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Hadron pair production in semi-inclusive electron positron annihilation process at twist-4

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Abstract Fragmentation functions are important quantities in describing the hadronization process in high energy reactions. They can induce various azimuthal modulations which can be measured to reveal correlations between transverse momenta and polarizations. Without introducing initial state uncertainties, electron positron annihilation process is known as an ideal process to investigate the fragmentation functions. In this paper, therefore, we calculate the hadron pair production in the semi-inclusive electron positron annihilation process $e^+ + e^- \to h_1 + h_2 + \bar{q} + X$ at twist-4 to study dihadron fragmentation functions. Here $\bar{q}$ denotes an antiquark that corresponds to a jet of hadrons in experiments. Together with single hadron fragmentation functions, dihadron fragmentation functions can provide additional ways to extract nucleon parton distribution functions from the semi-inclusive deeply inelastic scattering experiments with two detected final state hadrons. We calculate the differential cross section of the hadron pair production semi-inclusive electron positron annihilation process at twist-4 level. The calculation is carried out by using the collinear expansion method. We also calculate azimuthal asymmetries in terms of dihadron fragmentation functions. Contributions from four-quark correlator are also taken into account. Both the electromagnetic and weak interactions are considered in this paper.

1 Introduction

Fragmentation functions (FFs) are important quantities in describing the hadronization process in high energy reactions. The study of FFs or the distribution of hadrons produced in the fragmentation of a quark/gluon offers a great opportunity to understand the mechanism of hadronization and hadronic structures in certain high energy reactions. From a phenomenological point of view, they induce various azimuthal modulations which can be measured to reveal correlations between transverse momenta and polarizations. The single hadron production FFs have been widely investigated in the past several decades in the deeply inelastic scattering and electron positron annihilation processes. Hadron pair production FFs or DiFFs, meanwhile, have gained a great deal of attention in recent years. They were first introduced to describe the hadron pair production in a fragmenting jet at leading twist level in Refs. [1,2] and extended to the twist-3 level in Ref. [3]. One of the reasons why one studies DiFFs is that they are universal and can be factorized in high energy reactions. By extracting from the two-jet events in the electron positron annihilation process [4–7], conveniently, they can be used to study the nucleon structures, especially for the transversity distribution function which reveals the transversely polarized quarks in a transversely polarized nucleon [8–11]. Because the intrinsic transverse momentum of the quark can be integrated away and no transverse momentum dependent (TMD) functions are required. Another reason is that DiFFs are considered to be strongly related to the jet handedness and can be used to investigate the quark and/or gluon polarizations [4,11,12]. For example, DiFF $G_1^\perp$ is the difference of probabilities for a longitudinally polarized quark with opposite chiralities to produce a pair of unpolarized hadrons.

In the quantum field theoretical formulation, both FFs and DiFFs are defined via the corresponding quark–quark correlators or correlation functions which are matrices in the Dirac space depending on the hadron states. This suggests that they can be decomposed into different components expressed in terms of basic Lorentz covariants and scalar functions. These scalar functions which contain the information of hadron production mechanism are known as FFs and/or DiFFs. Previous discussion of DiFFs are limited to twist-3 or subleading twist level. In this paper, however, we extend the discussion to the twist-4 level in the semi-inclusive electron positron annihilation process. Semi-inclusive implies a back-to-back jet is also measured in addition to the hadron pair. The produced

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hadrons are spinless. Spin-dependent hadron production process is not considered in this paper. In the semi-inclusive electron positron annihilation process, we find that the only difference between the single hadron production and the hadron pair production is the nonperturbative correlator. In this case, it is straightforward to use the collinear expansion [13–16] which has been used widely in deeply inelastic scattering and annihilation processes [17–25] to calculate the hadron pair production in this paper. With the collinear expansion, we present a systematic calculation of the hadron-pair production in the annihilation process, both the electromagnetic (EM) and weak interactions are considered. We note that if only higher twist contributions are considered, weak and interference terms and corresponding results should not be included in this paper. In other words, only the EM interaction and/or higher twist contributions (twist-4) make sense at low-energy limit. As for weak interaction, future electron positron colliders, e.g., CEPC, FCC-ee, which are high-luminosity high-energy colliders provide unique precise measurements of Z, W and H bosons and the top quark, it must be considered. Of course, for the hadron production annihilation process, leading twist contributions dominate, higher twist ones are suppressed. In a word, we present a systematic calculation of the hadron-pair production in the annihilation process, which includes both the higher twist and weak results. Our calculation, for a new reaction, provide a set of measurable quantities for a better understanding of hadronization and quark flavor separation.

This paper is organized as follows. In Sect. 2 we present a brief introduction to formalism of the semi-inclusive electron positron annihilation process. In Sect. 3 we calculate the hadronic tensor at twist-4 level. Contributions from the four-quark correlator are involved. We present the differential cross section and azimuthal asymmetries in Sect. 4. Finally, we present the summary in Sect. 5

2 The formalism

To be explicit, we consider the tree-level semi-inclusive process $e^+ + e^- \to h_1 + h_2 + \bar{q} + X$ where $\bar{q}$ denotes an antiquark that corresponds to a jet of hadrons and h_1, h_2 denote outgoing hadrons in experiments, see Fig. 1. The differential cross section of this process is given by

$$d\sigma = \frac{\alpha_{em}^2}{2\pi^2 s Q^4} A_r L^r_{\mu\nu}(l_1, l_2) W_r^{\mu\nu}(p_1, p_2, k') \times \frac{d^3 p_1}{2E_1} \frac{d^3 p_2}{2E_2} \frac{d^3 k'}{2\pi 2E_k}, \tag{1}$$

where α_{em} is the fine structure constant, $s = Q^2 = q^2$ with $q = l_1 + l_2$, l_1, l_2 are momenta of the leptons, p_1, p_2 are

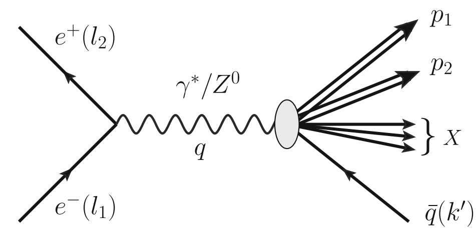

Fig. 1 Illustrating diagram for the $e^+ + e^- \to h_1 + h_2 + \bar{q} + X$ process

momenta of the outgoing hadrons.The symbol r can be $\gamma\gamma$, ZZ and γZ, for EM, weak and interference terms, respectively. A summation over r in Eq. (1) is understood, i.e. the total cross section is given by

$$d\sigma = d\sigma^{ZZ} + d\sigma^{\gamma Z} + d\sigma^{\gamma\gamma}. \tag{2}$$

A_r's are defined as

$$A_{\gamma\gamma} = e_q^2,$$
$$A_{ZZ} = \frac{Q^4}{\left[(Q^2 - M_Z^2)^2 + \Gamma_Z^2 M_Z^2\right] \sin^4 2\theta_W} \equiv \chi,$$
$$A_{\gamma Z} = \frac{-2e_q Q^2 (Q^2 - M_Z^2)}{\left[(Q^2 - M_Z^2)^2 + \Gamma_Z^2 M_Z^2\right] \sin^2 2\theta_W} \equiv \chi_{int}, \tag{3}$$

where e_q is the charge of a certain quark with flavor q, Γ_Z, M_Z are width and mass of Z^0 boson, θ_W is the Weinberg angle.

The leptonic tensors for different cases are given by

$$L^{\gamma\gamma}_{\mu\nu}(l_1, l_2) = l_{1\mu} l_{2\nu} + l_{1\nu} l_{2\mu} - (l_l \cdot l_2) g_{\mu\nu}, \tag{4}$$

$$L^{ZZ}_{\mu\nu}(l_1, l_2) = c_1^e \left[l_{1\mu} l_{2\nu} + l_{1\nu} l_{2\mu} - (l_l \cdot l_2) g_{\mu\nu}\right] + i c_3^e \varepsilon_{\mu\nu l_1 l_2}, \tag{5}$$

$$L^{\gamma Z}_{\mu\nu}(l_1, l_2) = c_V^e \left[l_{1\mu} l_{2\nu} + l_{1\nu} l_{2\mu} - (l_l \cdot l_2) g_{\mu\nu}\right] + i c_V^e \varepsilon_{\mu\nu l_1 l_2}, \tag{6}$$

where $c_1^e = (c_V^e)^2 + (c_A^e)^2$ and $c_3^e = 2 c_V^e c_A^e$. c_V^e and c_A^e are defined in the weak interaction current $J_\mu(x) = \bar{\psi}(x) \Gamma_\mu \psi(x)$ with $\Gamma_\mu = \gamma_\mu (c_V^e - c_A^e \gamma^5)$. Similar notations are also used for quarks where the superscript e is replaced by q. The corresponding hadronic tensor are given by

$$W^{\mu\nu}_{\gamma\gamma} = \sum_X \delta(q - p_1 - p_2 - k' - p_X) \langle 0 | J^\mu_{\gamma\gamma}(0) \times |p_1, p_2, k', X\rangle \langle p_1, p_2, k', X | J^\nu_{\gamma\gamma}(0) | 0\rangle, \tag{7}$$

$$W^{\mu\nu}_{ZZ} = \sum_X \delta(q - p_1 - p_2 - k' - p_X) \langle 0 | J^\mu_{ZZ}(0) \times |p_1, p_2, k', X\rangle \langle p_1, p_2, k', X | J^\nu_{ZZ}(0) | 0\rangle, \tag{8}$$

$$W^{\mu\nu}_{\gamma Z} = \sum_X \delta(q - p_1 - p_2 - k' - p_X) \langle 0 | J^\mu_{ZZ}(0)$$

