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ΔI = 1 "staggering" effect in even-even nuclei with effective triaxiality

M.S. Nadirbekov,[1,*] O.A. Bozarov,[1,†] and S.N. Kudiratov[1,‡]

[1] *Institute of Nuclear Physics, Tashkent, 100214, Uzbekistan*

Abstract. In this work, the model of the effective triaxiality of even-even nuclei with quadrupole and octupole deformations is applied to the description of the "staggering"-effect in the yrast band of the alternating parity energy spectrum. The components of the moment of inertia tensor depend on the angular part of the polar coordinates in the quadrupole-octupole space, the values of which are determined from the ground state of the nucleus, and they are expressed by the parameter ε_0. Within the framework of the proposed model, the staggering effect is well reproduced in even-even nuclei: ^{154}Sm, ^{158}Gd, ^{160}Dy, ^{170}Yb and ^{232}Th.

Keywords: quadrupole and octupole deformation, effective triaxiality, alternating parity energy spectrum, "staggering"-effect, triaxial rotor.

I. INTRODUCTION

Various well-studied types of deviation of the nuclear collective motion from the purely rotational motion are known [1]. As a result of these deviations in the structure of the nuclear rotation spectrum, high-order effects occur, such as "squeezing", "backbending" and "staggering" [2–5]

"Staggering" effects represent the branching of rotational bands in a sequence of states differing by several units of angular momentum I. For example, such ΔI=1, ΔI=2 and ΔI=4 "staggering" effects are observed in the energy bands of super-deformed nuclei [2, 5, 6]. These effects are very well known in even-even nuclei [1] and allow testing of various collective models [7]. The study of these fine effects in the structure of the collective interaction and the corresponding energy spectra of the nuclei suggests a complex behaviour of the collective properties. These collective modes represent complicated and diverse excitations involving many nucleons simultaneously, but can be described theoretically with a small number of degrees of freedom [1].

The use of discrete approximations of high-order derivatives of a given nuclear property as a function of a given physical quantity reveals various types of "staggering" effects that carry information about the subtle properties of the nuclear interaction and the corresponding high-order correlations in the collective dynamics of the system [8].

The appearance of a reflection of an asymmetric shape in atomic nuclei is associated in the geometric model with the manifestation of an octupole degree of freedom [9]. The main physical characteristic of a system with the manifestation of reflection asymmetry is associated with the violation of R- and P-symmetry. As mentioned above, these symmetries are broken individually, and the system remains invariant with respect to their product P·R^{-1} [1]. Then the spectrum of the system is characterized by the presence of energy bands in which the angular momenta have variable parity. Consequently, the negative parity band with the sequence of levels I$^\pi$= 1^-, 3^-, 5^-, 7^-, ..., which merges with the positive parity band with the sequence of levels I$^\pi$= 0^+, 2^+, 4^+, 6^+, . .., form a band with a sequence of levels I$^\pi$= 0^+ , 1^-, 2^+, 3^-, 4^+, 5^-, ... Such a band is observed in even-even nuclei of the rare earth region and in actinides [9–11]. In these bands of even-even nuclei, the odd-I and negative parity energy levels are shifted relative to the even- I energy levels of positive parity. That is, the level with angular momentum I is shifted relative to its neighbor with angular momentum I ±1 [4]. This quantity, usually called odd-even "staggering" or ΔI = ±1 "staggering", should disappear if the even and odd energy levels form a single band.

* Corresponding Author: mnadirbekov@yandex.ru
† bozarovoybek@gmail.com
‡ qudiratov@inp.uz

II. MODEL FORMALISM

The general theory of quadrupole and octupole excitation of even-even nuclei is determined by the Hamilton operator, which contains seven dynamical variables $\beta_2(\beta_2 \geq 0)$, $\gamma(0 \leq \gamma \leq \pi)$, $\beta_3(\beta_3 \geq 0)$, $\eta(0 \leq \eta \leq \pi)$, $\theta_1(0 \leq \theta_2 \leq 2\pi)$, $\theta_2(0 \leq \theta_2 \leq \pi)$, $\theta_3(0 \leq \theta_3 \leq 2\pi)$ [12]. Expressions for the kinetic energy operators $\hat{T}_{\beta_2}$ $\hat{T}_{\beta_3}$ and $\hat{T}_\gamma$ for β_2-, β_3-, γ-vibrations are given in the work [12]. And kinetic energy operator $\hat{T}_\eta$ for η-vibrations have the form [13]:

$$\hat{T}_\eta = -\frac{\hbar^2}{2B_3}\frac{1}{\beta_3^2}\left[\frac{\partial^2}{\partial\eta^2} + \frac{24\cos^2 2\eta - 6\cos 2\eta}{5 + 5\cos 2\eta + 8\cos^2 2\eta}\frac{\cos\eta}{\sin\eta}\frac{\partial}{\partial\eta}\right]. \tag{1}$$

Also $\hat{T}_{\text{rot}}$–rotational energy operator [12] and $V(\beta_2, \beta_3, \gamma, \eta)$–potential energy of β_2-, β_3-, γ- and η- vibrations.

