ELECTROSTATIC BEAM LINES FOR NEUTRINO AND MUON BEAMS AT TEVATRON AND YHK ENERGIES

G. Vesztergombi*
DPhPE, CEN-Saclay, 91191 Gif sur Yvette Cedex, France

1 - Introduction

Neutrino and muon beams provide unique opportunity at fixed-target hadronic machines to study weak and electromagnetic effects with a luminosity unattainable at collider machines. This makes fixed-target physics competitive despite relatively modest central mass energies. According to my knowledge, all the lepton (and also hadron) beams were made in the past and planned in the future using magnetic fields generated by normal or superconducting currents. This feature makes these beams more and more expensive with the increase of beam energy because one needs to hold together the parent particles along one (or more) kilometer(s) of the decay channel, due to the fact that the decay length is proportional to the energy. Figure 1 shows a typical case, the excellent muon line at the CERN-SPS which consists of 73 magnets consuming 6 MW power in dc and 2 in the pulsed mode.[1]

I know only one place where electric field plays a modest, but very important role, namely, in the beam extraction from the main ring of synchrotrons. Here we should like to demonstrate that most of the magnetic fields in muon or neutrino beam lines can be replaced by electrostatic fields of extremely simple construction. Knowing that 3×10^6 V/cm electric field can produce bending power equivalent to 1 Tesla = 10 kG magnetic field, it seems to be unbelievable that it is practical to manipulate beams with energies of many hundreds GeV by 100 kV/cm electric fields. The simple fact, however, that one can produce an electrostatic field cheaply along many kilometers can compensate its local weakness. Another advantage of the electrostatic field is that one can easily create such a configuration that produces simultaneous focusing in both horizontal and vertical directions practically independent of the particle energy.

2 - Longitudinal Versus Transverse Energy

The fact that intense and well collimated muon or neutrino beams can be produced by relatively weak electrostatic fields relies on two basic characteristics of their generation.

a) It is a general feature of hadronic interactions that the transverse momentum distribution of secondaries (mainly pions) is limited

$$d\sigma/dp_\perp \propto \exp\{-6p_\perp(\mathrm{GeV/c})\}.$$

This means that most of the secondaries have, e.g., $p_\perp < 500$ MeV/c. For simplicity, in what follows, we assume only pion secondaries.

b) The transverse momentum arising from the decay process is even more limited

$$p_\perp^\pi < 30 \text{ MeV/c for pions}$$

$$(p_\perp^K < 236 \text{ MeV/c for kaons}).$$

*On leave from Central Research Institute, Budapest, Hungary

Therefore it makes sense to introduce the notion of longitudinal and transversal energy for a particle having momenta $p_\parallel$ and $p_\perp$

$$E_\parallel = \sqrt{m^2 + p_\parallel^2} \approx p_\parallel$$

$$E_\perp = E - E_\parallel = \sqrt{m^2 + p_\parallel^2 + p_\perp^2} - E_\parallel$$

$$= E_\parallel \sqrt{1 + \frac{p_\perp^2}{E_\parallel^2}} - E_\parallel \approx E_\parallel\, \theta^2/2,$$

where $\theta \approx tg\theta = p_\perp/p_\parallel$. Some characteristic values are summarized in Table I. One can conclude that, e.g., the "enormous" 300 MeV/c transverse momentum of a 600 GeV pion can be decreased to zero if one displaces it in transverse electric field from a point with zero potential to a point with 75 kV potential. [The required potential difference for a similar ($p_\perp < 300$ MeV/c) 5 TeV beam[2] is merely 9 kV, which is less than the accelerating potential in the CRT of a normal TV set.] The $E_\perp(E_\parallel, p_\perp)$ relation for fixed $E_\perp$ values are shown in Fig. 2. In general, a particle with nominal (i.e., longitudinal) momentum $p_\parallel$ can be confined by field $U = E_\perp/e$ if it has transverse momentum

$$p_\perp < \sqrt{2p_\parallel U},$$

where e is the electric charge of the particle. In our unit e = 1.