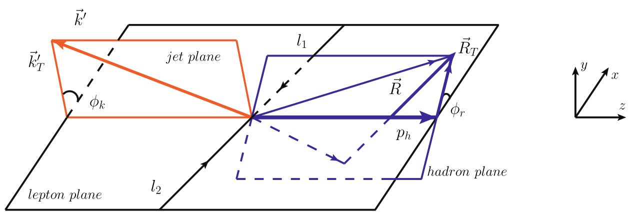

Fig. 2 Kinematics for the $e^+ + e^- \to h_1 + h_2 + \bar{q} + X$ process

$$\times |p_1, p_2, k', X\rangle\langle p_1, p_2, k', X| J^{\nu}_{\gamma\gamma}(0)|0\rangle, \quad (9)$$

where $J^{\mu}_{\gamma\gamma}(0) = \bar{\psi}(0)\gamma^{\mu}\psi(0)$ and $J^{\mu}_{ZZ}(0) = \bar{\psi}(0)\Gamma^{\mu}\psi(0)$. Although, we have shown EM, weak and interference terms for both the leptonic and hadronic tensors, we only present calculations of the weak interaction in the following context for simplicity. Other cases can be obtained in the similar way or by changing c_1, c_3 into $1, 0$ and c_V, c_A for EM and interference cases, respectively.

To describe the hadron pair production in the electron positron annihilation process, we define $p_h = p_1 + p_2$, $R = (p_1 - p_2)/2$ and introduce the frame, see Fig. 2, where momenta can be parameterized as

$$q = Q(1, 0, 0, 0), \quad (10)$$
$$l_1 = \frac{Q}{2}(1, -\sin\theta, 0, -\cos\theta), \quad (11)$$
$$p_h = (E_h, 0, 0, p_z), \quad (12)$$
$$R = (E_r, |R_T|\cos\phi_r, |R_T|\sin\phi_r, R_z), \quad (13)$$
$$k' = (E_k, |k'_T|\cos\phi_k, |k'_T|\sin\phi_k, k'_z). \quad (14)$$

We also introduce the following variables used in this paper,

$$z = \frac{2p_h\cdot q}{Q^2} = \frac{2p_1\cdot q}{Q^2} + \frac{2p_2\cdot q}{Q^2} = z_1 + z_2, \quad (15)$$
$$\xi = \frac{z_1}{z} = 1 - \frac{z_2}{z}. \quad (16)$$

In this case, the phase space factor can be rewritten as

$$\frac{d^3p_1}{2E_1}\frac{d^3p_2}{2E_2}\frac{d^3k'}{2E_k} = \frac{\pi}{8} z Q^2 dz dy d\phi_r dM_h^2 d\xi \frac{dk'_z}{2E_k} d^2k'_T. \quad (17)$$

Here we have used $d\Omega = 2dyd\phi_L = 4\pi dy$, with $y = p_h\cdot l_1/p_h\cdot q$, ϕ_L is the angle of lepton with respect to p_h, $d^2R_T = \xi(1-\xi)d\phi_r dM_h^2$, $M_h^2 = p_h^2 = (p_1+p_2)^2$. Thus, the differential cross section can be rewritten as

$$\frac{d\sigma}{dzdyd^2R_Td\xi d^2k'_T} = \frac{\alpha_{em}^2 z}{16\pi Q^4}\chi L_{\mu\nu}(l_1, l_2)W^{\mu\nu}(p_h, R, k'_T). \quad (18)$$

Here we have defined the hadronic tensor in terms of p_h, R and integrated over dk'_z,

$$W^{\mu\nu}(p_h, R, k'_T) = \int \frac{dk'_z}{2\pi 2E_k} W^{\mu\nu}(p_h, R, k'). \quad (19)$$

3 Hadronic tensor

From the previous section, we see that the cross section is given by the contraction of the leptonic tensor and the hadronic tensor. To obtain the cross section, we need the explicit expression of the hadronic tensor in the parton model. We calculate the hadronic tensor in the following context.

3.1 Hadronic tensor in the parton model

At the tree level of perturbative quantum chromodynamics (pQCD), in the parton model, we need to consider the series of diagrams illustrated in Fig. 3 where diagrams with exchange of j gluon(s) ($j = 0, 1, 2, \ldots$) are included. After the collinear expansion, the semi-inclusive hadronic tensor is obtained as

$$W_{\mu\nu}(p_h, R, k'_T) = \sum_{j,c}\tilde{W}^{(j,c)}_{\mu\nu}(p_h, R, k'_T), \quad (20)$$

where c denotes different cuts. The $\tilde{W}^{(j,c)}_{\mu\nu}$ is a trace of the collinear-expanded hard part and gauge invariant quark–j-gluon–quark correlator. The explicit expression are given by

$$\tilde{W}^{(0)}_{\mu\nu} = \frac{1}{2}\mathrm{Tr}\big[\hat{h}^{(0)}_{\mu\nu}\hat{\Xi}^{(0)}\big], \quad (21)$$
$$\tilde{W}^{(1,L)}_{\mu\nu} = -\frac{1}{4(p_h\cdot q)}\mathrm{Tr}\big[\hat{h}^{(1)\rho}_{\mu\nu}\hat{\Xi}^{(1)}_{\rho}\big], \quad (22)$$
$$\tilde{W}^{(2,M)}_{\mu\nu} = \frac{1}{4(p_h\cdot q)^2}\mathrm{Tr}\big[\hat{h}^{(2)\rho\sigma}_{\mu\nu}\hat{\Xi}^{(2,M)}_{\rho\sigma}\big], \quad (23)$$
$$\tilde{W}^{(2,L)}_{\mu\nu} = \frac{1}{4(p_h\cdot q)^2}\mathrm{Tr}\big[\hat{N}^{(2)\rho\sigma}_{\mu\nu}\hat{\Xi}^{(2)}_{\rho\sigma} + \hat{h}^{(1)\rho}_{\mu\nu}\hat{\Xi}^{(2\prime)}_{\rho}\big], \quad (24)$$

where we have omitted the arguments. The hard parts are given by

$$\hat{h}^{(0)}_{\mu\nu} = \Gamma^q_\mu \slashed{n}\Gamma^q_\nu/p_h^+, \quad (25)$$
$$\hat{h}^{(1)\rho}_{\mu\nu} = \Gamma^q_\mu \slashed{n}\gamma^\rho\slashed{\bar{n}}\Gamma^q_\nu, \quad (26)$$
$$\hat{N}^{(2)\rho\sigma}_{\mu\nu} = q^-\Gamma_\mu\gamma^\rho\slashed{n}\gamma^\sigma\Gamma_\nu, \quad (27)$$
$$\hat{h}^{(2)\rho\sigma}_{\mu\nu} = p_h^+\Gamma_\mu\slashed{\bar{n}}\gamma^\rho\slashed{n}\gamma^\sigma\slashed{\bar{n}}\Gamma_\nu/2. \quad (28)$$

The quark–j-gluon–quark correlators are given by

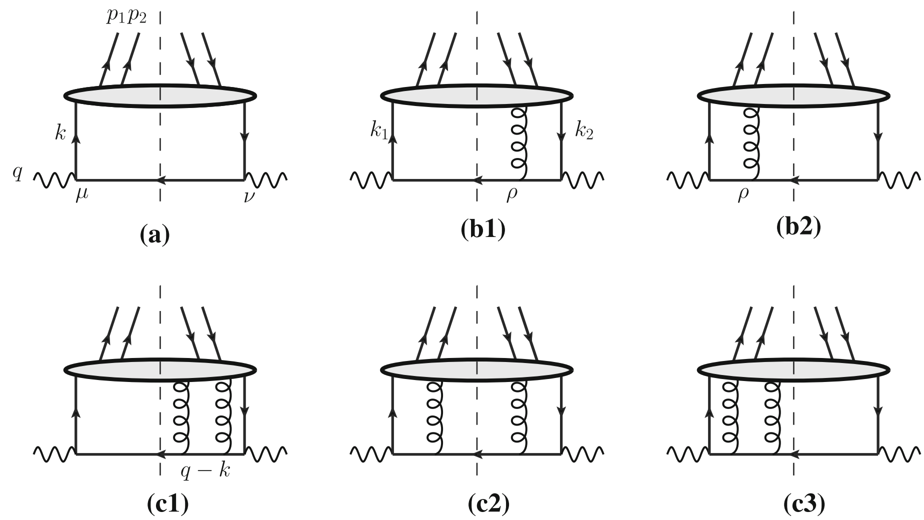

Fig. 3 The first few diagrams as examples of the considered diagram series with exchange of j-gluon(s) and different cuts. We see (**a**) $j=0$, (**b1**) $j=1$ and left cut, (**b2**) $j=1$ and right cut, (**c1**) $j=2$ and left cut, (**c2**) $j=2$ and middle cut, and (**c3**) $j=2$ and right cut, respectively

$$\hat{\Xi}^{(0)} = \sum_X \int \frac{p_h^+ d\zeta^- d^2\zeta_T}{2\pi} e^{-ip_h^+\zeta^-/z + ik_T\zeta_T} \times \langle 0|\mathcal{L}^\dagger(0,\infty)\psi(0)|p_h, R, X\rangle \times \langle p_h, R, X|\bar{\psi}(\zeta)\mathcal{L}(\zeta,\infty)|0\rangle, \quad (29)$$

