

Mathematical modeling of skyrmion shape deformation under uni-axial stresses

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Abstract. Skyrmion is a topologically stable spin texture and expected to be applied to the future computer memory. On the semiconductors such as FeGe and MnSi, the skyrmion configuration is stable in the sense that it is not strongly affected by a small variation of external stimuli such as temperature, magnetic field etc. In recent experiments, it was reported that the skyrmion shape is deformed from isotropic (or circular shape) to anisotropic (or elliptic shape) by an external mechanical stress. This shape deformation is caused by the so-called “strain-induced anisotropy (SIA)” of Dzyaloshinskii-Moriya interaction (DMI). In this presentation, we study the reason why this SIA appears in the DMI coefficient on the basis of Finsler geometry modeling technique by introducing a microscopic strain field τ , which is caused by or interacts with the applied external mechanical force.

Keywords: *Skyrmions, DM interaction, Strain-induced anisotropy, Finsler geometry, Monte Carlo*

1. Introduction

Skyrmions form the so-called skyrmion crystal (skx) phase, in which the spin texture is stable against external stimuli such as small fluctuations of temperature, magnetic field, etc. [1, 2, 3]. It is also shown that 2D skx is stable on elastic surfaces with small in-plane and out-of-plane fluctuations [4]. These types of stability are expected to be useful for the memory application. For this reason, the mechanical property of skyrmions has been studied intensively and extensively. In Ref.[5], it was experimentally shown that the mechanical stress can be used to annihilate/create skyrmions. This implies that there exists a non-trivial interaction between material elasticity and electron spin. Indeed, the experimental results shown in Ref.[6] indicate that the shape of skyrmion changes from circular to elliptical shapes under a uniaxial mechanical stress. The reason for this anisotropic shape deformation was also reported in [7] such that DMI coefficient D becomes direction dependent $D \rightarrow (D_x, D_y)$, $D_x \neq D_y$, which is called “strain-induced anisotropy (SIA)”. Moreover, it was also reported that the origin of this SIA of DM interaction comes from its spin-orbit interaction or magnetoelastic coupling. However, the magnetoelastic coupling is very complex, and therefore it is useful to obtain a simple effective theory for a certain intuitive understanding of the reason for the anisotropy in the DMI coefficient.



It is reported that in the Finsler geometry (FG) model for membranes the surface tension γ and/or the bending rigidity κ effectively change from constants to position-and-direction dependent ones [8]. Indeed, these material parameters, in the FG model, vary just like functions inside the material depending on the internal structure such as liquid crystal (LC) in liquid crystal elastomer (LCE) or polymer chains in polymeric materials [9]. Especially, the direction dependence of these material parameters is crucial to the mechanical properties such as the so-called soft-elasticity or the non-isotropic shape deformation. Therefore, it is natural to consider that the DMI coefficient D also becomes direction-dependent if a certain internal structure plays a role in modifying the derivative interaction of DMI via the Finsler metric. Therefore, it is worthwhile to study the reason why D becomes direction-dependent by FG modeling technique.

2. Model

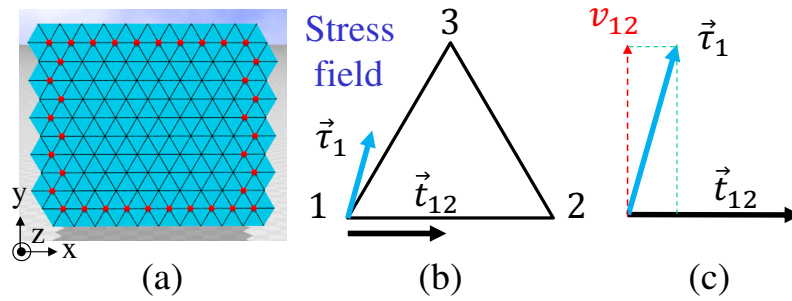


Figure 1. (a) Triangular lattice with periodic boundary conditions, (b) strain field τ_1 , and (c) the Finsler length v_{12} along the direction of \vec{e}_{12} . The small squares in (a) are the boundary vertices (BV), outside of which are vertices to be identified with BV at the opposite side by PBC.

