UNIVERSALITY OF MULTIPLICITY AND TRANSVERSE MOMENTUM DISTRIBUTIONS IN THE FRAMEWORK OF PERCOLATION OF STRINGS*

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In the framework of string percolation, the transverse momentum distributions in heavy ion collisions at all centralities and energies are shown to follow an universal behaviour, the shape of the distributions depending essentially on the transverse density of strings. We find that the relative suppression of intermediate and high p_{T} production in central nucleus–nucleus collisions has the some origin as the narrowing of multiplicity distributions, the clustering of strings. The clustering of strings also explains naturally the dependence on the centrality of the transverse momentum fluctuations and the strength of the two and three body Bose–Einstein correlations.

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1. Introduction

Recent experimental results from the Relativistic Heavy Ion Collider (RHIC) [1–4] show that the inclusive high p_T hadron production is strongly suppressed compared to the scaling with the number of binary nucleon–nucleon collisions, N_{coll}, expected on the basis of the factorization theorem for hard processes in perturbative QCD (pQCD) [5]. One possible explanation of this suppression is the predicted quenching of produced jets in hot quark–gluon matter [6–7]. Alternative explanations have been pointed out as a consequence of gluon saturation in the Color Glass Condensate (CGC) [8–9] or the percolation of strings [10–13]. These two approaches explicate also the weak dependence of the multiplicity per participant on the number

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of participants [9–10, 14–15] shown by the different RHIC collaborations. In addition, both pictures lead to a relation between the mean multiplicity and the mean transverse momentum [9–10, 12] and to a scaling law for the $p_{\rm T}$ distributions [8, 10, 13] which explains at least partially the high $p_{\rm T}$ suppression mentioned above. In this paper, we describe the explanation given in the framework of percolation of strings.

Multiparticle production is currently described in terms of color strings stretched between the partons of the projectile and the target. These strings decay into new ones by sea q–$\overline{q}$ production, and subsequently hadronize to produce observed hadrons. Color strings may be viewed as small areas in the transverse space, πr_0^2, with $r_0 \simeq 0.2$–0.25 fm, filled with the color field created by the colliding partons. With increasing energy and/or atomic number of the colliding particles, the number of exchanged strings grows, and they start to overlap, forming clusters, very much like disks in continuum two-dimensional percolation theory. Each cluster of several individual strings behaves as a single string with a higher color field, and with energy momentum that correspond to the sum of the energy momenta of the overlapping strings. At a certain critical density $\eta_{\rm c} \simeq 1.18$–1.5 a macroscopical cluster appears which marks the percolation phase transition [16]. For nuclear collisions, this density corresponds to $\eta = N_{\rm S}\, S_1/S_A$ where $N_{\rm S}$ is the total number of strings, each of area S_1. S_A corresponds to the nuclear overlapping area. For central AA collisions $b=0 \quad S_A = \pi R_A^2$.

The percolation theory governs the geometrical pattern of the string clustering. Its observable implications, however, require the introduction of some dynamics in order to describe the behaviour of the cluster formed by several overlapping strings. We assume that a cluster of n strings behaves as a single string with a high color field $\vec{Q}_n$ corresponding to the vectorial sum of the color charge of each individual $\vec{Q}_1$ string. The resulting color field covers the area S_n of the cluster. As $\vec{Q}_n^2 = (\sum_1^n \vec{Q}_1)^2$, and the individual string colors may be oriented in an arbitrary manner respective to one another, the average $\vec{Q}_{1i}\,\vec{Q}_{1j}$ is zero, so $\vec{Q}_n^2 = n\,\vec{Q}_1^2$. $\vec{Q}_n$ depends also on the area S_1 of each individual string that comes into the cluster, as well on the total area of the cluster S_n,

$$Q_n = \sqrt{\frac{nS_n}{S_1}}\, Q_1 \,. \tag{1}$$

S_n corresponds to the total area occupied by n disks. Notice that if the strings are just touching each other $S_n = nS_1$ and $Q_n = nQ_1$ so the strings are independent of each other. On the contrary, if they fully overlap, $S_n = S_1$ and $Q_n = \sqrt{n}\,Q_1$. Knowing the color charge Q_n, one can compute the multiplicity μ_n and the mean transverse momentum $\langle p_{\rm T}^2\rangle_n$ of the particles produced by a cluster of n strings. One finds [10–11]

$$\mu_n = \sqrt{\frac{nS_n}{S_1}}\,\mu_1\,, \quad \langle p_{\mathrm{T}}^2\rangle_n = \sqrt{\frac{nS_1}{S_n}}\,\langle p_{\mathrm{T}}^2\rangle_1\,, \tag{2}$$

where μ_1 and $\langle p_{\mathrm{T}}^2\rangle_1$ are the mean multiplicity and mean p_{T}^2 of particles produced by a simple string. In the saturation limit, all the strings overlap into a single cluster that approximatively occupies the whole interaction area, one gets the following universal scaling law that relates the mean transverse momentum and the multiplicity per unit rapidity and unit transverse are [10, 12]

$$\langle p_{\mathrm{T}}^2\rangle_{AA} = \frac{S_1}{S_{AA}}\,\frac{\langle p_{\mathrm{T}}^2\rangle_1}{\mu_1}\,\mu_{AA}\,. \tag{3}$$

