

UNIVERSITY OF SOUTHERN DENMARK

Natural Models of Inflation

On The Composite Nature of The Inflaton

MASTER THESIS

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Particle Physics & Cosmology

October 1, 2012

Abstract

This project explore the possibility that the inflaton, the particle responsible for inflation, could be a composite state. The inflaton is traditionally modelled as a fundamental scalar, but only one possible fundamental scalar have yet been discovered, making an alternative attractive. Working in different particle setups I will show that inflation from a composite inflaton is a valid alternative to the fundamental scalar inflaton and that it might even be a more natural way to explain an inflationary era.

I show that inflation with a composite inflaton can arise from technicolor models and from pure Yang - Mills theories. Furthermore the scale of the effective description of such theories are the Grand Unification scale, insuring a valid effective description through the inflationary era. I describe three models, at this point two of them are working theories of inflation, with one of them not yet excluded by experiments.

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1 Introduction

1.1 Paradigm of Cosmology

The ultimate question in natural science must be "*Where does it all come from?*". What differs the branches of natural science is merely the area of understanding that are imposed when searching. The most fundamental of these natural sciences might be physics, as any other branch can in principle be derived from the physical principles of nature. What physicist believe in I have neither time, space, nor the knowledge to cover it all, but I will sketch a few things that will be come relevant for this project. The project is partly concerned with particle physics and partly with cosmology, and even though one might think that there is no relations between the two, due to the huge difference in size of the objects in question, there is probably as large a correlation as one can imagine.

I will not go into details about quantum field theory, the theory which describes the smallest building blocks of nature, other than say that as temperature increases in the universe more and more of quantum field theory becomes relevant, and the interplay of particles have huge impact on how the universe evolves. For an introduction to quantum field theory I suggest the work by Peskin and Schroeder [1].

For cosmology I will give a short recap of the evolution of the universe, but I will not go into details on how this theory is supported by observations. Moreover there are many factors I will neglect. It might be of some help to recall to figure 1 during the following.

Imagine that we could travel backwards in time, starting from today. As we move further and further back we experience some of the great events in the evolution of the universe. Today the universe as a whole is flat and contains about 4 % bright matter ¹, 23 % dark matter and 73 % dark energy. The last two contributions yet to be explained. This makes our universe dominated by dark energy today, which expands the universe as Hubble discovered in 1929 [3]. We see some structure at large scales that I will explain later.

Moving on we see galaxies being formed by gravity and further back we see even stars being formed this way. Until now the universe has been relatively bright, as stars have been shining for us, but when we reach a time where the stars did not shine the universe becomes totally dark, atleast almost. There is some light with long wavelenght called cosmic microwave background radiation (CMB). At this point it is not in the visible spectrum but it will be, this era is called the dark ages for obvious reasons. Before star formation the only elements formed are hydrogen and helium.

When we move along we come the point where the CMB is formed, as well as neutral hydrogen and helium. As we have been moving further and further

¹atoms, electrons, neutrinos, quarks, gauge bosons and so on.

back the temperature has been rising and the universe is now so small and hot that the photons of the CMB are so energetic and dense that they can split hydrogen into a proton and an electron. On the contrary, when moving forward in time, a proton combines with an electron and emits a CMB photon. The same happens for the electrons orbiting helium. This period is called recombination.

We are now in a non-transparent universe with protons, electrons photons and helium nuclei. We have moved to around 380.000 of what later came to be know as Earth years after the big bang, or about 13.3 billion years backwards in time. This is the furthest back we can see using photons as a messengers, since the universe from here on will be non-transparent. Moving further we also see that neutrinos decouple from matter, leaving a cosmic neutrino background.

3 minutes before the big bang we observe what is called the big bang nucleosynthesis, where the nuclei of helium is formed from protons and neutrons. This event and the story of moving closer to the big bang can be fund in a pedagogical book by Steven Wienberg named "*The First Three Minutes*" [4].

We are now nearly at the big bang and we have come to a crossroads of theories. In the standard big bang the universe becomes increasingly denser and hotter ending up in what looks like an singularity of spacetime. However, as this project will focus on, inflation is an alternative. For the last part of this section I will assume inflation and be starting at the beginning of inflation and move forward in time.

The universe is very small and dense, quantum fluctuations are happening all the time, and a new particle, never seen before, arrives; the inflaton. This particle can make the universe expand very rapidly, and it will, stretching everything, and making the universe cold. This rapid expansion leaves its marks on the universe and a lot of it can be observed in CMB today.

Even the quantum fluctuation of the tiniest of objects gets stretched so much that they today are seeds for large scale structure formation. At the end of inflation the inflaton will decay into other particles rapidly increasing the temperature to a point where the other processes of the hot big bang theory can commence. This very short, very violent era of inflation solves some of the problems that the standard big bang theory has, but it is still consistent with experiments.

Before inflation we do not have sure clues to what happened, our best guess at this point is first a unification of the standard model forces into a grand unification theory, and after this only string theory may explain the interplay between gauge forces and gravity.

Inflation is the general history that I will working with for this project.

1.2 General Conventions

throughout the project there is some notation and convention that is used repetitively.

- Indices i and j run from 1 to 3 and denote the spacial part of for instance the metric, where as μ and ν includes the time component 0 and therefore

runs from 0 to 3. Furthermore for repeated indices Einstein summation is used. The derivative ∂_μ denote the derivative vector $\frac{\partial}{\partial x^\mu}$.

- Natural units are taken for granted. So $c = \hbar = k_B = 1$, where c is the speed of light, \hbar is the reduced Planck constant and k_B is Boltzmann's constant. In these units the Planck mass, M_P takes the value $1.22 \cdot 10^{19} GeV$. Due to Einstein's field equation of general relativity the reduced Planck mass, M_{Pl} , is often used $M_{Pl} = 2.43 \cdot 10^{18} GeV$.
- When using Feynmann diagrams the time direction of future is going to the right, so the diagram can be read as ordinary text.
- Conventions on the meaning of "big bang" are many. The different meanings I will use is a theory with or without inflation. The big bang theory or standard big bang is a theory without inflation, where as big bang, reheating or hot big bang is in a theory with inflation. big bang in this sence is before inflation and hot big bang is equivalent with reheating, which is after inflation.