Here we replace the variables γ and η with their effective values γ_{eff} and η_{eff} [12, 14, 15], and variables β_2 and β_3 are dynamic. In this case, the rotational energy operator $\hat{T}_{\text{rot}}$ depends on the effective values of γ_{eff} and η_{eff} through projections of moments of inertia [12].

Now we write Schrödinger equation in polar coordinates σ $(0 \leq \sigma \leq \infty)$ and ε $(-\frac{\pi}{2} \leq \varepsilon \leq \frac{\pi}{2})$ [16]:

$$\left\{-\frac{\hbar^2}{2B}\left[\frac{\partial^2}{\partial\sigma^2} + \frac{1}{\sigma\partial\sigma} + \frac{\partial^2}{\sigma^2\partial\varepsilon^2}\right] + \hat{T}_{\text{rot}} + W(\sigma,\varepsilon) - E_I^{\pm}\right\}\Phi_I^{\pm}(\sigma,\varepsilon) = 0. \tag{2}$$

A solution of the equation (2) we will find in the form

$$\Phi_I^{\pm}(\sigma,\varepsilon,\theta) = F_{I\tau}^{\pm}(\sigma,\varepsilon,)\varphi_{I\tau}^{\pm}(\theta). \tag{3}$$

Equation for $\varphi_{I\tau}^{\pm}(\theta)$ be present

$$\left(\hat{T}_{rot} - \epsilon_{I\tau}^{\pm}\right)\varphi_{I\tau}^{\pm}(\theta) = 0. \tag{4}$$

here $\epsilon_{I\tau}$-energy of triaxial rotor [17], the quantum number τ labels the different eigenvalues $\epsilon_{I\tau}^{\pm}$ of $\hat{T}_{\text{rot}}$ corresponding to a given angular momentum I. General solution of the equation (4) very complicated. Therefore to solve equation (4) we assume, that the components of the moment of inertia $\mathcal{J}_i$ takes following form in polar coordinates:

$$\mathcal{J}_1 = 8B\sigma^2\left[\cos^2\varepsilon_0\sin^2\left(\gamma_{\text{eff}} - \frac{2\pi}{3}\right) + \sin^2\varepsilon_0\left(\frac{3}{2}\cos^2\eta_{\text{eff}} + \sin^2\eta_{\text{eff}} + \frac{\sqrt{15}}{2}\sin\eta_{\text{eff}}\cos\eta_{\text{eff}}\right)\right], \tag{5}$$

$$\mathcal{J}_2 = 8B\sigma^2\left[\cos^2\varepsilon_0\sin^2\left(\gamma_{\text{eff}} - \frac{4\pi}{3}\right) + \sin^2\varepsilon_0\left(\frac{3}{2}\cos^2\eta_{\text{eff}} + \sin^2\eta_{\text{eff}} - \frac{\sqrt{15}}{2}\sin\eta_{\text{eff}}\cos\eta_{\text{eff}}\right)\right], \tag{6}$$

$$\mathcal{J}_3 = 8B\sigma^2\left(\cos^2\varepsilon_0\sin^2\gamma_{\text{eff}} + \sin^2\varepsilon_0\sin^2\eta_{\text{eff}}\right). \tag{7}$$

The capability of the triaxial quadrupole-octupole rotor model for description the structure of the lowest positive- and negative-parity levels in the spectra of heavy even-even nuclei is considered in Ref. [12], where $\varphi_{I\tau}^{\pm}(\theta)$ has the form as in [12].

We take potential energy $W(\sigma,\varepsilon)$ in the vicinity of the minimum $\sigma_0, \pm\varepsilon_0$ accept as [16]

$$W(\sigma,\varepsilon) = V(\sigma) + \frac{C_\varepsilon}{2\sigma^2}(\varepsilon \mp \varepsilon_0)^2, \tag{8}$$

then the variables are (2) separated and $F_I^{\pm}(\sigma,\varepsilon)$ divided into factors:

$$F_I^{\pm}(\sigma,\varepsilon) = f_I^{\pm}(\sigma)\chi(\varepsilon \mp \varepsilon_0). \tag{9}$$

Equation for $\chi(\varepsilon \mp \varepsilon_0)$:

$$\frac{\partial^2\chi_\nu(\varepsilon \mp \varepsilon_0)}{\partial\varepsilon^2} + \frac{2B}{\hbar^2}\left[\pm\epsilon_\nu - \frac{C_\varepsilon}{2}(\varepsilon \mp \varepsilon_0)\right]\chi(\varepsilon \mp \varepsilon_0) = 0, \tag{10}$$

where ϵ_ν are eigenvalues and $\chi(\varepsilon \mp \varepsilon_0)$ are eigenfunctions of equation (10), C_ε - elasticity constant with respect to ε-vibrations.

