https://doi.org/10.1051/epjconf/202430404006

Single Folding Potential Calculations in ^{141}Pr

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Abstract. In this study, Skyrme and Gogny forces have been used to describe interactions for Hartree-Fock (HF) and Hartree-Fock-Bogoliubov (HFB) calculations, considering all nucleons to evaluate nucleon densities. The nucleon densities of the ^{141}Pr were calculated by using the Skyrme-HF (SHF) method with Woods-Saxon Potential (SHF-WS) and with Harmonic Oscillator Potential (SHF-HO), HFB method with Skyrme (HFB-S) and with Gogny (HFB-G) interactions. In these calculations, the effects of the three-body force have been accounted for in both Skyrme and Gogny forces through a density-dependent term necessary to describe various properties of nuclei and nuclear matter. The root-mean-square (rms) radii of proton, neutron, and charge densities of the nucleus were calculated using the density-dependent Skyrme-type effective nucleon-nucleon (NN) interaction in the HF approach. The Folding potential, developed to describe nucleon-nucleus (n-Pr) elastic scattering data, was obtained by single folding potential to calculate reaction cross-section calculations for ^{141}Pr target. The calculated cross-sections, obtained using density data from four different models, were compared with each other and with experimental data from the literature for analysis and interpretation of the results.

1 Introduction

The ground state properties of nuclei, especially nucleon densities, are generally calculated using Skyrme and Gogny forces. The nucleon densities of ^{141}Pr have been calculated by using;

- Hartree-Fock-Skyrme (based on the Woods-Saxon Potential) (SHF-WS)
- Hartree-Fock-Skyrme (based on the Harmonic Oscillator Potential) (SHF-HO)
- Hartree-Fock-Bogolyubov-Skyrme (HFB-S)
- Hartree-Fock-Bogolyubov-Gogny (HFB-G)

methods. The obtained nucleon densities for the ^{141}Pr have been compared with each other and then used to calculate the interaction potential of elastic scattering of neutrons from ^{141}Pr. These obtained potentials are compared with theoretical potentials which are available in the literature and then used in the TALYS nuclear reaction code to get differential cross sections of ^{141}Pr(n,el) reaction. The obtained cross section is also compared with both theoretical calculations and experimental values.

1.1 Hartree-Fock Approximation

The Hartree-Fock (HF) is a useful approximation used in calculating the ground state properties of spherical nuclei [1–3]. HF calculation codes developed using this and derived approaches have been in use for several years. In the HF approach, equations are solved using one iteration. In these approaches, harmonic oscillator or Wood-Saxon wave functions are recommended for the wave function considered for iteration [4].

The nucleon, whether proton or neutron, density ρ is generated based on these wave functions. Then, iterations are made between the intensity and $U(\rho)$ energy potential for ψ wave function.

The Skyrme force is the most effective of the forces used for the calculations used in the HF approximation [5, 6]. In this way, very important nuclear quantities such as nuclear radius, density distributions and surface thickness can be calculated. Some calculation codes have been developed by applying the HF approximation to the specified Skyrme force by fitting through the least squares method. These computational codes have also been adapted to include studies of ground state correlations and nuclear excitation.

1.2 Skyrme Forces

Skyrme forces are considered the best phenomenological force used to calculate some properties of the nucleus, especially its ground states. The Skyrme interaction used in the calculation of nuclear properties is defined in its sim-

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plest form as follows:

$$\vec{V}_{skyrme} = \sum_{i<j} \vec{V}(i, j) + \sum_{i<j<k} \vec{V}(i, j, k). \quad (1)$$

While, the first term defines interaction of two-bodies, the second term defines that of three-bodies.

1.3 Skyrme-HFB

Many properties of nuclei can be explained using a model of independent particles moving at an average potential [7]. Using the Skyrme-HF model [8], a single particle potential (which depends on the distribution of nucleons in the nucleus) can be obtained from the two-body interaction for individual particles. This calculation is done using the variation principle with Slater determinants as trial wave functions (e.g. harmonic oscillator or Woods-Saxon wave functions).

The most general wave functions in Skyrme-HFB theory consist of independently moving quasiparticles and are determined using the variational principle [9, 10]. These wave functions are obtained by taking into account as many correlations as possible in the static approximation of a single particle.

1.4 Gogny Forces

Another self-compatible field method is the Gogny model using Gogny interaction [11]. Gogny force is the interaction of two non-divergent particles and its parameters are arranged according to experimental data. The Gogny force is widely used in determining the properties of nuclear structure requiring mean field calculations. The parameters of the force are determined according to experimental data of infinite nuclei and mean field calculations in accordance with the properties of infinite nuclear matter.

The effect of the three-body force is taken into account in the calculations, where both Skyrme and Gogny forces are used. This is achieved through a density-dependent term that is necessary to describe various properties of the nucleus and nuclear matter [12, 13].