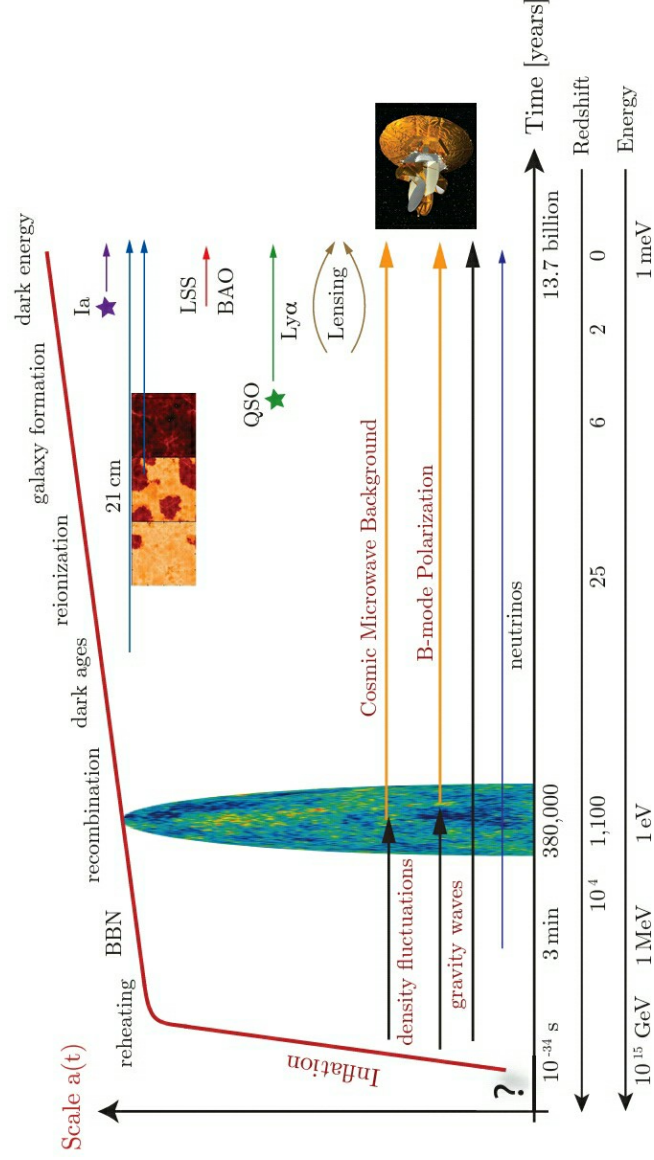


Figure 1: Key events in the evolution of the universe within the inflation paradigm. Some of the features I will use are the observables from inflation, e.i. the density perturbations and the possibility of gravitational waves from the rapid expansion during inflation. These have impact on CMB and it is from here they are observed [2].

2 Problems With The Big Bang

*We shall have to evolve
problem-solvers galore
since each problem they solve
creates ten problems more.*

Piet Hein

The aim for any physicist is to understand nature and in the field of cosmology the focus is the evolution of the universe in general. I will in this section present one of the great theories of cosmology of the twentieth century together with some of the problems that it implies.

2.1 Standard Big Bang

When we look around in everyday life some come to think; "Where did it all come from?". This is the foundation for a chain of research fields; ecosystems, anatomy, cell biology, bio chemistry, chemistry, nuclear physics and particle physics. But without a universe to occupy these things they might not have any meaning at all. This is where cosmology comes to aid, with theories of where and when it all began and how it began, the most dominant being the big bang theory.

The universe is growing and has always done so, for as far back as one can observe. At some point there must have been a point in time where the universe was of minimum size. In the big bang theory this point was around 13,7 billion years ago, where the universe was of infinitesimal size. Starting from this point the universe evolved, starting with a "big bang" from which space and time came to exist and later the particle content of the particle physics standard model.

To be able to describe how the universe evolved after a big bang, it is necessary to define a metric for spacetime. To do this we need some assumptions and they are not nearly as complicated as their implications. The first assumption could in principle be made by almost anyone. No matter where we look at the sky we always see the same picture; stars, galaxies, clusters and so forth. All over the universe seems to be exactly the same no matter in which direction we look. This leads to the assumption that the universe seen from Earth is isotropic.

The next assumption is a bit more philosophical, but still reasonable. If large scale structures look isotropic from Earth would they do so at any other place in the universe? It does not seem reasonable to assume that Earth is a special place in the universe², so we might assume that no matter where we are,

²Besides that it is the only place we know sustains life.

the universe must be isotropic. Hence the universe is everywhere isotropic and therefore homogeneous.

One last assumption could be made for simplicity; further assuming that only the space component of the metric depends on time it is possible to derive the Friedmann - Robertson - Walker (FRW) metric³;

$$ds^2 = -dt^2 + a^2(t) \left(\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2(\theta)d\phi^2) \right).$$

As formulated here the metric describes an object in comoving coordinates, meaning that an object that is not moving, besides the expansion, will have constant coordinates, r, θ and ϕ . The constant k is the curvature constant, taking the values $-1, 0$, and 1 for negatively, flat or positively curved spacetime respectively. Left is only the scaling factor, $a(t)$. This is what links comoving coordinates to physical coordinates, given a comoving distance r the physical distance is $R = a(t)r$.

When the metric for spacetime is in place, it is natural to investigate how this is linked to the content of the universe through Einsteins field equation:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi GT_{\mu\nu}.$$

Here $T_{\mu\nu}$ is the energy-momentum tensor, $R_{\mu\nu}$ the Ricci tensor and R is the scalar curvature[5].

Plugging in the FRW metric and the assumption of everywhere isotropy, such that the energy-momentum tensor is diagonal, $T_{\mu\nu} = \text{diag}(\rho, p, p, p)$, one can derive the Friedmann equations, two coupled differential equations;

$$\left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi G\rho}{3} - \frac{k}{a^2}, \quad (2.1)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p). \quad (2.2)$$

Here ρ is the total energy density of the universe and p its pressure. These equations let one calculate how the scaling factor evolves with time depending on the compositions of matter in the universe. The scaling factor is directly linked to the physical size of the universe as will be clear shortly.

2.2 Problems with Big Bang

Even though the big bang theory has explained many things in nature, for instance the early formation of Hydrogen and Helium and the existence of CMB, there are some problems with the theory.

³This was done independently by Friedmann, Lemaître, Robertson and Walker in the 1920s and 1930s

2.2.1 The Horizon Problem

Taking a look at a photon traveling through spacetime. This is done following a line called null geodesic with $ds^2 = 0$. In flat space, and only taking the radial part, this implies that $dr = \frac{dt}{a(t)}$. Now it is rather simple to calculate the comoving distance, d_h a photon can travel from time t_0 to time t_1 [2],

$$d_h = \int_{t_0}^{t_1} dr = \int_{t_0}^{t_1} \frac{dt}{a(t)} = \int_{a(t_0)}^{a(t_1)} \frac{d \ln(a)}{Ha},$$

here $H \equiv \frac{\dot{a}}{a}$ is the Hubble parameter. The physical distance from the beginning of the universe and til today is what is called the particle horizon, it is the distance a photon can travel in this time interval. Remember that the physical distance is $D_h = a(t)d_h$ at time t .

Then length scale $(Ha)^{-1}$ is what is called the comoving Hubble radius⁴. Particles separated by more than the Hubble radius can not communicate now due to their relative speed, where as particles separated by the particle horizon have never been able to communicate.

With the Friedmann equations one can derive the following behavior of $(Ha)^{-1}$ [6],

$$(Ha)^{-1} = H_0^{-1} a^{\frac{1}{2}(1+3\omega)}, \quad (2.3)$$

where ω is a dimensionless number with $p = \omega\rho$. This equations-of-state parameter, ω , incodes the effects of different kinds of substance, for radiation $\omega = \frac{1}{3}$, ordinary matter have $\omega = 0$, and even the cosmological constant have a equation-of-state parameter, $\omega = -1$.

This implies that the comoving particle horizon scales as:

$$d_h \propto \begin{cases} a & \text{for radiation dominated} \\ a^{\frac{1}{2}} & \text{for matter dominated.} \end{cases}$$

The important aspect here is that the comoving particle horizon is monotonically increasing with time. As the Hubble sphere expands, more of the universe becomes visible. Hence with only matter and radiation dominated epochs, events entering the horizon today was not connected when CMB photons decoupled.

But the universe must have been very homogeneous at decoupling to explain the homogeneity seen in the CMB spectra. So if different parts of the universe were not causally connected then, how can it be that they share such similarities as observed?