This relation is similar to the one obtained in the framework of CGC [8]. The comparison with all the experimental data for all kind of collisions at all available energies show a reasonable agreement [12]. Some small deviations occur and were expected due to the approximations done in obtaining (3) and the corrections to (3) due to the energy-momentum conservation.

In the limit of high density μ, one obtains

$$\left\langle \frac{nS_1}{S_n}\right\rangle = \frac{\eta}{1-\exp(-\eta)} \equiv \frac{1}{F(\eta)^2}\,, \tag{4}$$

and equations (2) are

$$\mu = N_{\mathrm{strings}}\;F(\eta)\mu_1\,, \quad \langle p_{\mathrm{T}}^2\rangle = \frac{1}{F(\eta)}\,\langle p_{\mathrm{T}}^2\rangle_1\,, \tag{5}$$

where μ and $\langle p_{\mathrm{T}}^2\rangle$ are the total multiplicity and mean momentum and N_{strings} is the total number of strings created in the collision. Notice that as $N_{\mathrm{strings}} \simeq N_A^{4/3}$, where N_A is the number of wounded nucleons, and $F(\eta) \simeq N_A^{-1/3}$ we obtain $\mu \simeq N_A$, *i.e.* the saturation of the multiplicity per participant. The detailed comparison with the experimental RHIC data at $\sqrt{s} = 130$ and 200 GeV show a good agreement. At LHC are predicted around 1800 charged particles per unit rapidity in central Pb–Pb collision what is lower than most of the predictions of other models.

The dependence of the transverse momentum fluctuations on the number of participants can be naturally understood from Eq. (2). At low density, most of the particles are produced by individual strings with the same $\langle p_{\mathrm{T}}\rangle_1$ so the fluctuations are small. Similarly, at large density above the percolation critical point, there is essentially only one cluster formed by most of the strings created in the collision and therefore fluctuations are not expected either. Instead, the fluctuations are expected to be maximal just below the percolation critical density, where there are cluster formed by very different number of strings, with different size, and therefore with different $\langle p_{\mathrm{T}}\rangle_n$. Indeed the comparison with RHIC and SPS data shows a good agreement [14–15].

2. Multiplicity and transverse momentum distributions

The multiplicity distribution can be expressed [17] as a superposition of Poisson distribution with different mean multiplicities

$$P\,(n) = \int dN\, W(N)\, P(N,n)\,. \tag{6}$$

The Poisson distribution $P\,(N,n) = \frac{\exp(-N)\,N^n}{n!}$, $N = \langle n\rangle$, represents the cluster fragmentation, while the weight factor $W\,(N)$ reflects the cluster size distribution. This quantity has contributions due to both the nuclear structure and the parton distribution inside the nucleon.

Concerning the p_{T} distribution, one needs the distribution $f(x, m_{\mathrm{T}})$ for each string or cluster and the cluster size distribution $W(x)$. For $f(x, m_{\mathrm{T}})$ we assume the Schwinger formula $f\,(x, m_{\mathrm{T}}) = \exp(-m_{\mathrm{T}}^2\, x)$. In this formula x is related to the string tension or equivalently to the mean transverse mass of the string. Therefore, we can write for the total m_{T} distribution

$$f(m_{\mathrm{T}}) = \int W(x)\; f(x, m_{\mathrm{T}})\,. \tag{7}$$

$W(x)$ is well approximated by gamma distribution

$$W(x) = \frac{\gamma}{\Gamma(k)}\,(\gamma x)^{k-1}\,\exp(-\gamma x)\,. \tag{8}$$

In fact, in peripheral heavy ion collisions, the density of strings is small and therefore there is no overlapping. The cluster size distribution in this case is peaked at low values. As the centrality increases, the density of strings also increases, so there is more and more overlapping among the strings. The cluster size distribution is strongly modified. The increase of centrality can be seen as a transformation of the cluster size distribution of the type

$$W(N) \longrightarrow \frac{N\;W(N)}{\langle N\rangle} \longrightarrow \ldots \frac{N^k\;W(N)}{\langle N^k\rangle} \longrightarrow \ldots\,. \tag{9}$$

This kind of transformation was studied by Jona-Lasinio related to the renormalization group in probabilistic theory [18] and correspond to a size-biasing transformation changing the size of one string by cluster size and the corresponding associated variables, μ and $\langle p_{\mathrm{T}}\rangle$.