$$\hat{\Xi}^{(1)}_\rho = \sum_X \int \frac{p_h^+ d\zeta^- d^2\zeta_T}{2\pi} e^{-ip_h^+\zeta^-/z + ik_T\zeta_T} \times \langle 0|\mathcal{L}^\dagger(0,\infty)D_\rho(0)\psi(0)|p_h, R, X\rangle \times \langle p_h, R, X|\bar{\psi}(\zeta)\mathcal{L}(\zeta,\infty)|0\rangle, \quad (30)$$

$$\hat{\Xi}^{(2M)}_{\rho\sigma} = \sum_X \int \frac{p_h^+ d\zeta^- d^2\zeta_T}{2\pi} e^{-ip_h^+\zeta^-/z + ik_T\zeta_T} \times \langle 0|\mathcal{L}^\dagger(0,\infty)D_\rho(0)\psi(0)|p_h, R, X\rangle \times \langle p_h, R, X|\bar{\psi}(\zeta)D_\sigma(\zeta)\mathcal{L}(\zeta,\infty)|0\rangle, \quad (31)$$

$$\hat{\Xi}^{(2')}_\rho = \sum_X \int \frac{p_h^+ d\zeta^- d^2\zeta_T}{2\pi} e^{-ip_h^+\zeta^-/z + ik_T\zeta_T} \times p_h^\sigma \langle 0|\mathcal{L}^\dagger(0,\infty)D_\rho(0)D_\sigma(0)\psi(0)|p_h, R, X\rangle \times \langle p_h, R, X|\bar{\psi}(\zeta)\mathcal{L}(\zeta,\infty)|0\rangle, \quad (32)$$

$$\hat{\Xi}^{(2)}_{\rho\sigma} = \sum_X \int \frac{p_h^+ d\zeta^- d^2\zeta_T}{2\pi} ip_h^+ d\eta^- e^{-ip^+\zeta^-/z + ik_T\zeta_T} e^{-ip_h^+\eta^-/z} \times \langle 0|\mathcal{L}^\dagger(\eta,\infty)D_\rho(\eta)D_\sigma(\eta)\mathcal{L}^\dagger(0,\eta^-)\psi(0) \times |p_h, R, X\rangle\langle p_h, R, X|\bar{\psi}(\zeta)\mathcal{L}(\zeta,\infty)|0\rangle. \quad (33)$$

where $D_\rho = i\partial_\rho - gA_\rho$ are the transverse covariant derivative, $\mathcal{L}(0, y)$ is the gauge link. To obtain the gauge link, one should consider all the gluon exchanging diagrams. However, we only show first few diagrams as examples up to twist-4 in this paper. Higher contributions are neglected for simplicity. The argument ζ in the quark filed operator ψ and gauge link represents $(0, \zeta^-, \vec{\zeta}_T)$. We note that the leading power contribution of $\tilde{W}^{(j)}_{\mu\nu}$ is twist-$(j+2)$. Therefore the second term in Eq. (24) has no contribution up to twist-4 because of of the factor p^σ in the definition of $\hat{\Xi}^{(2')}_\rho$ given by Eq. (32). The leading power contribution of this term is twist-5.

3.2 Decomposition of correlators

In the jet production semi-inclusive electron positron annihilation process, there is no helicity flip, which implies only the chiral even quantities are involved. We only need to consider the γ^α- and the $\gamma^5\gamma^\alpha$-terms in the decomposition of correlators in terms of gamma-matrices, such as $\hat{\Xi}^{(0)} = \gamma^\alpha \Xi^{(0)}_\alpha + \gamma^5\gamma^\alpha \tilde{\Xi}^{(0)}_\alpha + \cdots$. Here $\Xi^{(0)}_\alpha$, $\tilde{\Xi}^{(0)}_\alpha$ are coefficient functions which can be obtained by

$$\Xi^{(0)}_\alpha = \frac{1}{4}\mathrm{Tr}[\gamma^\alpha \hat{\Xi}^{(0)}], \quad (34)$$

$$\tilde{\Xi}^{(0)}_\alpha = \frac{1}{4}\mathrm{Tr}[\gamma^\alpha\gamma^5 \hat{\Xi}^{(0)}]. \quad (35)$$

We see that they are respectively a vector and an axial-vector and can be further decomposed according to their Lorentz transformation properties in terms of the basic Lorentz covariants constructed from basic variables at hand. The coefficient functions are therefore expressed as the sum of the basic Lorentz covariants multiplied by scalar functions which are known as the DiFFs. From previous discussion, we see only $\bar{n}_\alpha$, n_α, $k_{T\alpha}$ and $R_{T\alpha}$ as well as some scalars can be used to construct Lorentz covariants. For example, we have

$$\hat{\Xi}^{(0)} = \gamma^\alpha (\bar{n}_\alpha A + n_\alpha B + k_{T\alpha} C + R_{T\alpha} D) + \cdots, \quad (36)$$

where A, B, C, D are scalar functions. We see that A, C, D and B are leading twist, twist-3 and twist-4 functions, respectively. In keeping with conventions, we rename $A \sim D_1$, $B \sim D_3$, $C \sim D^\perp$ and $D \sim D^\triangleleft$. Inserting those dimension coefficients, we therefore obtain Eq. (37). Similar method of decomposing the correlator can be found in Refs.

[23,24,26]. The other DiFFs can be obtained in the similar way, we present them in the following.

Based on the previous discussion and conventions given in Refs. [1,3], we present the decomposition of correlators at twist-4 level. Most of the DiFFs given below are new, which were not included in previous references. For the quark–quark correlator, the coefficient functions are decomposed as

$$z\Xi^{(0)}_{\alpha} = \bar{n}_\alpha p_h^+ D_1 + R_{T\alpha} D^{\sphericalangle} + k_{T\alpha} D^{\perp} + n_\alpha \frac{M_h^2}{p_h^+} D_3, \quad (37)$$

$$z\tilde{\Xi}^{(0)}_{\alpha} = \bar{n}_\alpha p_h^+ \frac{\tilde{k}\cdot R}{M_h^2} G_1^{\perp} - \tilde{R}_{T\alpha} G^{\sphericalangle} - \tilde{k}_{T\alpha} G^{\perp} + n_\alpha \frac{\tilde{k}\cdot R}{p_h^+} G_3, \quad (38)$$

where $\tilde{k}\cdot R = \varepsilon_T^{Rk} = \varepsilon_T^{\mu\nu} R_{T\mu} k_{T\nu}$ with $\varepsilon_T^{\mu\nu} = \varepsilon^{\alpha\beta\mu\nu}\bar{n}_\alpha n_\beta$. In this paper, we use D and G to denote chiral even DiFFs. The notations are same to single hadron production FFs. However, only DiFFs are considered in this paper and notations should not be misunderstood. Superscripts $\sphericalangle$, $\perp$ are respectively used to denote the R_T- and k_T-dependent DiFFs. Subscript 3 denotes the twist-4 functions. The coefficient functions obtained from quark–gluon–quark correlator are given by

$$\begin{aligned} z\Xi^{(1)}_{\rho\alpha} &= \bar{n}_\alpha R_{T\rho} p_h^+ D_d^{\sphericalangle} + \bar{n}_\alpha k_{T\rho} p_h^+ D_d^{\perp} \\ &+ g_{T\rho\alpha} M_h^2 D_{3d} + i\varepsilon_{T\rho\alpha} \tilde{k}\cdot R D'_{3d} \\ &+ R_{T\langle\rho} R_{T\alpha\rangle} D_{3d}^{\sphericalangle} + k_{T\langle\rho} k_{T\alpha\rangle} D_{3d}^{\perp} + \langle R_{T\rho} k_{T\alpha}\rangle D_{3d}^{\times}, \end{aligned} \quad (39)$$

$$\begin{aligned} z\tilde{\Xi}^{(1)}_{\rho\alpha} &= i\bar{n}_\alpha \tilde{R}_{T\rho} p_h^+ G_d^{\sphericalangle} + i\bar{n}_\alpha \tilde{k}_{T\rho} p_h^+ G_d^{\perp} \\ &+ i\varepsilon_{T\rho\alpha} M_h^2 G_{3d} + g_{T\rho\alpha} \tilde{k}\cdot R G'_{3d} \\ &+ \frac{i}{2} R_{T\{\rho} \tilde{R}_{T\alpha\}} G_{3d}^{\sphericalangle} \\ &+ \frac{i}{2} k_{T\{\rho} k_{T\alpha\}} G_{3d}^{\perp} + \frac{i}{2}\{R_{T\{\rho} \tilde{k}_{T\alpha\}}\} G_{3d}^{\times}. \end{aligned} \quad (40)$$

where we have used the following notations for convenience,

$$R_{T\langle\rho} R_{T\alpha\rangle} = R_{T\rho} R_{T\alpha} - \frac{R_T^2}{2} g_{T\rho\alpha}, \quad (41)$$

$$\langle R_{T\rho} k_{T\alpha}\rangle = R_{T\rho} k_{T\alpha} + k_{T\rho} R_{T\alpha} - R_T\cdot k_T g_{T\rho\alpha}, \quad (42)$$

$$R_{T\{\rho} \tilde{R}_{T\alpha\}} = R_{T\rho} \tilde{R}_{T\alpha} + R_{T\alpha} \tilde{R}_{T\rho}, \quad (43)$$

$$\{R_{T\{\rho} \tilde{k}_{T\alpha\}}\} = R_{T\rho} \tilde{k}_{T\alpha} + R_{T\alpha} \tilde{k}_{T\rho} + k_{T\rho} \tilde{R}_{T\alpha} + k_{T\alpha} \tilde{R}_{T\rho}. \quad (44)$$

Superscripts $'$, $\times$ are also used to mark certain DiFFs.

