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Exact classical approach to the electron's self-energy and anomalous g-factor

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Abstract – An exact classical approach to the calculation of electron's self-energy and anomalous g-factor is reported. The electron's intrinsic dynamics, related electrodynamics and occurrence of anomalous magnetic moment are completely determined. A unique regularization of the electromagnetic field scalar potential underlying all results is derived. A fundamental transcendental equation satisfied by the electron's anomalous g-factor is obtained, with solution $a_e = 0.0011596521800027(65)$, matching the experimentally measured value reported in the literature to 0.59 parts per trillion. Field representation of the electron intrinsic and orbital dynamics in atoms is discussed.

Introduction. – The anomalous magnetic moment and the intrinsic dynamics of non-composite particles have been considered as unique features of the quantum field theory since the beginning of its development [1–7]. The electron's anomalous magnetic moment and its fundamental properties are the first to be studied and realized (see ref. [8] and references therein). With the aid of quantum electrodynamics the value of corresponding g-factor was predicted with a stunning accuracy [8–15], leaving no space for mistrusting the mathematical framework of quantum theory. On the other hand, the quest to calculate and measure as precisely as possible the anomalous magnetic moments of the remaining two charged leptons, the muon and tau, is still open. The muon's anomalous magnetic moment [16,17] is still puzzling the community aiming to reduce the gap between theory and experiment [18–24], with the most recent result reported in ref. [25]. Having a very short lifetime and being the massive ones among all leptons, measuring and predicting the anomalous magnetic moment of tau is a challenging task requiring great efforts [26–32]. Although, there is a serious discrepancy between theory and experiment, such efforts may have the potential to shed more light on the actual contribution of the higher-order hadronic terms.

The microscopic electrodynamics underlying the occurrence of anomalous component in the intrinsic magnetic moment of the charged leptons is indispensably related to a singularity-free radial dependence of the corresponding effective mass density and self-energy [1,3,33–38]. The latter are believed to be partially addressable only by the methods of quantum theory [39–44], with no classical analog. Moreover, within the semiclassical approaches of the quantum mechanics, like those used to study multielectron systems, the evaluation of self-interactions and self-energy still poses a challenge [45,46]. Therefore, the quest for a (semi-)classical regularized electrodynamics quantifying the self-interaction, self-energy and anomalous magnetic moment of charged leptons is far from being clear cut.

The present letter reports an exact classical approach quantifying the self-interaction, self-energy and anomalous g-factor of the electron. A non-probabilistic description of the electron's intrinsic dynamics and related electrodynamics is discussed in detail. Essential regularization of the electromagnetic field scalar potential underlying all

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results and allowing the effect of self-interaction to be accounted for with removable singularities is obtained. The electron's anomalous g-factor is calculated and compared with the theoretical and experimental results available in the literature. The obtained value agrees with the most recently reported experimental value to 5.9 parts in 10^{-13}, surpassing by accuracy the latest theoretical prediction of the quantum field theory.

Theoretical framework. – Henceforth, all mathematical representations are narrowed down to the framework of classical and semi-classical relativistic mechanics and electrodynamics. We find it convenient to omit the four-vector convention and represent all physical quantities and equations using the standard three-dimensional vector formalism.

General notations and representations. Consider an isolated electron, with rest frame of reference $\mathbf{R}$, rest mass m_e and electric charge $\bar{e} = -e$, where e denotes the elementary charge. Let $r_{ce} = \alpha\bar{\lambda}_{ce}$ denote the electron's electromagnetic radius at rest, where α is the fine structure constant and $\bar{\lambda}_{ce}$ is the corresponding reduced Compton wavelength. Let ρ_e and ρ_{m_e} be the electron's charge and rest mass densities defined within the spatial domain $\Omega_{ce} \in \mathbb{R}^3$, with boundary $\partial\Omega_{ce}$ and volume V_{ce}. The boundary $\partial\Omega_{ce}$ is considered as smooth and spherically symmetric with radius r_{ce}. In accordance with the experimental observations [47,48] the distribution of charge and mass in Ω_{ce} is considered as isotropic, with $\rho_e\rho_{m_e}^{-1} = em_e^{-1}$.