2 Nucleon Densities

The nucleon densities, which depend on the wave function of the state and the occupation probability w of that state, are given by:

$$\rho_q\left(\overrightarrow{R}\right) = \sum_{\beta\in q} w_\beta \Psi^\dagger_\beta(\overrightarrow{r})\Psi_\beta(\overrightarrow{r}), \quad (2)$$

where q indicates neutron or proton density and β denotes the state. Ψ_β represents the single particle wave function of the β state in the SHF method, while it represents the quasi-particle wave function of the quasi-particle β state in the HFB-S method. Using Eq. (2), the rms radii of nucleon densities can be evaluated by;

$$r_q = \left\langle r_q^2\right\rangle^{1/2} = \left[\frac{\int r^2\rho_q(r)dr}{\int \rho_q(r)dr}\right]^{1/2}. \quad (3)$$

The nucleon densities by the HFB-S and HFB-G methods are evaluated directly using the TALYS nuclear reaction code and one can find details in [14].

3 Single Folding potential

Elastic scattering data can be analyzed using the HF code, in which the calculated values replace the phenomenological potential. Double Folding potential is used in the nucleus-nucleus interaction[15, 16] .

$$V_f(R) = N \int\int \rho_1(r_1)\rho_2(r_2)V_{NN}(s)dr_1dr_2, \quad (4)$$

where ρ_1 and ρ_2 are the nucleon densities of interacting nuclei, R is the distance between the centers of mass of these nuclei, and N is the normalization factor. The relative vector ($\vec{s} = \vec{R} + \vec{r}_1 + \vec{r}_2$) indicates the distance between the interacting nucleon pair. $V_{NN}(s)$ gives the effective nucleon-nucleon (NN) interaction. During nucleon-nucleus scattering, the displacement of two interacting nucleons is called "knock-on exchange" [17, 18]. This event is included in the Double Folding integral as follows

$$V'_{NN} = V_{NN} + J_{00}(E)\delta(s) \quad (5)$$

The $V_{NN}(s)$ component used in our calculations has the following form

$$V_{NN}(s) = \left[7999\frac{e^{-4s}}{4s} - 2134\frac{e^{-2.5s}}{2.5s}\right]\text{MeV}, \quad (6)$$

while $J_{00} = -146$ MeV.fm^3

The nucleon-nucleus reaction is called the standard Single-fold optical model and Single Folding potential [15, 19], and is expressed as given below

$$V_{SF}(R) = \int \rho(r)V_{NN}\left(|\vec{R} - \vec{r}|\right)d\vec{r}. \quad (7)$$

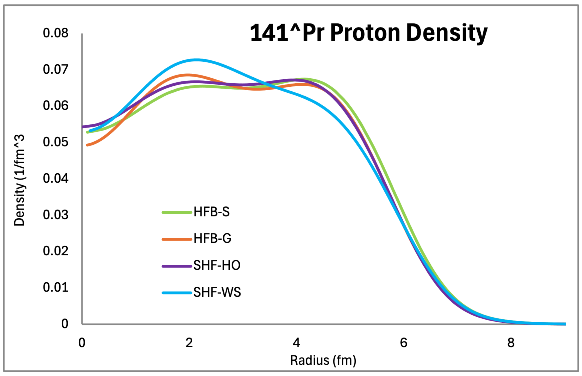
Here,

$$V_{NN}\left(|\vec{R} - \vec{r}|\right) = \int dr\rho(r)V(s). \quad (8)$$

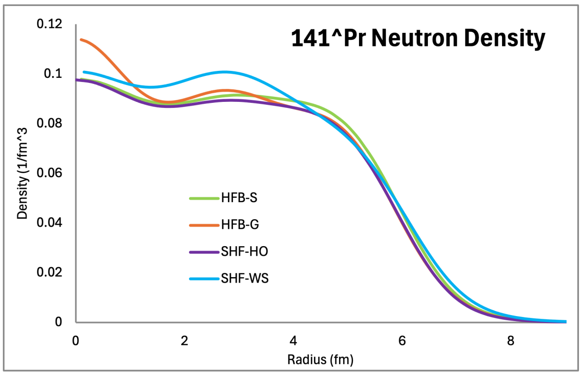
The V interaction potential depends on the density because the folding integral is taken depending on the density of both interacting nuclei and nucleon. The nucleon-nucleus interacting potential includes effects that depend only on the density of a single nucleus. When the phenomenological potential mentioned in the Single Folding integral is included, the density effects of the other particle are neglected, while the $V_{NN}(s)$ potential is not directly related to the density of the nucleus[20].

4 Results

Three codes, namely HF code [21] , HAFOMN code [22] and TALYS nuclear reaction code [14] were used to calculate nucleon densities of ^{141}Pr. Calculated values are given in Figure 1. All methods give similar results and one can see that SHW-WS results slightly differ from other methods.

