

Exploring entanglement using a hardware quantum computer simulation

Lorena Solväng^{*} , Jesper Haglund  and Marcus Berg 

Department of Engineering and Physics, Karlstad University, Karlstad, Sweden

E-mail: lorena.solvang@kau.se



Abstract

We have developed an activity using simulation hardware called the Quantum Teleportation & Superdense Coding toolkit. The toolkit contains classical electronic components, such as circuit boards and cables, that mimic the behaviour of quantum gates. The activity was designed to be accessible to upper-secondary school students who are not familiar with the mathematical formalism often used for teaching quantum mechanics. Groups of upper-secondary school students that have visited our university during outreach initiatives have participated in the activities, and we report on our experiences of introducing the toolkit for this group of students.

Keywords: quantum physics, entanglement, simulation hardware, upper-secondary school students

1. Introduction

The 2022 Nobel Prize laureates Alain Aspect, John Clauser and Anton Zeilinger have brought public attention to quantum physics, one of the cornerstones of modern physics. They each conducted experiments that exhibited the property of *quantum entanglement*, a fundamental aspect of

quantum physics [1, 2]. Fundamental principles of quantum physics have given rise to new technologies such as quantum computing and quantum communication. Quantum computers are predicted to save substantial time and minimize inaccuracies in certain kinds of data processing [3].

To ensure informed use and new development of quantum technologies, a well-educated ‘quantum workforce’ is needed [4–7]. In many countries, learning about quantum physics starts at the upper-secondary school level [8, 9], and topics concerning quantum technologies seem to be fascinating to young people who often see those topics as ‘cool science’ [8, 10]. However, students are more attracted by conceptual issues, such as wave-particle duality or Schrödinger’s cat, than

^{*} Author to whom any correspondence should be addressed.



Original content from this work may be used under the terms of the [Creative Commons Attribution 4.0 licence](#). Any further distribution of this work must maintain attribution to the author(s) and the title of the work, journal citation and DOI.

by solving the Schrödinger equation [8]. Research also suggests that teaching and learning quantum physics is challenged by the complexity of the mathematical formalism of quantum physics and by quantum phenomena that go against students' classically-grounded intuitions [8–13].

Teaching materials developed for introducing quantum physics in upper-secondary school rarely include teaching of quantum entanglement [7], despite the fact that experts consider it one of the fundamental concepts of quantum physics [8, 14]. One possible strategy to engage students in learning about entanglement, a strategy that is already appreciated in teaching of quantum physics, is teaching by using simulations [15, 16]. Recently, a tutorial on quantum computing was constructed and evaluated [17]. The study shows that before engaging with the tutorial, students experienced challenges with the difference between an N -bit classical and N -qubit quantum computer.

2. Theoretical background

For a general introduction to quantum physics that includes entanglement, see e.g. Susskind and Friedman [18] and for quantum computing, see e.g. the tutorial mentioned above [17]. In this section, we provide a few brief remarks about a few specific concepts that we explore in our activity, aimed at readers who already know a little bit of quantum physics.

2.1. Entanglement

That two particles are in an entangled state means that the state of one particle is correlated with the state of the other particle, regardless of the distance between them [1, 19]. This implies that an entangled two-particle state is not merely a description of the individual particles, but the complete description of the combined system of the two particles. Susskind and Friedman describe entanglement as being 'the quantum mechanical generalization of correlation' [18]. Entangled systems display correlations found in quantum physics that are not found in classical physics. Those correlations are not merely hypothetical relations, they have already been observed in a large number

of experiments (e.g. in the experiments conducted by the 2022 Nobel Prize laureates since the 1960s) [2].

2.2. Quantum bits and superposition

In classical computers, digital information is transferred in data units called bits. A bit is the smallest unit of digital information, representing a binary value of either 0 or 1, or sound on/off, etc. Multiple bits represent more complex data such as characters, numbers, images, and videos. In contrast, quantum computers primarily use quantum bits, or qubits for short, which are typically two-level systems—'the quantum version of the two-state system' [18]. While some quantum computing architectures use higher-dimensional units (e.g. qutrits or qudits) [20, 21], qubits remain the most widely used building blocks in current quantum computing implementations.