3 - Motion in Transversal Electric Field

Beam focusing means that one is able to diminish the transverse component of the particle momentum by the application of transverse electric field. The equation of motion can be written as

$$(1) \qquad \frac{dp_\perp}{dt} = e\varepsilon$$

$$(2) \qquad \frac{dp_\parallel}{dt} = 0 \to p_\parallel = \text{const.}$$

which means in our case that

$$X_\parallel = Z = ct$$

because on one hand we can neglect the particle mass and on the other hand the relation

$$\sqrt{v_\parallel^2 + v_\perp^2} \approx v_\parallel$$

is valid with high precision. Rewritting Eq. (1) in convenient units one gets

$$\frac{dp_\perp[\mathrm{eV/c}]}{dZ[\mathrm{cm}]} = -\varepsilon[\mathrm{V/cm}],$$

where the minus sign ensures that the transverse momentum of a particle entering at an angle of θ_0 will decrease during the crossing of the electric field region.

3.1 - Motion in homogenous field

As is well known from classical mechanics, in a homogenous electric field ε_0 one gets a parabolic trajectory. The Z-coordinate of maximal radial displacement defined by the condition, that $p_\perp = 0$

$$Z_H^* = p_\perp/\varepsilon_0.$$

Figure 3 shows an example with $E_\parallel = 600$ GeV/c, $p_\perp = 300$ MeV/c and $\varepsilon_0 = 30$ kV/cm. The maximal radial displacement is equal to 2.5 cm. The peak is reached at a distance $Z = 10^4$ cm = 100 m.

3.2 - Motion in 1/r radial electric field

In practice, one can effect easily the field

$$\varepsilon(r) = -\varepsilon_0 \frac{r_0}{r}$$

between two conducting tubes, where ε_0 denotes the field strength at the surface of the inner tube. The corresponding potential

$$\phi(r) = -r_0 \varepsilon_0 \ln \frac{r_0}{r}.$$

One must solve the equation

$$\frac{dp_r}{dZ} = \frac{d(E_\parallel \theta)}{dt} = -\varepsilon_0 \frac{r_0}{r}.$$

For simplicity, we neglect the eventual azimuthal component, i.e., $p_\phi = 0$.

The r_0/r term can be eliminated by using energy conservation

$$\Delta E_\perp = \frac{1}{2} E_\parallel \left(\theta_0^2 - \theta^2\right) = \phi(1) - \phi(0)$$
$$= -r_0 \varepsilon_0 \ln \frac{r_0}{r}$$

which gives

$$r_0/r = \exp\left\{-\frac{1}{2}\frac{E_\parallel}{r_0\varepsilon_0}\left(\theta_0^2 - \theta^2\right)\right\}$$

by substitution

$$\frac{d\theta}{dZ} = -\frac{\varepsilon_0}{E_\parallel}\exp\left\{-\frac{1}{2}\frac{E_\parallel}{r_0\varepsilon_0}\left(\theta_0^2 - \theta^2\right)\right\}.$$

It can be solved formally

$$Z = \frac{E_\parallel r_0}{\varepsilon_0}\exp\left\{\frac{1}{2}\frac{E_\parallel}{\varepsilon_0 r_0}\theta_0^2\right\}\int_{\theta_0}^{\theta} - \exp\left\{-\frac{1}{2}\frac{E_\parallel}{\varepsilon_0 r_0}\theta^2\right\} d\left(\sqrt{\frac{E_\parallel}{\varepsilon_0 r_0}}\right)\theta.$$