Let the electron be characterized by an effective rest mass

$$M_e = \int_{\Omega_{ce}} \rho_{M_e} \mathrm{d}v, \tag{1}$$

where the effective mass density $\rho_{M_e} = \rho_{M_e}(r)$ is a smooth function over $r \in (0, +\infty)$. Here, r is time-independent radial parameter.

Let $\mathbf{u}_e$, with unit vector $\boldsymbol{\kappa}$ and magnitude $|\mathbf{u}_e| = u_e$, be the electron's relative velocity with respect to the observational rest frame of reference $\mathbf{O}$ and $\mathbf{p}_e = \gamma_e m_e \mathbf{u}_e$ be its momentum, where γ_e is the corresponding Lorentz factor. Furthermore, let $\mathbf{r}_e$, with $|\mathbf{r}_e| = r_e$, be an intrinsic field vector oscillating about the origin of $\mathbf{R}$, with angular velocity $\boldsymbol{\omega}_e$ and areal velocity

$$\mathbf{f}_e = \frac{1}{2}(\mathbf{r}_e \times \tilde{\mathbf{u}}_e), \tag{2}$$

where $\tilde{\mathbf{u}}_e$ is the corresponding tangential velocity, with $|\tilde{\mathbf{u}}_e| = \tilde{u}_e$, $\tilde{\mathbf{u}}_e = \boldsymbol{\omega}_e \times \mathbf{r}_e$ and $\tilde{\mathbf{u}}_e \cdot \mathbf{u}_e = 0$. Here, r_e and $\tilde{u}_e$ are time-independent conjugate quantities and $\bar{\lambda}_{ce}$ is not an intrinsic wavelength characterizing the electron at rest. It represents the lower bound in the range of the electron's observable radius r_e. Since the areal velocity is intrinsic, we have the constraint

$$r_e\tilde{u}_e = \bar{\lambda}_{ce}c, \tag{3}$$

where c is the light speed in vacuum. We would like to point out, furthermore, that the magnitude of relative velocity $\mathbf{u}_e$ is intrinsic and not an average quantity. Therefore, in the Lorentz boost it remains invariant.

From eq. (2) there follows that the dynamics of $\mathbf{r}_e$ underlie the occurrence of angular magnetic moment

$$\boldsymbol{\mu}_e = -\frac{1}{2}r_e \int_{\Omega_{ce}} G_e \mathbf{j}_e \mathrm{d}v, \tag{4}$$

where $\mathbf{j}_e = \rho_e r_e \boldsymbol{\omega}_e$ is a pseudovector associated to the charge density current $\mathbf{J}_e = \gamma_e\rho_e\mathbf{u}_e$. Here, G_e is the integrand g-factor, where the latter reads

$$g_e = \frac{2}{V_{ce}} \int_{\Omega_{ce}} G_e \mathrm{d}v, \qquad G_e = \frac{e\rho_{M_e}}{m_e\rho_e}. \tag{5}$$

Since $g_e = 2(1 + a_e)$, we further have $M_e = m_e(1 + a_e)$, where a_e is the electron's anomalous g-factor.

Electromagnetic field. The pseudodensity current $\mathbf{j}_e$ occurs not only in $\mathbf{O}$ but also in $\mathbf{R}$ and satisfies $\gamma_e\mathbf{j}_e \cdot \mathbf{J}_e = |\mathbf{J}_e|^2$. The free electron cannot be at rest with respect to any observer and $u_e = \tilde{u}_e$. Since $\dot{\mathbf{u}}_e$ and $\dot{\boldsymbol{\omega}}_e$ are zero vectors the scalar $\varphi_e = \varphi_e(r)$ and vector $\mathbf{A}_e = \mathbf{A}_e(r)$ potentials of the electromagnetic field in $\mathbf{O}$ do not depend explicitly on time and according to the Lorentz transformations, we have

$$\varphi_e(r) = \gamma_e\eta_e\psi_e(r), \qquad \mathbf{A}_e(r) = 2\gamma_e\frac{\mathbf{u}_e}{c^2}\psi_e(r), \tag{6}$$