A rough estimate tells that there were around 10^6 disconnected regions of the universe inside what corresponds to today's particle horizon at recombination [5]. In Figure 2 are shown the light cone from today and from two distinct measurements of CMB. The two measurements can be related to two different points in space at the time of recombination. In the figure it is seen that with the standard big bang, the two CMB events have never been in causal contact, hence it is strange that they seem that much alike. This is called The Horizon Problem

⁴From Hubble's law it is the distance at which the universe expands with the speed of light

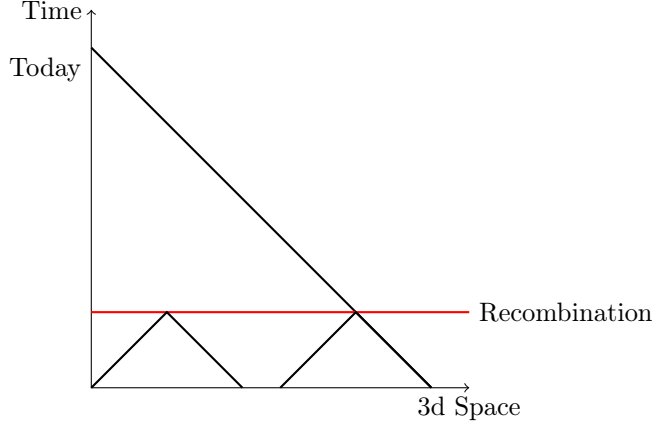


Figure 2: Figure of the light cones from today to two distinct events at recombination. We see that the part of the universe at Big Bang ($t = 0$) that could affect each event are non-overlapping.

2.2.2 The Flatness Problem

CMB measurements have given incredible information about the universe, as both the homogeneity and the curvature of spacetime can be derived from CMB observations. The current observations points to a nearly flat universe [7].

Knowing the curvature of the universe today enable us to estimate the curvature as it would have been in the past in order to explain today's curvature. One can rewrite equation (2.1) to;

$$\Omega_k(a) \equiv 1 - \Omega(a) = \frac{-k}{(aH)^2}, \quad (2.4)$$

where $\Omega = \frac{\rho}{\rho_{crit}}$ with $\rho_{crit} = 3H^2$ and Ω_k is the density parameter for curvature, measuring the effect of curvature. If we again look at how this quantity change with time, I find;

$$\Omega_k(a) \propto \begin{cases} a^2 & \text{for radiation dominated} \\ a & \text{for matter dominated.} \end{cases}$$

Again I find that it is monotonically decreasing when I go backwards in time. So with the observed curvature of today estimates of the curvature at key moments can be made [2]. Below are estimates for the time at the Big Bang nucleosynthesis (BBN), the GUT era and the Planck scale;

$$\begin{aligned} \Omega_k(t_{BBN}) &\leq \mathcal{O}(10^{-16}) \\ \Omega_k(t_{GUT}) &\leq \mathcal{O}(10^{-55}) \\ \Omega_k(t_{Planck}) &\leq \mathcal{O}(10^{-61}). \end{aligned}$$

This is a case of extreme fine-tuning, getting the curvature correct within 61 digits seems very unlikely, this is The Flatness Problem. It would be better if this near flatness occurred naturally from some mechanism.

2.2.3 Further Problems

Both the Horizon and the Flatness Problem can be solved by fine-tuning the initial conditions; making the curvature as small as needed, and assuming that the universe was as homogeneous as needed for CMB to decouple to the spectra seen today.

I now take a closer look at the inhomogeneities of CMB which is of the order 10^{-5} compared to the mean [6]. Any inhomogeneity in the matter density at recombination is gravitational unstable, meaning that a small fluctuation in density will create areas of stronger gravitational pull on other matter and therefore increase the fluctuation.

So the inhomogeneities seen today are larger than the ones observed in the CMB, and extrapolating even further back, matter densities must have been extremely homogeneous, but how can this be if they were not causally connected? This can also be solved by initial conditions, however the precision needed seems unreasonable. This problem is not the same as the Horizon Problem. The Horizon problem was about how the universe could be homogeneous, whereas this problem is about the size of the inhomogeneities.

Related to the same gravitational instability is the large-scale structure formation problem. As described any region with a higher matter density will tend to grow even higher, this is the basic mechanism to formation of any non-uniformly distributed matter in the universe. Stars are initially formed from gravitational instability, and so are galaxies and galaxy clusters. But as with all of them there is the need for some initial region with higher density. As I will show inflation can give rise to this initial seed.

A problem of a more theoretical kind comes from Grand Unification Theories. Assuming a Grand Unification Theory is found, a symmetry breaking of this, when lowering the energy of the universe, can produce magnetic monopoles as described by Ryden in [6]. Taking the estimate from Ryden for granted there would be approximately 10^{82} magnetic monopoles per cubic meter at the time of GUT. Within seconds⁵ magnetic monopoles would dominate the universe. But how is it that these are not observed today then? This is called the Monopole Problem and it could in principle be applied to other exotic particles than just magnetic monopoles.

⁵ Actually within $10^{-16}s$.

3 Inflation as a Solution

*Do not give up! It is not over.
The universe is balanced.
Every set-back bears with it the seeds of a come-back.*
Steve Maraboli

When examining both the Horizon and the Flatness Problem from section 2 I see a similarity; both of them only become a problem if the comoving Hubble distance, $(Ha)^{-1}$, is strictly increasing. This motivates a period with a new mechanism called inflation, which solves the problems by changing that exact similarity.

3.1 Basic Idea of Inflation

To solve the Flatness and Horizon Problem, I need nothing else than assume a period with decreasing Hubble radius;

$$\frac{d}{dt} \frac{1}{Ha} < 0.$$

This requirement can be restated in many equivalent ways each with its own interpretation. The following requirements for inflation are all equivalent, I will shortly show why;

$$\frac{d}{dt} \frac{1}{Ha} < 0, \tag{3.1}$$

$$\rho + 3p < 0, \tag{3.2}$$

$$\ddot{a} > 0, \tag{3.3}$$

$$\omega < -\frac{1}{3}. \tag{3.4}$$

The first equation states that the comoving Hubble sphere must decrease, (3.3) states that the size of the universe must accelerate, and (3.2) provides a constraint on the kind of substance that can drive inflation. For instance it can not be ordinary matter since its pressure is positive and (3.2), for positive density, states that the substance driving inflation have negative pressure. Equation (3.4) is merely a rewriting of this.

I defined inflation as a period of decreasing Hubble radius and this is the starting point for the equivalence of the above equations. That the expansion of the universe must accelerate is easily seen from the decreasing Hubble radius;

$$0 > \frac{d}{dt} \frac{1}{Ha} = \frac{d}{dt} \frac{1}{\dot{a}} = -\frac{\ddot{a}}{\dot{a}^2},$$

and hence $\ddot{a} > 0$. Recalling equation (2.2),

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p),$$

it is easy to see that $\rho + 3p < 0$ in an accelerated expansion. Further, with $p = \omega\rho$, it is clear that $\omega < -\frac{1}{3}$ is the exact same statement.

A simple example of inflation, which can be used as a template, is made by taking $\omega = -1$. In this example equation (2.3) gives that H^{-1} is constant, and hence H is constant too. Since $H = \frac{\dot{a}}{a}$ the scaling factor must evolve as $a \propto e^{Ht}$. This is why inflation some times is referred to as a period of exponential expansion. Since $\omega < -\frac{1}{3}$ in all inflationary models, it is safe to assume that in any model the evolution is close to this, and the Hubble parameter is nearly constant.

In figure 3, I have tried to illustrate, in a simple manner, how inflation can be understood. During inflation the comoving horizon decreases, so that large comoving scales no longer fit inside. What this means is that some fluctuation with large wavelength, for instance large scale density fluctuation, will no longer be seen as a wave, but only as a density increase, since only part of the wave is within the comoving horizon. As inflation continues, small and smaller scales exits the horizon. When inflation has done its part the hot big bang scenario takes over, expanding the comoving horizon as in the standard big bang. This will make larger and larger scales re-enter the horizon, so that they again become relevant for physics inside the horizon⁶. An illustration including fluctuations with different wavelength can be found in figure 5, which might make my point more clear.