The maximal radial displacement corresponds to $\theta = 0$

$$Z_r^* = \sqrt{\frac{E_\parallel r_0}{\varepsilon_0}}\exp\left\{\frac{1}{2}\frac{E_\parallel \theta_0^2}{\varepsilon_0 r_0}\right\}\frac{\sqrt{2\pi}}{2}\int_{-y_0}^{y_0}\frac{1}{\sqrt{2\pi}}e^{-1/2y^2}dy$$
$$< \sqrt{\frac{\pi E_\parallel r_0}{2\varepsilon_0}}\exp\left\{+\frac{y_0^2}{2}\right\},$$

where

$$y = \theta \cdot \frac{E_\parallel}{\varepsilon_0 r_0} = \frac{p_\perp}{\sqrt{E_\parallel \varepsilon_0 r_0}},$$

e.g., $y_0 = 0.5\times10^{-3}\sqrt{6\times10^{11}/3\times10^4} \approx 2.2$ for $\theta_0 = 0.5$ mrad $E_\parallel = 600$ GeV, $\varepsilon_0 = 30$ kV/cm and $r_0 = 1$ cm. Other typical values are summarized in Table II. It is obvious that at the same maximal field strength $Z_r^* > Z_H^*$ because the focusing power of the 1/r fields decreases with r, and therefore one consumes the remaining $p_\perp$ more and more slowly. The advantage of this configuration is evident for neutrino beam production because this means that the neutrinos will be produced most of the time from pions whose polar angle is more inclined toward zero at decay than at their emission point. One can get the homogeneous case as a limit $r_0 \to \infty$

$$Z_r^*(r_0 \to \infty) = \frac{E_\parallel r_0}{\varepsilon_0} e^{+\delta_0}\frac{1}{2}\int_{-y_0}^{y_0} e^{-\delta}dy =$$
$$= \frac{E_\parallel r_0}{\varepsilon_0}\cdot 1 \cdot \frac{1}{2}\cdot 2y = \frac{p_\perp}{\varepsilon_0},$$

where $\delta = y^2/2$ goes to zero.

4 - The Confining Tube

4.1 - Confining Characteristics of an Ideal Tube

The ideal device consists of two concentric metal tubes with radii r and R. Here we assume that the inner tube is completely transparent. Applying voltage difference $U = \phi(R) - \phi(r)$ across the two tubes, energy conservation ensures that all the particles having $E_\perp < U$ [the corresponding $(p_\parallel, p_\perp)$ region is shown in Fig. 2] will oscillate inside the outer tube. Their trajectory will be linear inside the inner tube and some kind of elongated parabola between the tubes. This device is **confining** but not **focusing**, because the particles emitted from a single point will not be recollected in another point (even for identical $E_\parallel$ energies) if they have different entrance angle θ_0 into the electric field region, due to the fact that Z_r^* is dependent on θ_0.

The energy conservation argument is valid for $p_\phi \neq 0$ case, too. $p_\phi \neq 0$ complicates only the trajectory by changing the planar motion into a 3-dimensional one.

This confinement works horizontally and vertically **simultaneously.** It can be continued infinitely, allowing more effective use of parent particles. This is extremely useful for muons because practically all the muons will be confined *until the end of the tube irrespective of their* energy. This is based on the fact that the transversal energy $E_\perp^\mu$ of the decay muon generally will be, in first approximation, E_μ/E_π times less than $E_\perp^\pi$, because the $p_\perp$ contribution from the decay processes is negligible.

4.2 - The Confining Tube Construction

The beam pipe construction is complicated by the fact that the inner tube should be transparent. Due to the extremely small crossing angles $0 < \theta_0 < 0.5$ mrad, even with a wall thickness of only 10 microns, a particle with a typical θ_0 value of 0.1 mrad would have to traverse $L = 10^{-3}$ cm/10^{-4} rad = 10 cm of material. In a long beam line hundreds of such crossings can occur. The effect of the inner tube can be produced by wires stretched parallel to the axis at a distance r. The wire diameter d_W is not a very critical parameter. The overall transparency depends on the number of wires, N_W, arranged along the perimeter. For example, the transparency factor will be

$$\varepsilon_{TRANS} = 1 - \frac{N_W d_W}{2r\pi} \approx 1 - \frac{3 \times 1 \text{ mm}}{2 \times 5 \text{ mm} \times 3.14} \approx 90\%$$

in case of $N_W = 3$, $d_W = 1$ mm, $r = 0.5$ cm.