Now we will write an equation for $f_I^{\pm}(\sigma)$:

$$-\frac{\hbar^2}{2B}\left[\frac{\partial^2 F_I^{\pm}(\sigma)}{\partial\sigma^2} + \frac{\partial F_I^{\pm}(\sigma)}{\sigma\partial\sigma}\right] + \left[\frac{\hbar^2\epsilon_{I\tau}^{\pm}}{4B\sigma^2} + V(\sigma) \mp \frac{\hbar^2}{4B}\frac{\epsilon_\nu}{\sigma^2} - E_I^{\pm}\right]f_I^{\pm}(\sigma) = 0. \tag{11}$$

The potential $V(\sigma)$ can assume

$$V(\sigma) = V_0 \left(\frac{\sigma}{\sigma_0} - \frac{\sigma_0}{\sigma} \right)^2. \tag{12}$$

Then the equation (11) will take the form:

$$\left[\frac{\partial^2}{\partial \sigma^2} + \frac{\partial}{\sigma \partial \sigma} - \frac{\epsilon^{\pm}_{I\tau}}{2\sigma^2} - 2g\frac{\sigma^2}{\sigma_0^4} + \frac{4g}{\sigma_0^2} - \frac{2g}{\sigma^2} \pm \frac{\epsilon_\nu}{2\sigma^2} + \frac{2BE^{\pm}_I}{\hbar^2} \right] f^{\pm}_I(\sigma) = 0,$$

where

$$g = \frac{BV_0\sigma_0^2}{\hbar^2}.$$

Enter new variable

$$x = \frac{\sqrt{2g}}{\sigma_0^2}\sigma^2.$$

Then, we obtain following hypergeometric differential equation

$$\left[x\frac{\partial^2}{\partial x^2} + (2s^{\pm} + 1 - x)\frac{\partial}{\partial x} - \left(s^{\pm} + \frac{1}{2} - \tilde{E} \right) \right] W^{\pm}_I(x) = 0, \tag{13}$$

here

$$s^{\pm} = \sqrt{\frac{\epsilon^{\pm}_{I\tau} + 4g \mp \epsilon_\nu}{8}}.$$

Wave functions of the equation (13) are confluent hypergeometric function $W^{\pm}_I(-n, 2s^{\pm} + 1, x)$, $n = 0, 1, 2, ...$ – quantum number of σ-vibrations. Thus wave functions of the equation (11) we can write

$$f^{\pm}_{In\tau}(x) = N_\sigma x^{s^{\pm}} e^{-\frac{x}{2}} W^{\pm}_I(-n, 2s^{\pm} + 1, x)/ \tag{14}$$

N_σ -normalization factor. Eigenvalues of the equation (13) are

$$E^{\pm}_{nI\tau} = \hbar\omega \left(2n + \sqrt{\frac{\epsilon^{\pm}_{I\tau} + 4g \mp \epsilon_\nu}{2}} + 1 - \sqrt{2g} \right). \tag{15}$$

Energy excited collective states:

$$\triangle E^{\pm}_{nI\tau} = \hbar\omega \left(2n + \sqrt{\frac{\epsilon^{\pm}_{I\tau} + 4g \mp \epsilon_\nu}{2}} - \sqrt{\frac{4g - \epsilon_0}{2}} \right).$$

We introduce the notation

$$\Delta^{\pm}_\nu = 4g \mp \epsilon_\nu, \qquad \Delta_0 = 4g - \epsilon_0.$$

And

$$\triangle E^{\pm}_{nI\tau} = \hbar\omega \left(2n + \sqrt{\frac{\epsilon^{\pm}_{I\tau} + \Delta^{\pm}_\nu}{2}} - \sqrt{\frac{\Delta_0}{2}} \right) \tag{16}$$

Energy levels of the excited collective states in presented approximation determined by parameters: $\hbar\omega$ (in keV), $\Delta^{\pm}_\nu$ (dimensionless), Δ^{+}_0 (dimensionless). ε_0 (in degree), γ_{eff} (in degree) and η_{eff} (in degree).

