SUPERCONVERGENCE RELATION AND PARITY VIOLATING ANALOGUE OF GDH SUM RULE

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Abstract

Sum rule of superconvergence type for parity violating amplitudes (p.v. analogue of Gerasimov-Drell-Hearn sum rule) is considered. Elementary processes initiated by polarized photons in the lowest order of electroweak theory are calculated as an examples illustrating the validity of the p.v.sum rule. The class of parity violating photon-induced processes for proton (nucleon) targets is considered in the frame of effective low energy theories and phenomenological models based on p.v. nucleon-nucleon and nucleon-meson effective interactions. Assuming the saturation of p.v. sum rule the possibility to limit the range of the parameters used in phenomenological models and badly known from existing experimental data is discussed.

The GDH sum rule [1] and its verification is recently intensively studied [2, 3] and considered as a clean and important test of the spin dynamics and the knowledge of the nucleon spin structure especially in the resonance region. First direct data for real photons taken at MAMI [3] are especially important in understanding the subject and new data also with higher energies are now available and expected in future from ELSA, GRAAL, CEBAF and Spring-8 [4, 14]. The experiments based on intense polarized beams of photons [3, 6] give also the opportunity to test the weak (parity violating) part of photon-hadron interactions. As it was argued in the paper [7] the polarized photon asymmetry in π^+ photo-production near the threshold could be a good candidate to measure p.v. pion-nucleon coupling h^1_π. The knowledge of p.v. couplings in nucleon-meson and nucleon-nucleon forces is a very important point for understanding physics of nonleptonic weak p.v. hadronic interactions. In addition the $\gamma\rho\pi$ and $\gamma\Delta N$ p.v. couplings, poorly known, can also play role in photon-induced reactions.
The h^1_π coupling has been also detected in the more complex atomic systems [8, 9]. The extraction of h^1_π from atomic experiments is however more difficult due to poor understanding of many-body systems with correlations and the disagreement between ^{18}F and ^{133}Cs based experiments is seen.
Recently a set of sum rules for parity violating part of Compton amplitudes has been derived in [10]. These new sum rules might be useful in checking the consistency of various theoretical approaches used in a description of low energy hadronic structure. The p.v. amplitudes for two different cases have been considered : for polarized photons scattered off unpolarized target and for unpolarized photons and polarized target particles. In this

talk I will concentrate on the first possibility (polarized photons and unpolarized target). Let us define by $f_h^{(-)\gamma}$ and σ_h^T the amplitude and the relevant total cross section averaged over target particle spin, respectively. h indicates photon helicity eigenvalue and ω is the photon energy in the laboratory system. As it was shown in [10] the superconvergence relation for amplitude $f_h^{(-)\gamma}$:

$$\frac{f_h^{(-)\gamma}(\omega)}{\omega}|_{\omega\to\infty} \to 0 \quad (1)$$

leads to p.v.analogue of GDH sum rule in the following form:

$$\int_{\omega_{th}}^{\infty} \frac{\sigma_h^T - \sigma_{-h}^T}{\omega'} d\omega' = 0 \quad (2)$$

To verify the p.v. sum rule (2) the simple examples of the elementary parity violating processes have been considered: the photon-neutrino reaction into W boson and electron, the photon-electron reaction with neutrino and W boson and with electron and Z^0 boson production. It is straightforward to verify that the integral in p.v. analogue of GDH sum rule 2, calculated in the lowest order of perturbation electroweak theory is zero for the difference of the cross sections averaged over lepton target spin as well as for left- and right-handed lepton separately. In the lowest order of the perturbation theory the integral in p.c. GDH sum rule for lepton targets is also zero [1] but for p.v. sum rule must be zero in any order of perturbation theory. Let us now discuss the more interesting nucleon (proton) target case. The one of the most interesting feature of GDH sum rule for nucleon targets is a quick saturation of the GDH integral. The dominant contribution (about 90%) to the GDH sum rule proceeds from the photon's energy range from the threshold up to 0.55 GeV [2, 3, 4, 14]. To discuss the saturation effect for p.v. sum rule the following saturation criterion quantity F is defined:

$$F = \left| \frac{\int_{\omega_{th}}^{\omega_{sat}} \frac{\sigma_h^T - \sigma_{-h}^T}{\omega'} d\omega}{\int_{\omega_{th}}^{\omega_{sat}} \frac{|\sigma_h^T - \sigma_{-h}^T|}{\omega'} d\omega'} \right|. \quad (3)$$

The smallness of F indicates that the relatively large contributions from the differences of the cross sections cancel and changes sign in the photon energy range defined by ω_{sat}.