An important difference between a classical system and a quantum system is that a quantum system has the possibility of superposition, which is the ability to be in (what in a given basis appears to be) multiple states at the same time until it is measured [5]. The behaviour of a quantum object, such as a single qubit, is probabilistic when measured. The state of a single qubit can be represented using basis states, denoted using 'bra-ket notation' as $|0\rangle$ and $|1\rangle$, or using vector notation as $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ [5]. When a qubit is measured, the result is either 0 or 1 and each possible outcome is measured with a certain probability. These two probabilities must, like all probabilities, sum up to 1, i.e. there is a 100% chance that *some* outcome is measured. Measuring 0 is in standard quantum physics expressed as saying that the qubit has 'collapsed' to the $|0\rangle$ state, and measuring 1 that it has collapsed to the $|1\rangle$ state.

For a system of many (N) qubits, the result of measurement is either 0 or 1 for each qubit and any superposition of all 2^N combinations of single-qubit basis states is allowed. For example, for a two-qubit system ($N = 2$) there are four outcomes: 00, 01, 10 and 11. Using the bra-ket notation, the four quantum basis states corresponding

to these four measurement outcomes can be represented as $|00\rangle$, $|01\rangle$, $|10\rangle$ and $|11\rangle$, and a general state is any superposition of these, where all four possible measurement probabilities sum to 1, i.e. 100%.

3. The hardware used in the activity

For the activity with the students, we have used the Quantum Teleportation & Superdense Coding toolkit [22, 23]. The toolkit was developed by the company Phase Space Computing in Linköping, Sweden.

As mentioned in the beginning of the paper, the toolkit contains classical electronic components, that for the purposes of our activity exactly mimic the behaviour of quantum gates [23]. In any simulation, it is important to distinguish what is being simulated (here, a real quantum computer) and the implementation of the simulation (here, a set of classical electrical circuit components). In most of this text, and in the activity with the students, we discuss the simulation as if it were the real thing. In our experience, the simulation works just like a quantum computer is expected to, so we will make no attempt to explain the electronic internals of the classical circuits. What we will instead explain is how ‘a quantum computer is expected to work’.

The functions of these components are described below in the same order as they were described to the students participating.

3.1. The source, the switch, and the measurement device

Figure 1 displays a circuit where a source is connected to a switch and to a measurement device. The source is a button marked ‘SAMPLE’ connected to a power socket. The 2-way selector switch has two positions labeled as $|0\rangle$ and $|1\rangle$. The measurement device is an analog display.

When the switch’s input state is $|0\rangle$, and we push ‘SAMPLE’ on the source, the pointer on the measurement device points left, indicating ‘the measurement device displays 0’. When the input state is $|1\rangle$ the measurement device points right, indicating ‘the measurement device displays 1’. In a quantum system, measurements give

specific results based on the probabilities of different quantum states. When we repeat measurements many times, we can see the correct pattern of results. Because of this, the analogue display in our simulation should always show either 0 or 1, just like a quantum circuit.

3.2. Demonstration set-up, step 1: the *H*-gate, and then two of them

Figure 2 shows a circuit where a Hadamard gate is connected between the switch and the measurement device. A Hadamard gate (*H*-gate) turns an input state of $|0\rangle$ or $|1\rangle$ into a specific superposition of $|0\rangle$ and $|1\rangle$.

When the input state is $|0\rangle$, the probability of the measurement device connected after the *H*-gate displaying 0 is 50% and the probability of the measurement device displaying 1 is also 50% (this is called the sum state, $\begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$). When the input state is $|1\rangle$, the measurement device still shows the two possible outputs 0 and 1 with 50% chance each (this is called the difference state $\begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix}$). Hence, by just looking at the measurement device in this arrangement, it will be impossible to know the input state.

