PHENOMENOLOGY OF SUPERSYMMETRY

Pierre Fayet
Laboratoire de Physique Théorique de l'Ecole Normale Supérieure*
24 rue Lhomond, 75231-Paris cedex 05, France

ABSTRACT : We present the mass spectrum of a supersymmetric theory of particles and discuss the relations between the mass spectrum and the couplings of the new gauge boson U. We study the phenomenological consequences of such theories : new spin-0 and spin-1/2 leptons and quarks, gluinos and R-hadrons, effects of the U boson and of the gravitino, in particular in astrophysics.

RESUME : Nous présentons le spectre de masse d'une théorie supersymétrique des particules, et discutons ses relations avec les couplages du nouveau boson de jauge U. Nous étudions les conséquences phénoménologiques de telles théories : nouveaux leptons et quarks de spin 0 et 1/2, gluinos et R-hadrons, effets du boson U et du gravitino, en particulier en astrophysique.

*Laboratoire Propre du Centre National de la Recherche Scientifique, associé à l'Ecole Normale Supérieure et à l'Université de Paris-Sud.

Supersymmetry relates bosons to fermions, associating particles differing by 1/2 unit of spin[1]. It leads to the introduction of new particles such as heavy spin-0 partners for the leptons and quarks, and spin-1/2 partners for the photon and gluons, named the photino and gluinos[2]. It also leads to relations between particles already appearing in ordinary gauge theories : the spin-0 Higgs bosons associated with the spontaneous breaking of the gauge symmetry are now related to the corresponding massive gauge bosons. Supersymmetry is the natural framework, both for the study of spontaneously broken gauge theories of interactions, and for the introduction of gravitation in particle physics.

We shall review some of the main consequences of supersymmetric theories of particles, in particular in astrophysics. (See also ref. 3 for a sometimes more detailed discussion.) Before that, we first present the mass spectrum of a very simple supersymmetric theory, and its relations to the couplings of a new neutral gauge boson.

1. SYMMETRY BREAKING AND MASS SPECTRUM

After the spontaneous breaking of the gauge symmetry the particle content of a supersymmetric theory of weak, electromagnetic and strong interactions can be summarized as follows :

	Spin-1	Spin-1/2	Spin-0
Massless gauge multiplets	Photon Gluons	Photino Gluinos	
Massive gauge multiplets	Intermediate gauge bosons ($W^{\pm}$, Z, U)	Dirac heavy fermions (L_{W+}, L_{W-}, L_Z, L_U) (+ antiparticles)	Higgs bosons ($w^{\pm}$, z, u)
		Leptons Quarks	Spin-0 leptons Spin-0 quarks

Table 1 : Particle content of a supersymmetric theory of weak, electromagnetic and strong interactions.

Each Dirac lepton or quark is associated with two spin-0 leptons or quarks, while the photino and gluinos are a singlet and an octet of neutral Majorana spin-1/2 particles, respectively. To every neutral (or charged) massive gauge boson is associated one neutral (or charged) massive Higgs boson, and one (or two) heavy Dirac fermion(s).

In Table 1 we have extended the gauge group to SU(3) x SU(2) x U(1) x U(1), introducing a new neutral gauge boson U. This one is associated under supersymmetry with the Dirac fermion L_U and the ordinary neutral Higgs boson, here denoted by u. This extension is necessary if one wants the spontaneous breaking of the supersymmetry to generate large masses for spin-0 leptons and quarks at the tree approximation[4]. (Alternately many authors consider a different approach, in which spin-0 leptons and quarks would acquire large masses from radiative corrections ; this necessitates additional symmetry breaking scales in the theory.)

As long as supersymmetry is conserved the mass spectrum is the one given in fig. 1.

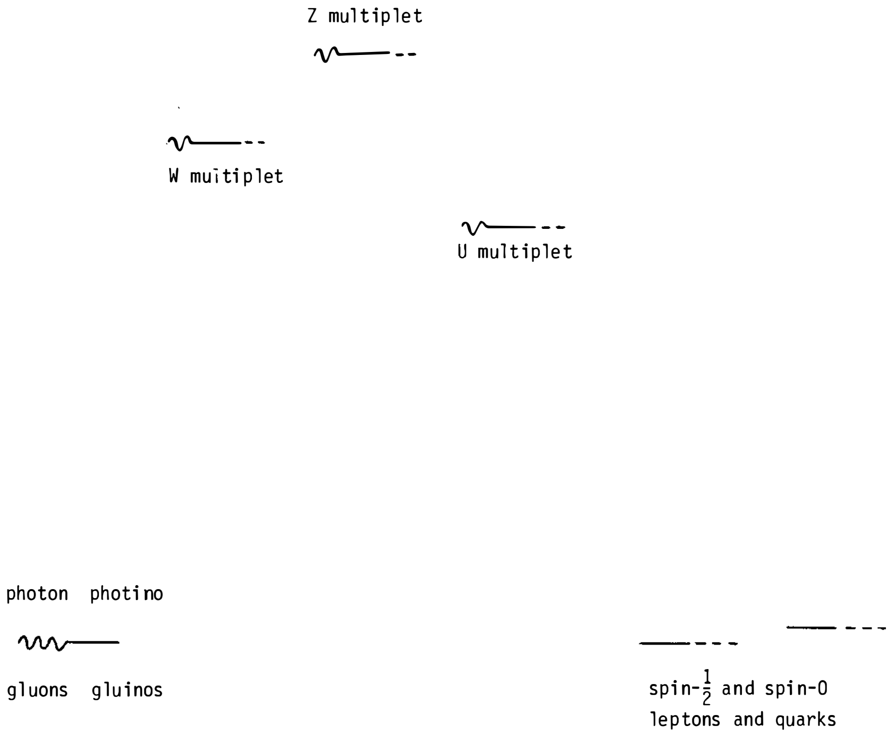

Fig. 1 : Mass spectrum of the supersymmetric SU(3) x SU(2) x U(1) x U(1) theory of particles. We have represented the new gauge boson U as heavy, but it may also be light ; this would be the case if the new U(1) gauge coupling constant g" is very small.