The wires can be fixed by thin supporting rings to withstand the electrostatic repulsion forces. The outer ring with diameter R = 4 cm can be made from any metal, strong enough to hold the vacuum. Vacuum is required in order to minimize a) $p_\perp$ generation by Coulomb scattering; and b) avoid electric dischanges in the gaseous atmosphere.

The field of the wire "tube" will be a little bit different from the ideal one, due to the induced charges in the outer metal tube. The field of induced charges can be easily calculated by introducing a mirror charge at $R' = R^2/r$ distance [Fig. 4(a)]. In case of more wires the number of mirror charges should be increased respectively [Fig. 4(b)]. The effect of mirror charges depends on the $\rho = R/r$ ratio, for example, if $\rho = 4$ the field contribution of mirror charges in the center will be about 6% ($\approx 1/\rho^2$), which is negligible in the first approximation.

Of course this system is confining only for particles whose charges have the correct sign. Opposite charged particles will be diverted. The sign selection for muons is provided automatically, but neutrinos and anti-neutrinos can be mixed if the pion sign selection was not performed previously.

5 - Some Possible Beam Configurations

One can generate wide or narrow band beams depending on the selection of the parent beam source. In this section we are assuming a pion source whose lateral dimensions are smaller than 1 cm and that all the particles have $E_\perp < U$ where U is the potential applied to the confining tube. The "beam energy" $E_\parallel$ is left free. It will be specially defined if its role is important for a given configuration.

5.1 - Neutrino Beams

The simple neutrino beam line shown in Fig. 5(a) consists practically of only a single confining tube. The beam intensity depends on the pion source, tube length, shielding width and the applied confining potential. In order to get more elongated oscillations for parent particles (which means less divergence for neutrinos) it seems to be preferable to use as large an outer radius as possible providing smaller field strength at a given potential difference.

5.2 - Combined Muon and Neutrino Beam ν - Tagging

In our high energy limit the muon and neutrino from a given decay arrive at the same time to the same plane in Z, but different in (x, y). If one uses thin shielding (order of 10 meters) to kill only the surviving hadrons one can use the previous arrangement for studying muon and neutrino interactions simultaneously. Neutrino tagging is feasible because the almost 100% confining of muons ensures that if there is a ν-interaction the muon also will be observable. The muon sign determines uniquely the neutrino type.

If one uses a narrow band pion-beam the knowledge of the muon energy determines the neutrino energy. Thus, for example, the neutral current studies could be carried out with precise knowledge of incoming neutrino parameters. In the TeV energy region the weak cross section is large enough to observe a considerable number of neutrino interactions even in the case of pion beams of modest intensity. Thus there will not be intensity problems for muon detection.

5.3 - Monochromatic Muon Beam

For a dedicated muon beam [Fig. 5(b)] one needs a separator bending magnet after the hadron absorber. If one uses permanent magnets the field can be maintained without continuous energy consumption. The muon beam energy (and so the **polarization** in case of narrow band pion beam) can be selected by inserting a greater or smaller number of pairs of permanent magnet modules. (Sign change also requires a rearrangement of permanent magnets.)

The halo can be eliminated almost completely by scraping toroids constructed from iron discs magnetized in a defocusing manner. It is enough to place scraping magnets only in the first half of the selected muon line because the once confined muons will remain within the confining tube independently of its length.

5.4 - Multi-Band Muon Beam

For measurement of the structure function ratio $R = \sigma_L/\sigma_T$ and electroweak interferences one requires at least two measurements with different monochromatic beam energies or polarizations. This means on one hand increased systematic errors and on the other hand waste of most of the muons diverted from the selected muon line. There are two possibilities to overcome these difficulties

a) The detector includes an incoming muon beam analyzer at the end of the beam line, as in the case of neutrino tagging;

b) Multi-band selective confining is shown in Fig. 6 for the case of the so-called trichromatic arrangement. It provides tagged muons with polarizations

$$\lambda_+ = +0.8, \ \lambda_0 = 0, \ \lambda_- = -0.8.$$

It can be built from standardized elements: 8 bending magnets and 8 confining tubes. The halo scraping is performed after the unification of the 3 components.