where $\eta_e = 1 + \beta_e^2$, with $\beta_e = u_e c^{-1}$. The function $\psi_e(r)$ is regularized to the origin of $\mathbf{R}$ with respect to $\mathbf{O}$ and reads

$$\psi_e(r) = \frac{\bar{e}}{4\pi\varepsilon_o r}\left(1 - e^{-\frac{\gamma_e r}{(1+a_e)\lambda_{ce}}}\right), \tag{7}$$

where ε_o is the vacuum permittivity. The function given in eq. (7) satisfies the classical field equation

$$\Delta_r\psi_e(r) - \chi_e^2\phi_e(r) = 0, \tag{8}$$

where Δ_r is the radial part of the Laplace operator in spherical coordinates and $\chi_e = \gamma_e((1 + a_e)\lambda_{ce})^{-1}$ is a scaling constant. For $u_e < c$, both functions in eq. (8) satisfy the boundary conditions

$$\psi_e(r), \phi_e(r) = \begin{cases} 0, & r \to \infty, \\ \dfrac{\gamma_e\bar{e}}{4\pi\varepsilon_o(1 + a_e)\lambda_{ce}}, & r \to 0. \end{cases} \tag{9}$$

The scale constant gives the lower bound of the electromagnetic field spatial frequency with respect to an observer in $\mathbf{O}$. The function $\phi_e(r)$ is associated to a Yukawa potential contributing to the electron's self-energy, with scaling constant $\gamma_e c((1+a_e)h)^{-1}$, where h is Planck's constant. Therefore, we have $\chi_e = \gamma_e m_e c((1 + a_e)h)^{-1}$ showing that within the quantum theory the self-interaction would be governed by an off shell photon with effective mass $m_e(1 + a_e)^{-1}$. Comparison between the Coulomb and regularized potentials is shown in fig. 1.

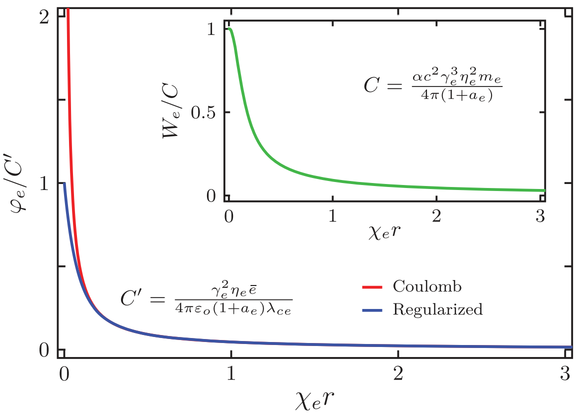

Fig. 1: Graphical representations of the regularized electromagnetic field scalar potential and Coulomb counterpart depicted for comparison. The corresponding electromagnetic field energy as a function of the distance from the origin of electron's rest frame of reference is depicted in the inset.

For an observer in $\mathbf{O}$ the dependence on r of the electromagnetic field potentials given in eq. (6) is confined within the plane perpendicular to the electron's relative velocity. Accordingly, the Coulomb and Lorenz gauge are satisfied trivially and the electromagnetic field potentials do not obey the Liénard-Wiechert representation [49–52]. The electromagnetic field does not propagate at speed c independently of the electron and it does not classify as an on shell coupling between the electric and magnetic fields. The system will exhibit neither spontaneous emission nor absorption of photons. The electromagnetic field is confined to the electron with Umov-Poynting vector $\mu_o^{-1}(\mathbf{E}_e \times \mathbf{B}_e) = \varepsilon_o E_e^2 \mathbf{u}_e$, where $\mathbf{E}_e$ is the corresponding electric field, $\mathbf{B}_e$ is the magnetic one, μ_o is the vacuum magnetic permeability and $\varepsilon_o E_e^2$ is the corresponding energy density. Integrating the latter over $\mathbb{R}^3$ by accounting for the representation $E_e = |\nabla_r \varphi_e(r)|$, we obtain the energy of the electromagnetic field. We have