Introducing (8) into (6) and (7) we obtain

$$\frac{\Gamma\,(n+k)}{\Gamma\,(n+1)\,\Gamma(k)}\;\frac{\gamma'^k}{(1+\gamma')^{n+k}}=\int\limits_0^\infty dN\;\frac{e^{-N}N^n}{n!}\;\frac{\gamma'}{\Gamma(k)}\,(\gamma'N)^{k-1}\,\exp\left(-\gamma'N\right), \tag{10}$$

$$\left(1+\frac{m_{\mathrm{T}}^2}{\gamma}\right)^{-k}=\int\limits_0^\infty dx\;\exp\left(-m_{\mathrm{T}}^2x\right)\frac{\gamma}{\Gamma(k)}\,(\gamma x)^{k-1}\,\exp\left(-\gamma x\right). \tag{11}$$

The distribution obtained in (10) is the well known negative binomial distribution, whose mean value and dispersion verify

$$\langle n\rangle=\langle N\rangle=\frac{k}{\gamma'}\,,\quad \frac{\langle N^2\rangle-\langle N\rangle^2}{\langle N\rangle^2}=\frac{1}{k}\,,\quad \frac{\langle n^2\rangle-\langle n\rangle^2}{\langle n\rangle^2}=\frac{1}{k}+\frac{1}{\langle n\rangle}\,. \tag{12}$$

The corresponding values for (11) are

$$\langle x\rangle=\frac{k}{\gamma}\quad,\quad \frac{\langle x^2\rangle-\langle x\rangle^2}{\langle x\rangle^2}=\frac{1}{k}\,. \tag{13}$$

As we have said before, the increase of centrality can be seen as a process of size-biasing [18–20] which modifies the cluster size distribution, increasing the value of k and redefining the multiplicity and mean transverse momentum of the particles created by the new clusters. Indeed, the invariance of the weight function under the transformation $x\to\lambda\,x$ and $\gamma\to\gamma/\lambda$ where $\lambda=F(\eta)$ due to (5), leads to the changes $m_{\mathrm{T}}^2\to m_{\mathrm{T}}^2\,\lambda$ and $\gamma'\to\gamma'/\lambda$ in the transverse mass and multiplicity distributions, respectively. According to these considerations, $k\sim 1/F(n)$. On the other hand, the fact that the p_{T} distribution becomes harder as the energy increases has been taken into account by assuming $k\sim\mu$ where $\mu=1/(1-\sqrt{\alpha_s})$ [13].Therefore the p_{T} distribution is

$$\frac{A}{\left(\gamma+F(\eta)\,m_{\mathrm{T}}^2\right)^k}\,. \tag{14}$$

We use

$$k=\mu\left(c_1+\frac{c_2}{F(\eta)}\right) \tag{15}$$

for $\eta>0.2$. With the values $c_1=3/2$ and $c_2=1$ is obtained a good agreement with the data on π^0 production for central Au–Au and peripheral Au–Au collisions at $\sqrt{s}=200$ GeV as it is seen in Fig. 1. In order to obtain the values for the densities η, we have computed the number of created strings using a Monte-Carlo code based on the quark-gluon string model.

The comparison for π, k and p production is also good [13]. The WA98 data on Pb–Pb central and peripheral collisions at SPS energies are also reproduced. In Fig. 2 we show our results for d-Au central and minimum bias collisions. We observe less suppression than the one obtained in Au–Au collisions, although more than the general trend of experimental data.

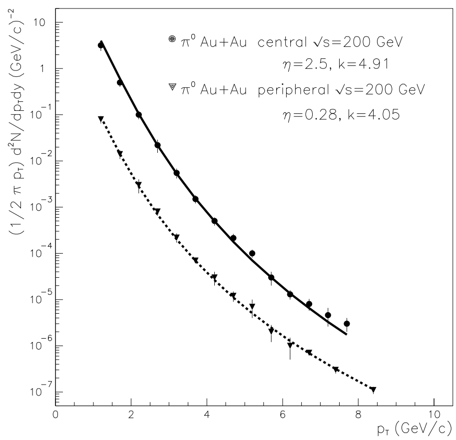

Fig. 1. Comparison between our results and experimental data from Au–Au central and peripheral collisions at $\sqrt{s} = 200$ GeV.