3.1.1 Solutions

With this new epoch of an expanding universe it is fairly simple to solve the problem from section 2.2.

Monopoles Assuming that magnetic monopoles were created before inflation, an estimate of the number density could be of the order $10^{82}m^{-3}$. If inflation made the Hubble radius e^{100} times larger, the number density would have diluted to around $10^{-49}m^{-3}$ or one monopole for every 15 cubic parsec. Then it is no wonder that magnetic monopoles never have been observed[6].

Horizon The Horizon Problem was a problem of disconnected parts of cosmos. With inflation all of the universe can be connected before inflation and via inflation still have the size it has today. This implies that information entering the horizon now have been disconnected from this part since inflation, however it was connected before inflation making the universe homogeneous. From figure 4 it is seen how inflation effectively extends the

⁶Remember that only events inside the horizon can even hope to interact due to the speed of expansion.

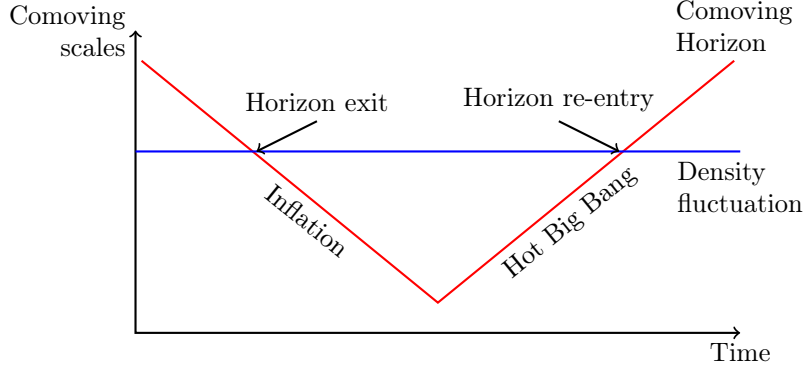


Figure 3: Illustration of the comoving horizon during inflation and after. The horizontal line (blue) is a density fluctuation at some constant scale, as the comoving horizon decreases during inflation, the fluctuation will at some point exit the horizon. It will stay outside the horizon until the hot big bang have expanded the universe so that the fluctuation again can fit inside the comoving horizon [2].

conformal time axis backwards in time, so that inflation starts at $\tau_i = -\infty$ where $a = 0$ [2], this makes the light cones of the CMB events connected with almost every point before inflation.

Flatness Equation (2.4) and a decreasing Hubble radius implies that inflation drives space time towards a more flat universe. So before inflation the universe might or might not be flat but inflation will make it flat and Ω_k can increase until today from this flat universe.

Another aspect of inflation worth noticing is the formation of large structures in the universe [5]. From astro physics one learn that stars form from Jeans instable dust clouds, but not only stars form in this way. Take a static universe and let there be some density fluctuation of matter at a location x . The slightly stronger gravitational potential at x will tend to pull more matter towards it, further increasing its mass and hence potential strength, this is what is called gravitational instability. Now let the universe expand, gravitational instability is still valid, but now the expansion counteracts a free collapse. As with Jeans instability the density fluctuation at x can form larger matter formations depending on its initial size. If it is too small, the expansion will prevent it from growing, but if it is large enough it can form large structures, like galaxies, clusters or even larger structures. The expansion of the universe will for structure formation act like gas pressure does it for star formation.

But where do these initial seeds come from? As with the star formation the Jeans mass is very high if no perturbations are present inside the Jeans radius. So seeds are needed for a reasonable mass range, and the same holds for large structure formation. Inflation can provide such seeds.

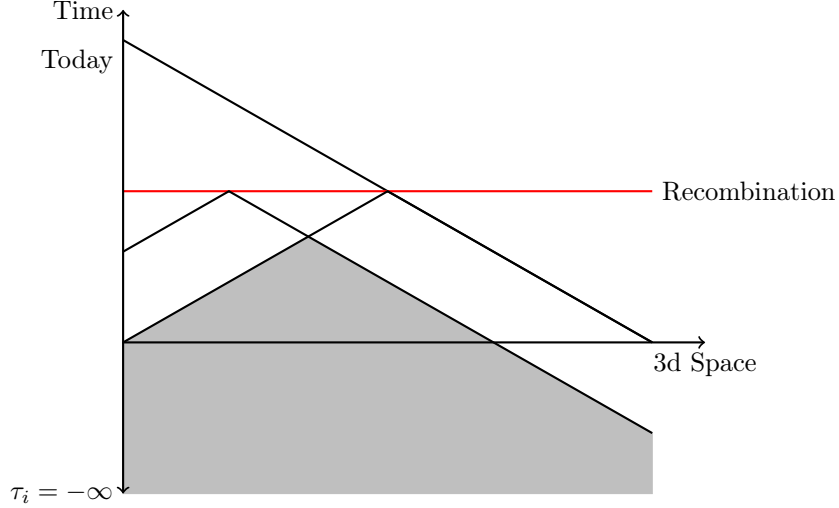


Figure 4: Sketch of the inflationary solution to The Horizon Problem. Inflation extends the conformal time backwards until $\tau_i = -\infty$, this makes the two CMB events connected on the grey patch. This patch extends downwards to infinity.

In a following section (section 3.2) I will talk about the inflaton, the field possibly responsible for inflation. The inflaton is described by a quantum field theory in which empty space is not that empty. Quantum fluctuations are taking place all the time. Everywhere, particles are popping in and out of vacuum. This gives rise to a momentarily density fluctuation of particles at that point. But the instance the particles are created it vanishes again, so classically there is no over density, since over a short time interval there is a total of zero particles. However during inflation these fluctuations becomes important. The rapid expansion of inflation stretches the field fluctuations, giving them an exponentially growing wave length. Since matter perturbations are linked to metric perturbations, or curvature perturbations, the fluctuations in the field implies perturbations in the curvature. Later in section 7.2, I find that curvature perturbations are frozen outside the Hubble sphere. So as the wave length of the field fluctuations are stretched, it will at some point exit the horizon, and therefore curvature perturbations will be frozen until re-entering the horizon. Since quantum fluctuations creates all wavelengths, curvature perturbations are present at all scales [5]. In figure 5 I have illustrated how different fluctuation scales exits the horizon during inflation, and, then later on re-enters, affecting the physics within the horizon. This includes both large scale structure and the cosmic microwave background. Density fluctuations within the horizon can affect physics and therefore provide seeds for large scale structure formation.

Once curvature perturbations are frozen in outside the horizon the classical time mean is no longer zero, hence inflation have made quantum fluctuations macroscopic classical perturbations. This has implications. Once the perturbations re-enter the horizon, it will start to affect macroscopic matter through gravity and the Jeans instability mentioned above. So the unavoidable quantum fluctuations of the inflaton field can, during the history of the universe, provide seeds for large structure formation, so that the universe has the structure observed today.