$$W_e(r) = \alpha c^2 \gamma_e^2 \eta_e^2 m_e \frac{\bar{\lambda}_{ce}}{2r}\left(2 - 4e^{\frac{\gamma_e r}{(1+a_e)\lambda_{ce}}} + 2e^{2\frac{\gamma_e r}{(1+a_e)\lambda_{ce}}} + \frac{\gamma_e r}{(1+a_e)\lambda_{ce}}\right)e^{-2\frac{\gamma_e r}{(1+a_e)\lambda_{ce}}},$$

where

$$\lim_{r\to 0} W_e(r) = \frac{\alpha c^2 \gamma_e^3 \eta_e^2 m_e}{4\pi(1+a_e)}. \tag{10}$$

The dependence on r is depicted in the inset of fig. 1.

The limits given in eqs. (9) and (10) underline that for $u_e < c$ the radial dependence of the electromagnetic field has a removable singularity ensuring the electron's self-energy takes finite value. Moreover, at high momentum scale the theory remains consistent, since the limit $u_e \to c$ corresponds to an open system.

Self-energy. – Introducing the electron's self-energy in accordance with the theoretical framework set in the previous section, we apply the Hamiltonian formalism.

The Hamiltonian. Since the system is closed, with the action of no additional fields, the electron exhibits no exchange of energy and hence momentum. Consequently, neither external nor net self-forces [52–54] are acting on the electron. The system's energy remains purely kinetic, with Hamiltonian not depending explicitly on time. However, the considered system definitely exhibits a type of self-interaction, with energy that is not directionally specific. The corresponding Hamiltonian reads

$$H_e = \gamma_e m_e c^2 + \Sigma_e, \tag{11}$$

where Σ_e is the electromagnetic self-energy. In particular, Σ_e equals the spatial average over the domain Ω_{ce} of the interaction energy $\bar{e}\varphi_e$ describing the electron's electromagnetic self-interaction in $\mathbf{O}$. Moreover, the electron's self-energy is an effective kinetic-energy generated by the corresponding self-interaction yielding larger mass density $\rho_{M_e} > \rho_{m_e}$, with ρ_{M_e} satisfying eq. (1). In the absence of self-interaction $\rho_{M_e} = \rho_{m_e}$ and the system is electrically neutral. Thus, we have

$$\Sigma_e = \gamma_e c^2 \int_{\Omega_{ce}} \rho_{M_e} - \rho_{m_e} \mathrm{d}v, \tag{12}$$

where

$$\begin{aligned}\rho_{M_e} &= \rho_{m_e}\left(1 + \eta_e \frac{r_{ce}}{r}\left(1 - e^{-\frac{\gamma_e r}{(1+a_e)\lambda_{ce}}}\right)\right)\\ &= \rho_{m_e}\left(1 + \frac{\eta_e \bar{e}}{m_e c^2}\psi_e\right).\end{aligned} \tag{13}$$

The self-energy term given in eq. (12) is intrinsic and not a potential energy of a gradient field. Represented within the mathematical framework of quantum theory, it will remain invariant with respect to the electron's orbital state in many-body systems. That may be of benefit to the researchers studying multi-electron systems with the aid of computational methods that fail to account for the self-interactions without generating errors, see, for example, the case of Kohn-Sham density functional theory [46,55,56]. Moreover, having an exact value of a scaling constant regularizing the electrons interactions in solids, like χ_e from the cutoff term $\phi_e(r)$ represented explicitly in eq. (7) regularizing the electromagnetic field potentials in eq. (6), is a key point in facilitating the successful application of time-dependent density functional theory in studying electric polarization-switching mechanism in ferroelectric systems [57,58].

The Hamiltonian density. The Hamiltonian given in eq. (11) has a corresponding density related to the dynamics of the field vector $\mathbf{r}_e$. According to eqs. (3) and (13), for $r = r_e$, we have

$$\mathcal{H}_e = c^2 \rho_{m_e}\left(\gamma_e + \frac{\alpha}{m_e c}\mathcal{P}_e\right), \tag{14}$$

where

$$\mathcal{P}_e = m_e \gamma_e \eta_e \tilde{u}_e\left(1 - e^{-\frac{\gamma_e c}{2\pi(1+a_e)\tilde{u}_e}}\right)$$

Table 1: Theoretical and experimental values of the electron's anomalous g-factor. The result predicted by the regularized classical electrodynamics (CED) discussed in the present study is given in the second row. The third and fourth rows show some of the recent results based on renormalized quantum electrodynamics (QED) and included hadronic and weak contributions (QED$^+$), respectively. In the fourth row only the uncertainty arising from the fine structure constant is included. The last row shows the most recent experimental result.