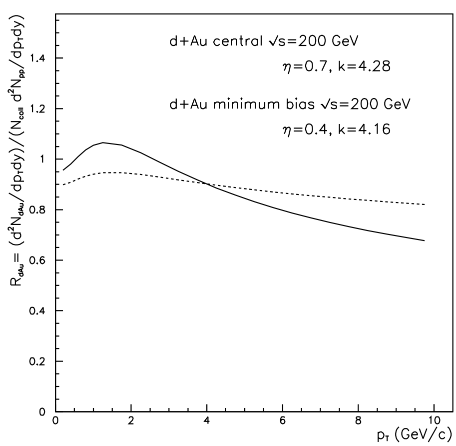

Fig. 2. Our results for d-Au central and minimum bias collisions at $\sqrt{s} = 200$ normalized to our results for p–p minimum bias collisions multiplied by the number of collisions.

3. Conclusions

The general trend of our results agree with the obtained independently in [21] using similar ideas. We have obtained a reasonable description of the p_{T} distributions by means of universal scaling. The high p_{T} suppression in this approach is a consequence of the clustering of strings. This effect is also able to describe the centrality dependence of p_{T} fluctuations. We are aware of the limitations of our approach, in particular the exponential formula used for the cluster fragmentations, that neglects the hard p_{T} tail. This can be at the origin of the differences with d-Au data. We obtain suppression of the tail of multiplicity distributions.

Our approach has similarities with the CGC. In both approaches the initial state interactions — gluon saturation in the CGC or clustering of strings in the percolation approach — produce a suppression of the p_{T} distributions. In both approaches there is a suppression of high p_{T} and multiplicities. On the contrary, in the framework of the jet quenching phenomena, the energy loss of the jet produces additional soft gluons that would fragment into hadrons increasing the multiplicities, unless strong shadowing occurs in the gluon structure functions.

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REFERENCES

[1] K. Adcox *et al.*, PHENIX Collaboration, *Phys. Rev. Lett.* **B561**, 82 (2003); S. Mioduszewski, PHENIX Collaboration, *Nucl. Phys.* **A715**, 199 (2003).

[2] C. Adler *et al.*, STAR Collaboration, *Phys. Rev. Lett.* **90**, 032301 (2003); G.J. Kunde, STAR Collaboration, *Nucl. Phys.* **A715**, 189 (2003).

[3] B.B. Back *et al.*, PHOBOS Collaboration, `nucl-ex/0302015`.

[4] I. Arsene *et al.*, BRAHMS Collaboration, *Phys. Rev. Lett.* **91**, 072305 (2003).

[5] J.C. Collins, D.E. Soper, G. Sterman, *Nucl. Phys.* **B308**, 831 (1988).

[6] M. Gyulassy, I. Vitev, X.N. Wang, *Phys. Rev. Lett.* **86**, 2537 (2001).

[7] R. Baier, Y.L. Dokshitzer, A.H. Muller, S. Peigné, D. Schiff, *Nucl. Phys.* **B438**, 291 (1997).

[8] J. Schaffner-Bielich, D. Kharzeev, L. McLerran, R. Venugopalan, *Nucl. Phys.* **A705**, 494 (2002).

[9] D. Kharzeev, E. Levin, L. McLerran, *Phys. Lett.* **B561**, 93 (2003).

[10] M.A. Braun, F. del Moral, C. Pajares, *Phys. Rev.* **C65**, 024907 (2002); *Nucl. Phys.* **A715**, 791c (2003).

[11] M.A. Braun, C. Pajares, *Phys. Rev. Lett.* **85**, 4864 (2001).

[12] J. Dias de Deus, E.G. Ferreiro, C. Pajares, R. Ugoccioni, `hep-ph/0303220`.

[13] J. Dias de Deus, E.G. Ferreiro, C. Pajares, R. Ugoccioni, `hep-ph/0304068`.

[14] E.G. Ferreiro, F. del Moral, C. Pajares, `hep-ph/0303137`.

[15] J. Dias de Deus, A. Rodrigues, `hep-ph/0308011`.

[16] N. Armesto, M.A. Braun, E.G. Ferreiro, C. Pajares, *Phys. Rev. Lett.* **77**, 3736 (1996); M. Nardi, H. Satz, *Phys. Lett.* **B442**, 14 (1998).

[17] M. Kutschera, K. Werner, *Z. Phys.* **C45**, 91 (1989).

[18] G. Jona-Lasinio, *Nuovo Cim.* **B26**, 99 (1975).

[19] J. Dias de Deus, C. Pajares, C.A. Salgado, *Phys. Lett.* **B408**, 417 (1997); *Phys. Lett.* **B407**, 335 (1997); *Phys. Lett.* **B409**, 474 (1997).

[20] S. Hegyi, *Phys. Lett.* **B411**, 321 (1997); J. Dias de Deus, C. Pajares, *Phys. Lett.* **B442**, 395 (1998).

[21] J. Ranft, R. Engel, S. Roesler, *Nucl. Phys.* **B** (*Proc. Suppl.*) **122**, 392 (2003); F.W. Bopp *et al.*, *Acta Phys. Pol. B* **35**, 303 (2004), these proceedings.