The spontaneous breaking of supersymmetry is triggered by an additional singlet chiral superfield, which is electrically neutral but couples to the U. It describes a Majorana spinor ζ together with its spin-0 partner. The massless spin-1/2 goldstino field associated with the spontaneous breaking of supersymmetry is obtained by mixing ζ with the heavy Dirac fermion associated with the U particle. We find the following mass spectrum :

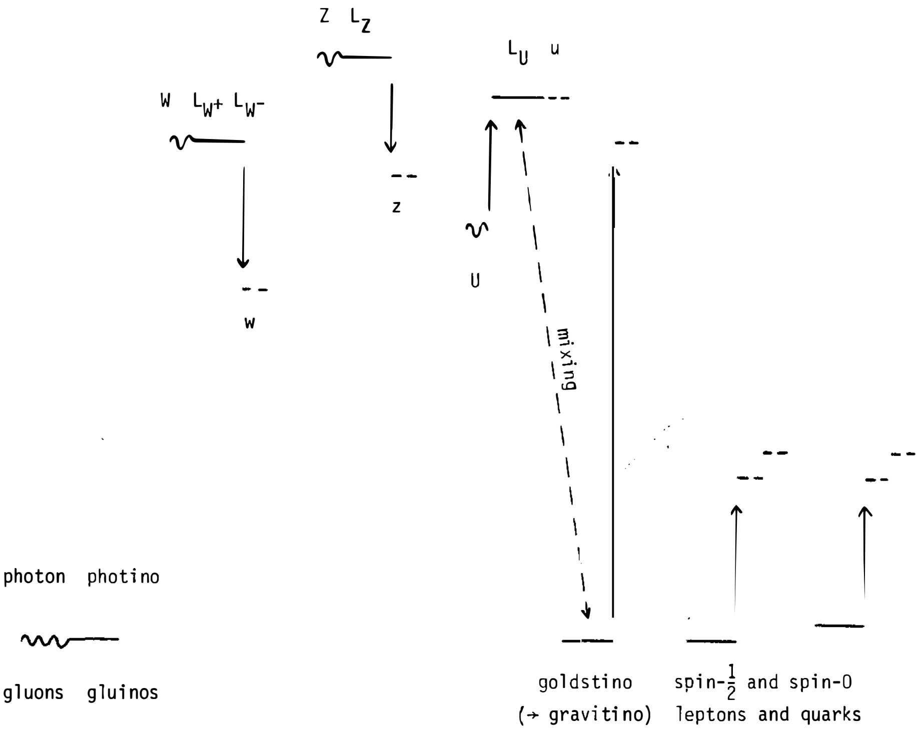

Fig. 2 : Mass spectrum of a spontaneously broken supersymmetric theory of particles. Even if the U boson is very light (g" very small), both the Dirac fermion L_U and the standard Higgs boson u would still be heavy.

We have assumed, for simplicity, that the two Higgs doublets of the theory acquire equal vacuum expectation values (x = 1), so that there is no Z-U mixing. The photino and goldstino fields are then orthogonal combinations of neutral spinor fields, i.e.

$$< \text{photino} \mid \text{goldstino} > \; = \; 0 \qquad (1)$$

Alternately if $x \neq 1$ the Z and U mix, the goldstino is a linear combination of several neutral spin-1/2 fields including the photino, and the two charged Dirac fermions L_W+ and L_W- are no longer degenerate in mass with the charged W. Their masses verify the relation :

$$m^2(L_W+) + m^2(L_W-) \; = \; 2\, m^2(W) \qquad (2)$$

The mass^2-splittings between bosons and fermions are determined by the (gauge or Yukawa) couplings of the goldstino field, i.e. also by the couplings of the bosonic partner of the goldstino. As a result we find relations between the mass spectrum and the new neutral current J_U coupled to the gauge boson U. In the simplest case J_U is purely axial, and the two spin-0 particles (s_f and t_f) associated with each Dirac lepton and quark (f) are degenerated in mass, up to radiative corrections :

$$J_U \text{ axial} \Leftrightarrow m_{s_f} \; = \; m_{t_f} \qquad (3)$$

In that case parity is conserved for all processes involving spin-1/2 and spin-0 leptons and quarks as well as the photon and gluons, photino and gluinos.

The axial part of the U current is determined by the gauge invariance of the Yukawa couplings responsible for quark and lepton masses. It is universal for all leptons and quarks which transform like left-handed doublets and right-handed singlets. (Alternately, "mirror" leptons and quarks, which would transform like right-handed doublets and left-handed singlets, would also have a universal axial coupling to the U boson, but with the opposite sign.) The vector part of the U current is parametrized by means of an angle ϕ , i.e. :

$$\begin{cases} J_{U\,\text{Left}} \sim (1 - \cos\phi) \\ J_{U\,\text{Right}} \sim -(1 + \cos\phi) \end{cases} \qquad (4)$$

We may have different angles ϕ_e, ϕ_μ, ϕ_τ for the three lepton sectors, as long as they do not mix. However, we have only one angle ϕ_q for the quark sector, owing to the gauge invariance of the Yukawa couplings inducing the mass matrix responsible for the mixing of the three generations of quarks.

There exist linear relations between the boson-fermion mass^2-splittings in the different multiplets of supersymmetry. In particular let us consider a lepton or quark, f, together with its spin-0 partners, s_f and t_f. The corresponding mass^2-splitting is related to the mass^2-splitting between the $W^\pm$ and the charged Higgs boson $w^\pm$ associated with it under supersymmetry :

$$\begin{cases} m^2(s_f) - m^2(f) = (1 - \cos\phi_f)\ \dfrac{m^2(W^\pm) - m^2(w^\pm)}{4} \\ m^2(t_f) - m^2(f) = (1 + \cos\phi)\ \dfrac{m^2(W^\pm) - m^2(w^\pm)}{4} \end{cases} \quad (5)$$

This implies that

$$\left(\frac{m^2(s_f) + m^2(t_f)}{2}\right)^{1/2} \lesssim \frac{1}{2}\, m(W^\pm) \sim 40\ \mathrm{GeV}/c^2 \quad (6)$$

These relations, valid in a large class of models, should not, however, be considered as necessary consequences of supersymmetry alone. If indeed there exist spin-0 leptons and quarks lighter than $\sim$ 40 GeV/c^2, this would have very important phenomenological implications.

