverge when $t=0$ and vanish when t is infinitely large while the scalar field vanishes at the initial epoch. Bulk viscosity in the model decreases with time leading to inflationary model. The model will be useful in the discussion of structure formation in the early universe in scalar-tensor cosmology.

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