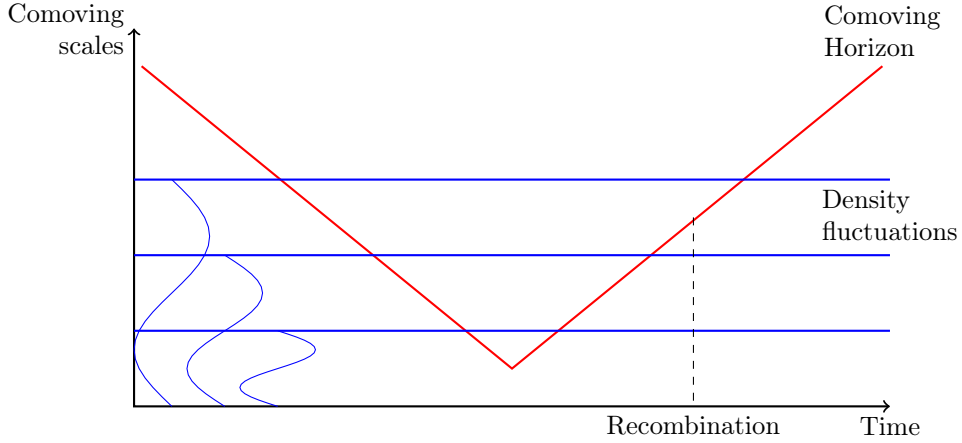


Figure 5: Sketch of fluctuations as they exit the horizon and re-enters later on. At the time of recombination only a part of the fluctuation spectrum have re-entered the horizon. Only fluctuation modes that are inside the horizon can affect the process of last scattering. The 3 modes drawn are by far not the only ones. Fluctuations are created on all scales, and hence form a continuous spectrum of wavelengths. This briefly explains the so called Sachs-Wolfe effect, that the temperature fluctuations of CMB are almost constant with scale. This means that no matter at what scale measurements are taken, the fluctuations will be of the same order *e.i.* there was a density fluctuation with that wavelength. However at small scales peaks in the CMB spectrum will occur due to micro effects.

3.1.2 History of Inflation

The inflationary paradigm have been around for some time now, and with more and more observations aimed on inflation, more and more become convinced that inflation might be the way the universe evolved although there is still a lot to learn. Inflation theory have, as many other theories, a long period of development before it became widely accepted. Credit for the original inflation

theory is assigned to Alan Guth, who in 1980 published his article "*Inflationary Universe: A Possible Solution to the Horizon and Flatness Problems*" [8] in which a new epoch, called inflation, in the evolution of the universe was introduced. Guth knew that the standard model of particle physics was not valid to arbitrary scales, so instead of beginning the universe with a standard big bang he postulated inflation, and that from the end of inflation could start working on particle physics, hence the standard model only have to work up to temperatures around the end of inflation.

However an inflating universe was already proposed by de Sitter back in 1917, this was however not driven by a particle like Guth's theory, but is was an empty space only featuring a cosmological constant. It was introduced to show that Einsteins general theory of relativity did not necessarily lead to a static universe as Einstein wanted.

An inflation theory like Guth's was present in Soviet in 1979, developed by Alexei Starobinsky. In short he showed that an inflating universe could evolve from quantum corrections and that this era could solve some of the problems in cosmology, and that it furthermore made some changes to the CMB. Starobinsky's theory was only published in Soviet, due to the way history unfolded at that time, and there is no indications that Guth knew of Starobinsky's work.

One year after Guth's article Andrei Linde answered with "*A new Inflationary Universe Scenario: A Possible Solution of the Horizon, Flatness, Homogeneity, Isotropy and Primordial Monopole Problems*" [9]. Here Linde solved some of the problems that originated from Guth's theory. One of the main differences is that Guth assumed the universe to be divided into bubbles, and that these bubbles at the end of inflation are filled by inflatons. Linde solved this problem by letting the inflaton decay into standard model particles when it at its final stage oscillates in the minimum of its potential, this last process is called reheating.

Inflation is generally modelled by a particle called the inflaton, which traditional is a fundamental scalar particle. In Linde's article the inflaton is constantly refereed to as the Higgs particle. That the two particles are so much alike motivates several things, trying to use solutions to problems involving the Higgs particle to solve similar problems in inflation, as well as trying to use the standard model Higgs particle as the inflaton [10]. I will in this project be working on the first part, namely trying to resolve the problem of a fundamental scalar particle by means of models designed for the same problem with the Higgs particle.

3.2 The Inflaton

I now turn focus at a more technical aspect of inflation; the inflaton, the particle that supposedly should drive inflation. In high energy physics the idea is always to explain new phenomena with some elementary particle, as was it with electricity. Light and all other matter can be explained by a "few" elementary particles. This procedure is right now applied to dark matter, where scientists are searching for one, or maybe several, new particles that could explain dark

matter. Here I will introduce some of the basic ideas of how the inflaton could drive inflation. Different kinds of inflation stem from different kinds of inflatons and some basic definitions of models will be introduced.

3.2.1 The Single Scalar Inflaton

The simplest framework for inflation is single scalar field inflation, in which the inflaton is a scalar particle with self interactions via its potential. I will take this as my starting point and then apply changes for this to explain other models.

If inflation should couple to gravity, a general action for the inflaton could be:

$$S = S_{EH} + S_\phi,$$

here S_{EH} is the Einstein Hilbert action⁷ and S_ϕ is the action for the inflaton. In the Einstein frame, which is the frame where only a minimal coupling to gravity is present, a general action will be

$$S = \int d^4x \sqrt{-g} \left[\frac{M_P^2}{2} R + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right]. \quad (3.5)$$

Since I am working in curved space time I need a metric, $g_{\mu\nu}$, and g is its determinant, R is the scalar curvature, ϕ the inflaton field and $V(\phi)$ its potential.

First I examine if this really can drive inflation by checking if one of the requirements for inflation is met. The easiest one to check is (3.4). To do this I need both pressure and density of the inflaton expressed in terms of ϕ , this can be done through the energy-momentum tensor, $T_{\mu\nu}$. By definition I have

$$T_{\mu\nu} = - \frac{2}{\sqrt{-g}} \frac{\delta S_\phi}{\delta g^{\mu\nu}},$$

and by calculation the variation of $\sqrt{-g}$ one can get:

$$T_{\mu\nu} = 2 \frac{\delta \mathcal{L}_\phi}{\delta g^{\mu\nu}} - g_{\mu\nu} \mathcal{L}_\phi. \quad (3.6)$$

Assuming the FRW metric again equation (3.6) simplifies to [2]:

$$\begin{aligned} p_\phi &= \frac{1}{2} \dot{\phi}^2 - V(\phi) \\ \rho_\phi &= \frac{1}{2} \dot{\phi}^2 + V(\phi). \end{aligned} \quad (3.7)$$

These calculations are not done here. If the potential dominates the inflaton Lagrangian it is possible to have negative pressure and it is even possible to have $\omega = \frac{p}{\rho} < -\frac{1}{3}$. With that said, a scalar field, ϕ , can explain inflation from a particle physics point of view.

⁷The action proposed by Hilbert from which Einsteins field equation can be derived.

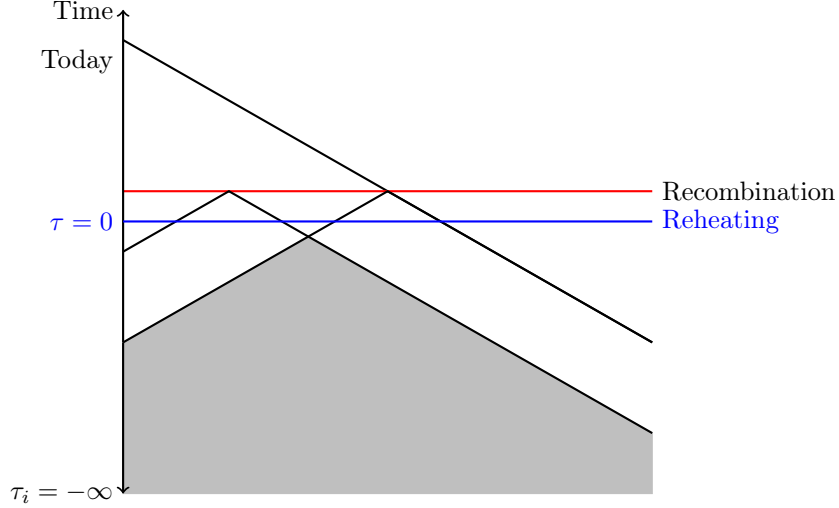


Figure 6: The light cones from the two CMB events are connected due to inflation, but at the time of reheating they are not. Therefore reheating can be seen as resembling the standard big bang, just without the singularity.