2. NEW SPIN-0 AND SPIN-1/2 LEPTONS AND QUARKS :

a) Spin-0 leptons and quarks

Spin-0 leptons are unstable and decay extremely quickly into the corresponding lepton by emission of a photino or goldstino. A pair of spin-0 leptons could be produced in e^+e^- annihilation, then decay[3,5] according to

$$\begin{aligned} e^+e^- &\to \text{Pair of spin-0 leptons} \\ &\to \text{Non coplanar pair } (e^+e^-,\ \mu^+\mu^- \text{ or } \tau^+\tau^-) \\ &\quad + 2 \text{ unobserved photinos or goldstinos.} \end{aligned} \quad (7)$$

Systematic searches for spin-0 leptons have been carried out at PETRA and, more recently, at PEP[6]. They lead to the following lower limits :

$$\begin{aligned} &m(\text{spin-0 electrons}) > 16\ \mathrm{GeV}/c^2 \\ &m(\text{spin-0 muons}) > 16\ \mathrm{GeV}/c^2 \\ &m(\text{spin-0 taus}) > 15\ \mathrm{GeV}/c^2 \end{aligned} \quad (8)$$

One can also search for unstable spin-0 quarks. They would decay into ordinary quarks by emission of a photino or goldstino or, more frequently, of a gluino. Constraints on their masses are expected to be available soon. Let us now briefly discuss the possible existence of a new class of spin-1/2 and spin-0 leptons and quarks.

b) Mirror families of quarks and leptons

The extra U(1) gauge group has been introduced in order to make spin-0 leptons and quarks heavier than their spin-1/2 partners. This has the drawback of generating γ^5 anomalies, since the U boson has a universal axial coupling with all ordinary leptons and quarks. In view of cancelling them we shall have to introduce additional fields. This will lead us to consider, in particular, heavy mirror leptons and quarks, which transform like right-handed doublets and left-handed singlets of the weak interaction gauge group. (See ref. 7 for a discussion of mirror particles in the framework of N = 2 extended supersymmetric theories.) The U boson has, again, a universal axial coupling, but now of the opposite sign, with all mirror leptons and quarks. Formulas (4, 5) can be extended to the mirror sector, with an additional minus sign. We have, in particular :

$$[m^2(s_{mirror}) + m^2(t_{mirror}) - 2m^2(f_{mirror})] = - [m^2(s_f) + m^2(t_f) - 2m^2(f)] < 0 \tag{9}$$

Mirror fermions would be unstable and decay extremely quickly into mirror spin-0 leptons and quarks, by emission of a photino, goldstino, or gluino.

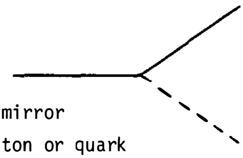

Fig. 3 : Diagrams responsible for the decays of heavy mirror spin-$\frac{1}{2}$ leptons and quarks.

Most mirror spin-0 leptons and quarks would be short-lived (e.g., $t_{\mu M} \rightarrow t_{eM}\, \bar{\nu}_{eM}\, \nu_{\mu M}$, induced by the exchange of the fermionic partner of the W), but some may be long-lived or even stable. Present limits on mirror spin-0 lepton masses should be of the order of 15 GeV/c^2 ; and we may assume that this is also true for ordinary and mirror spin-0 quarks. Then formula (9) implies the following lower limit :

$$m(\text{mirror fermions}) \gtrsim 15 \text{ GeV/c}^2 \sqrt{2} \simeq 21 \text{ GeV/c}^2 \tag{10}$$

3. GLUINOS AND R-HADRONS

Gluinos are a color-octet of neutral spin-1/2 particles associated with the gluons under supersymmetry. They are massless at the tree approximation, although they may acquire a mass by radiative corrections, the value of this mass being model-dependent[8]. Gluinos may combine with quarks, antiquarks and gluons to give new color singlet hadronic states named R-hadrons[9] ; see also G.R. Farrar's lecture at this Rencontre. R-hadrons would be unstable and decay into ordinary hadrons by emission of a photino or goldstino (without charged lepton). Their estimated lifetime is short, i.e. $\sim 10^{-12}$ - 10^{-15} s in the simplest case, for a mass of 1.2 GeV/c^2. It depends strongly, both on the masses of the R-hadrons, and of those of spin-0 quarks, denoted by m_s ; one has :

$$\Gamma(\text{R-hadron} \rightarrow \text{photino} + \text{hadrons}) \sim \alpha\, \alpha_s \, \frac{m^5(\text{R-hadron})}{m_s^4} \qquad (11)$$

$$\Gamma(\text{R-hadron} \rightarrow \text{goldstino} + \text{hadrons}) \sim \frac{e_g^2}{4\pi}\, \alpha_s \, \frac{m^5(\text{R-hadron})}{m_s^4} \qquad (12)$$

In the latter formula $e_g\sqrt{2}$ denotes the Yukawa coupling constant of a goldstino to a quark line. It is proportional to the new U(1) gauge coupling constant g". The parameter e_g can be eliminated in favor of the parameter d which measures the scale of the spontaneous breaking of the supersymmetry (or, equivalently, of the mass acquired by the gravitino, when supersymmetry is realized locally)[3] :

$$\frac{e_g^2}{m_s^4} \simeq \frac{1}{d^2} = \frac{4\pi}{3}\, \frac{G_{\text{Newton}}}{m^2_{\text{gravitino}}} \qquad (13)$$

Therefore :

$$\Gamma(\text{R-hadron} \rightarrow \text{goldstino} + \text{hadrons}) \sim \frac{\alpha_s}{d^2}\, m^5(\text{R-hadron}) \qquad (14)$$

One can search for the missing energy carried away by the emitted photinos and goldstinos, in calorimeter experiments. From the results of the Caltec-Stanford experiment at Fermilab[10] one gets, assuming an R-hadron mass of $\sim$ 2 GeV/c^2,

$$\sigma(pN \rightarrow R\bar{R} + X) \lesssim 40\ \mu b \qquad \text{at} \quad \sqrt{s} \simeq 27 \text{ GeV} \qquad (15)$$

Therefore R-hadron masses should be at least of the order of 1.5 or 2 GeV/c^2.

Much stronger limits on the production cross section of R-hadrons can be obtained from beam dump experiments, provided an additional hypothesis on spin-0 quark masses is made. The photino cross sections are given by [11] :

$$\sigma(\text{photino} + \text{nucleon} \to \text{photino} + \text{hadrons}) \simeq .7\ 10^{-38}\text{cm}^2\ E(\text{GeV})\left(\frac{m_s}{40\ \text{GeV}/c^2}\right)^{-4} \quad (16)$$

$$\sigma(\text{photino} + \text{nucleon} \to \text{gluino} + \text{hadrons}) \simeq 100\ 10^{-38}\text{cm}^2\ E(\text{GeV})\left(\frac{m_s}{40\ \text{GeV}/c^2}\right)^{-4} \quad (17)$$

The goldstino cross sections have also been evaluated. They are proportional to the quantity $1/d^2$ appearing in formula (13).