6 - Conclusion

In perspective, the electrostatic beam confinement has a number of advantages: flexible and simple beam line construction, exact confinement along unlimited length without significant energy consumption. It can provide beams of unique quality: tagged neutrinos or muon beams without halo. Of course, the realization in practice requires more careful studies and optimization for each individual case.

Acknowledgement

I am grateful to A. Benvenuti, N. Doble, A. Staube, G. Smadja, and J. Zsembéry for clarifying discussions and especially to A. Montag for careful reading of this manuscript.

References

1. R. Cliff and N. Doble, CERN/EA/74-A.
2. A. De Rujula, G. Charpak, S. Glashow, and R. R. Wilson, Harvard preprint HUTP-83/A019, March 1983. The "snout" of GEOTRON could be easily built using a "low-voltage" electrostatic confining tube.

Table 1

$p_{/\!/} \simeq E_{/\!/}$ [GeV]	$p_\perp$ [GeV/c]	θ [mrad]	$E = E_{/\!/}(1 + \frac{\theta^2}{2})$ [GeV]	$E_\perp = \frac{1}{2} E_{/\!/} \theta^2$
50	0.3	6.0	$50(1+18*10^{-6})$	900 keV
100	0.3	3.0	$100(1+4.5*10^{-6})$	450 keV
500	0.3	0.6	$500(1+18*10^{-8})$	90 keV
1000	0.3	0.3	$1000(1+4.5*10^{-8})$	45 keV
5000	0.3	0.06	$5000(1+18*10^{-10})$	9 keV
500	0.6	1.2	$500(1+72*10^{-8})$	360 keV
1000	0.6	0.6	$1000(1+18*10^{-8})$	180 keV
5000	0.6	0.12	$5000(1+72*10^{-10})$	36 keV

Table 2

$E_{/\!/}$ [GeV]	$p_\perp$ [GeV/c]	$Z^*_{HOMOGEN}$[m]	$y_o = \frac{p_\perp^2}{E_{/\!/}\varepsilon_o r_o}$	$Z^*_{[1/r]}$	$r^{MAX}_{[1/r]}$
50	0.3	100	7.8	$1.6*10^{13}$ m	$\sim10^{13}$ cm
500	0.3	100	2.45	1000 m	20 cm
5000	0;3	100	0.78	124 m	1.35 cm
500	0.6	200	4.9	$6*10^5$ m	$1.6*10^5$ cm
5000	0.6	200	1.56	470 m	3.3 cm