3.2.2 Inflaton and the Universe

I will here summarize how inflation works.

In conformal time the universe began at $\tau = -\infty$. The universe was at first hot and dense, but after around 10^{-33} seconds inflation starts, expanding the universe by at least a factor of e^{60} . The expansion is adiabatic, so the temperature drops dramatically during inflation, in some models with a factor one million due to the exponential expansion. The particle responsible for the expansion, the inflaton, loses a lot of its energy to this expansion via friction (this will be shown shortly), and as the inflaton loses energy inflation comes to a hold. The inflaton is now at its potential minimum, oscillating. When it does so this the inflaton will decay into other particles and at the end into standard model particles, this period of decay is called reheating. Due to all the new particles the temperature of the universe rises again, to the value before inflation, and the hot big bang can commence in its normal way, with matter and radiation dominated eras.

A sketch including reheating can be found in figure 6, showing how reheating can be seen as the analog of the standard big bang. Figure 6 can be compared with figure 2 and 4. In Figure 7 is a sketch of the inflaton field as it rolls down its potential. Around ϕ_{CMB} the fluctuations in the inflaton field, needed to explain CMB, are created. Here after the inflaton rolls down, and ends up in the minimum, producing reheating.

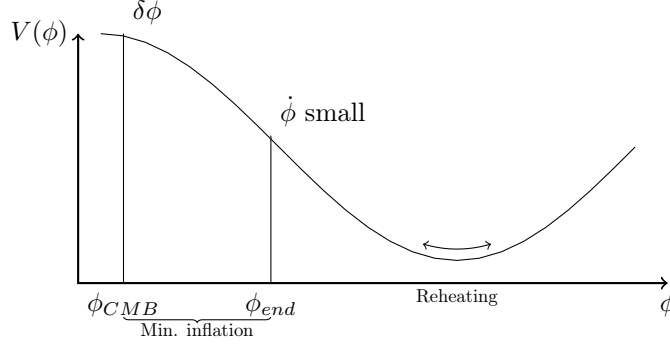


Figure 7: Starting from left, the fluctuations of the inflaton field, that later will be temperature fluctuations in CMB, are created. After this the inflaton slowly rolls down the potential until inflation ends. Here after reheating takes place at the minimum of the potential, where the inflaton decays to other particles.

3.2.3 Slow-Roll Inflation

With a possible particle to produce inflation, inflation models can be categorized into different kinds, slow-roll inflation is one of special interest due to its popularity. To classify slow-roll inflation it is more convenient to use the so-called slow-roll parameters, ϵ and η . To define these I first derive the equation of motion from equation (3.5) assuming the FRW metric,

$$0 = \partial^\mu \partial_\mu \phi + 3 \frac{a^2 \dot{a}}{a^3} \partial^0 \phi + \frac{\partial}{\partial \phi} V(\phi).$$

This is the equation of motion for the scalar field, written in a more familiar way I have,

$$\ddot{\phi} + 3H\dot{\phi} - \frac{\nabla^2 \phi}{a^2} + V_{,\phi} = 0 \quad (3.8)$$

Here it is seen that the inflaton is exposed to "friction" from the term with $\dot{\phi}$, it states that the field experiences friction due to the expansion of the universe. To estimate how large this friction is I can combine equation (2.2) and (3.7) and get,

$$\dot{H} + H^2 = \frac{-8\pi G}{3} (\dot{\phi}^2 - V(\phi)). \quad (3.9)$$

Assuming that the Hubble parameter is almost constant during inflation, for a domination potential, the Hubble friction term in (3.8) is significant, and therefore $\ddot{\phi}$ in (3.8) can be neglected. So if the inflaton field varies slowly from start, it will slowly roll down its potential, hence the name slow-roll inflation. To parameterize inflation in this formalism I will use the slow-roll parameters,

ε and η . Since \dot{H} carries information about inflation, it is natural to construct a parameter from this. Equation (3.9) can be written as,

$$\frac{\ddot{a}}{a} = \frac{8\pi G V(\phi)}{3} (1 - \varepsilon),$$

where $\varepsilon = \frac{\dot{\phi}^2}{V}$ is the first slow-roll parameter. This can be rewritten into $\varepsilon = \frac{\dot{H}}{H^2}$. Knowing that inflation is a period of accelerated expansion is equivalent to say that:

$$\boxed{\varepsilon < 1 \Rightarrow \text{Inflation}}$$

This statement will be of significant importance to us later on, since this is the easiest way to check when inflation ends. The first slow-roll parameter states that the potential should be dominant and that the acceleration of the universes expansion is positive.

Neglecting $\ddot{\phi}$ is just an assumption that this should be small compared to the other terms in (3.8). This can also be rewritten into a requirement for inflation. Since I would like $\ddot{\phi} \ll H\dot{\phi}$, I might as well require that

$$\eta = \frac{\ddot{\phi}}{H\dot{\phi}} \ll 1.$$

This is the second slow-roll parameter, which states that it is a system of high friction. With ε and η I can decide whether or not the inflaton field is in a state to produce inflation. It is however more convenient to use the definition of the slow-roll parameters in terms of the inflaton potential:

$$\begin{aligned} \varepsilon_V &= \frac{M_P^2}{2} \left(\frac{V_{,\phi}}{V} \right)^2 \\ \eta_V &= M_P^2 \frac{V_{,\phi\phi}}{V}. \end{aligned} \tag{3.10}$$

From now on I will drop the subscript V for the potential slow-roll parameters, so with ε and η I mean the ones from (3.10). There is still the same requirements on these parameters to obtain inflation.

I have shown that during inflation the scaling factor roughly goes as an exponential, and more precise it evolves as $\exp(Ht)$. From this fact I am able to define another quantity, \mathcal{N} , describing the number of e-foldings the scaling factor experiences during inflation. This means that with \mathcal{N} e-foldings, the scaling factor becomes $e^{\mathcal{N}}$ times larger. This simply defines \mathcal{N} as

$$\mathcal{N} \equiv \ln \left(\frac{a_{end}}{a_{ini}} \right),$$

where I have the scaling factor when inflation starts and when it ends. Taking the exponential growth of the scaling factor for granted, the number of e-foldings can be written as[2]

$$\mathcal{N} = \int_{t_{ini}}^{t_{end}} H dt = \frac{1}{M_P^2} \int_{\phi_{end}}^{\phi_{ini}} \frac{V}{V_{,\phi}} d\phi, \tag{3.11}$$

where M_P has been introduced so that \mathcal{N} is dimensionless.

In short inflation is an era where the potential is relatively flat, and the field can roll down slowly, increasing ε from some starting value less than one to the point where inflation ends at $\varepsilon = 1$. When inflation runs successfully I can check the duration by analyzing the size of \mathcal{N} , to ensure that the universe is inflated enough to explain today's observations.

3.2.4 Higgs Inflation

One model that looks a lot like the model I will be working on later in this project is Higgs Inflation in its form by F. Bezrukov and M. Shaposhnikov in [10]. With the motivation that both the Higgs particle and the inflaton initial were to be a fundamental scalar why not let the Higgs boson work as the inflaton? The work led to the conclusion that with only a minimal coupling to gravity the Higgs bosons self coupling would create too strong fluctuations compared to the ones observed. This can be solved with a non-minimal coupling to gravity, in which case, with the standard model Lagrangian \mathcal{L}_{SM} , the Lagrangian for inflation is

$$\mathcal{L} = \mathcal{L}_{SM} - \frac{M^2}{2}R - \xi H H^\dagger R, \quad (3.12)$$

H is the Higgs field, R the scalar curvature, ξ the new coupling constant and $M \leq M_P$. With a new non-minimal coupling calculations become harder, but a conformal transformation of the field can remove the coupling resulting in a non-minimal kinetic term. Redefining the field will also remove this, making calculations fairly simple afterwards. The framework for inflation in the Higgs inflation scenario will be useful later on, since the non-minimal coupling will do the same for composite inflation as it does for Higgs inflation.