The metric tensor is defined as $g_{T\mu\nu} = g_{\mu\nu} - \bar{n}_\mu n_\nu - \bar{n}_\nu n_\mu$. Here we add a subscript d to denote DiFFs defined via quark–gluon–quark correlator $\hat{\Xi}^{(1)}_\rho$.

Up to twist-4, we only need the leading power contributions from $\hat{\Xi}^{(2)}_{\rho\sigma}$ and $\hat{\Xi}^{(2,M)}_{\rho\sigma}$, i.e. the $\bar{n}_\alpha$-terms.

$$\begin{aligned} z\Xi^{(2)}_{\rho\sigma\alpha} &= p_h^+ \bar{n}_\alpha \Big[g_{T\rho\sigma} M_h^2 D_{3dd} + i\varepsilon_{T\rho\sigma} \tilde{k}\cdot R D'_{3dd} \\ &+ R_{T\langle\rho} R_{T\sigma\rangle} D_{3dd}^{\sphericalangle} + k_{T\langle\rho} k_{T\sigma\rangle} D_{3dd}^{\perp} \\ &+ \langle R_{T\rho} k_{T\sigma}\rangle D_{3dd}^{\times} \Big], \end{aligned} \quad (45)$$

$$\begin{aligned} z\tilde{\Xi}^{(2)}_{\rho\sigma\alpha} &= p_h^+ \bar{n}_\alpha \Big[i\varepsilon_{T\rho\sigma} M_h^2 G_{3dd} + g_{T\rho\sigma} \tilde{k}\cdot R G'_{3dd} \\ &+ \frac{i}{2}\Big(R_{T\{\rho} \tilde{R}_{T\sigma\}} G_{3dd}^{\sphericalangle} + k_{T\{\rho} k_{T\sigma\}} G_{3dd}^{\perp} \\ &+ \{R_{T\{\rho} \tilde{k}_{T\sigma\}}\} G_{3dd}^{\times} \Big) \Big], \end{aligned} \quad (46)$$

where we use dd in the subscript to denote DiFFs defined via quark–gluon–gluon–quark correlator $\hat{\Xi}^{(2)}_{\rho\sigma}$. We require the decomposition of $\hat{\Xi}^{(2,M)}_{\rho\sigma}$ takes exactly the same form as that of $\hat{\Xi}^{(2)}_{\rho\sigma}$. We just add an additional superscript M to distinguish them from each other and omit the equations here. From Eqs. (37)–(46), we see that the decomposition of Ξ and that of $\tilde{\Xi}$ have exact one to one correspondence. For each D, there is a G corresponding to it. They always appear in pairs. Because of the Hermiticity of $\hat{\Xi}^{(0)}$ and $\hat{\Xi}^{(2,M)}_{\rho\sigma}$, the DiFFs defined via them are real. For those defined via $\hat{\Xi}^{(1)}_\rho$ and $\hat{\Xi}^{(2)}_{\rho\sigma}$, there is no such constraint so that they can be complex.

From the QCD equation of motion, $\gamma\cdot D\psi = 0$, we can relate the quark–j-gluon–quark correlators to the quark–quark correlator. This implies not all the DiFFs shown above are independent. Instead of giving a detailed derivation here, we just show the main steps for obtaining these relationships. Since correlators are gauge invariant because of gauge links, we first show the equation of the gauge link $\mathcal{L}$,

$$\frac{\partial}{\partial y^-}\mathcal{L}(z,y) = \mathcal{L}(z,y) igA^+(y). \quad (47)$$

For simplicity, we $\langle\psi(0)\rangle\langle\bar{\psi}(z)\rangle$ to denote the correlator $\hat{\Xi}^{(0)}$. Similar conventions apply to other correlators. Multiplying $\langle\psi(0)\rangle\langle\bar{\psi}(z)\rangle$ by k^+ from the left gives,

$$k^+\langle\psi(0)\rangle\langle\bar{\psi}(z)\rangle = -\langle\psi(0)\rangle\langle\bar{\psi}(z)D^+(z)\rangle, \quad (48)$$

where $D^+(z) = i\partial_{z^-} - gA^+(z)$. If we insert γ_T^ρ into the correlator, and repeating the derivation, we obtain

$$\begin{aligned} k^+\langle\gamma_T^\rho\psi(0)\rangle\langle\bar{\psi}(z)\rangle &= -\frac{1}{2}\Big(\langle\psi(0)\rangle\langle\psi(z)D^+\bar{\not{n}}\not{n}\gamma_T^\rho\rangle \\ &+ \langle\gamma_T^\rho\not{n}\bar{\not{n}}D^+\psi(0)\rangle\langle\psi(z)\rangle \Big). \end{aligned} \quad (49)$$

Here we have used $1 = \frac{1}{2}(\bar{\not{n}}\not{n} + \not{n}\bar{\not{n}})$. Using $\gamma^\mu = \not{n}\bar{n}^\mu + \bar{\not{n}}n^\mu + \gamma_T^\mu$, the QCD equation of motion $\not{D}(z)\psi(z) = 0$ can be rewritten as

$$D^+\not{n}\psi = -\left(D^-\not{n} + \gamma_T\cdot D_T\right)\psi, \tag{50}$$

$$\bar{\psi}\overleftarrow{D}^+\not{\bar{n}} = -\bar{\psi}\left(\overleftarrow{D}^-\not{\bar{n}} + \gamma_T\cdot\overleftarrow{D}_T\right). \tag{51}$$

Substituting Eqs. (50)–(51) into (49), we have

$$\begin{aligned}k^+\langle\gamma_T^\rho\psi(0)\rangle\langle\bar{\psi}(z)\rangle = &-\frac{1}{2}\Big(\langle\psi(0)\rangle\langle\psi(z)D_T^\rho\not{\bar{n}}\rangle\\ &+\langle\not{\bar{n}}D_T^\rho\psi(0)\rangle\langle\psi(z)\rangle\Big)\\ &+\frac{1}{2}i\varepsilon_T^{\rho\sigma}\Big(\langle\psi(0)\rangle\langle\psi(z)D_{T\sigma}\not{\bar{n}}\gamma^5\rangle\\ &-\langle\not{\bar{n}}\gamma^5D_{T\sigma}\psi(0)\rangle\langle\psi(z)\rangle\Big).\end{aligned} \tag{52}$$

If one inserts $\gamma_T^\rho\gamma^5$ in the correlator, one will obtain the dual relation, i.e

$$\begin{aligned}k^+\langle\gamma_T^\rho\gamma^5\psi(0)\rangle\langle\bar{\psi}(z)\rangle = &-\frac{1}{2}\Big(\langle\psi(0)\rangle\langle\psi(z)D_T^\rho\not{\bar{n}}\gamma^5\rangle\\ &+\langle\not{\bar{n}}\gamma^5D_T^\rho\psi(0)\rangle\langle\psi(z)\rangle\Big)\\ &+\frac{1}{2}i\varepsilon_T^{\rho\sigma}\Big(\langle\not{\bar{n}}D_{T\sigma}\psi(0)\rangle\langle\psi(z)\rangle\\ &-\langle\psi(0)\rangle\langle\psi(z)D_{T\sigma}\not{\bar{n}}\rangle\Big).\end{aligned} \tag{53}$$

We obtain the relationships for the transverse components of the correlators $\Xi_T^{(0)\rho}$ and $\tilde{\Xi}_T^{(0)\rho}$. It is convenient to rewrite them in a unified form,

$$k^+\Xi_T^{(0)\rho} = -g_T^{\rho\sigma}\,\mathrm{Re}\,\Xi_{\sigma+}^{(1)} - \varepsilon_T^{\rho\sigma}\,\mathrm{Im}\,\tilde{\Xi}_{\sigma+}^{(1)}, \tag{54}$$

$$k^+\tilde{\Xi}_T^{(0)\rho} = -g_T^{\rho\sigma}\,\mathrm{Re}\,\tilde{\Xi}_{\sigma+}^{(1)} - \varepsilon_T^{\rho\sigma}\,\mathrm{Im}\,\Xi_{\sigma+}^{(1)}. \tag{55}$$

To obtain Eqs. (54)–(55), we have utilized that two terms in each parenthesis in Eqs. (52) and (53) are conjugate to each other (translation should be used to prove this). Substituting those correlators shown in Eqs. (37)–(40) into Eqs. (54) and (55) leads the following relationships between twist-3 DiFFs,

$$\frac{1}{z}(D^{\sphericalangle} - iG^{\sphericalangle}) = -(D_d^{\sphericalangle} - G_d^{\sphericalangle}), \tag{56}$$

$$\frac{1}{z}(D^{\perp} - iG^{\perp}) = -(D_d^{\perp} - G_d^{\perp}), \tag{57}$$

where coefficients (e.g., R_T, k_T) have been reduced.