The goldstino coupling constant e_g is expected to be, at most, comparable with e/2 or e, and could be much smaller. Then the interaction rate of the goldstino is, at most, comparable with the interaction rate of the photino[11]. (Even if this were not true, the constraint obtained by taking only the photino cross sections into consideration would still remain essentially valid). From the BEBC beam dump experiments we get[12] :

$$2\sigma(pN \to R\bar{R} + X)\ .\ \sigma_i < 2\ 10^{-66}\ \text{cm}^{-4} \qquad \text{at} \quad \sqrt{s} = 27\ \text{GeV} \quad (18)$$

With a rough estimate $< E > \sim 30$ GeV/c^2 for the average energy of each of the two emitted photinos (i.e. $2 < E > \sim 60$ GeV/c^2 for the average missing energy), we get from formula (16) the limit :

$$\sigma(pN \to R\bar{R} + X) \quad < \quad 6\ \mu b\ \left(\frac{m_s}{40\ \text{GeV}/c^2}\right)^4 \quad (19)$$

If R-hadrons are light enough so they can be reexcited by photinos in the final state, formula (17) should be used and we get the much stronger limit :

$$\sigma(pN \to R\bar{R} + X) \quad < \quad 40\ \text{nb} \times \left(\frac{m_s}{40\ \text{GeV}/c^2}\right)^4 \quad (20)$$

(The actual limit is somewhat less constraining owing to the threshold factor associated with the reexcitation of an R-hadron in the final state ; to estimate this one should make a more detailed analysis of the momentum spectrum of photinos and goldstinos produced in R-hadron decays.) A limit as constraining as $\sim$ 40 nb would imply that R-hadrons must be heavier than $\sim$ 4 GeV/c^2 ; see ref. 13 for an estimate of R-hadron production cross sections in terms of their masses.

4. PHENOMENOLOGY OF THE NEW GAUGE BOSON U

a) Neutral current processes

The exchange of the new neutral gauge boson U, in addition to the usual Z, may lead to large deviations from the successful neutral current phenomenology of the standard model. Most of the time we shall assume for simplicity that there is no Z-U mixing effect, i.e. that x = 1. Then we get the mass relations :

$$\left\{\begin{array}{l} \dfrac{g^2}{8m_W^2} = \dfrac{g^2 + g'^2}{8m_Z^2} = \dfrac{G_F}{\sqrt{2}} \\[2ex] \dfrac{g''^2}{8m_U^2} = \dfrac{G_F}{\sqrt{2}}\, r^2 \end{array}\right. \tag{21}$$

in which r < 1 or r = 1, depending on whether or not a singlet Higgs field induces an additional contribution to the U mass. The effects of Z and U exchanges can be described by the effective Lagrangian density

$$\mathcal{L}_{eff} = 2G_F \sqrt{2} \left[J_Z^2 + r^2 \frac{m_U^2}{m_U^2 + q^2} J_U^2\right] \tag{22}$$

The factor $m_U^2/m_U^2 + q^2$ originates from the U boson propagator. It is equal to 1 in the local limit ($|q| << m_U$), but almost vanishes if the U is very light.

The expression of the U current is[14] :

$$\begin{aligned} J^\mu_U = & - \tfrac{1}{4}(1 - \cos\phi_e)(\bar{\nu}_{eL}\gamma^\mu \nu_{eL} + \bar{e}_L \gamma^\mu e_L) + \tfrac{1}{4}(1 + \cos\phi_e)\, \bar{e}_R \gamma^\mu e_R \\ & - \tfrac{1}{4}(1 - \cos\phi_q)(\bar{u}_L \gamma^\mu u_L + \bar{d}_L \gamma^\mu d_L) + \tfrac{1}{4}(1 + \cos\phi_q)(\bar{u}_R \gamma^\mu u_R + \bar{d}_R \gamma^\mu d_R) \\ & + \ldots \end{aligned} \tag{23}$$

b) Neutrino scattering experiments

$\overset{(-)}{\nu}_\mu$-nucleon and $\overset{(-)}{\nu}_\mu$-electron scattering experiments can be parametrized in terms of the six quantities[3,14] :

$$\left\{\begin{array}{l} u_L = \frac{1}{2} - \frac{2}{3}\sin^2\theta + \frac{1}{8}(1 - \cos\phi_\mu)(1 - \cos\phi_q)\dfrac{m_U^2}{m_U^2 + q^2}\, r^2 \\[2ex] d_L = -\frac{1}{2} + \frac{1}{3}\sin^2\theta + \frac{1}{8}(1 - \cos\phi_\mu)(1 - \cos\phi_q)\dfrac{m_U^2}{m_U^2 + q^2}\, r^2 \\[2ex] u_R = -\frac{2}{3}\sin^2\theta - \frac{1}{8}(1 - \cos\phi_\mu)(1 + \cos\phi_q)\dfrac{m_U^2}{m_U^2 + q^2}\, r^2 \end{array}\right.$$

$$\left\{ d_R = \frac{1}{3}\sin^2\theta - \frac{1}{8}(1-\cos\phi_\mu)(1+\cos\phi_q)\frac{m_U^2}{m_U^2+q^2}r^2 \right. \tag{24}$$

$$\left\{ \begin{aligned} g_V &= -\frac{1}{2} + 2\sin^2\theta - \frac{1}{4}(1-\cos\phi_\mu)\cos\phi_e\frac{m_U^2}{m_U^2+q^2}r^2 \\ g_A &= -\frac{1}{2} + \frac{1}{4}(1-\cos\phi_\mu)\frac{m_U^2}{m_U^2+q^2}r^2 \end{aligned} \right. \tag{25}$$

The results of $\overset{(-)}{\nu}_\mu$-nucleon scattering experiments[15] imply that the additional U-exchange contributions to u_L and d_L cannot be too large, while the constraints on u_R and d_R are less restrictive. Altogether, we find :

$$\begin{aligned} &(1-\cos\phi_\mu)\frac{m_U^2}{m_U^2+q^2}r^2(1-\cos\phi_q) \lesssim 1/2 \\ &(1-\cos\phi_\mu)\frac{m_U^2}{m_U^2+q^2}r^2(1+\cos\phi_q) \lesssim 1 \end{aligned} \tag{26}$$

This implies

$$(1-\cos\phi_\mu)\frac{m_U^2}{m_U^2+q^2}r^2 \lesssim 3/4 \tag{27}$$

and one of the three factors at least must be smaller than 1.

i) $(1-\cos\phi_\mu) \lesssim 1/2$ would mean that the new neutral current is mostly V + A, at least in the muon sector.

ii) $m_U^2/m_U^2 + q^2 \lesssim 1/2$ would mean that the U is light compared with the momentum transfer in the experiments considered.

iii) $r^2 \lesssim 1/2$ would mean that a Higgs singlet gives an additional contribution to the U mass.