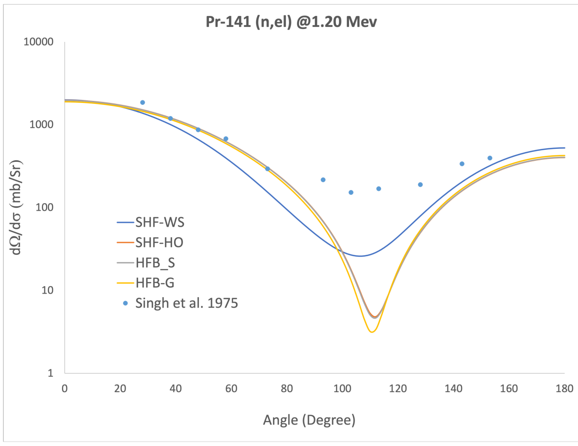


(a)

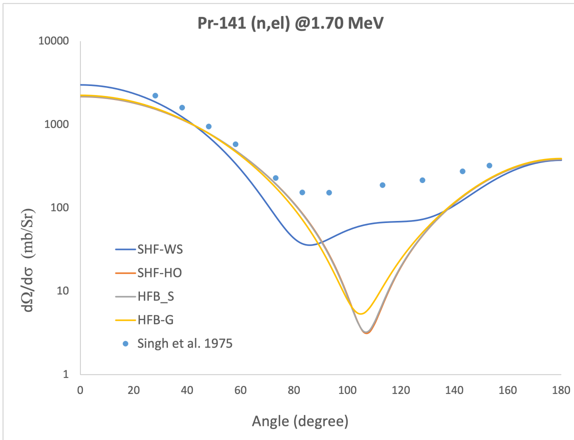


(b)

Figure 1. The nucleon densities of ^{141}Pr.



(a)



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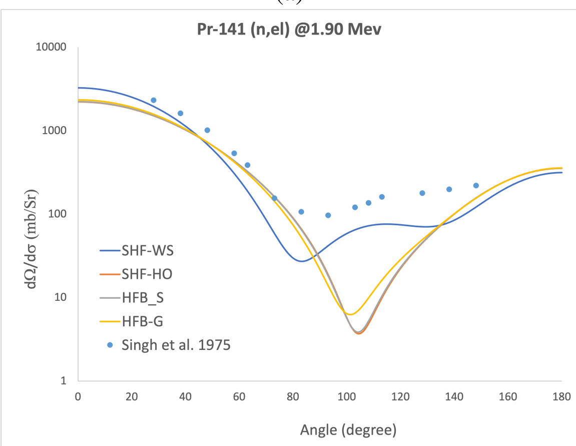
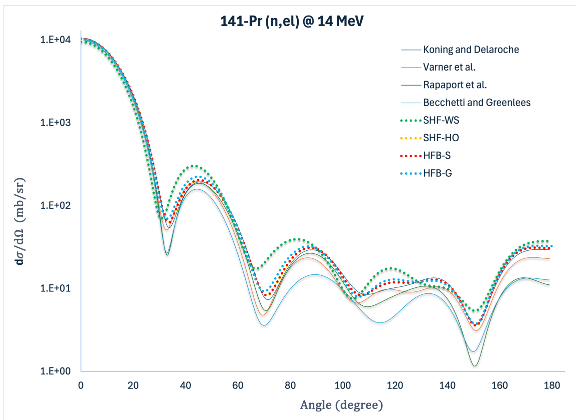


(c)



(d)

Figure 2. The ^{141}Pr(n,el) reaction cross section at a) 1.2 MeV b) 1.7 MeV c) 1.9 MeV and d) 14 MeV .

The nucleon densities were used in Single Folding potential calculations to obtain the nuclear interaction potential for the ^{141}Pr (n,el) reaction. The calculated potentials are given in Table 1 with comparison to theoretical values which are available in the literature. Only, values for 14 MeV neutron energy values are given in Table 1.

The obtained potential values were used in the TALYS nuclear reaction code to evaluate ^{141}Pr(n,el) differential cross sections at 1.2, 1.7, 1.9 and 14 MeV. The cross section values are compared in Figure 2 graphically.

5 Conclusion

The results obtained in this study can be listed as follows:

1. Similar results are obtained if nucleon densities are calculated in four different models.

Table 1. Nuclear interaction potential values for ^{141}Pr(n,el) reaction at 14 MeV.

	Theoretical	Potential (MeV)	Potential (MeV)	This work This work
	[23]	-44.964	-47.8521	SHW-WS
^{141}Pr (n,el)	[24]	-46.577	-43.9269	SHF-HO
@ 14 MeV	[25]	-46.301	-43.9754	HFB-S
	[26]	-47.905	-44.7173	HFB-G

2. For elastic scattering reactions created with neutrons, interaction potentials can be calculated by using nucleon densities in the Single Folding method. Instead of the Double Folding method, which is generally used for nucleus-nucleus interactions, the Single Folding method used for nucleon-nucleus interactions gives results that are very compatible with other theoretical values found in the literature.

3. It is seen that compatible results are obtained in the interaction potentials calculated for the projectile energies for which experimental data are available.

4. By using this method, it has been seen that interaction potentials for all nuclei can be obtained depending on the projectile energy, and in future studies, new interaction potential equations will be obtained thanks to the fitting process that depends on both the energy as well as on Z and N.

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