Methods	a_e	Reference
CED	0.0011596521800027(65)	eq. (17)
QED	0.00115965218178(77)	[11]
QED$^+$	0.001159652181606(229)	[13]
Experiment	0.00115965218059(13)	[15]

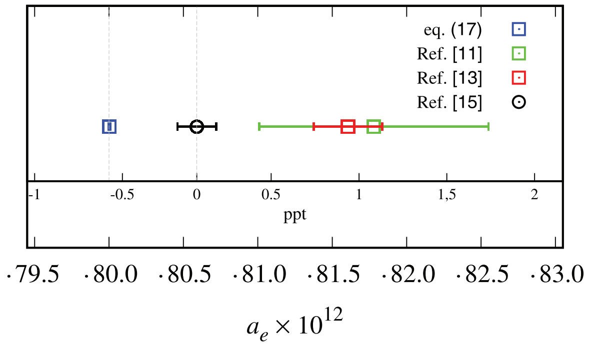

Fig. 2: Calculated and measured (black circle) values of the electron's anomalous g-factor. Only the most recent results are depicted, with data provided in table 1. For the sake of clarity, the first ten digits on the left-hand side with respect to the decimal point are replaced by a central dot symbol.

is the generalized momentum. From eq. (14) we obtain the equations of motion as

$$\tilde{u}_e = \int_{\Omega_{ce}} \frac{\partial \mathcal{H}_e}{\partial \mathcal{P}_e} \mathrm{d}v, \qquad \dot{\mathcal{P}}_e = 0, \tag{15}$$

describing the electron's intrinsic dynamics.

The anomalous g-factor. – Integrating over the domain Ω_{ce} in eq. (15), we get

$$\tilde{u}_e = \alpha c, \qquad \eta_e = 1 + \alpha^2, \qquad \gamma_e^{-1} = \sqrt{1-\alpha^2}. \tag{16}$$

Taking into account the explicit representation of the effective mass density given in eq. (13), from eq. (5) we obtain a transcendental equation satisfied by the electron's anomalous g-factor. We have

$$a_e = 3\eta_e \left(\frac{1}{2} - \left(\frac{1 - e^{-\frac{\alpha\gamma_e}{2\pi(1+a_e)}} \left(1 + \frac{\alpha\gamma_e}{2\pi(1+a_e)}\right)}{\left(\frac{\alpha\gamma_e}{2\pi(1+a_e)}\right)^2} \right) \right). \tag{17}$$

Along with the successful quantum theory calculations [12,13], eq. (17) represents a unique classical equation predicting the electron's anomalous g-factor. The calculated value is given in table 1 along with the most recent results found in the literature, where for the fine structure constant we obtain $\alpha = 137.0359990849004(3)^{-1}$. The graphical representation of the data given in table 1 is shown in fig. 2. It is worth noting that according to NIST [59] the value of fine structure constant is $\alpha = 137.035999084^{-1}$. We would like to point out that for $\alpha = 137.035999206^{-1}$ [60], the computed value of a_e matches the experimental one only to a few parts per billion.

To the best of our knowledge, the value of the anomalous g-factor given in the second row in table 1 represents the most accurate theoretical results reported by far.

Field representations. – Hereon quantum mechanical representations of immense relevance are used by exception for complementary purpose.