We can use the similar way to obtain these relationships for the minus components of $\Xi_\alpha^{(0)}$ and $\tilde{\Xi}_\alpha^{(0)}$, they are given by

$$\begin{aligned}2k^{+2}\,\Xi_-^{(0)} &= k^+\left(g_T^{\rho\sigma}\,\Xi_{\rho\sigma}^{(1)} + i\varepsilon_T^{\rho\sigma}\,\tilde{\Xi}_{\rho\sigma}^{(1)}\right)\\ &= -g_T^{\rho\sigma}\,\Xi_{\rho\sigma+}^{(2,M)} + i\varepsilon_T^{\rho\sigma}\,\tilde{\Xi}_{\rho\sigma+}^{(2,M)},\end{aligned} \tag{58}$$

$$\begin{aligned}2k^{+2}\,\tilde{\Xi}_-^{(0)} &= k^+\left(g_T^{\rho\sigma}\,\tilde{\Xi}_{\rho\sigma}^{(1)} + i\varepsilon_T^{\rho\sigma}\,\Xi_{\rho\sigma}^{(1)}\right)\\ &= -g_T^{\rho\sigma}\,\tilde{\Xi}_{\rho\sigma+}^{(2,M)} + i\varepsilon_T^{\rho\sigma}\,\Xi_{\rho\sigma+}^{(2,M)}.\end{aligned} \tag{59}$$

From Eqs. (58) and (59), we can relate twist-4 DiFFs defined via correlators $\hat{\Xi}^{(0)}$, $\hat{\Xi}^{(1)}$ and $\hat{\Xi}^{(2,M)}$ as,

$$D_3 = zD_{-3d} = -z^2D_{+3dd}^M, \tag{60}$$

$$G_3 = zG'_{-3d} = -z^2G_{+3dd}^{M\prime}, \tag{61}$$

where $D_{-3d}\equiv D_{3d}-G_{3d}$, $G_{-3d}\equiv G_{3d}-D_{3d}$, $D_{+3dd}^M\equiv D_{3dd}^M+G_{3dd}^M$ and $G_{+3dd}^{M\prime}\equiv G_{3dd}^{M\prime}+D_{3dd}^{M\prime}$. The unified relationships given in Eqs. (56)–(57) and (60)–(61) are important in the calculation of the hadronic tensor. They guarantee that the hadron tensor satisfies current conservation law.

3.3 Hadronic tensor at twist-4

In this part we calculate the leading twist, twist-3 and twist-4 hadronic tensor in turn based on the previous calculation. First of all, we calculate the leading twist one.

The leading twist contributions only come from the quark–quark correlator $\hat{\Xi}^{(0)}$. To calculate them we use

$$\mathrm{Tr}\left[\hat{h}_{\mu\nu}^{(0)}\not{\bar{n}}\right] = -\frac{4}{p_h^+}\left(c_1^q g_{T\mu\nu} + ic_3^q\varepsilon_{T\mu\nu}\right), \tag{62}$$

$$\mathrm{Tr}\left[\hat{h}_{\mu\nu}^{(0)}\gamma^5\not{\bar{n}}\right] = \frac{4}{p_h^+}\left(c_3^q g_{T\mu\nu} + ic_1^q\varepsilon_{T\mu\nu}\right), \tag{63}$$

and Eqs. (37)–(38). Substituting them into Eq. (21) we obtain the leading twist hadronic tensor,

$$\begin{aligned}z\tilde{W}_{t2\mu\nu} = &-2\Big[c_1^q g_{T\mu\nu} + ic_3^q\varepsilon_{T\mu\nu}\Big]D_1\\ &+2\Big[c_3^q g_{T\mu\nu} + ic_1^q\varepsilon_{T\mu\nu}\Big]\frac{\tilde{k}\cdot R}{M_h^2}G_1^\perp.\end{aligned} \tag{64}$$

We find that $\tilde{W}_{t2\mu\nu}$ satisfies the current conservation $q^\mu\tilde{W}_{t2\mu\nu} = q^\nu\tilde{W}_{t2\mu\nu} = 0$.

Twist-3 contributions come from both the quark–quark correlator $\hat{\Xi}^{(0)}$ and the quark–gluon–quark correlator $\hat{\Xi}_\rho^{(1)}$. We first calculate these contributions from the quark–quark correlator $\hat{\Xi}^{(0)}$. Here we use

$$\mathrm{Tr}\left[\hat{h}_{\mu\nu}^{(0)}\not{k}\right] = \frac{4}{p^+}\left(c_1^q k_{\{\mu}n_{\nu\}} + ic_3^q\tilde{k}_{[\mu}n_{\nu]}\right), \tag{65}$$

$$\mathrm{Tr}\left[\hat{h}_{\mu\nu}^{(0)}\gamma^5\not{k}\right] = -\frac{4}{p^+}\left(c_3^q k_{\{\mu}n_{\nu\}} + ic_1^q\tilde{k}_{[\mu}n_{\nu]}\right), \tag{66}$$

where k denote k_T, R_T, $\tilde{k}_T$ and $\tilde{R}_T$. Using Eqs. (37)–(38) and substituting them into Eq. (21) we obtain

$$
\begin{aligned}
z\tilde{W}^{(0)}_{t3\mu\nu} = &-\frac{2}{p_h^+}\Big[\big(c_1^q k_{T\{\mu}n_{\nu\}} + ic_3^q \tilde{k}_{T[\mu}n_{\nu]}\big)D^{\perp} \\
&+ \big(c_1^q R_{T\{\mu}n_{\nu\}} + ic_3^q \tilde{R}_{T[\mu}n_{\nu]}\big)D^{\sphericalangle}\Big] \\
&+ \frac{2}{p_h^+}\Big[\big(c_3^q \tilde{k}_{T\{\mu}n_{\nu\}} - ic_3^q k_{T[\mu}n_{\nu]}\big)G^{\perp} \\
&+ \big(c_1^q \tilde{R}_{T\{\mu}n_{\nu\}} - ic_3^q R_{T[\mu}n_{\nu]}\big)G^{\sphericalangle}\Big]. \qquad (67)
\end{aligned}
$$

For the twist-3 contribution from the quark–gluon–quark correlator $\hat{\Xi}^{(1)}_\rho$, we have

$$
\mathrm{Tr}\big[\hat{h}^{(1)\rho}_{\mu\nu}\bar{\not n}\big] = -8\big(c_1^q g^{\rho}_{T\mu}\bar{n}^{\nu} + ic_3^q \varepsilon^{\rho}_{T\mu}\bar{n}^{\nu}\big), \qquad (68)
$$

$$
\mathrm{Tr}\big[\hat{h}^{(1)\rho}_{\mu\nu}\gamma^5\bar{\not n}\big] = +8\big(c_3^q g^{\rho}_{T\mu}\bar{n}^{\nu} + ic_1^q \varepsilon^{\rho}_{T\mu}\bar{n}^{\nu}\big), \qquad (69)
$$

Using Eqs. (39)–(40) and substituting them into Eq. (22), we obtain

$$
\begin{aligned}
z\tilde{W}^{(1)L}_{t3\mu\nu} = &\frac{2p_h^+}{p_h\cdot q}\Big[\big(c_1^q k_{T\mu}\bar{n}_{\nu} - ic_3^q \tilde{k}_{T\mu}\bar{n}_{\nu}\big)D_d^{\perp} \\
&+ \big(c_1^q R_{T\mu}\bar{n}_{\nu} - ic_3^q \tilde{R}_{T\mu}\bar{n}_{\nu}\big)D_d^{\sphericalangle}\Big] \\
&- \frac{2p_h^+}{p_h\cdot q}\Big[\big(c_1^q k_{T\mu}\bar{n}_{\nu} + ic_3^q \tilde{k}_{T\mu}\bar{n}_{\nu}\big)G_d^{\perp} \\
&+ \big(c_1^q R_{T\mu}\bar{n}_{\nu} + ic_3^q \tilde{R}_{T\mu}\bar{n}_{\nu}\big)G_d^{\sphericalangle}\Big]. \qquad (70)
\end{aligned}
$$

The complete twist-3 hadronic tensor is the sum of all the twist-3 contributions, i.e, $\tilde{W}_{t3\mu\nu} = \tilde{W}^{(0)}_{t3\mu\nu} + \tilde{W}^{(1)L}_{t3\mu\nu} + \left(\tilde{W}^{(1)L}_{t3\nu\mu}\right)^*$. Using Eqs. (56)–(57), (67) and (70), we eliminate the non-independent DiFFs and obtain the complete hadronic tensor at twist-3.

$$
\begin{aligned}
z\tilde{W}_{t3\mu\nu} = &-\frac{2}{p_h\cdot q}\Big[\big(c_1^q k_{T\{\mu}\bar{q}_{\nu\}} + ic_3^q \tilde{k}_{T[\mu}\bar{q}_{\nu]}\big)D^{\perp} \\
&+ \big(c_1^q R_{T\{\mu}\bar{q}_{\nu\}} + ic_3^q \tilde{R}_{T[\mu}\bar{q}_{\nu]}\big)D^{\sphericalangle}\Big], \\
&+ \frac{2}{p_h\cdot q}\Big[\big(c_3^q \tilde{k}_{T\{\mu}\bar{q}_{\nu\}} - ic_3^q k_{T[\mu}\bar{q}_{\nu]}\big)G^{\perp} \\
&+ \big(c_1^q \tilde{R}_{T\{\mu}\bar{q}_{\nu\}} - ic_3^q R_{T[\mu}\bar{q}_{\nu]}\big)G^{\sphericalangle}\Big], \qquad (71)
\end{aligned}
$$

where $\bar{q} = q - 2p_h/z$. It it can be shown that $\tilde{W}_{t3\mu\nu}$ satisfies the current conservation $q^{\mu}\tilde{W}_{t3\mu\nu} = q^{\nu}\tilde{W}_{t3\mu\nu} = 0$.