However, an important aspect of superposition is that it is not *inherently* random: if we would add another *H*-gate in series, we would recover the original input. For example, if we send in $|0\rangle$ and pass it through *two H*-gates as in figure 3, this always gives 0 when measured. In other words, the state after the two *H*-gates is completely deterministic, with 100% probability to measure 0, if $|0\rangle$ was sent in.

For readers familiar with matrix multiplication, the *H*-gate can be represented as

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \Rightarrow H \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix},$$

$$H \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix}$$

which is the above discussion expressed mathematically: *H* maps $|0\rangle$ to the sum state and *H* maps $|1\rangle$ to the difference state. Mathematically,

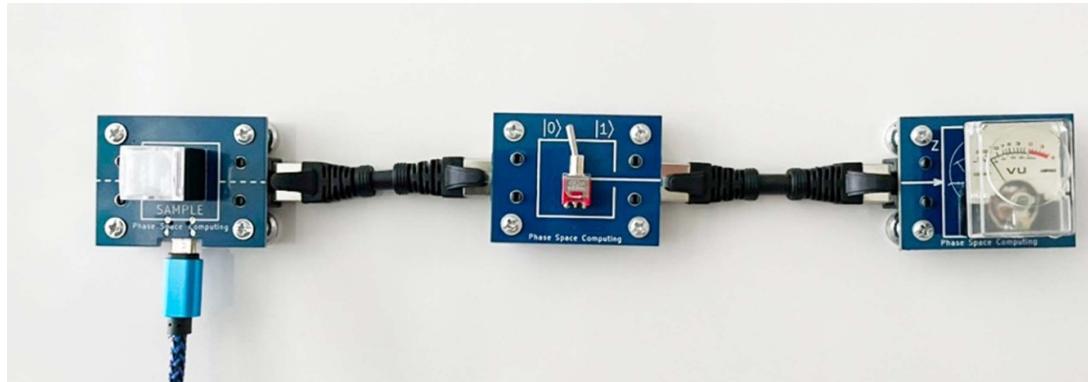


Figure 1. The source (left) is connected to a switch (middle) and to a measurement device (right).

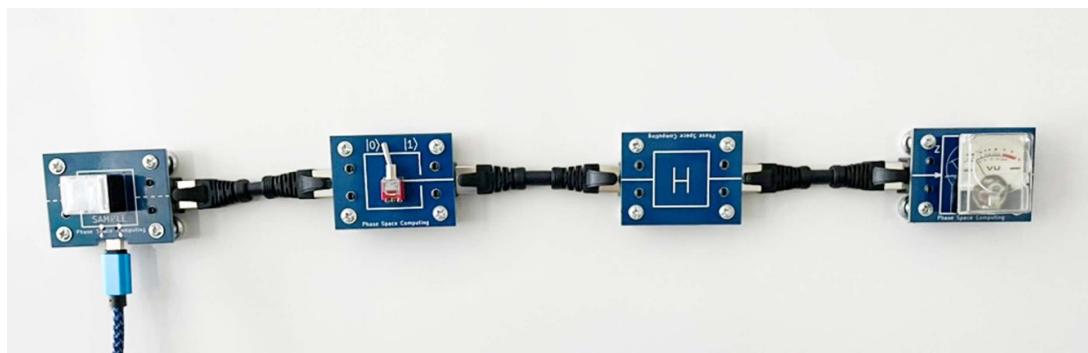


Figure 2. The H -gate (marked with an H) connected between the switch and the measurement device.