We start with the continuous form of Hamiltonian for DMI, which is given by

$$S_{\text{DM}} = \frac{1}{2} \int \sqrt{g} d^2x g^{ab} \vec{e}_a \cdot \sigma \times \frac{\partial \sigma}{\partial x_b}, \quad (1)$$

where $\vec{\sigma} (\in S^2: \text{unit sphere})$ denotes the spin variable, and $x_a (a=1, 2)$ are a local coordinate on the 2-dim surface, of which the position is denoted by a three-dimensional vector $\vec{r} (\in \mathbf{R}^3)$, where z component is fixed to zero (Fig. 1(a)). The symbol $\vec{e}_a (= \partial \vec{r} / \partial x_a)$ is the tangential vector of the surface along the positive direction of x_a . The matrix g^{ab} and g are the inverse and determinant of Finsler metric g_{ab} , respectively. The discrete form of g_{ab} is given by $g_{ab} = \begin{pmatrix} 1/v_{12}^2 & 0 \\ 0 & 1/v_{13}^2 \end{pmatrix}$. In this expression, v_{ij} is a Finsler length and is defined by

$$v_{ij} = \sqrt{1 - |\vec{t}_{ij} \cdot \vec{\tau}_i|^2} + v_0, \quad \vec{t}_{ij} = \vec{\ell}_{ij} / \ell_{ij}, \quad \vec{\ell}_{ij} = \vec{r}_j - \vec{r}_i, \quad (2)$$

where \vec{t}_{ij} is the unit tangential vector from the vertices i to j corresponding to \vec{e}_a in Eq. (1). The symbol $\tau_i (\in S^1: \text{unit circle})$, on every vertex (Fig. 1(b)), denotes "internal strain field" corresponding to displacement in atomic position. The constant v_0 in v_{ij} is fixed to $v_0 = 1.5$, which not the cutoff in this case. Compared to the case that v_0 is treated as a cutoff, anisotropy in v_{ij} is expected to be small. The strain field $\vec{\tau}$ has values on the unit circle S^1 , because we are interested only in its directional change for the first-order approximation. The sign of $\vec{\tau}$ should be defined by whether the external force is tensile or compressible, however, we simply assume

that the roles of $\vec{\tau}$ and $-\vec{\tau}$ are the same because of the definition of v_{ij} in Eq. (2). The unit of Finsler length v_{12} along the direction of \vec{t}_{12} is shown in Fig. 1(c).

Now, let us introduce the discrete Hamiltonian S by including the exchange interaction S_0 of spins and the Zeeman energy S_B such that

$$S(\sigma, \tau) = \lambda S_0 + \delta S_{DM} - S_B - S_{\text{ext}}, \quad S_0 = \sum_{ij} (1 - \vec{\sigma}_i \cdot \vec{\sigma}_j), \quad (3)$$

$$S_{DM} = \sum_{ij} d_{ij} \vec{t}_{ij} \cdot \vec{\sigma}_i \times \vec{\sigma}_j, \quad S_B = \sum_i \vec{\sigma}_i \cdot \vec{B}, \quad \vec{B} = (0, 0, B), \quad S_{\text{ext}} = \sum_i (\vec{\tau}_i \cdot \vec{f})^2,$$

where \sum_{ij} denotes the sum over all bonds ij connecting the nearest neighbor vertices i and j . The triangular lattice on the xy plane ($\subset \mathbf{R}^3$) is used to define these energy functions (Fig. 1(a)). The coefficients d_{ij} ($= d_{ji}$) in S_{DM} are given by

$$d_{12} = \frac{1}{6} \left(\frac{v_{12}}{v_{13}} + \frac{v_{21}}{v_{23}} \right), \quad d_{23} = \frac{1}{6} \left(\frac{v_{23}}{v_{21}} + \frac{v_{32}}{v_{31}} \right), \quad d_{31} = \frac{1}{6} \left(\frac{v_{31}}{v_{32}} + \frac{v_{12}}{v_{13}} \right), \quad (4)$$

where v_{ij} is Finsler length defined on the three edges of triangle Δ_{123} (Fig.1(b)). The detailed information on how to obtain the discrete d_{ij} will be reported elsewhere.

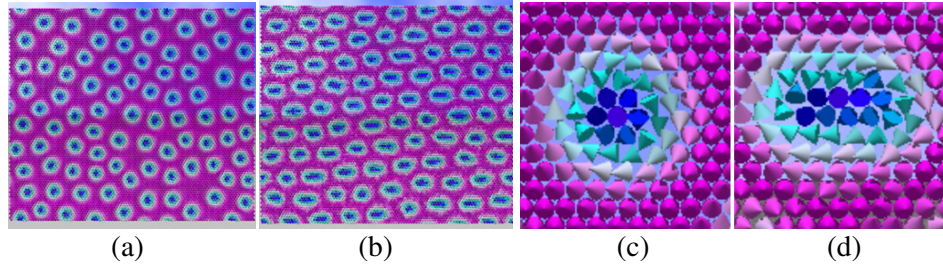


Figure 2. Snapshots of skyrmions in (a) isotropic with zero external force $f = 0$ and (b) anisotropic skyrmions with finite f , and (c), (d) their enlarged snapshots, The spins are represented by cones, which are pointing to the spin direction.