$r_o = 1$ cm, $\varepsilon_o \doteq \varepsilon\ (r=r_o) = 30$ kV/cm

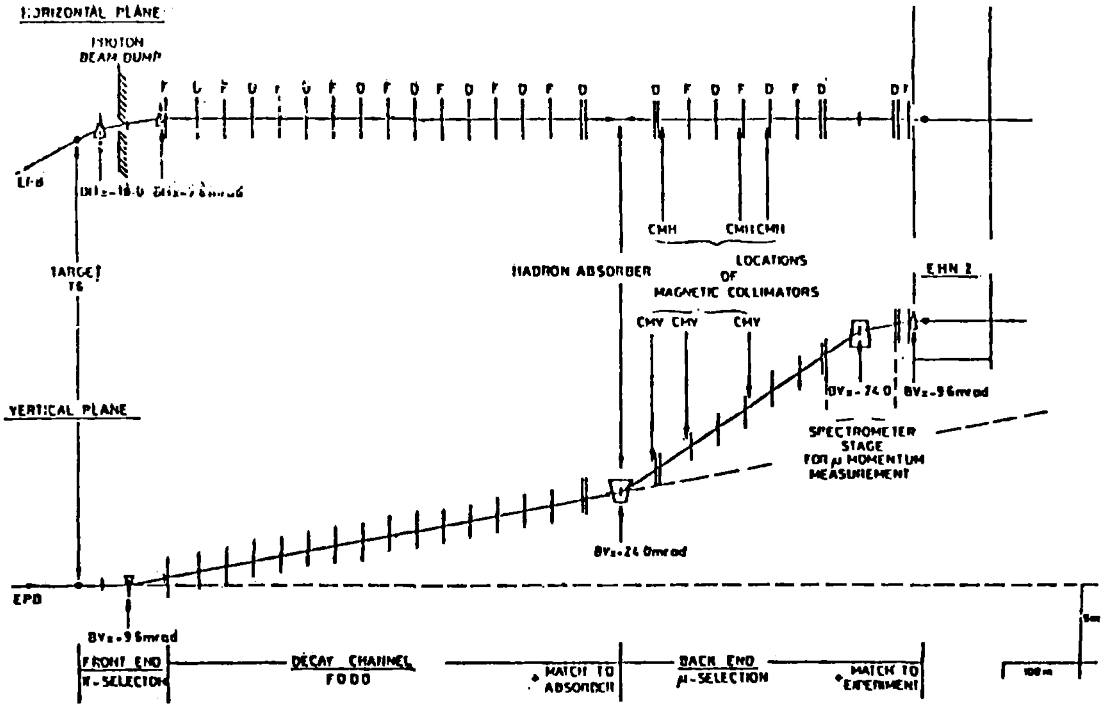

Fig. 1. The muon beam line at CERN.

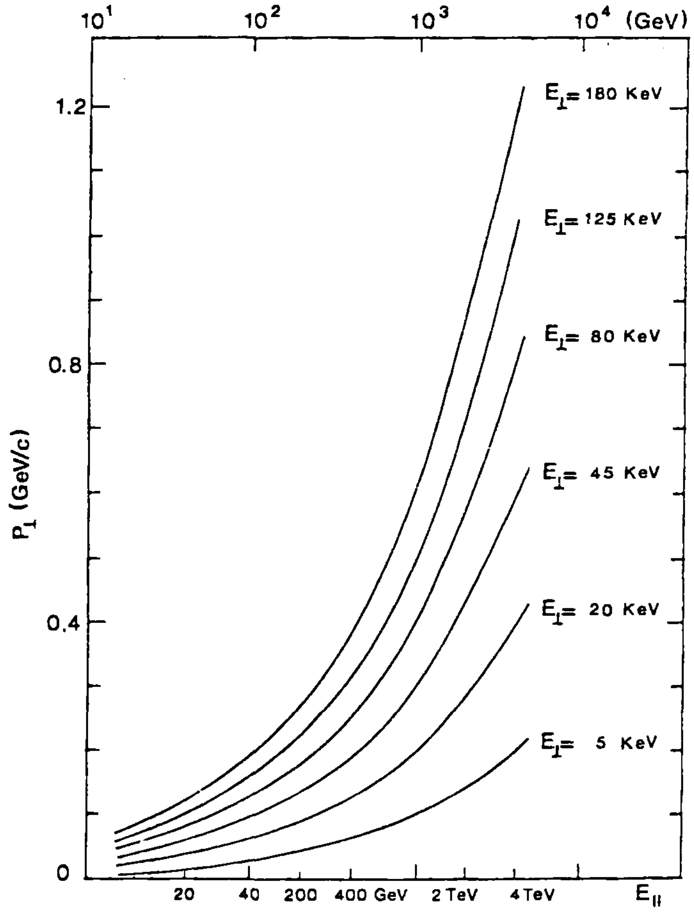

Fig. 2. Particles with momenta ($E_{\parallel}$, $p_{\perp}$) will be confined by voltage U applied to the confining tube if their transverse energy $E_{\perp} < eU$.