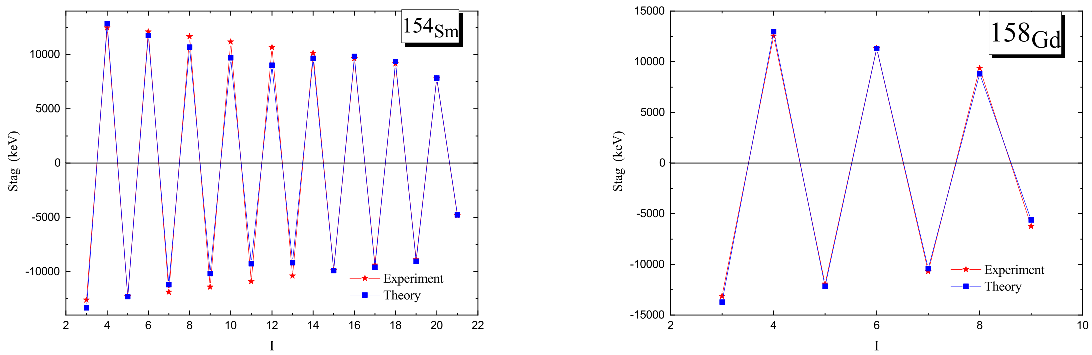

FIG. 1. Theoretical and experimental [18] behavior ΔI = 1 ”staggering” effect for ^{154}Sm (left) with adjusted model parameters: $\hbar\omega$ = 808.55 keV, Δ_0^+ = 222.26, Δ_0^- = 288.72, Δ_1^+ = 195.16, Δ_1^- = 212.69, $\varepsilon_0 = 4.35^0$, $\gamma_{\rm eff} = 5.42^0$, $\eta_{\rm eff} = 9.02^0$. RMS = 39.32 keV; and ^{158}Gd (right) with adjusted model parameters: $\hbar\omega$ = 388.61 keV, Δ_0^+ = 31.38, Δ_0^- = 79.16, Δ_1^+ = 49.63, Δ_1^- = 54.27, $\varepsilon_0 = 4.9^0$, $\gamma_{\rm eff} = 5.49^0$, $\eta_{\rm eff} = 51.03^0$. RMS = 32.21 keV.

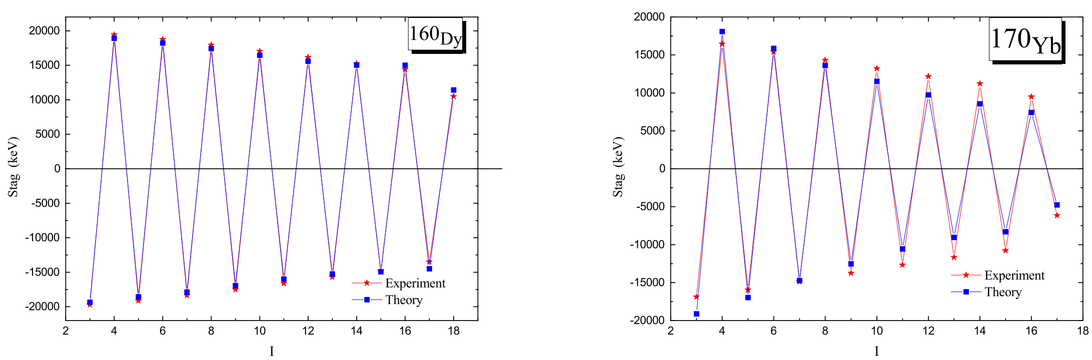

FIG. 2. The same as in Fig. 1, but for ^{160}Dy (left): $\hbar\omega$ = 801.87 keV, Δ_0^+ = 145.98, Δ_0^- = 204.99, Δ_1^+ = 129.28, Δ_1^- = 137.05, $\varepsilon_0 = 37.84^0$, $\gamma_{\rm eff} = 134.09^0$, $\eta_{\rm eff} = 23.46^0$. RMS = 61.71 keV; and ^{170}Yb (right): $\hbar\omega$ = 633.43 keV, Δ_0^+ = 64.12, Δ_0^- = 110.14, $\varepsilon_0 = 42.24^0$, $\gamma_{\rm eff} = 0.32^0$, $\eta_{\rm eff} = 89.92^0$. RMS = 99.55 keV..