3. Monte Carlo results

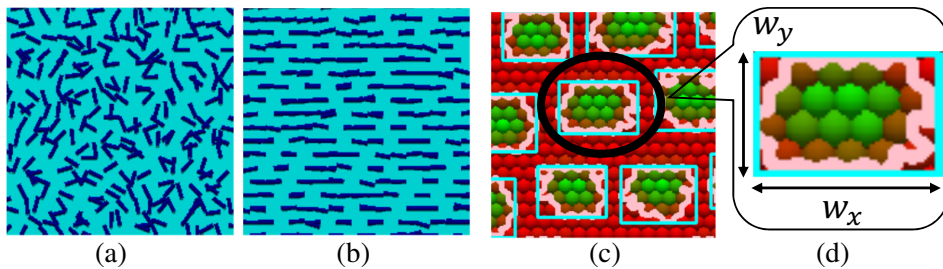


Figure 3. Snapshots of internal strain field $\vec{\tau}$ corresponding to (a) isotropic sky phase with $\vec{f} = (0, 0)$ and (b) anisotropic skyrmions with finite $\vec{f} = (f, 0)$. (c) Snapshots for the graphical measurement of the vertical and horizontal widths W_x , W_y of skyrmion and (d) its enlarged view, where the spins are represented by spheres for simplicity.

The dynamical variables to be integrated out in the partition function are σ and τ . The lattice size assumed in the simulations is $N = 10000$. Snapshots of sky phase for the external forces

$\vec{f} = (0, 0)$ and $\vec{f} = (f, 0)$ (Figs. 2(a),(b)), and their enlarged views (Figs. 2(c),(d)) are shown. The external force f is applied into the horizontal direction, along which the skyrmion shape deforms to be oblong. Internal strain fields τ_i corresponding to $\vec{f} = (0, 0)$ and $\vec{f} = (f, 0)$ (Figs. 3(a),(b)) are random and aligned along the horizontal direction, respectively. To measure the anisotropy of skyrmions, the horizontal and vertical sizes W_x and W_y are graphically measured (see Figs. 3(c),(d)). The anisotropy $\delta = (1 - W_y/W_x)/(1 + W_y/W_x)$ is calculated using these W_x and W_y , and the distribution of δ is plotted in Figs. 4(a),(b). The same technique is applied to the snapshots of skyrmions of simulation data reported in Ref.[6]. The “Exp” represent the reported data for experimental measurements in Ref.[6]. We find that the results are almost comparable. The mean value of the coefficients (D_x, D_y) of DMI can be obtained and will be reported elsewhere including the detailed information of parameters used in the simulations.

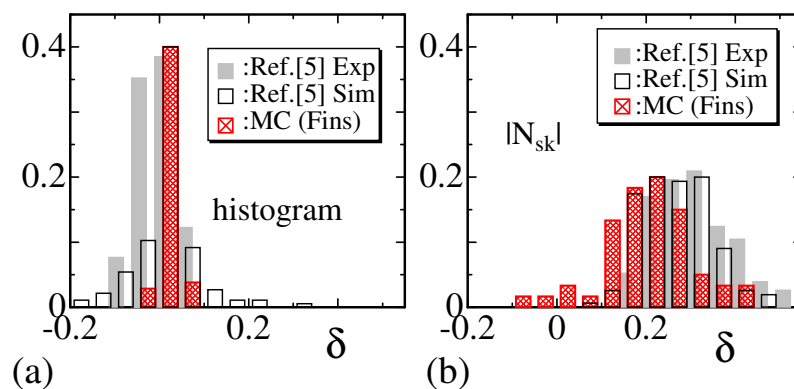


Figure 4. Distribution of $\delta = (1 - W_y/W_x)/(1 + W_y/W_x)$ for (a) isotropic and (b) anisotropic configurations in the skx phase. The symbol Exp denotes the data in [6], MC (Fins) denotes the results obtained by a graphical measurement of the simulation data, Sim is also obtained by applying the same technique to the data reported in [6].

4. Concluding remarks

This paper reports Finsler geometry (FG) modeling technique for the skyrmion shape deformation caused by an external tensile force, and that the Monte Carlo results are consistent with the reported experimental data. In the FG model, the internal strain field τ_i is introduced in addition to the spin degrees of freedom. The direction of this τ_i is controlled by the external tensile force \vec{f} . This allows us to understand intuitively why and how the DMI coefficient effectively depends on the force direction. The ferromagnetic interaction S_F can also be modified by Finsler metric, and a lot of numerical studies are necessary to find where the anisotropy of magnetoelastic coupling comes from.

Acknowledgment

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