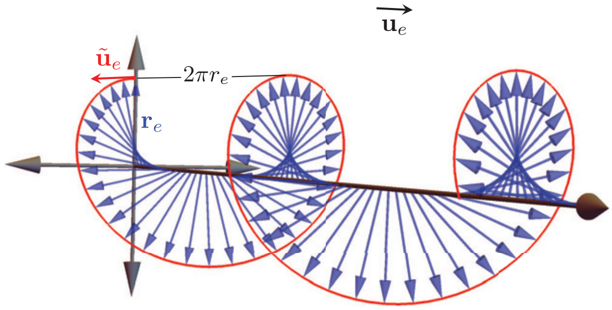

Fig. 3: Graphical representation of a right circular polarization of the electron's intrinsic field vector $\mathbf{r}_e$ (blue arrows) along the axis of relative motion of the electron's rest frame of reference. The corresponding helix (red line) has a pitch $2\pi r_e$, since for a free electron $\tilde{u}_e = u_e$.

Intrinsic magnetic moment. The magnetic moment presented in eq. (4) is intrinsic. It is a classical representation of the electron's spin magnetic moment with included anomalous component. The anomalous component occurs as a result of the regularized self-interaction (see eq. (12)). In particular, substituting the explicit representation of G_e from eq. (5) in eq. (4), we get

$$\boldsymbol{\mu}_e = -\frac{1}{2} g_e \mu_B \boldsymbol{\kappa}, \tag{18}$$

where μ_B is the Bohr magneton. Here, stepping closer to the frontier of quantum theory, we take into account the equality $\bar{\lambda}_{ce} m_e c = \hbar$.

The intrinsic magnetic moment is related to neither rotation nor spinning of the electron's electric charge or rest mass, with $\mathbf{R}$ remaining inertial. It occurs since the electron's intrinsic field vector $\mathbf{r}_e$ is oscillating with angular frequency $\omega_e = \tilde{u}_e r_e^{-1}$, see eq. (2). Therefore, as the origin of $\mathbf{R}$ is moving relatively to an observer, with velocity $\mathbf{u}_e$, the electron appears as a circularly polarized traveling wave of amplitude $A_e = r_e$ and carrying rest mass M_e, see fig. 3. Accordingly, the electron's dynamics in $\mathbf{O}$ is associated to the vector field $\boldsymbol{\Phi}_e(\mathrm{x}) = A_e \Phi_e(\mathrm{x}) \mathbf{n}_e$, where

$\mathrm{x} \in \mathbb{R}^{1,3}$ is the position four-vector of the origin of electron's rest frame of reference and $\mathbf{n}_e \in \mathbb{C}^3$ is the field's unit vector. Here, at a certain point in spacetime the phase factor $\Phi_e(\mathrm{x})$ of the propagator satisfies the Klein-Gordon equation

$$\left(\Box + \frac{M_e^2 c^2}{\hbar^2}\right) \Phi_e(\mathrm{x}) = 0. \tag{19}$$

The electron's intrinsic field vector is represented by the vector field in $\mathbf{O}$ according to the relation $\mathbf{r}_e = \sqrt{2}\mathrm{Re}\{\mathbf{\Phi}_e(\mathrm{x} = ct)\}$, where $\mathbf{n}_e = (\frac{1}{\sqrt{2}}, \pm\frac{i}{\sqrt{2}}, 0)$.

Effective mass-energy equivalence. Taking into account eqs. (11) and (12), we obtain the effective mass-energy relation

$$\mathcal{E}_e = \gamma_e M_e c^2, \tag{20}$$

where for the electromagnetic self-energy we get

$$\Sigma_e = a_e \gamma_e m_e c^2.$$

Here we take into account that $M_e = m_e(1 + a_e)$. From eqs. (19) and (20) there follows that the electron is characterized by the effective momentum $P_e = \gamma_\alpha M_e u_e$ satisfying the energy-momentum relation

$$\mathcal{E}_e = \sqrt{M_e^2 c^4 + P_e^2 c^2}. \tag{21}$$

Moreover, the effective rest mass, energy and momentum related by eq. (21) are physical characteristics of the spinor field $\Psi_e(x_\mu)$ that effectively accounts for the discussed self-interaction and satisfies the Dirac equation

$$(i\hbar\gamma^\mu \partial_\mu - M_e c)\Psi_e(x_\mu) = 0. \tag{22}$$

In contrast to the Dirac spinor in standard quantum theory the spinor field in eq. (22) accounts for the anomalous component in the electron's spin magnetic moment.