The Heavy Baryon Chiral Perturbation Theory (HBχPT) is a natural candidate for the low energy theory of strong interaction. As a first example of the p.v. photon scattering off nontrivial, composite targets we consider the p.v. Compton amplitude on proton calculated in the frame of HBχPT. According to [11] the p.v. Compton amplitude can be written in CMS as follows:

$$\begin{aligned} M_{h_f,h_i}^{(-)s_f,s_i}(\vec{k},\vec{k}') &= \overline{N_{s_f}}[F_1\vec{\sigma}\cdot(\hat{\vec{k}}+\hat{\vec{k}}')\vec{\epsilon_i}\cdot\overline{\vec{\epsilon}_f'} - F_2(\vec{\sigma}\cdot\overline{\vec{\epsilon}_f'}\hat{\vec{k}}'\cdot\vec{\epsilon_i}+\vec{\sigma}\cdot\vec{\epsilon_i}\hat{\vec{k}}'\cdot\overline{\vec{\epsilon}_f'}) \\ &- F_3\hat{\vec{k}}\cdot\overline{\vec{\epsilon}_f'}\hat{\vec{k}}'\cdot\vec{\epsilon_i}\vec{\sigma}\cdot(\hat{\vec{k}}+\hat{\vec{k}}') - iF_4\vec{\epsilon_i}\times\overline{\vec{\epsilon}_f'}\cdot(\hat{\vec{k}}+\hat{\vec{k}}')]N_{s_i}, \end{aligned} \quad (4)$$

and the interesting quantity for discussing p.v. sum rule and superconvergence relation is $F_4 = -\frac{1}{2}f_{h=+1}^{(-)\gamma}$. The calculations based on HBχPT analysis in NLO provides value of

[1]The GDH integral in QED is zero in the lowest order of perturbation series; the anomalous magnetic moment of electron contributes in higher order α^3

the coefficient F_4 as follows:

$$F_4^{NLO} = -\frac{e^2 g_A h_\pi^1 \mu_n}{8\sqrt{2}\pi^2 M F_\pi}(\omega - \frac{m_\pi^2}{\omega} arcsin^2(\frac{\omega}{m_\pi})). \quad (5)$$