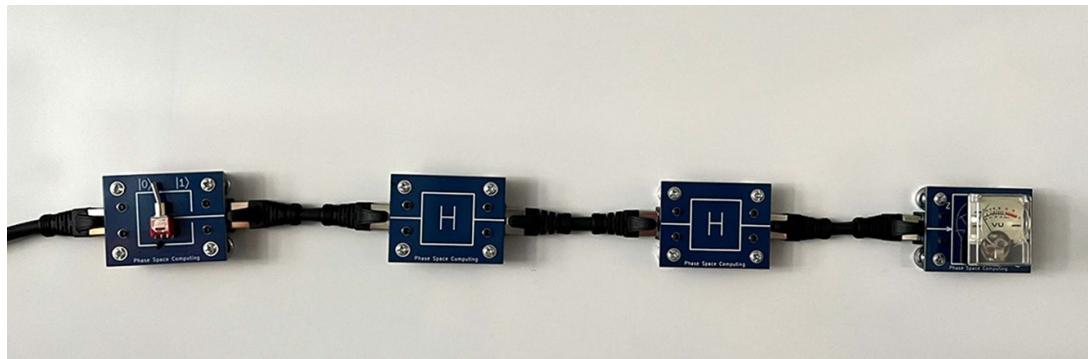


Figure 3. Two H -gates between the switch and the measurement device.

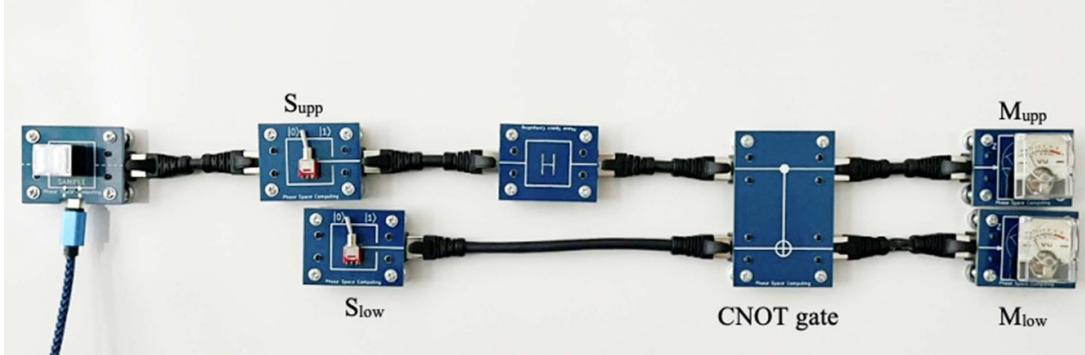


Figure 4. A CNOT-gate is connected in a circuit with a source (left), two switches (S_{upp} and S_{low}), an H -gate (marked with H) and two measurement devices (M_{upp} and M_{low}).

two sequential H -gates can be represented by multiplying the H matrix by itself. This combination acts as the identity matrix, i.e. does nothing at all:

$$H \cdot H = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

We emphasize that we do not expect typical upper-secondary school students to use this mathematical representation. In the activity, we discuss the physics with them in words, and would not recommend teachers to introduce the mathematical formalism at the upper-secondary level. The mathematical details here are only to help the reader who is not familiar with quantum gates.

The main discussion point here is: two H -gates in succession act completely deterministically. From a classical point of view, it is tempting to call a single H -gate a ‘randomizer’. If we did, we would have to call the second H -gate a ‘de-randomizer’. But as students can themselves conclude after a moment’s thought, making a truly random sequence deterministic by some subsequent operation is impossible. So, both words are problematic! However, we find that they can help encourage discussion of randomness in quantum physics with this demonstration as a concrete example. The conclusion should be that quantum physics is evidently not ‘random’ in the everyday sense of that word. As Susskind writes ‘Yes, you heard me correctly—the time evolution of the state [...] is deterministic.’

3.3. Demonstration set-up, step 2: the CNOT-gate

Figure 4 shows a component which simulates a CNOT-gate, connected in a circuit with a source, two switches (one on the upper line, denoted by S_{upp} and one on the lower line, denoted by S_{low}), an H -gate (marked with H) connected after the switch S_{upp} , and two measurement devices (denoted by M_{upp} and M_{low}). If $|0\rangle$ enters the upper input of the CNOT-gate, it is ‘off’, and when $|1\rangle$ enters the upper input of the CNOT-gate, it is ‘on’. When the CNOT-gate is on, it flips the input coming from the lower switch (S_{low}). ‘Flips’ means to change the state from $|0\rangle$ to $|1\rangle$, or from $|1\rangle$ to $|0\rangle$. It is possible to describe also the CNOT-gate by matrices as above, but that requires the concept of *tensor product*. Here we will instead use only words, which is how we discuss it with the students.