Atomic orbitals. From eqs. (3) and (16), we calculate the free electron field vector magnitude. We have $r_e = r_B$, where r_B is the Bohr radius. Moreover, expanding in power series the Lorentz factor in eq. (20), we get

$$M_e c^2 + \frac{\alpha^2}{2} M_e c^2 + \frac{3\alpha^4}{8} M_e c^2 + \cdots,$$

where the second term is the non-relativistic oscillation energy of the free electron field vector. Omitting the anomalous component it equals the absolute value of the ground-state energy of the hydrogen atom. Therefore, the ground state of a hydrogen atom preserves the amplitude of the free electron field vector. Consequently, the corresponding total angular momentum equals the spin one. As a result of the proton-electron interaction the electron's net relative velocity equals zero and the dynamics of $\mathbf{r}_e$ appears as not directionally specific. At any instant of time, instead of a helix (see fig. 3), the oscillation of the electron field vector $\mathbf{r}_e$ indicates a point on a sphere representing the first hydrogen atomic orbital, see fig. 4. When excited, the magnitude and dynamics of the electron field vector change obtaining different orbital shape and hence electron cloud.

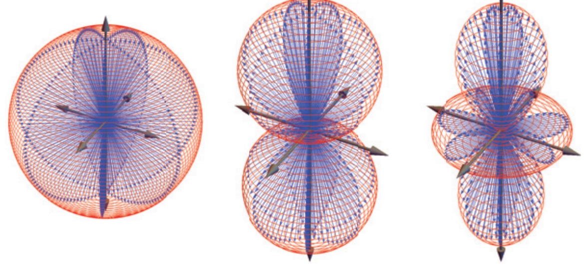

Fig. 4: Graphical representation of the angular dynamics of the electron field vector $\mathbf{r}_{nlm}$ in the hydrogen atom, see eq. (23). The vector's magnitude is normalized to unity and the density of arrows is reduced for the sake of clarity.

According to the current picture, the electron cloud does not represent a moving around the nucleus point-like charge and mass with motion exclusively stabilized by the uncertainty of corresponding position and momentum. The electron's charge and mass only seemingly orbit around the atomic nucleus. In particular, the atomic shells and subshells appear as different modes of oscillation of $\mathbf{r}_e = \mathbf{r}_e(\theta, \varphi)$ related to the stationary wave function $\Phi_e(\varrho, \theta, \varphi) = R_e(\varrho) Y_e(\theta, \varphi)$, where ϱ, θ and φ are the radial coordinate, polar and azimuthal angles, respectively. The associated vector field is $\mathbf{\Phi}_e(\theta, \varphi) = A_e Y_e(\theta, \varphi)\mathbf{n}_e$, where $\mathbf{n}_e$ is the radial unit vector. The stationary wave function satisfies the Helmholtz equation

$$\left(\Delta + \zeta^2(\varrho)\right) \Phi_e(\varrho, \theta, \varphi) = 0, \tag{23}$$

where $\zeta^2(\varrho) = 2m_e\hbar^{-2}(E - U(\varrho))$, E is the total energy of the oscillator and $U(\varrho) = -m_e c^2 r_{ce} \varrho^{-1}$ is its non-regularized potential energy. Note that as in eq. (19), the delta operator in eq. (23) is associated only to the kinetics of $\mathbf{r}_e$ and not to the motion of electron's charge and mass. Therefore, in the corresponding Schrödinger equation the momentum operator of the electron will be associated only to the dynamics of its field vector. Moreover, the hydrogen wave functions will be related to the probability of finding the electron's field vector pointing at a certain direction in space and having a particular magnitude. The position and momentum associated to the field vector will be naturally related via the Heisenberg uncertainty relations.