3.2.5 Other Models

I'll briefly go through some alternative ideas to scalar inflation, all based on a TASI lecture series [2, p. 36].

Small Field With the constraint on ε one can calculate some "distance" the field ϕ has evolved, $\Delta\phi$. If this change is less than M_p the model is called a small field model. One great aspect of small field models is that the gravitational waves produced by inflation are so small that they can not be observed today. This is good since, to date, there have not directly been observed any gravitational waves. One way to obtain a small field model is from some higher scale gauge theory. When this theory spontaneously breaks it can produce the conditions for small field inflation. A historical example is the Coleman-Wienberg potential:

$$V(\phi) = V_0 \left(\left(\frac{\phi}{M} \right)^4 \left(\ln \left(\frac{\phi}{M} \right) - \frac{1}{4} \right) + \frac{1}{4} \right). \quad (3.13)$$

I will later use a potential very similar to this when working on Glueball Inflation in section 5.

Large Field It could be that the model for inflation changed more than required for small field models, in this case the model is called a large field model. In general it produces large gravitational waves, in which case I would expect them to observe these in the near future by Planck. An example of a potential that produces large field inflation is from a group of models called chaotic inflation models, here the potential takes the form $V(\phi) = \lambda_p \phi^p$.

Non-Minimal Coupling In most theories it is assumed that the inflaton couples minimally to the graviton⁸. There is however no reason why it could not couple strongly to the graviton. This can in some cases even be an advantage as will be seen later on in section 4. In most of the theories with a non-minimal coupling, a transformation of the Lagrangian can remove the strongly coupled term thereby making calculations easier. A general setup for this statement in a single field theory can be seen in section 4.2.2.

Non-Canonical Kinetic Terms Until now all that can change a inflation model is the inflaton potential, this is however not the case in general. In quantum field theory one of the first things to learn is the kinetic term for a scalar field, $\frac{1}{2} \partial_\mu \phi \partial^\mu \phi$. It is possible to have a kinetic term that is not on this form, a so called non-canonic kinetic term. Let a Lagrangian be written in the form $\mathcal{L} = X - V$ where $X = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi$ is the kinetic part and V is the potential. A non-canonical theory then in general have a Lagrangian of the form, $\mathcal{L} = F(\phi, X) - V$, where F is some function. In most cases this new type of kinetic terms can be removed by a redefinition, this will be the case in section 4. Inflation can be obtained from these non-canonical kinetic terms in models called k -inflation [11], where it is not the potential but the kinetic terms of the inflaton that produces inflation.

Different Inflavons As a last example there is the possibility to have multiple inflaton fields, i.e. more than one particle driving inflation. This kind of theory is generally more complicated, since one has to work with more than one field, and there is possibilities for interactions between all new particles [12, 13]. It is however very natural to consider more than one inflaton; to describe an atom certainly more than one fundamental particle are used, and even more than one composite particle.

⁸The hypothetical particle that should propagate gravity.

4 A Composite Inflaton

*Some say that the universe is made so that when we
are about to understand it, it changes
into something even more incomprehensible.
And then there are those who say that this has already happened.*

Douglas Adams

In this section I will explore the proposal by P. Channuie, J. Jørgensen and F. Sannino, that the inflaton could be a composite state of some higher scale gauge theory[14]. In the previous section I showed that a new fundamental scalar field could drive inflation, making the universe expand as wanted. But to this day only one possible fundamental scalar particles has been found in nature; the 125 GeV Higgs. For any other scalar particle, it has decomposed into fermions at higher energies. That there have not been found any fundamental scalars is not just a bad thing, maybe the initial idea to explain inflation and the Higgs mechanism via a fundamental scalar field was not correct, but without fundamental scalar fields renormalization does not require arbitrary fine tuning of for instance mass operators [1]. This is a good thing for inflation since in the first place inflation was needed for the exact reason that extreme fine tuning was unnatural.

With theories like Technicolor no fundamental scalars are assumed, but instead uses a fermionic condensate from a higher scale gauge sector to play the role as scalar field. This is the motivation for the following section. The main idea is to form a condensate from a higher gauge sector and let this composite state be the inflaton. What I then would like to see, is if inflation still works successfully.

4.1 Minimal Walking Technicolor

If inflation is to be understood, I need to go beyond the standard model. For this part Minimal Walking Technicolor (MWT) will be the stage at which the play will unfold. In short MWT extends the standard model by a new gauge sector at some higher energy scale. Where the standard model have the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$, MWT adds a $SU(2)$ gauge group and two new fermions transforming with respect to the 2 index symmetric representation of $SU(2)$. This opens for new ways to resolve both the Higgs problem and inflation through the new gauge sector. As a working full theory details on MWT can be found in [15], but I will now make a short recap of the things needed for inflation.

Minimal Walking Technicolor was originally designed to solve a problem coming from a fundamental Higgs boson, namely the divergence of the mass operator for the Higgs field. The work in [15] is aimed at solving this problem.

At first glance the problems for the scalar Higgs particle and for the scalar inflaton are the same, which justifies the direct use of MWT for the inflation sector instead of the Higgs sector. For now the goal is to form a low energy effective Lagrangian, \mathcal{L}_{MWT} , at the inflation scale. The "smallest" Technicolor model passing precision tests is the MWT theory with 2 colors and 2 massless fermions in the 2 index symmetric representation of the color gauge group, $SU(2)$. This theory has a global $SU(4)$ symmetry that later will be broken to $SO(4)$ dynamically. The two new fermion fields will be denoted U and D for techni-up and techni-down quark, and the condensate that breaks the global symmetry is of the form $\langle \bar{U}U + \bar{D}D \rangle$, this breaks 9 of the 15 generators of $SU(4)$ leaving a $SO(4)$ global symmetry.

With the knowledge of this breaking a more elegant way of describing the fields is in the form of a particle matrix, M . Since 9 generators will be broken, these can be chosen a priori, call them X^a with $a = 1, \dots, 9$. For each of these there will be a Goldstone pseudo scalar and a scalar partner, Π^a and $\tilde{\Pi}^a$. The condensate will be modelled by a scalar field, σ , and its pseudo scalar partner, Θ . The field σ will in turn be associated with the composite inflaton. Now the particle matrix takes the form

$$M = \left[\frac{\sigma + i\Theta}{2} + \sqrt{2} \left(i\Pi^a + \tilde{\Pi}^a \right) X^a \right] E$$

where there is summation over a and E is given by

$$E = \begin{pmatrix} 0 & \mathbb{1}_{2 \times 2} \\ \mathbb{1}_{2 \times 2} & 0 \end{pmatrix}.$$

With M an effective Lagrangian can be formed for MWT [15];

$$\mathcal{L}_{MWT} = \frac{1}{2} \text{Tr} (D_\mu M D^\mu M^\dagger) - \mathcal{V}(M) + \mathcal{L}_{ETC}. \quad (4.1)$$

here D_μ is the covariant derivative build from the gauge group generators at the effective scale⁹. In equation (4.1) there is a term \mathcal{L}_{ETC} , this is the Lagrangian of an extended Technicolor model. To make MWT work the way it does, model builders need to assume a new gauge sector above MWT, but details of this theory is not necessary for this. In fact the extended Technicolor theory for the Minimal Composite Inflation decouples from the theory of interest [14]. The $SU(4)$ invariant potential \mathcal{V} is given by

$$\begin{aligned} \mathcal{V}(M) = & \frac{-m^2}{2} \text{Tr} (MM^\dagger) + \frac{\lambda}{4} \text{Tr} (MM^\dagger)^2 + \\ & \lambda' \text{Tr} (MM^\dagger MM^\dagger) - 2\lambda'' (\det(M) + \det(M^\dagger)). \end{aligned}$$

Since this potential has a non-trivial minimum, M obtains a vacuum expectation value, VEV. Since the only concern is about σ all other fields in M are set to zero to simplify calculations.