As described above, the H -gate creates a superposition, which, as shown in figure 4 enters the CNOT-gate in the upper (‘control’) input. We can (incompletely) characterize the quantum state in the upper line after the H -gate by saying that the probability of having $|0\rangle$ entering the upper input of the CNOT-gate (so it is off) is 50% and the probability of $|1\rangle$ entering (so it is on) is also 50%. (This description of the state after the H -gate is incomplete, as explained above, since the sum state and the difference state both have the 50%–50% property. However, the distinction between them is not crucial for the argument that follows.) In this language, we see that the CNOT-gate is

‘on’, i.e. will flip the lower input, 50% of the time. More concretely, when both switches are set to $|0\rangle$, as shown in figure 4, the output from the H -gate after S_{upp} is $|0\rangle$ 50% of the time. In this case, the lower line will remain $|0\rangle$ and the measurement devices M_{upp} and M_{low} display 0 and 0. The other 50% of the time, the output from the H -gate is $|1\rangle$, so the CNOT-gate is ‘on’, i.e. it will flip the lower input from $|0\rangle$ to $|1\rangle$. In this alternative, the measurement devices M_{upp} and M_{low} display 1 and 1. In summary, when both upper and lower lines have their input states set to $|0\rangle$, the two measurement devices display either 0 and 0 (collapse to a basis state $|00\rangle$) or 1 and 1 (collapse to a basis state $|11\rangle$). These outputs, 00 and 11, are examples of perfect correlation between the states in the upper and lower lines. In particular, the outputs 01 and 10 are never obtained, unless the components malfunction.

Similarly to above, we can talk about the H -CNOT combination as an ‘entangler’ [2]: the input is two uncorrelated (un-entangled) quantum bits, and they emerge correlated (entangled). Looking forward, we will then introduce the CNOT- H ordering, that we can call a ‘de-entangler’, similarly to how two H -gates became a ‘de-randomizer’.

But first, in the demonstration we ask the students: have you heard of Einstein’s critique of quantum physics? Einstein argued that quantum mechanics is incomplete because it appears to allow for ‘spooky action at a distance’—the idea that particles can instantaneously affect each other’s states, even when separated by large distances, which would have contradicted the theory of relativity. In this activity, we make Einstein’s critique concrete, as follows.

Assume we describe the joint quantum state in the two lines before M_{upp} and M_{low} as saying it does not itself ‘know’ whether it is in $|00\rangle$ or $|11\rangle$, but only decides when measured. Then, if we separate M_{upp} and M_{low} by a distance of say 100 m, using longer cables than in figure 4, how do they ‘know’ to give such correlated results? It seems as if they would need to ‘communicate’ instantaneously. However, in this classical simulation, this cannot be simulated in real time: if we would film the longer-cable circuit with nanosecond time resolution, the quantum correlations would appear delayed. As Susskind and

Friedman [18] write: ‘This problem is not a problem for quantum mechanics. It is a problem for simulating quantum mechanics with a classical Boolean computer.’ But for practical purposes in our demonstration, this deficiency of the classical simulation is never detected. (The Nobel prize experiments show that the correlation exists at distances greater than 100 m, but there it is usually not interpreted as causal influence.)

In our activity, we use the simulation to help students understand these concepts. By observing the behaviour of the entangled states and the measurement outcomes, students can see how quantum mechanics predicts correlations that classical physics cannot explain. In particular, they see that some correlations only become evident when physically separated measurements are compared.