The solutions of eq. (23) are orthogonal polynomials associated to the eigenstates of the hydrogen atom. We have $\Phi_e(\varrho, \theta, \varphi) \to \Phi_{nlm}(\varrho, \theta, \varphi) \equiv \Lambda_{lm} R_{nl}(\varrho) Y_l^m(\theta, \varphi)$, where $R_{nl}(\varrho)$ are the radial wave functions and $Y_l^m(\theta, \varphi)$ the spherical harmonics. Moreover, n, l and m are the prime, orbital and magnetic quantum numbers, respectively. The oscillator's energy is

$$E_n = -\frac{m_e c^2 r_{ce}}{2n^2 r_B}.$$

For all n and l, the field's amplitude $A_e \to A_{nl}$ is the peak amplitude equal to the value of ϱ at the global maximum of the function $|\varrho R_{nl}(\varrho)|$. Furthermore, $Y_e(\theta,\varphi) \to \Lambda_{lm} Y_l^m(\theta,\varphi)$, where for all l and m the normalization constant Λ_{lm} preserves the peak amplitude. In particular, for $m,\theta = \{0,0\},\{\pm l, 0.5\pi\}$, we have the constraint $\Lambda_{lm}|Y_l^m(\theta,0)| = 1$. Thus, for the electron field vector we have $\mathbf{r}_e \to \mathbf{r}_{nlm} = A_{nl}\mathrm{Re}\{\Lambda_{lm}Y_l^m(\theta,\varphi)\}\mathbf{n}_e$, where the corresponding field reads $\mathbf{\Phi}_e(\theta,\varphi) \to \mathbf{\Phi}_{nlm}(\theta,\varphi) = A_{nl}\Lambda_{lm}Y_l^m(\theta,\varphi)\mathbf{n}_e$.

The angular dynamics related to three different atomic orbitals, with $A_{10} = r_B$, $A_{21} = 4r_B$ and $A_{32} = 9r_B$, is depicted in fig. 4.

Summary and conclusions. – The present latter reports an exact classical approach quantifying the electron's self-interaction, self-energy, spin and anomalous magnetic moments. The presented mathematical framework demonstrates that the classical and quantum representations of the electron's intrinsic dynamics are very closely interconnected and that the corresponding intrinsic and anomalous magnetic moments are not exclusive features of the quantum theory.

In particular, a framework of essential classical representations of the electron's effective rest mass and intrinsic magnetic moment are given (see eqs. (1) to (5)). To this aim an inherent field vector underlying the electron's wave-like dynamics is introduced. Furthermore, a unique regularization of the electromagnetic field scalar potential that removes the radial singularity in the system is presented, see eq. (7). Consequently, unique equations of motion and a transcendental equation describing the electron's intrinsic dynamics, predicting the Bohr radius and the electron's anomalous g-factor are derived, see eqs. (15) and (17), respectively. The electron's effective rest mass and self-energy are exactly calculated, showing that the rest energy of an electrically charged particle is larger than the rest energy of electrically neutral particle of the same rest mass, see eq. (20). Furthermore, the electron's anomalous g-factor is calculated with stunning precision obtaining the most accurate and only classical value reported by far, see table 1 and fig. 2. In addition, the semiclassical and effective Dirac spinor fields describing a self-interacting electron are derived, see eqs. (19) and (22), respectively. A fundamental advantage of the presented fields is that they directly account for the occurrence of anomalous component in the electron's intrinsic magnetic moment and are more massive to the degree defined by eq. (20). Moreover, a field representation of the atomic orbitals is obtained, proposing that each of the latter represents a particular oscillation of the electron field vector closely related to the stationary wave function of the hydrogen atom, see eq. (23).

The reported approach has the potential to address the occurrence of anomalous magnetic moment of the remaining massive leptons and the non-composite particles in general. Applied to study the muon self-interaction it predicts a highly accurate result for the corresponding anomalous g-factor. The calculated value matches the most recent experimental one reported in the literature to about 0.43 ppb. These results lie beyond the scope of the present letter and will be discussed in a separate paper.

The approach may be built on and implement into the framework of quantum field theory to influence our understanding about the inter-relationship between classical and quantum physics beyond the corresponding principle.

It is worth emphasizing, furthermore, that the obtained regularization of the electromagnetic field scalar potential may significantly facilitate the optimization of some computational methods in solid state physics making them self-consistent, especially methods studying multi-electron systems that fail to address the self-interactions without the consideration of additional corrections.

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