$b \to s\ell^+\ell^-$ in the high q^2 region at two-loops

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We report on the first analytic NNLL calculation for the matrix elements of the operators O_1 and O_2 for the inlusive process $b \to X_s l^+ l^-$ in the kinematical region $q^2 > 4m_c^2$, where q^2 is the invariant mass squared of the lepton-pair.

1 Introduction

In the Standard Model, the flavor-changing neutral current process $b \to X_s l^+ l^-$ only occurs at the one-loop level and is therefore sensitive to new physics. In the kinematical region where the lepton invariant mass squared q^2 is far away from the $c\bar{c}$-resonances, the dilepton invariant mass spectrum and the forward-backward asymmetry can be precisely predicted using large m_b expansion, where the leading term is given by the partonic matrix element of the effective Hamiltonian

$$\mathcal{H}_{eff} = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{10} C_i(\mu) O_i(\mu). \tag{1}$$

We neglect the CKM combination $V_{us}^* V_{ub}$ and the operator basis is defined as in [1]. In [2] we published the first analytic NNLL calculation of the high q^2 region of the matrix elements of the operators

$$O_1 = (\bar{s}_L \gamma_\mu T^a c_L)(\bar{c}_L \gamma^\mu T^a b_L), \qquad O_2 = (\bar{s}_L \gamma_\mu c_L)(\bar{c}_L \gamma^\mu b_L)\,, \tag{2}$$

which dominate the NNLL amplitude numerically. Earlier these results were only available analytically in the region of low q^2 [3, 4].

2 Calculations

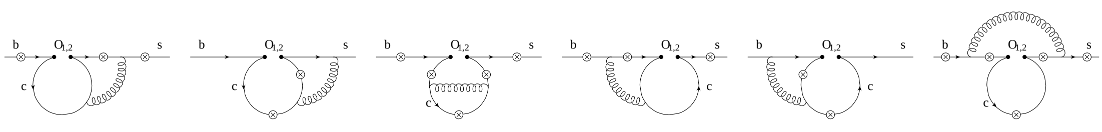

Figure 1: Diagrams that have to be taken into account at order α_s. The circle-crosses denote the possible locations where the virtual photon is emitted (see text).

The diagrams contributing at order α_s are shown in Figure 1. We set $m_s = 0$ and define $\hat{s} = q^2/m_b^2$ and $z = m_c^2/m_b^2$, where q is the momentum of the virtual photon. After reducing occurring tensor-like Feynman integrals [5] the remaining scalar integrals can be further reduced to master integrals using integration by parts (IBP) identities [6]. Considering the region $\hat{s} > 4z$, we expanded the master integrals in z and kept the full analytic dependence in $\hat{s}$.

For power expanding Feynman integrals we use a combination of *method of regions* [7] and *differential equation techniques* [8, 9]: Consider a set of Feynman integrals $I_1, \ldots, I_n$ depending on the expansion parameter z and related by a system of differential equations obtained by differentiating I_α with respect to z and applying IBP identities:

$$\frac{d}{dz} I_\alpha = \sum_\beta h_{\alpha\beta} I_\beta + g_\alpha, \tag{3}$$

where g_α contains simpler integrals which pose no serious problems. Expanding both sides of (3) in ϵ, z and $\ln z$

$$I_\alpha = \sum_{i,j,k} I_{\alpha,i}^{(j,k)} \epsilon^i z^j (\ln z)^k, \qquad h_{\alpha\beta} = \sum_{i,j} h_{\alpha\beta,i}^{(j)} \epsilon^i z^j, \qquad g_\alpha = \sum_{i,j,k} g_{\alpha,i}^{(j,k)} \epsilon^i z^j (\ln z)^k, \tag{4}$$

and inserting (4) into (3) we obtain algebraic equations for the coefficients $I_{\alpha,i}^{(j,k)}$

$$0 = (j+1) I_{\alpha,i}^{(j+1,k)} + (k+1) I_{\alpha,i}^{(j+1,k+1)} - \sum_\beta \sum_{i'} \sum_{j'} h_{\alpha\beta,i'}^{(j')} I_{\beta,i-i'}^{(j-j',k)} - g_{\alpha,i}^{(j,k)}. \tag{5}$$

This enables us to recursively calculate higher powers of z of I_α, once the leading powers are known. In practice this means that we need the $I_{\alpha,i}^{(0,0)}$ and sometimes also the $I_{\alpha,i}^{(1,0)}$ as initial condition to (5). These initial conditions can be computed using method of regions. A non trivial check is provided by the fact that the leading terms containing logarithms of z can be calculated by both method of regions and the recurrence relation (5).

The summation index j in (4) can take integer or half-integer values, depending on the specific set of integrals I_α. In order to determine the possible powers of z and $\ln(z)$ we used the algorithm described in [9].

3 Results

In order to get accurate results we keep terms up to z^{10}. Our results agree with the previous numerical calculation [10] within less than 1% difference. The impact of our results on the perturbative part of the high q^2-spectrum [3]

$$R(\hat{s}) = \frac{1}{\Gamma(\bar{B} \to X_c e^- \bar{\nu}_e)} \frac{d\Gamma(\bar{B} \to X_s \ell^+ \ell^-)}{d\hat{s}} \tag{6}$$

is shown in Figure 2 (left), where we used the same parameters as in [2]. The finite bremsstrahlung corrections calculated in [4] are neglected. From Figure 2 (left) we conclude that for $\mu = m_b$ the contributions of our results lead to corrections of the order $10\% - 15\%$. Integrating $R(\hat{s})$ over the high $\hat{s}$ region, we define

$$R_{\text{high}} = \int_{0.6}^{1} d\hat{s}\, R(\hat{s}). \tag{7}$$

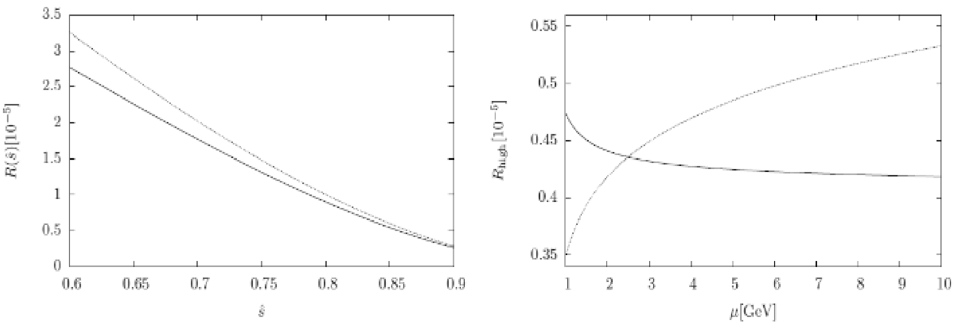

Figure 2: Perturbative part of $R(\hat{s})$ (left) and R_{high} (right) at NNLL. The solid lines represents the NNLL result, whereas in the dotted lines the order α_s corrections to the matrix elements associated with $O_{1,2}$ are switched off. In the left figure we use $\mu = m_b$. See text for details.

Figure 2 (right) shows the dependence of the perturbative part of R_{high} on the renormalization scale. We obtain

$$R_{\text{high,pert}} = (0.43 \pm 0.01(\mu)) \times 10^{-5}, \tag{8}$$

where we determined the error by varying μ between 2 GeV and 10 GeV. The corrections due to our results lead to a decrease of the scale dependence to 2%.

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