3.4. Demonstration set-up, step 3: dense coding

In the circuit shown in figure 5, we have added to the H -CNOT circuit in figure 4 two classically controlled components, the I/X -gate and I/Z -gate, and also CNOT- and H -gates. Also, we always put the measurement devices M_{upp} and M_{low} and the end of the circuit, i.e. the far right.

The switches on the I/X - and I/Z -gates make it possible to choose by hand between gates represented by the identity matrix I (that does nothing to the state, as explained above) and the matrices X and Z , respectively. Hence the notation I/X and I/Z .

In matrix notation, an X -gate is represented as the X Pauli matrix:

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \Rightarrow X \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$

$$X \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

i.e. it acts like a classical NOT-gate on the basis states, flipping $|0\rangle$ to $|1\rangle$ and $|1\rangle$ to $|0\rangle$. Similarly, a Z -gate is represented as the Z Pauli matrix:

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \Rightarrow Z \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix},$$

$$Z \begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$$

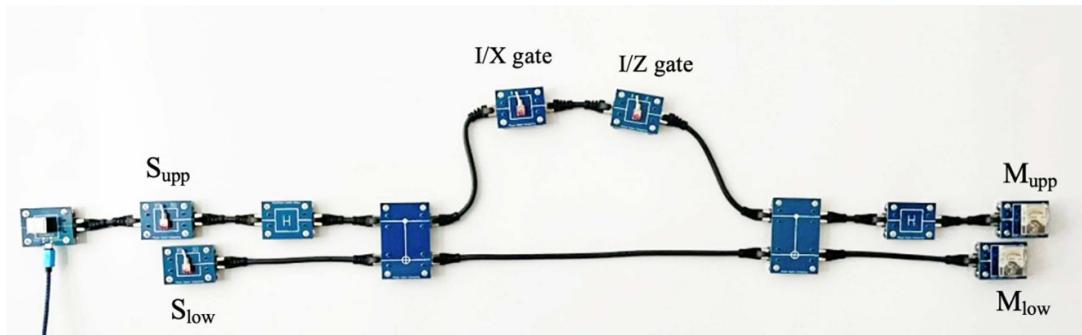


Figure 5. Simulation of a simple quantum computer circuit: dense coding.

so it flips the sum state to the difference state, and conversely. Since multiplying the matrices as X times Z versus Z times X gives different results, the ordering of the gates in the circuit matter.

Like for the CNOT-gate above, in the activity we gave a description solely in words, but interested readers can check the following mathematically by applying the relevant matrices. In fact, we will now ‘zoom out’ and treat the entire circuit as a black box, to make two points concerning input and output. Set the switches S_{upp} and S_{low} both to $|0\rangle$, as in figure 5. In the demonstration, when the switch on the I/X -gate is positioned to I (identity operation, i.e., do nothing to what comes in), the lower measurement device (M_{low}) will show 0, and when the switch is positioned to X , M_{low} will show 1. When the switch on the I/Z -gate is positioned to I (identity), the upper measurement device (M_{upp}) will show 0, and when the switch is positioned to Z , M_{upp} will show 1. This is supposed to illustrate, without going through mathematical details, that:

1. Using the I/X - and I/Z -gates, we can adjust what we see on *both* measurement devices M_{upp} and M_{low} , even though the I/X - and I/Z -gates are both located on the *upper* channel. The fact that M_{upp} and M_{low} provide four distinct measurements means they can encode four values (00,01,10,11), or two classical bits, of information, even though they are both located on a single line. Roughly speaking, this particular quantum circuit can handle twice as much information per bit line as a classical computer, i.e. the coding of information is twice as ‘dense’ internally, hence the

name of this quantum circuit: ‘dense coding’. If this would be true for each qubit, then a quantum computer with 100 qubits would correspond to $2^{100} \approx 10^{30}$ classical bits. Evaluating the performance of a real quantum computer is much more involved than this oversimplified estimate, but the *exponential* advantage over a classical computer is representative of algorithms adapted to quantum computing.