The results of ν_μ-e and $\bar{\nu}_\mu$-e scattering experiments can also be used to give constraints on the U boson, which are similar to the ones given by eq. (26, 27)[14].

When looking for deviations from the predictions of the standard model it is particularly important to remember that the exchanges of a light U boson would modify the neutral current phenomenology at lower values of the momentum transfer ($|q^2| \lesssim m_U^2$) only, the higher-$|q^2|$ phenomenology remaining unchanged.

c) e^+e^- annihilation

If the new neutral current is mostly V + A (i.e. for $|1 - \cos\phi| \lesssim 1/2$) it has no large effects on neutrino cross sections, but it can modify the $e^+e^- \to e^+e^-$, $\mu^+\mu^-$ or $\tau^+\tau^-$ annihilation cross sections[16]. Both the total cross section and the forward-backward asymmetry could be affected. The asymmetry can be expressed in terms of the parameter h_{AA}, which is equal to $g_A^2 = 1/4$ in the standard model, for $s >> m_Z^2$. Here h_{AA} is given by the following expression, independent of cos ϕ :

$$h_{AA} = \frac{1}{4}\left(\frac{m_Z^2}{m_Z^2 - s} + \frac{m_U^2}{m_U^2 - s} r^2\right) \tag{28}$$

From there one gets the new (simplified) expression of the asymmetry :

$$A \simeq -\frac{3}{2}\frac{G_F s}{8\pi\sqrt{2}\alpha}\left[\frac{m_Z^2}{m_Z^2 - s} + \frac{m_U^2}{m_U^2 - s} r^2\right] \tag{29}$$

The standard model predicts $A \simeq -9\%$ at $\sqrt{s} = 34$ GeV. If the U is heavy the predicted asymmetry would be multiplied by about $(1 + r^2)$, i.e. it would be doubled for r = 1. This is well in agreement with the Tasso result (A = -16.1 ± 3.2%), but it seems excluded by the results of the Jade, Cello and Mark J experiments, which report lower values of the asymmetry[16].

If $A \simeq 2(A_{standard})$ is excluded, this will imply that r is smaller than 1, or that the U boson is relatively light. To illustrate this, if we assume that r = 1 we find that a result such as

$$-16\% < A < -5\% \tag{30}$$

would imply

$$m_U < 20 \text{ GeV/c}^2 \tag{31}$$

In addition, limits on cos ϕ can be deduced from the values of the total cross sections, by using the expression

$$h_{VV} = (2\sin^2\theta - \tfrac{1}{2})^2 \frac{m_Z^2}{m_Z^2 - s} + \left(\frac{\cos\phi}{2}\right)^2 \frac{m_U^2}{m_U^2 - s} r^2 \tag{32}$$

such limits are not yet very constraining (e.g., $h_{VV} < .12 \Rightarrow r|\cos\phi| < .7$).

If r = 1 we may soon be forced to conclude that the U, if it exists, must be relatively light. It is then possible to search for it as a relatively narrow resonance in e^+e^- annihilation[17]. The electronic width is given by :

$$\Gamma(U \to e^+e^-) = \frac{G_F\, m_U^3}{24\pi\sqrt{2}} (1 + \cos^2 \phi_e)\, r^2 \qquad (33)$$

$$= 110\ \mathrm{eV/c^2}\ m_U(\mathrm{GeV})^3, \quad \text{if } \cos \phi_e = 0,\ r = 1 \qquad (34)$$

A 20 GeV/c^2 U would have a partial width $\sim$ 1 MeV/c^2. Whenever a scan for narrow resonances has been performed no such object has been found. This is the case, in particular, in most of the 1 to 7.6 GeV/c^2 mass interval, as well as in the T region. A systematic search would allow one to discover or eliminate a U particle having r = 1 and a mass in the energy range considered.

d) Production of a light U in ψ and T decays

We have discussed elsewhere the lifetime and decay modes of the U ; and its production in ψ, T, positronium and kaon decays, as well as beam dump experiments[17]. The latter[18] exclude the existence of a U boson in the 1 to 7 MeV/c^2 mass range, since in that case the decay $U \to e^+e^-$ should have been observed.

A U lighter than 1 MeV/c^2 either escapes or decays into $\nu\,\bar{\nu}$ pairs. There is also a very rare decay mode $U \to 3\,\gamma$ (brought to my attention by H. Faissner) with a branching ratio

$$B(U \to 3\,\gamma) \sim \alpha^3 \left(\frac{\cos \phi}{1 - \cos \phi}\right)^2 \qquad (35)$$

If the U mass is between $2m_e$ and $2m_\mu$ the U will decay into $\nu\,\bar{\nu}$ or e^+e^- with branching ratios of about 60% and 40% respectively, assuming $\cos \phi = 0$. We shall use, in the following analysis, the estimate

$$B(U \to \text{unobserved neutrals}) \gtrsim 60\% \qquad (36)$$

for $m_U < 2m_\mu$.