Correlators $\hat{\Xi}^{(0)}$, $\hat{\Xi}^{(1)}_{\rho}$ and $\hat{\Xi}^{(2)}_{\rho\sigma}$ all have contributions to twist-4 hadronic tensor. We first calculate contributions from quark–quark correlator $\hat{\Xi}^{(0)}$ and use

$$
\mathrm{Tr}\big[\hat{h}^{(0)}_{\mu\nu}\not n\big] = \frac{8}{p_h^+}c_1^q n_{\mu}n_{\nu}, \qquad (72)
$$

$$
\mathrm{Tr}\big[\hat{h}^{(0)}_{\mu\nu}\gamma^5\not n\big] = -\frac{8}{p_h^+}c_3^q n_{\mu}n_{\nu}, \qquad (73)
$$

and Eqs. (37)–(38). Substituting the twit-4 terms into Eq. (21) we have

$$
z\tilde{W}^{(0)}_{t4\mu\nu} = \frac{4M_h^2}{(p_h^+)^2}c_1^q n_{\mu}n_{\nu}D_3 - \frac{4\tilde{k}\cdot R}{(p_h^+)^2}c_3^q n_{\mu}n_{\nu}G_3. \qquad (74)
$$

To calculate the twist-4 contributions from quark–gluon–quark correlator $\hat{\Xi}^{(1)}_{\rho}$, we use

$$
\begin{aligned}
\mathrm{Tr}\big[\hat{h}^{(1)\rho}_{\mu\nu}\gamma^{\alpha}\big] = &4c_1^q\big[2n_{\mu}\bar{n}_{\nu}g_T^{\rho\alpha} + g_{T\mu\nu}g_T^{\rho\alpha} - g^{\{\rho}_{T\mu}g^{\alpha\}}_{T\nu}\big] \\
&- 4ic_3^q\big[2n_{\mu}\bar{n}_{\nu}\varepsilon_T^{\rho\alpha} + g^{\rho}_{T\mu}\varepsilon^{\alpha}_{T\nu} + g^{\alpha}_{T\nu}\varepsilon^{\rho}_{T\mu}\big], \qquad (75)
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{Tr}\big[\hat{h}^{(1)\rho}_{\mu\nu}\gamma^5\gamma^{\alpha}\big] = &4ic_1^q\big[2n_{\mu}\bar{n}_{\nu}\varepsilon_T^{\rho\alpha} + g^{\rho}_{T\mu}\varepsilon^{\alpha}_{T\nu} + g^{\alpha}_{T\nu}\varepsilon^{\rho}_{T\mu}\big] \\
&- 4c_3^q\big[2n_{\mu}\bar{n}_{\nu}g_T^{\rho\alpha} + g_{T\mu\nu}g_T^{\rho\alpha} - g^{\{\rho}_{T\mu}g^{\alpha\}}_{T\nu}\big]. \qquad (76)
\end{aligned}
$$

Using twist-4 DiFFs given in Eqs. (39)–(40) and Eq. (22), we have

$$
\begin{aligned}
z\tilde{W}^{(1)L}_{t4\mu\nu} = &-\frac{4M_h^2}{p_h\cdot q}c_1^q n_{\mu}n_{\nu}D_{-3d} + \frac{4\tilde{k}\cdot R}{p_h\cdot q}c_3^q n_{\mu}n_{\nu}G'_{-3d} \\
&+ \frac{1}{p_h\cdot q}\big[c_1^q k_{T\langle\mu}k_{T\nu\rangle} + ic_3^q k_{T\{\mu}\tilde{k}_{T\nu\}}\big]D^{\perp}_{-3d} \\
&+ \frac{1}{p_h\cdot q}\big[c_1^q R_{T\langle\mu}R_{T\nu\rangle} + ic_3^q R_{T\{\mu}\tilde{R}_{T\nu\}}\big]D^{\sphericalangle}_{-3d} \\
&+ \frac{1}{p_h\cdot q}\big[c_1^q \langle R_{T\mu}k_{T\nu}\rangle + ic_3^q \{R_{T\{\mu}\tilde{k}_{T\nu\}}\}\big]D^{\times}_{-3d}. \qquad (77)
\end{aligned}
$$

It is convenient to divide the contributions from quark–gluon–gluon–quark correlator $\hat{\Xi}^{(2)}_{\rho\sigma}$ into two parts, one is the middle-cut part and the other is the left- and right-cut part. We first consider the middle-cut part and use the superscript M to distinguish it from the others. Using

$$
\mathrm{Tr}\big[\hat{h}^{(2)\rho\sigma}_{\mu\nu}\bar{\not n}\big]p_h^+ = -8c_1^q p_{h\mu}p_{h\nu}g_T^{\rho\sigma} - 8ic_3^q p_{h\mu}p_{h\nu}\varepsilon_T^{\rho\sigma}, \qquad (78)
$$

$$
\mathrm{Tr}\big[\hat{h}^{(2)}_{\mu\nu}\gamma^5\gamma^{\alpha}\big]p_h^+ = 8c_3^q p_{h\mu}p_{h\nu}g_T^{\rho\sigma} + 8ic_1^q p_{h\mu}p_{h\nu}\varepsilon_T^{\rho\sigma}, \qquad (79)
$$

we obtain

$$
\begin{aligned}
z\tilde{W}^{(2)M}_{t4\mu\nu} = &-\frac{4M_h^2}{(p_h\cdot q)^2}c_1^q p_{h\mu}p_{h\nu}D^{M}_{+3dd} \\
&+ \frac{4\tilde{k}\cdot R}{(p_h\cdot q)^2}c_3^q p_{h\mu}p_{h\nu}G^{M\prime}_{+3dd}. \qquad (80)
\end{aligned}
$$

To obtain the contributions from the left- and right-cut parts, we use

$$\mathrm{Tr}\big[\hat{N}^{(2)\rho\sigma}_{\mu\nu}\not{n}\big] = +\frac{4(p_h\cdot q)}{p_h^+}c_1^q\Big[g_T^{\rho\sigma}g_{T\mu\nu} + g^{\rho}_{T[\mu}g^{\sigma}_{T\nu]}\Big] - \frac{4(p_h\cdot q)}{p_h^+}ic_3^q\Big[g^{\rho}_{T\mu}\varepsilon^{\sigma}_{T\nu} - g^{\sigma}_{T\nu}\varepsilon^{\rho}_{T\mu}\Big], \tag{81}$$

$$\mathrm{Tr}\big[\hat{N}^{(2)\rho\sigma}_{\mu\nu}\gamma^5\not{n}\big] = -\frac{4(p_h\cdot q)}{p_h^+}c_3^q\Big[g_T^{\rho\sigma}g_{T\mu\nu} + g^{\rho}_{T[\mu}g^{\sigma}_{T\nu]}\Big] + \frac{4(p_h\cdot q)}{p_h^+}ic_1^q\Big[g^{\rho}_{T\mu}\varepsilon^{\sigma}_{T\nu} - g^{\sigma}_{T\nu}\varepsilon^{\rho}_{T\mu}\Big], \tag{82}$$

and obtain

$$z\tilde{W}^{(2)L}_{t4\mu\nu} = \frac{2M_h^2}{p_h\cdot q}\left[c_1^q g_{T\mu\nu} + ic_3^q\varepsilon_{T\mu\nu}\right]D_{-3dd}, \tag{83}$$

where $D_{-3dd} = D_{3dd} - G_{3dd}$. Summing over all the twist-4 contributions and using Eqs. (60)–(61) to eliminate the non-independent DiFFs yields

$$\begin{aligned} z\tilde{W}_{t4\mu\nu} = {} & \frac{4M_h^2}{p_h\cdot q}c_1^q\bar{q}_\mu\bar{q}_\nu D_3 - \frac{4\tilde{k}\cdot R}{p_h\cdot q}c_3^q\bar{q}_\mu\bar{q}_\nu G_3 \\ & + \frac{4M_h^2}{p_h\cdot q}\left[c_1^q g_{\mu\nu} + ic_3^q\varepsilon_{T\mu\nu}\right]\mathrm{Re}D_{-3dd} \\ & + \frac{2}{p_h\cdot q}c_1^q\Big[k_{T\langle\mu}k_{T\nu\rangle}\mathrm{Re}D^{\perp}_{-3d} \\ & + R_{T\langle\mu}R_{T\nu\rangle}\mathrm{Re}D^{\sphericalangle}_{-3d} + \langle R_{T\mu}k_{T\nu}\rangle\mathrm{Re}D^{\times}_{-3d}\Big] \\ & - \frac{2}{p_h\cdot q}c_3^q\Big[k_{T\{\mu}\tilde{k}_{T\nu\}}\mathrm{Im}D^{\perp}_{-3d} \\ & + R_{T\{\mu}\tilde{R}_{T\nu\}}\mathrm{Im}D^{\sphericalangle}_{-3d} + \{R_{T\{\mu}\tilde{k}_{T\nu\}}\}\mathrm{Im}D^{\times}_{-3d}\Big]. \end{aligned} \tag{84}$$

It it can be shown that $\tilde{W}_{t4\mu\nu}$ satisfies the current conservation $q^\mu\tilde{W}_{t4\mu\nu} = q^\nu\tilde{W}_{t4\mu\nu} = 0$.

In this part we obtain the complete hadronic tensor up to twist-4 level. We show the leading twist hadronic tensor in Eq. (64) and show the twist-3 hadronic tensor in Eq. (71). The twist-4 hadronic tensors are given in Eq. (84). All these hadronic tensors satisfy the current conservation law.