2. The output is completely deterministic, with no probabilistic aspect whatsoever. The settings on the I/X - and I/Z -gates determine the output in a completely predictable way, given a specific input. This provides an additional, and more applied, counterexample to the vague idea that quantum physics is ‘inherently random’, by illustrating that a quantum computer in fact takes classical input, and can produce classical deterministic output. In the ‘internals’ of the quantum circuit, however, there are entangled states with correlations that can only be described quantum-mechanically. They are then ‘de-entangled’ by the CNOT-H combination.

To better understand how quantum measurements work, imagine using a Hadamard gate on a single qubit. This gate puts the qubit into an equal mixture of the $|0\rangle$ and $|1\rangle$ states. When we measure the qubit, after applying the Hadamard gate, we get either 0 or 1, each happening 50% of the time. If we repeat this many times, we see a pattern that matches these probabilities. This shows that while each measurement is random, the overall results follow a predictable pattern.

4. Impressions from implementation of the activity and educational reflections

The activity was held at Karlstad University by the first and third authors with around 200 upper-secondary school students divided in eight groups in four consecutive sessions. At this stage we have not collected data on the participating students' interaction with the activity, or their learning gains or attitudes toward the activity, but would like to share some of our impressions from conducting the activity.

One impression from implementation of the activity is that the Quantum Teleportation & Superdense Coding toolkit's components are easy to connect and to use, hence it can directly be used in class. However, components are rather small, so the students need to be quite close to see the components' positions and the results on the measuring device. Our groups of 20–25 students were not small enough for all students to see the components in every detail. Maybe smaller groups would be preferable to involve more of the quiet students.

Most of the students were not familiar with the mathematical formalism often used for teaching quantum mechanics. In our activity, no complex mathematics is needed to get a grip of what is being simulated, leading to conceptual thoughts about what quantum physics is and to its possible applications.

In line with earlier research [1, 9, 15, 16], we recognize the educational aid of visualising quantum mechanics phenomena at upper-secondary school level and that combining the affordances of both physical and virtual experiments can strengthen learning [13, 24]. The simulation of quantum states and the measurement process allows students to visualise the counter-intuitive nature of quantum phenomena, such as superposition and entanglement. In addition, we believe that, in contrast to 'pure' computer visualisation, this kind of hands-on simulation tools increases students' engagement, which may lead to a better understanding of the simulated phenomena. While there are many tools such as the IBM Quantum Composer [25] that are excellent for visualising and simulating quantum circuits, the physical interaction with the Quantum Teleportation & Superdense Coding toolkit provides a tangible experience that can

enhance learning. Using both physical and virtual tools together can enhance the learning experience, providing students with multiple ways to explore quantum concepts.

Data availability statement

No new data were created or analysed in this study.

Acknowledgment

We thank Ljungbergsfonden for funding the purchase of the Quantum Teleportation & Superdense Coding toolkit.

ORCID iDs

Lorena Solvang  <https://orcid.org/0000-0002-4790-6032>

Jesper Haglund  <https://orcid.org/0000-0003-4997-2938>

Marcus Berg  <https://orcid.org/0000-0001-9611-2450>

Received 20 November 2024, in final form 14 March 2025

Accepted for publication 26 March 2025

<https://doi.org/10.1088/1361-6552/adc5d2>

References

- [1] Dür W and Heusler S 2014 Visualization of the invisible: the qubit as key to quantum physics *Phys. Teach.* **52** 489–92
- [2] The Nobel Prize 2022 (available at: www.nobelprize.org/prizes/physics/2022/press-release/)
- [3] de Souza Pimenta R C and Bezerra A T 2023 Revisiting semiconductor bulk hamiltonians using quantum computers *Phys. Scr.* **98** 045804
- [4] Hughes C, Finke D, German D-A, Merzbacher C, Vora P M and Lewandowski H J 2022 Assessing the needs of the quantum industry *IEEE Trans. Educ.* **65** 592–601
- [5] Greinert F, Müller R, Bitzenbauer P, Ubben M S and Weber K-A 2023 Future quantum workforce: competences, requirements, and forecasts *Phys. Rev. Phys. Educ. Res.* **19** 010137
- [6] Nita L, Mazzoli Smith L, Chancellor N and Cramman H 2023 The challenge and opportunities of quantum literacy for future education and transdisciplinary