Strong experimental limits on radiative decays of the ψ now exist[19] and preliminary results concerning the T have been presented at this Conference[20]. Comparing the experimental limit

$$B(\psi \to \gamma + \underset{\text{lighter than 1 GeV/c}^2}{\text{unobserved neutrals}}) < 1.4\ 10^{-5} \qquad (37)$$

with the theoretical expectation for a U lighter than $2m_\mu$,

$$B(\psi \to \gamma + U_{\hookrightarrow \text{nothing}}) \sim (5 \text{ or } 6)\ 10^{-5}\ x^2\, r^2 \quad (60\% \text{ to } 100\%) \qquad (38)$$

we find the constraint

$$x^2 \; r^2 \lesssim .5 \tag{39}$$

In a similar way, a limit $\sim 10^{-4}$ on the analogous branching ratio of the T, when compared with the expected rate

$$B(T \to \gamma + U_{\hookrightarrow \text{nothing}}) \sim 3 \; 10^{-4} \; \frac{r^2}{x^2} \; (60\% \text{ to } 100\%) \tag{40}$$

would imply

$$\frac{r^2}{x^2} \lesssim .6 \tag{41}$$

Formulas (39, 41), if confirmed, would allow one to eliminate the existence of a very light U boson having r = 1, whatever the value of x is. (This, however, relies on the hypothesis (36) ; if cos ϕ were close to one most U's would decay into e^+e^-, $\mu^+\mu^-$ or $q\bar{q}$ pairs, rather than neutrino pairs). It would be useful to study not only the decays ψ or $T \to \gamma$ + nothing, but, also the decays into $\gamma \; e^+e^-$, or $\gamma \; \mu^+\mu^-$.

5. PHENOMENOLOGY OF THE GRAVITINO

a) The massive gravitino

The supersymmetry generator Q_α and the generator of spacetime translations P^μ satisfy the algebra[1)]

$$\{Q, \bar{Q}\} = -2 \not{P} \tag{42}$$

with

$$P^\mu = \int T^{\mu o}(x) \; d^3x \tag{43}$$

$$Q_\alpha = \int J^o{}_\alpha(x) \; d^3x \tag{44}$$

When supersymmetry is realized locally (supergravity[21)]) the theory is invariant under local spacetime transformations and includes general relativity. The spin-2 graviton couples to the conserved energy momentum tensor $T^{\mu\nu}$ while its superpartner, the spin -3/2 gravitino, couples to the conserved supersymmetry current $J^\mu{}_\alpha$. The strength of both couplings is fixed by the extremely small constant

$$\kappa = (8\pi \; G_{Newton})^{1/2} \simeq 4 \; 10^{-19} \; (\text{GeV}/c^2)^{-1} \tag{45}$$

The spontaneous breaking of global supersymmetry generates a massless spin-1/2 goldstino. As soon as supersymmetry is realized locally the Goldstone fermion field is eliminated by the super-Higgs mechanism while the gravitino acquires a mass $m_{3/2}$. It is given by

$$m_{3/2} = \kappa \frac{d}{\sqrt{6}} \quad (46)$$

in which the parameter d measures the magnitude of the spontaneous breaking of supersymmetry, at the global level.

b) Lifetime of the gravitino, and constraints on a short-lived gravitino

A massive gravitino would be unstable and decay into any boson-fermion pair, provided the decay is energetically allowed.

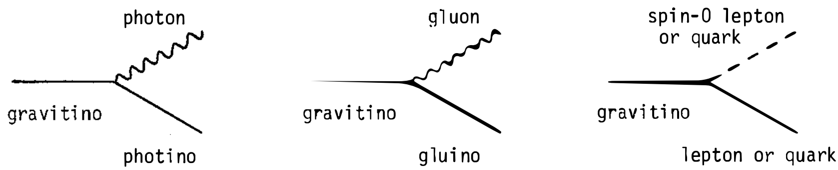

Fig. 4 : Vertices which may be responsible for gravitino decay.

Assuming that the gravitino is heavier than the photino(*) we find, as an order-of-magnitude estimate, the decay rate

$$\Gamma(\text{gravitino} \to \gamma + \text{photino}) \sim \frac{1}{6} G_{Newton}\, m_{3/2}^3 \quad (47)$$

A 1 GeV/c^2 gravitino would decay into γ + photino, while a heavier gravitino would have many other decay channels. This gives :

$$\text{for } m_{3/2} \sim 1\ GeV/c^2 \qquad \tau_{3/2} \sim 10^{15}\ s(\sim 3\ 10^7\ y) \quad (48)$$

$$\text{for } m_{3/2} \sim 10^5\ GeV/c^2 \qquad \tau_{3/2} \sim 10^{-2}\ sec. \quad (49)$$

(*)Alternately the photino, if heavier than the gravitino, would decay into γ + gravitino ; cf. G.R. Farrar's lecture, and refs. 22, which suggest that the photino might have a mass of 14 eV/c^2, or $\sim$ 100 eV/c^2.

If $m_{3/2}$ were larger than $\sim$ 1 GeV/c^2, almost none of the primordial gravitinos would be surviving now, and the gravitino contribution to the present energy density of the universe would be negligible.

Heavy gravitinos, however, might have modified the expansion rate of the early universe, and could have led to too large a value for the primordial helium abundance. This does not happen if gravitinos are sufficiently heavy (i.e. $m_{3/2}$ larger than $\sim 10^5$ GeV/c^2), so they decay very early ($\tau_{3/2}$ shorter than $\sim 10^{-2}$ s). In that case the gravitino contribution to the expansion rate when the n/p ratio freezes out is negligible, and the helium abundance is not affected.

Therefore gravitinos should be, either very heavy ($\gtrsim 10^5$ GeV/c^2) and short-lived, or on the contrary very light and stable or quasistable. Before discussing constraints on gravitinos of the latter type we need to know more about their interactions.

c) Interaction cross sections of the gravitino

The amplitudes involving one gravitino are gravitational amplitudes proportional to κ. The corresponding cross sections or decay rates are proportional to G_{Newton}. This does not mean, however, that they are necessarily negligible, since the gravitino mass may be very small, and one can show that the interaction cross sections, or decay rates, involving a very light gravitino with polarization $\pm$ 1/2 are proportional to

$$\frac{G_N}{m_{3/2}^2} = \frac{3}{4\pi\ d^2} \tag{50}$$

More precisely a very light gravitino behaves essentially like the massless goldstino of globally supersymmetric theories[3,23]. Whether or not the corresponding cross sections and decay rates are sizeable depends on two things :

i) the value of the parameter d ; if d is relatively small the gravitino may have interactions comparable in strength with weak interactions, or even larger ; on the other hand if d is large the gravitino interactions are negligible ; d large does not imply that the boson-fermion mass^2-splitting Δm^2 itself has to be large : one may have $d \gg \Delta m^2$, provided the new U(1) gauge coupling constant g" is very small.

ii) low-energy theorems tell us that the amplitudes involving one gravitino (goldstino) generally vanish at low energy ; then one can expect cross sections and decay rates involving a light gravitino to be suppressed ; however, this is not the case, owing to the photon-photino or gluon-gluino exact, or approximate, mass degeneracy ; as a result, the dominant processes involving the gravitino at low energies are :

$$\text{gravitino} + A \leftrightarrow \text{photino} + B \qquad (51)$$

and, also, the production of gravitino-antiphotino pairs. They can be computed in terms of the matrix element of the electromagnetic current :

$$< B|\ J^{\mu}_{el}|A > \qquad (52)$$

The equivalence between the gravitino and goldstino aspects is valid for the total amplitudes. It involves non trivial properties of individual diagrams, for example those illustrated in fig. 5.