3.4 Contributions from the four-quark correlator

In the previous calculations, we only consider the contributions from quark–j-gluon–quark correlators. In fact, up to twist-4, there are also contributions from diagrams involving the four-quark correlator [27]. The four-correlator for the hadron pair production is define as

$$\begin{aligned} \hat{\Xi}^{(0)}_{(4q)}(k_1,k,k_2) = {} & \frac{g^2}{8}\int\frac{d^4y}{(2\pi)^4}\frac{d^4y_1}{(2\pi)^4}\frac{d^4y_2}{(2\pi)^4} \\ & \times e^{-ik_1y+i(k_1-k)y_1-i(k_2-k)y_2} \\ & \times\sum_X\langle 0|\bar{\psi}(y_2)\mathcal{L}^\dagger(0,y_2)\psi(0)|p_h,R,X\rangle \\ & \times\langle p_h,R,X|\bar{\psi}(y)\mathcal{L}(y,y_1)\psi(y_1)|0\rangle. \end{aligned} \tag{85}$$

Here g is the strong coupling constant. Some example of the four-quark diagrams are shown in Fig. 4. We note that if the cut is given at the middle we have contributions from $e^+e^-\to p_1p_2gX$ (gluon jet). If the cut at the left and/or right, we have contributions from $e^+e^-\to p_1p_2\bar{q}X$ (quark jet). Both of them contribute to the hadron pair production in the electron positron annihilation process, in this case we consider them together.

It can be shown that gauge links included in the correlators given by Eq. (85) are obtained by taking the multiple gluon scattering into account [20,24]. The hadronic tensor $W^{(g)}_{4q\mu\nu}$ for both the quark and gluon jet cases can be written as the unified form

$$\hat{W}^{(g/q)}_{4q\mu\nu} = \frac{1}{p_h\cdot q}\int d\hat{z}d\hat{z}_1d\hat{z}_2h^{g/q}_{4q}\Big[\big(c_1^q g_{T\mu\nu} + ic_3^q\varepsilon_{T\mu\nu}\big)\hat{C}_s + \big(c_3^q g_{T\mu\nu} + ic_1^q\varepsilon_{T\mu\nu}\big)\hat{C}_{ps}\Big], \tag{86}$$

Here letter with hat, e.g. $\hat{z}$, is used to distinguish variables from ones used before. $\hat{C}_s$ and $\hat{C}_{ps}$ are these correlators considered here. They can also be written as a unified form

$$\begin{aligned} \hat{C}_j = {} & \int\frac{d^2k'_T}{(2\pi)^2}\int d^4k_1d^4kd^4k_2\delta\left(\hat{z} - \frac{p_h^+}{k^+}\right)\delta(k_1^+\hat{z}_1 - p_h^+) \\ & \times\delta(k_2^+\hat{z}_2 - p_h^+)(2\pi)^2\delta^2(\vec{k}_T + \vec{k}'_T)\hat{\Xi}^{(0)}_{(4q)j} \\ & \times(k_1,k,k_2;p_h.R), \end{aligned} \tag{87}$$

where $j = s, ps$. The corresponding $\hat{\Xi}^{(0)}_{(4q)s}$ and $\hat{\Xi}^{(0)}_{(4q)ps}$ are given by

$$\begin{aligned} \hat{\Xi}^{(0)}_{(4q)s} = {} & \frac{g^2}{8}\int\frac{d^4y}{(2\pi)^4}\frac{d^4y_1}{(2\pi)^4}\frac{d^4y_2}{(2\pi)^4}e^{-ik_1y+i(k_1-k)y_1-i(k_2-k)y_2} \\ & \times\sum_X\Big\{\langle 0|\bar{\psi}(y_2)\not{n}\psi(0)|p_h,R,X\rangle \\ & \times\langle p_h,R,X|\bar{\psi}(y)\not{n}\psi(y_1)|0\rangle \\ & \times + \langle 0|\bar{\psi}(y_2)\gamma^5\not{n}\psi(0)|p_h,R,X\rangle \\ & \times\langle p_h,R,X|\bar{\psi}(y)\gamma^5\not{n}\psi(y_1)|0\rangle\Big\}, \end{aligned} \tag{88}$$

$$\hat{\Xi}^{(0)}_{(4q)ps} = \frac{g^2}{8}\int\frac{d^4y}{(2\pi)^4}\frac{d^4y_1}{(2\pi)^4}\frac{d^4y_2}{(2\pi)^4}e^{-ik_1y+i(k_1-k)y_1-i(k_2-k)y_2}$$

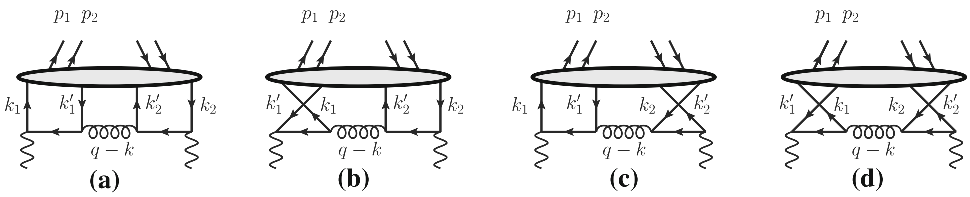

Fig. 4 The first four of the four-quark diagrams where no multiple gluon scattering is involved. In (**a**), we have $k'_1 = k_1 - k$ and $k'_2 = k_2 - k$; in (**b**) we have the interchange of k_1 with k'_1; in (**c**) we have the interchange of k_2 with k'_2; in (**d**) we have both interchanges of k_1 with k'_1 and k_2 with k'_2

$$\times \sum_X \Big\{ \langle 0|\bar\psi(y_2)\gamma^5 \not n \psi(0)|p_h, R, X\rangle \times \langle p_h, R, X|\bar\psi(y)\not n\psi(y_1)|0\rangle + \langle 0|\bar\psi(y_2)\not n\psi(0)|p_h, R, X\rangle \times \langle p_h, R, X|\bar\psi(y)\gamma^5\not n\psi(y_1)|0\rangle \Big\}. \quad (89)$$

For simplicity, we have omitted gauge links in Eqs. (88)–(89).

In the hadronic tensor Eq. (86), $h_{4q}^{g/q}$ denotes the sum of all the hard parts.

$$h_{4q}^g = \frac{\hat z \hat z_B^3 \delta(\hat z - \hat z_B)}{(\hat z_1 - \hat z_B + i\epsilon)(\hat z_2 - \hat z_B - i\epsilon)} + \frac{\hat z_B^2/\hat z_1 \hat z_2 \delta(\hat z - \hat z_B)}{(1/\hat z_1 + i\epsilon)(1/\hat z_2 - i\epsilon)} - \frac{\hat z_B^3/\hat z_2 \delta(\hat z - \hat z_B)}{(\hat z_1 - \hat z_B + i\epsilon)(1/\hat z_2 - i\epsilon)} - (1 \leftrightarrow 2)^*, \quad (90)$$

$$h_{4q}^{qL} = \frac{\hat z \hat z_B^3 \delta(\hat z_1 - \hat z_B)}{(\hat z - \hat z_B - i\epsilon)(\hat z_2 - \hat z_B - i\epsilon)} - \left(\frac{1}{\hat z_2} \to \frac{1}{\hat z} - \frac{1}{\hat z_2}\right) - \frac{\hat z \hat z_B^3 \delta(\hat z_1 + \hat z_B - \frac{\hat z_1 \hat z_B}{\hat z})}{(\hat z - \hat z_B - i\epsilon)(\hat z_2 - \hat z_B - i\epsilon)} + \left(\frac{1}{\hat z_2} \to \frac{1}{\hat z} - \frac{1}{\hat z_2}\right), \quad (91)$$

where $\hat z = \hat z_B = p_h^+/k^+$, $h_{4q}^{qR}(\hat z_1, \hat z, \hat z_2) = h_{4q}^{qL*}(\hat z_2, \hat z, \hat z_1)$. Summing over all the hard parts yields $h_{4q} = h_{4q}^{qL} + h_{4q}^{qR} + h_{4q}^g$.

As for the quark–j-gluon–quark correlators, we decompose $\hat C_s$ and $\hat C_{ps}$ in terms of the four-quark DiFFs,

$$\hat z \int d\hat z d\hat z_1 d\hat z_2 h_{4q} \hat C_s = M_h^2 D_{4q}, \quad (92)$$

$$\hat z \int d\hat z d\hat z_1 d\hat z_2 h_{4q} \hat C_{ps} = \tilde k \cdot R G_{4q}. \quad (93)$$

Substituting Eqs. (92)–(93) into Eq. (86) yields

$$\hat z \tilde W_{4q\mu\nu} = \frac{M_h^2}{p_h \cdot q}\left(c_1^q g_{T\mu\nu} + i c_3^q \varepsilon_{T\mu\nu}\right) D_{4q} + \frac{\tilde k \cdot R}{p_h \cdot q}\left(c_3^q g_{T\mu\nu} + i c_1^q \varepsilon_{T\mu\nu}\right) G_{4q}. \quad (94)$$

We see that they have the same modes as for the leading twist contributions. They lead to twist-4 modifications of the leading twist results.

4 Cross section and azimuthal asymmetries

In the previous section we obtained the complete hadronic tensor at twist-4 level. Contracting with leptonic tensor gives the cross section of the hadron pair production semi-inclusive electron positron annihilation process. We present the results in the following context.