Exploring entanglement using a hardware quantum computer simulation

problem-solving *Res. Sci. Technol. Educ.* **41** 564–80

[7] Brang M, Franke H, Greinert F, Ubben M S, Hennig F and Bitzenbauer P 2024 Spooky action at a distance? A two-phase study into learners' views of quantum entanglement *EPJ Quantum Technol.* **11** 33

[8] Henriksen E K, Bungum B, Angell C, Tellefsen C W, Frågåt T and Bøe M V 2014 Relativity, quantum physics and philosophy in the upper secondary curriculum: challenges, opportunities and proposed approaches *Phys. Educ.* **49** 678–84

[9] Vilarta Rodriguez L, van der Veen J T, Anjewierden A, van den Berg E and de Jong T 2020 Designing inquiry-based learning environments for quantum physics education in secondary schools *Phys. Educ.* **55** 065026

[10] Johansson A, Andersson S, Salminen-Karlsson M and Elmgren M 2018 “Shut up and calculate”: the available discursive positions in quantum physics courses *Cult. Stud. Sci. Educ.* **13** 205–26

[11] Bungum B and Selstø S 2022 What do quantum computing students need to know about quantum physics? *Eur. J. Phys.* **43** 055706

[12] Ireson G 2000 The quantum understanding of pre-university physics students *Phys. Educ.* **35** 15

[13] Montagnani S, Stefanel A, Chiofalo M L M, Santi L and Michelini M 2023 An experiential program on the foundations of quantum mechanics for final-year high-school students *Phys. Educ.* **58** 035003

[14] Weissman E Y, Merzel A, Katz N and Galili I 2022 Phenomena and principles: presenting quantum physics in a high school curriculum *Physics* **4** 1299–317

[15] Bondani M *et al* 2022 Introducing quantum technologies at secondary school level: challenges and potential impact of an online extracurricular course *Physics* **4** 1150–67

[16] Kohnle A, Baily C, Campbell A, Korolkova N and Paetkau M J 2015 Enhancing student learning of two-level quantum systems with interactive simulations *Am. J. Phys.* **83** 560–6

[17] Hu P, Li Y and Singh C 2024 Investigating and improving student understanding of the basics of quantum computing *Phys. Rev. Phys. Educ. Res.* **20** 020108

[18] Susskind L and Friedman A 2014 *Quantum Mechanics: The Theoretical Minimum* (Basic Books)

[19] Schroeder D V 2017 Entanglement isn't just for spin *Am. J. Phys.* **85** 812–20

[20] Wang Y, Hu Z, Sanders B C and Kais S 2020 Qudits and high-dimensional quantum computing *Front. Phys.* **8** 589504

[21] Kopf L, Hiekkämäki M, Prabhakar S and Fickler R 2023 Endless fun in high dimensions—A quantum card game *Am. J. Phys.* **91** 458–62

[22] Johansson N and Larsson J-Å 2019 Quantum simulation logic, oracles, and the quantum advantage *Entropy* **21** 1–76

[23] Phase Space Computing 2018 (available at: <https://phasespacecomputing.com/>)

[24] de Jong T, Linn M C and Zacharia Z C 2013 Physical and virtual laboratories in science and engineering education *Science* **340** 305–8

[25] IBM Quantum Composer 2023 (available at : <https://quantum.ibm.com/composer>)



Lorena Solvang is a graduate student in physics education at Karlstad University. Her research focuses on the use of digital technology, particularly GeoGebra, in physics education.



Jesper Haglund is a senior lecturer in physics education at Karlstad University. In his research, Jesper takes an interest in how digital technology can be used in physics education.



Marcus Berg is professor of physics at Karlstad University. He does research in theoretical high-energy physics and condensed matter physics, and is interested in physics education.