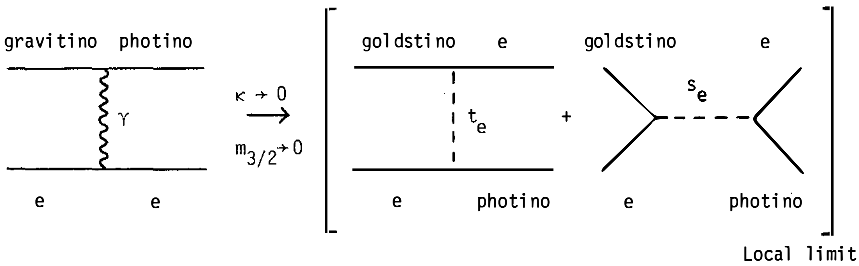

Fig. 5 : Diagrams representing the dominant contributions to the scattering of light gravitinos, or equivalently, goldstinos, on matter.

This leads to the following expression of the gravitino cross section[11] :

$$\sigma(\text{gravitino} + e \leftrightarrow \text{photino} + e) = \frac{16\pi\ \alpha\ G_{Newton}}{9m^2_{3/2}}\ m_e\ E \qquad (53)$$

$$\simeq .54\ 10^{-42}\text{cm}^2\ \left(\frac{m_{3/2}}{10^{-5}\ \text{eV}/c^2}\right)^{-2}\ E(\text{GeV}) \qquad (54)$$

which can be compared with the expected value of the ν_μ e scattering cross section for $\sin^2\theta = 1/4$, in the standard model :

$$\sigma(\nu_\mu e \to \nu_\mu\ e) \simeq 1.4\ 10^{-42}\text{cm}^2\ E(\text{GeV}) \qquad (55)$$

We have, roughly :

$$\sigma(\text{gravitino}) \simeq .4\ \sigma(\nu_\mu \text{ or } \bar{\nu}_\mu)\left(\frac{m_{3/2}}{10^{-5}\ \text{eV}/c^2}\right)^{-2} \qquad (56)$$

d) Particle physics limits on the gravitino mass

Gravitino-antiphotino pairs can be produced in the decays of heavy vector resonances such as the ψ and the Υ (or toponium)[3,23]. The branching ratio is given by

$$\frac{B(\text{onium} \rightarrow \text{inos})}{B(\text{onium} \rightarrow e^+e^-)} = \frac{G_{\text{Newton}}\, m^4_{\text{onium}}}{3\alpha\, m^2_{3/2}} \qquad (57)$$

From the study of the decay $\psi' \rightarrow \pi^+ \pi^- \psi(\rightarrow \text{nothing})$ one can deduce that the ratio (57) is smaller than .1, i.e.

$$m_{3/2} > 1.5\ 10^{-8}\ \text{eV/c}^2 \qquad (58)$$

A better limit could be obtained with the Υ , which is three times heavier than the ψ.

More generally, other processes where $\nu\,\bar{\nu}$ pairs can be radiated from a fermion line may also be used to give an upper limit on the radiation of a gravitino-antiphotino pair, and therefore a lower limit on the gravitino mass. For example a much stronger limit could be obtained from the study of the process $e^+e^- \rightarrow \gamma$ + (2 unobserved neutrals) at PETRA. From the cross sections given in ref. 11 we find that, if $m_{3/2} \lesssim 2.5\ 10^{-7}$ eV/c^2, the new-inos pair production would be $\gtrsim$ 600 times the $\nu_\mu\,\bar{\nu}_\mu$ pair production. This might already be excluded. Incidently, an upper limit on the process $e^+e^- \rightarrow \gamma$ photino antiphotino of the order of 60 times the $\gamma\,\nu_\mu\,\bar{\nu}_\mu$ production cross section would allow one to eliminate the existence of spin-0 electrons lighter than $m_W/2 \sim 40$ GeV/c^2.

e) Decoupling temperature of the gravitino and constraints on its mass

Using the values of the gravitino cross sections[11] one can evaluate their decoupling temperature T_d in terms of their mass. By comparing the interaction rate $n\,\sigma\,v \sim T^5/m^2_{3/2}$ with the expansion rate $t^{-1} \sim T^2$ one finds T_d as a function of the strength of the interaction (cf. ref. 24). From expression (56) of the gravitino $\leftrightarrow$ photino cross section, for gravitinos in the $\pm$ 1/2 polarization state, one gets

$$T_d(\text{gravitino}_{\pm 1/2}) \simeq T_d(\nu_\mu)\left(\frac{m_{3/2}}{10^{-5}\ \text{eV/c}^2}\right)^{2/3} \qquad (59)$$

while gravitinos in the $\pm$ 3/2 polarization state are so weakly interacting ($\sim G_{\text{Newton}}$) that they would have decoupled extremely early (or were never at equilibrium).

Formula (59) is a rough estimate, only valid for lower values of T_d. For

T > m(R-hadron) the reaction gravitino $\leftrightarrow$ gluino is more important than gravitino $\leftrightarrow$ photino, and keeps gravitinos in thermal equilibrium down to lower values of the temperature ; then (59) is expected to overestimate T_d by about an order of magnitude.