III. ΔI = 1 ”STAGGERING” EFFECT

Traditionally it has been assumed that the odd-even ”staggering” effect in octupole deformation bands starts with relatively high values of the ”staggering” effect at low spin values and then gradually decreases down to zero, thus indicating the gradual formation of a reflection band with an asymmetric shape. However, using the most recent based on experimental data in the field of actinides [18], it has been found that in lanthanides and in light actinides the odd-even “staggering” effect shows a ”zigzag” behaviour [19]. In other words, the quantity measuring the odd-even ”staggering” effect does not remain close to the vanishing value after reaching zero for the first time, but continues to oscillate in absolute value as I increases, forming a zigzag shape.

In Fig. (1)-(3) the theoretical and experimental [18] shows a ”zigzag” behaviour ΔI = 1 ”staggering” effect for even-even nuclei: ^{154}Sm, ^{158}Gd, ^{160}Dy, ^{170}Yb and ^{232}Th. The captions to these figures show the values of the fitted model parameters and the obtained RMS values (in keV) for the alternating-parity spectra for the above-mentioned even-even nuclei. Note that the RMS value (at ≤ 100 keV) is a good criterion for the applicability of different models [20]. All the uniform nuclei considered are well described by the proposed model. It is clear that a ”zigzag” behaviour of the ”staggering” effect and the disappearance of this effect doesn’t occur within the observed angular momentum range.

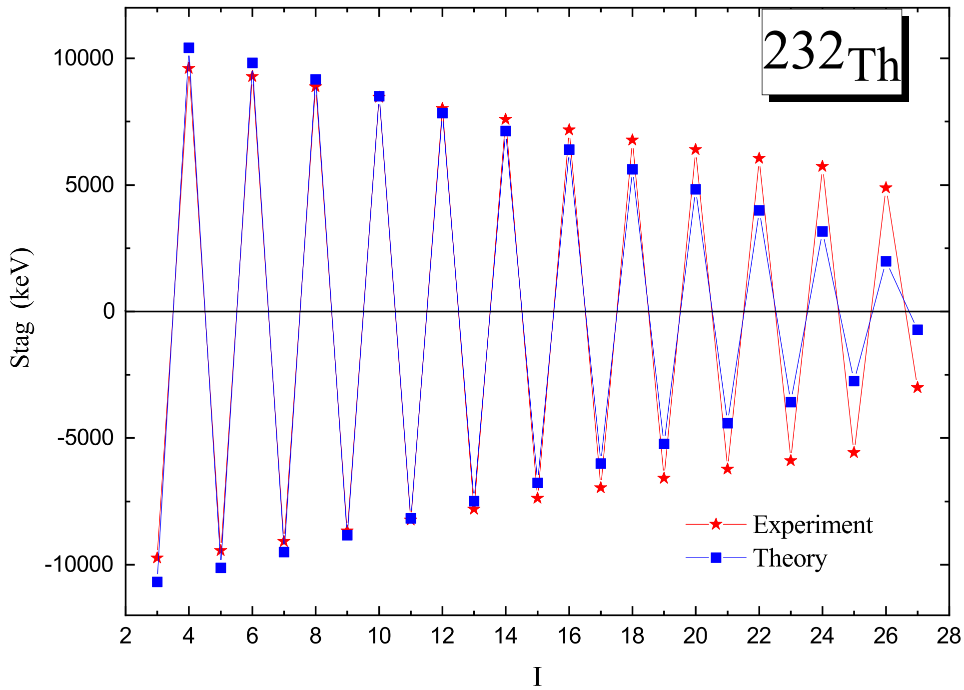

FIG. 3. The same as in Fig. 1, but for ^{232}Th: $\hbar\omega$ = 431.28 keV, Δ_0^+ = 207.4, Δ_0^- = 207.62, Δ_1^+ = 185.33, Δ_1^- = 222.26, $\varepsilon_0 = 32.23^0$, $\gamma_{\rm eff}$ = 119.08^0, $\eta_{\rm eff} = 35.77^0$. RMS = 61.42 keV.

IV. CONCLUSION

In this work, we propose a collective model formalism that incorporates the nonadiabatic coupling of axial quadrupole-octupole oscillations with the rotation of the triaxial quadrupole-octupole shape. The triaxial rotations are associated with opposite parities of states with odd and even values of collective angular momentum. The model thus provides a dynamic mechanism that can control the appearance of variable parity spectra in heavy even-even nuclei. Application of the model to yrast zones of variable parity in several rare earth and actinide nuclei shows a good reproduction of the corresponding experimental "staggering"-effect, and this effect is well reproduced in even-even nuclei: ^{154}Sm, ^{158}Gd, ^{160}Dy, ^{170}Yb and ^{232}Th. The effect studied at low values of the angular momentum of the level energy spectrum appears mainly due to a change in parity, while at high values of the angular momentum of the level energies it is due to the interaction between the rotation of the nucleus as a whole and the deformation of its shape.

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