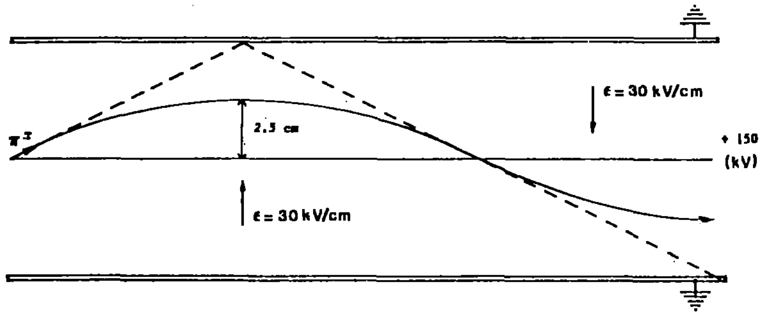

Fig. 3. Motion in homogeneous electric field.

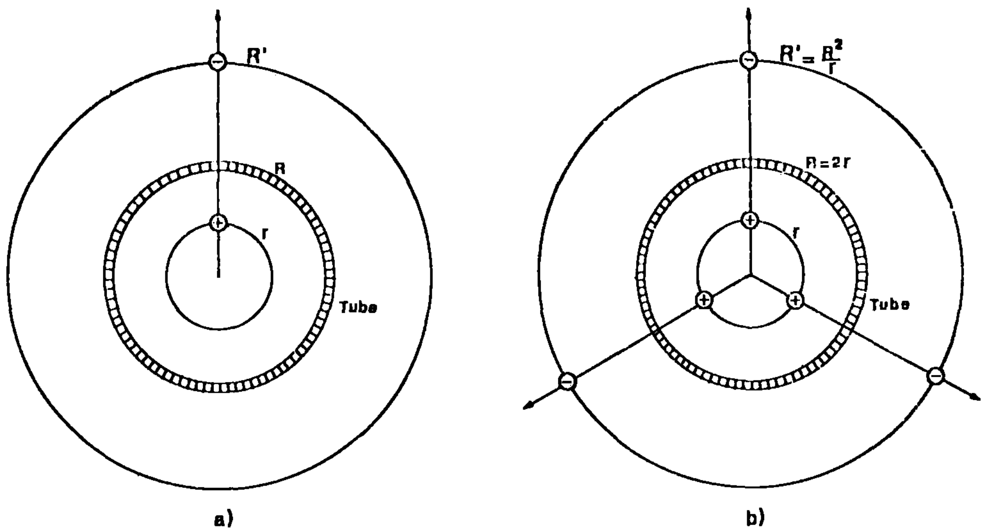

Fig. 4. Mirror charges in case of (a) one and (b) three off-center high-voltage wires.

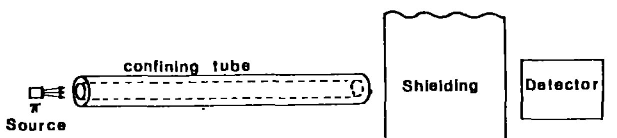

Fig. 5(a). Schematic view of a neutrino beam.

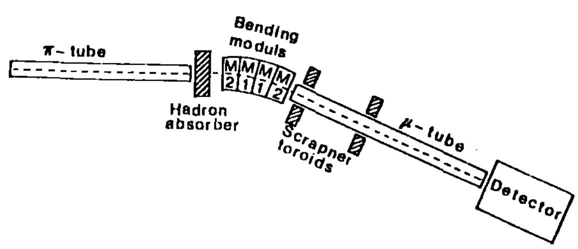

Fig. 5(b). Dedicated muon beam.

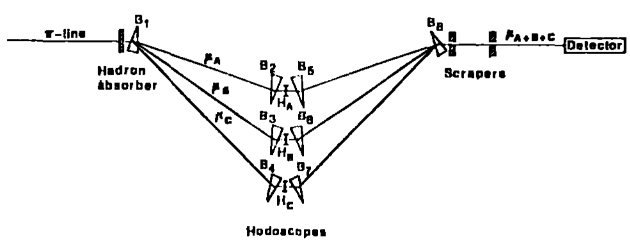

Fig. 6. Trichromatic muon beam.