It is easy to check that the high energy behavior of $\frac{F_4^{NLO}}{\omega}$ leads to constant and superconvergence relations (1) is violated and it is not guarantee that p.v. sum rule (2) holds. However a priori it is not excluded that p.v. sum rule might be satisfied if more corrections are taken into account in the frame of HBχPT (the p.v. amplitude F_4 contains only leading p.v. coupling h_π^1) as a working example we have considered low energy phenomenological model of pion photoproduction based on so-called pole approximation and effective Lagrangians [12]. The model discussed in [12] are relevant in low energy regime so we will limit ourselves to energy below 0.55 GeV. The contributions from high energy region have been assumed to be unimportant.
The p.v. cross sections of the polarized photon initiated π^+ and π^0 production are expressed by the sum of the p.v. coupling parameters represent strength of different meson's contribution to the processes, multiplied by formfactors (see [12]). In our calculations the ρ meson $(h_\rho^0, h_\rho^1, h_\rho^2)$, ω meson (h_ω^0, h_ω^1) and Δ (f_Δ) contributions have been taken into account. For π^+ production the important contribution leads from p.v. πNN coupling (h_π^1). In addition there are two extra contributions from Δ meson directly coupled to photon and nucleon (μ^*, $\gamma\Delta$N) and from interaction between photon, pion and ρ meson $(h_E, \gamma\rho\pi)$. The last two parameters are directly related to the p.v. photon-meson and photon-Δ-nucleon interactions while the previous ones are related to strong sector (p.v. meson-nucleon couplings). The direct p.v. photon-nucleon interaction is zero in the low energy limit. The knowledge about the values of all p.v. couplings is very limited. The strong sector meson-nucleon couplings can be calculated in different approaches and models as it is discussed in [13] in details. The eight sets of couplings which are summarized in Table 1 in [13] have been used by us to estimate the cross sections for pion photoproduction according [12] and to verify the quick saturation of p.v. sum rule. The μ^* and h_E parameters have been treated as a free parameters in the range limited by the experimental knowledge: $\mu^* \in (-15, 15)$ and $h_E \in (-17, 17)$ in units 10^{-7}. The contribution related to f_Δ parameter is very small in considered approach due to the fact that relevant formfactor is small and we fix this coupling as in [12] to be 10^{-7}. The predictions of the p.v. couplings listed in Table 1 from [13] are grouped in five columns depending on the method of calculations. The first two columns obey three sets of values, called in our notations DDH. They are based on the calculations from [14]. The third and fourth column (D) are related to the calculations done in [15]. The column 3 is updated with corrections comparing to column 4 as it is indicated by the range parameter K. The last set of couplings (column 5, KM) is based on HBχPT type calculations, [16]. Our analysis has been done in the following way. For every set of strong sector couplings the μ^* and h_E couplings have been drawed randomly from the allowed range of values and the differences of polarized photon cross sections have been calculated and used for estimation of saturation parameter F. The saturation hypothesis is expressed by the condition that $F < 0.1(10\%)$. The saturation condition limits the allowed values of parameters μ^* and h_E and the polarized photoproduction asymmetries for π^0 and π^+ have been calculated for the limited range of the couplings. In fig. 1 the predictions for polarized photon asymmetries for π^+ and π^0 production based on quick saturation assumption are shown for

one of the models (D with range parameter K=3, set 5 in Table 1) discussed in [13]. The asymmetries are relatively large and well constrained by saturation effect (shadowed regions).

Two of the considered sets of couplings (DDH second and third in Table 1 from [13]) do not satisfy saturation condition $F < 0.1$ which leads to the conclusions that the higher photon energy contribution is needed to fill p.v. sum rule and some structure in the difference of the polarized cross sections should be observed for photon energy higher than 0.55 GeV.

To summarize: We have examined the p.v. sum rule (p.v. analogue of GDH sum rule) formulated in [10] and discussed the possible quick saturation of the sum rule as it takes place in famous GDH sum rule measured on nucleon targets. The p.v. sum rule have been verified by straightforward calculations in the lowest order of electroweak theory for the photon induced processes with elementary lepton targets. The quick saturation hypothesis as we think related to the complexity of the target has been used to limit freedom in choice of μ^* and h_E couplings (badly constrained by experimental data) for the case of proton target. The saturation of the p.v. sum rule has been tested for eight different sets of p.v. nucleon-meson couplings and in six cases quick saturation has been found to be possible. The most constrained (by quick saturation condition) predictions for asymmetries for pion photo-production are showed.

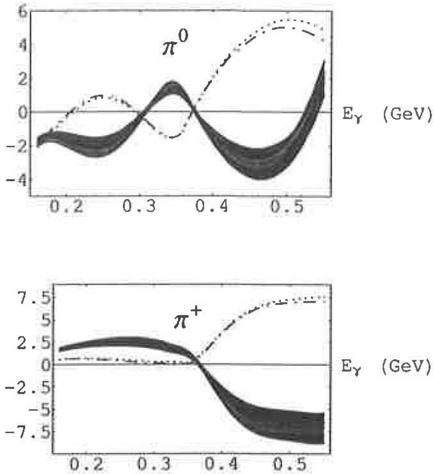

Figure 1: The pion p.v. asymmetries (in units 10^{-7}) calculated for model D (K=3) (set 5) according notation in Table 1 from [13]. The shadowed regions represent still possible freedom in choice of μ^* and h_E couplings constrained by quick saturation hypothesis. The darker and lighter part are related to different combinations on signs of the μ^* and h_E. The dashed lines correspond another possible choice of μ^* and h_E couplings not discussed in this talk

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