The complete differential cross section at twist-4 is given by

$$[d\sigma] = \frac{\alpha_{em}^2 \chi}{8\pi Q^2}\Big\{ T_1(y)\left(D_1 - \kappa_M \frac{D_{4q}}{z}\right) + T_2(y) k_{TM} R_{TM} \times \sin(\phi_r - \phi_k)\left(G_1^\perp - \kappa_M \frac{G_{4q}}{z}\right) - 2\kappa_M\left[T_3(y) k_{TM}\cos\phi_k D^\perp + T_3(y) R_{TM}\cos\phi_r D^\sphericalangle\right] - 2\kappa_M\left[T_4(y) k_{TM}\sin\phi_k G^\perp + T_4(y) R_{TM}\sin\phi_r G^\sphericalangle\right] + 4\kappa_M^2\left[2c_1^e c_1^q B(y)\frac{D_3}{z} + 2c_1^e c_3^q B(y) k_{TM} R_{TM}\sin \times(\phi_r - \phi_k)\frac{G_3}{z} - T_1(y)\mathrm{Re}D_{-3dd}\right] - 4\kappa_M^2 c_1^e c_1^q B(y)\left[k_{TM}^2\cos 2\phi_k \mathrm{Re}D_{-3d}^\perp + R_{TM}^2\cos 2\phi_r \mathrm{Re}D_{-3d}^\sphericalangle + k_{TM}R_{TM}\cos(\phi_r + \phi_k)\mathrm{Re}D_{-3d}^\times\right] - 4\kappa_M^2 c_1^e c_3^q B(y)\left[k_{TM}^2\sin 2\phi_k \mathrm{Im}D_{-3d}^\perp + R_{TM}^2\sin 2\phi_r \mathrm{Im}D_{-3d}^\sphericalangle + k_{TM}R_{TM}\sin(\phi_r + \phi_k)\mathrm{Im}D_{-3d}^\times\right]\Big\}, \quad (95)$$

where $[d\sigma] = d\sigma/dzdyd^2R_Td\xi d^2k'_T$. We also used $\kappa_M = M_h/Q$, $k_{TM} = |\vec k_T|/M_h$, $R_{TM} = |\vec R_T|/M_h$ and

$$T_1(y) = 2c_1^e c_1^q A(y) - c_3^e c_3^q C(y), \quad (96)$$

$$T_2(y) = 2c_1^e c_3^q A(y) - c_3^e c_1^q C(y), \quad (97)$$

$$T_3(y) = c_1^e c_1^q C(y)D(y) + c_3^e c_3^q D(y), \tag{98}$$
$$T_4(y) = c_1^e c_3^q C(y)D(y) - c_3^e c_1^q D(y) \tag{99}$$

with $A(y) = \frac{1}{2} - y + y^2$, $B(y) = 2y(1-y)$, $C(y) = 1-2y$ and $D(y) = \sqrt{y(1-y)}$ to simplify the expression. Contributions from four-quark correlator are involved in Eq. (95).

From Eq. (95), we can see there are sets of azimuthal modulations which can be measured in experiment and used to extract the corresponding DiFFs. To illustrate this we first present the definition of the azimuthal asymmetries, e.g.

$$\langle \sin\phi_k \rangle = \frac{\int [d\sigma] \sin\phi_k d\phi_k}{\int [d\sigma] d\phi_k}. \tag{100}$$

Other asymmetries can be defined in the similar way, we do not show them for simplicity. In this case, we can write down all the azimuthal asymmetries. The leading twist asymmetry is given by

$$\langle \sin(\phi_r - \phi_k) \rangle_2 = k_{TM} R_{TM} \frac{T_2(y) G_1^\perp}{2T_1(y) D_1}. \tag{101}$$

Here subscript 2 denotes the leading twist. The twist-4 correction of the leading twist asymmetry in Eq. (101) in the numerator is $\kappa_M^2 \left(-T_2(y)G_{4q} + 8c_1^e c_3^q B(y)G_3\right)/z$. We note that a summation of flavor q is explicit in the numerator and in the denominator, respectively. This applies also to all the results presented in the following of this paper. There are four twist-3 azimuthal asymmetries which are given by

$$\langle \cos\phi_k \rangle_3 = -\kappa_M k_{TM} \frac{T_3(y) D^\perp}{zT_1(y) D_1}, \tag{102}$$
$$\langle \cos\phi_r \rangle_3 = -\kappa_M R_{TM} \frac{T_3(y) D^\sphericalangle}{zT_1(y) D_1}, \tag{103}$$
$$\langle \sin\phi_k \rangle_3 = -\kappa_M k_{TM} \frac{T_4(y) G^\perp}{zT_1(y) D_1}, \tag{104}$$
$$\langle \sin\phi_r \rangle_3 = -\kappa_M R_{TM} \frac{T_4(y) G^\sphericalangle}{zT_1(y) D_1}, \tag{105}$$

where subscript 3 denotes the twist-3. There are six azimuthal asymmetries appearing at twist-4. They are

$$\langle \cos 2\phi_k \rangle_4 = -\kappa_M^2 k_{TM}^2 \frac{2c_1^e c_1^q B(y) \mathrm{Re} D_{-3d}^\perp}{zT_1(y) D_1}, \tag{106}$$
$$\langle \cos 2\phi_r \rangle_4 = -\kappa_M^2 R_{TM}^2 \frac{2c_1^e c_1^q B(y) \mathrm{Re} D_{-3d}^\sphericalangle}{zT_1(y) D_1}, \tag{107}$$
$$\langle \sin 2\phi_k \rangle_4 = -\kappa_M^2 k_{TM}^2 \frac{2c_1^e c_3^q B(y) \mathrm{Im} D_{-3d}^\perp}{zT_1(y) D_1}, \tag{108}$$
$$\langle \sin 2\phi_r \rangle_4 = -\kappa_M^2 R_{TM}^2 \frac{2c_1^e c_3^q B(y) \mathrm{Im} D_{-3d}^\sphericalangle}{zT_1(y) D_1}, \tag{109}$$
$$\langle \cos(\phi_r + \phi_k) \rangle_4 = \kappa_M^2 k_{TM} R_{TM} \frac{2c_1^e c_1^q B(y) \mathrm{Re} D_{-3d}^\times}{zT_1(y) D_1}, \tag{110}$$
$$\langle \sin(\phi_r + \phi_k) \rangle_4 = \kappa_M^2 k_{TM} R_{TM} \frac{2c_1^e c_3^q B(y) \mathrm{Im} D_{-3d}^\times}{zT_1(y) D_1}, \tag{111}$$

where subscript 4 denotes the twist-4.

If only the EM interaction is taken into account $c_V^{e,q} = 1, c_A^{e,q} = 0$, that is $c_1^{e,q} = 1, c_3^{e,q} = 0$ or $T_1(y) = 2A(y)$, $T_3(y) = C(y)D(y)$ and $T_2(y) = T_4(y) = 0$. In this case, we have

$$\langle \cos\phi_k \rangle_3 = -\kappa_M k_{TM} \frac{C(y)D(y) D^\perp}{2zA(y) D_1}, \tag{112}$$
$$\langle \cos\phi_r \rangle_3 = -\kappa_M R_{TM} \frac{C(y)D(y) D^\sphericalangle}{2zA(y) D_1}, \tag{113}$$
$$\langle \cos 2\phi_k \rangle_4 = -\kappa_M^2 k_{TM}^2 \frac{B(y) \mathrm{Re} D_{-3d}^\perp}{zA(y) D_1}, \tag{114}$$
$$\langle \cos 2\phi_r \rangle_4 = -\kappa_M^2 R_{TM}^2 \frac{B(y) \mathrm{Re} D_{-3d}^\sphericalangle}{zA(y) D_1}, \tag{115}$$
$$\langle \cos(\phi_r + \phi_k) \rangle_4 = \kappa_M^2 k_{TM} R_{TM} \frac{B(y) \mathrm{Re} D_{-3d}^\times}{zA(y) D_1}. \tag{116}$$

Only five azimuthal asymmetries are left. Asymmetries in Eqs. (101)–(116) can be measured to extract corresponding DiFFs.

5 Summary

In this paper, we calculate the hadron pair production in the semi-inclusive electron positron annihilation process at twist-4 level. Semi-inclusive implies the back-to-back jet is also measured in addition to the hadron pair. This process (jet production) is better than the double hadron (pair) production process because it does not introduce the extra uncertainties if the jet is seen as a(n) (anti)quark. It is then an ideal place to study the chiral even quantities (e.g. DiFFs). However, the shortcoming of this process is that it is impossible to study the chiral odd quantities since there is no helicity flip. The hadron (pair) production process has been discussed at leading twist, e.g. Refs. [7,28], in this paper we thus consider the jet production process at twist-4. Both the EM and weak interactions are considered. We calculate the cross section according to the collinear expansion method. It provides explicit expressions of the hadronic tensor at twist-4 level, see Eqs. (21)–(24), and the cross section can be easily obtained. We obtain one leading twist azimuthal asymmetry which has twist-4 corrections. Also, we have four twist-3 and six twist-4 azimuthal asymmetries. If only EM interaction is considered, two twist-3 and three twist-4 azimuthal asymmetries are left.

Electron positron annihilation is known as the cleanest process in studying the quark fragmentation and/or hadronization. Our calculation, considering the EM and weak interactions simultaneously, provides a set of measurable quantities for a better understanding of DiFFs, hadronization and even quark flavor separation.

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