A gravitino lighter than $\sim 10^{-2}$ eV/c^2 would decouple at $T \lesssim 100$ MeV/$c^2 \simeq m_\mu$, and would have the same effect as an additional two-component neutrino species for the abundance of helium in the universe. According to ref. 24, at most 4 light weakly interacting particles could be allowed. If ν_e, ν_μ, ν_τ and the photino were these four particles there would be no room left for the gravitino, as already discussed in ref. 3. This would imply the following lower limit on the gravitino mass :

$$m_{3/2} \gtrsim 10^{-2} \text{ eV}/c^2 \tag{60}$$

If the gravitino is heavier than $\sim 10^{-1}$ eV/c^2 it decouples before the quark-hadron phase transition. Various particle species annihilate, including finally the muons, heating up the photon and neutrino (and, presumably, photino) gas with respect to the gravitino gas, before neutrinos drop out of equilibrium, at $T \sim$ 1 MeV/c^2. Gravitinos are colder than neutrinos ; fewer of them are present when the n/p ratio freezes out (as nowadays). Such gravitinos have a negligible effect on the helium abundance. Similarly, the maximum mass they may have, without leading to an unacceptably large energy density for the universe, is larger than for ordinary neutrinos. If a (two-component) neutrino may be as heavy as ~ 100 eV/c^2, one finds (disregarding momentarily possible effects of photinos) :

$$m_{3/2} \lesssim 100 \text{ eV}/c^2 \, \frac{g_I(< T_d)}{g_I = \frac{43}{4}} \tag{61}$$

in which $g_I(< T_d)$ is the effective number of interacting degrees of freedom after gravitinos decouple, and g_I takes into account the photons, electrons and neutrinos, which remain at equilibrium down to $T \sim$ a few MeV/c^2. Formula (61) was used in ref. 25 to derive an upper limit of the order of 1 keV/c^2 for the gravitino mass.

One may also consider the effects of photinos. Let us assume, for simplicity, that they decoupled at a temperature T lower than m_μ (this is the case, unless spin-0 electrons are very heavy). Taking into account γ, ν_e, ν_μ, ν_τ and photinos we now have :

$$g_I = 2 + \frac{7}{8}(4 + 6 + 2) = \frac{25}{2} \tag{62}$$

This correction has little effect on the bound (61) for the gravitino mass.

Potentially more important is the fact that photinos may remain at equilibrium during the annihilation of e^+e^- pairs. These pairs would annihilate not only into photons but also into photinos and antiphotinos. Let us attempt to describe it in a more quantitative way. In the extreme situation for which photino interactions with electrons are relatively large, and photinos remain at equilibrium during most of the e^+e^- annihilation period, the usual formula giving the change of the number of photons in a co-moving volume

$$\frac{N_\gamma}{N_{\gamma o}} = \frac{4}{11} \tag{63}$$

is replaced by

$$\frac{N_\gamma}{N_{\gamma o}} = \frac{2 + \frac{7}{8}\, 2}{2 + \frac{7}{8}(4 + 2)} = \frac{15}{29} \tag{64}$$

e^+e^- pairs are now less efficient in heating up the photon gas with respect to the neutrino gas, since they heat up the photino gas as well. The neutrino temperature is higher than it would have been in the absence of photinos. It follows from entropy conservation that

$$T_\nu = \left(\frac{15}{29}\right)^{1/3} T_\gamma \tag{65}$$

The present number density of neutrinos of a given species is also higher :

$$n_{\nu o} = \frac{3}{4}\,\frac{15}{29}\, n_{\gamma o} \simeq 150\ \mathrm{cm}^{-3} \left(\frac{T_{\gamma o}}{2.7°\,\mathrm{K}}\right)^3 \tag{66}$$

In that case one gets a smaller limit on neutrino masses, i.e. ~ 70 eV/c^2 instead of ~ 100 eV/c^2.

Similarly, we find

$$\begin{cases} T_{\mathrm{gravitino}} = \left[\dfrac{15}{29}\,\dfrac{\frac{25}{2}}{g_I(<T_d)}\right]^{1/3} T_\gamma \\ T_{\mathrm{photino}} \simeq T_\gamma \end{cases} \tag{67}$$

$$\begin{cases} n_{\mathrm{gravitino}\ 0} = \dfrac{3}{4}\,\dfrac{15}{29}\,\dfrac{\frac{25}{2}}{g_I(<T_d)}\ 400\mathrm{cm}^{-3}\left(\dfrac{T_{\gamma o}}{2.7°\,\mathrm{K}}\right)^3 \\ n_{\mathrm{photino}\ 0} = \dfrac{3}{4}\ 400\mathrm{cm}^{-3}\left(\dfrac{T_{\gamma o}}{2.7°\,\mathrm{K}}\right)^3 \end{cases} \tag{68}$$

If indeed photinos decoupled at the end of the e^+e^- annihilation period the upper limit on their mass would be lowered down to $\sim$ 40 eV/c^2, while the upper limit on the gravitino mass obtained from formula (61) would have to be multiplied by $\sim$ .6.

A 1 keV/c^2 gravitino may have a decoupling temperature of a few tens of GeV, i.e. comparable to the masses of spin-0 quarks, denoted by m_s. More care is then required, since formulas such as (53, 54, etc.) are only valid for $T < m_s$. Other diagrams, where the gravitino couples to lepton-spin-0-lepton or quark-spin-0-quark pairs, soften the high-energy behaviour of the cross sections. One has

$$\sigma(\text{gravitino} \leftrightarrow \text{gluino}) \sim \begin{cases} \dfrac{G_N \alpha_s}{m_{3/2}^2} T^2 & \text{for } T < m_s \quad (69) \\[2ex] \dfrac{G_N \alpha_s}{m_{3/2}^2} \dfrac{m_s^4}{T^2} & \text{for } T > m_s \quad (70) \end{cases}$$

If spin-0 quarks are relatively light the cross section (69, 70) may be too small to keep gravitinos in equilibrium, even at $T \sim m_s$. In that case gravitinos would decouple very early, for T of the order of the unification mass or may be the Planck mass, when the number of interacting effective degrees of freedom was larger. In an SU(5)(x U(1)) supersymmetric theory this number is at least of the order of 400, but it can be much larger when other groups are chosen. Values as high as $\sim 10^4$ may not be unreasonable. This would give a less stringent limit on the mass of a quasistable gravitino

$$m_{3/2} \lesssim 100 \text{ keV}/c^2 \qquad (71)$$

To conclude this section about the gravitino, it appears that astrophysical considerations favour having a stable or quasistable gravitino in the 10^{-2} eV/c^2 $\rightarrow$ up to maybe 100 keV/c^2 mass range or, alternately, a short-lived one heavier than $\sim 10^5$ GeV/c^2.

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