⁹I will not need the exact form of the covariant derivative, but it can be found in [15].

The potential in σ , $V(\sigma)$, is rather easy to calculate, since $E = E^\dagger$ and $EE = \mathbb{1}_{4 \times 4}$,

$$V(\sigma) = \frac{-m^2}{2}\sigma^2 + \frac{\lambda}{4}\sigma^4 + \frac{\lambda'}{4}\sigma^4 - \frac{\lambda''}{4}\sigma^4. \quad (4.2)$$

The minimum of this potential corresponds to a VEV in σ of the size

$$\langle \sigma^2 \rangle = v^2 = \frac{m^2}{\lambda + \lambda' - \lambda''}$$

As one normally would do, the potential is investigated around its minimum. Here σ can be written as $\sigma = v + \phi$, where ϕ is some displacement from the minimum. With insertion of this in (4.2) I obtain:

$$\begin{aligned} V(\phi) &= \left(\frac{3}{2} - \frac{1}{2}\right) m^2 \phi^2 + \frac{\lambda + \lambda' - \lambda''}{4} \phi^4 \\ &\quad - \frac{1}{2} m^2 v^2 + \frac{\lambda + \lambda' - \lambda''}{4} v^4 \\ &\quad + (-m^2 + v m^2) \phi + (\lambda + \lambda' - \lambda'') v \phi^3 \\ &= \frac{1}{2} (2m^2) \phi^2 + \frac{\lambda + \lambda' - \lambda''}{4} \phi^4 + A(\phi) + V_0 \end{aligned} \quad (4.3)$$

here V_0 is the minimum of the potential and $A(\phi)$ is the terms with odd powers of ϕ .

As seen in equation (4.3) the field ϕ have a mass of $\sqrt{2}m$ and a self interaction through a ϕ^4 coupling of size $\frac{1}{4}(\lambda + \lambda' - \lambda'') \equiv \frac{1}{4}\kappa$. The original potential, (4.2), can be rescaled so that $A(\phi)$ is removed from (4.3), this is done by letting $\sigma \rightarrow \sigma - v$ in (4.2). The field, ϕ is what I will use as the inflaton from MWT, everything will however be formulated in terms of σ , which is equivalent by the argument above. That ϕ is a composite state is not immediate since it is formulated in an effective theory, but by comparing the fundamental and the effective theory, it is possible to derive that a condensation of fundamental techni-fermions can be modeled by the σ -field [15]. Moreover as a side comment it should not be too far fetched to say that in the fundamental MWT it was a composite state that broke the global symmetry, where as in the effective theory it is the scalar field σ , hence σ is an effective description of a condensate.

4.2 Minimal Composite Inflation

With the work by Bezrukov and Shaposhnikov[10], I would now like to explore how the composite scalar field from MWT could drive inflation. The difference from the former work is that, motivated by lack of fundamental scalars, we need to understand how composite states can drive inflation. MWT will here not only work as a underlying model but as an example of the general idea of a composite inflaton. Compared to [10], I will be working with the effective Lagrangian in which the scalar field originates from a condensate rather than being fundamental. Besides this the two inflation models will look very similar.

4.2.1 The Minimal Composite Inflation Action

First thing to do is to construct an action for the new inflation theory, Minimal Composite Inflation[14], MCI. There is now choice to make, will this new inflaton couple minimally to gravity or not? The minimally coupled theory will probably be more easily calculable but the non-minimal coupling will have one extra parameter from which inflation could work in this setup. Inspired by Higgs Inflation [10] a non-minimal coupling will be the way to go. Therefore the effective action for MCI is a sum of the Einstein - Hilbert action, the non-minimal coupling to gravity and the effective action for the new MWT model.

It should be noted here, that there is a hierarchy of theories. Starting at the lowest energy there is what looks like the normal standard model. This could be fundamental or it could be an effective description of a higher order theory with additional degrees of freedom. This theory, at some higher energy around the TeV scale, could be Minimal Walking Technicolor. When breaking the electroweak symmetry, the standard way is to use a fundamental scalar. However MWT solves the same problem with a condensate. From my point of view this makes it more natural by lack of fundamental scalars.

As already mentioned, MWT need an extension theory, ETC, at some even higher energy scale to work properly. At the inflationary scale, $\lesssim 10^{15} GeV$, there is a new theory; MCI. This again needs an extension theory, ECI, Extended Composite Inflation to work. This originates from the fact that MCI after all is exactly the same as MWT, just at some other scale. As can be seen the implications of working without fundamental scalars quickly manifests it self in increased complexity.

Picking up where I left, the action for Minimal Composite Inflation is

$$\mathcal{S}_{MCI} = \int d^4x \sqrt{-g} \left[-\frac{M_P^2}{2} R - \frac{1}{2} \xi Tr(MM^\dagger) R + \mathcal{L}_{MWT} \right],$$

where ξ is some new dimensionless coupling and \mathcal{L}_{MWT} is the effective Lagrangian for this high energy Minimal Walking Technicolor model I use to describe the composite inflaton. Since there is an extension of MWT a term in the Lagrangian for MWT describing this extension (see equation (4.1)) will be included in \mathcal{S}_{MCI} as well. However this extension, ECI, decouples as explained above [14] and hence there is no need to include it in further calculations.

With ECI decoupled from MCI I can concentrate on the inflaton part of the action. Dropping all other fields than σ I get

$$\mathcal{S}_{MCI,J} = \int d^4x \sqrt{-g} \left[-\frac{M_P^2 + \xi \sigma^2}{2} R + \frac{1}{2} g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma + \frac{m^2}{2} \sigma^2 - \frac{\kappa}{4} \sigma^4 \right]. \quad (4.4)$$

This action is in the Jordan frame, meaning that there is a direct non-minimal coupling term between the field in question and gravity. This is the simplest way I could construct the action I need, but when turning to the usability the Jordan frame is not suitable for inflation. The reason is that all parameters of inflation is defined from equation (3.5), which is in the Einstein frame, where

there is no direct non-minimal coupling to gravity. Since the physics do not depend on the which frame I use, equation (4.4) can then be transformed into the Einstein frame without any change of the physics. The new section will do this in a general framework.

4.2.2 Transforming Non-Minimally Coupled Terms

When working with physics, one often work with symmetry groups. One example is the Standard Model gauge group: $SU(3)_C \times SU(2)_L \times U(1)_Y$. Another symmetry could be the conformal symmetry or dilation symmetry. Physics is in many ways applied mathematics, but where mathematics deal with numbers, physicists deal with the consequences of these numbers. This makes a difference. When trying to calculate things in physics, in physics one calculate observables. Different kind of calculations can in fact be made to evaluate the same observable. For instance, nature does not care whether one measure speed in miles per hour, meters per second or any other unit, however the numbers in front makes a difference every different calculation. Taking a more relevant example, when calculations are done in field theory, what matters is not the form of the Lagrangian, but rather the form of the action. This does not mean that any Lagrangian can be used and that all of them will describe the same physics, but ones we have the Lagrangian, we can make transformations which preserve the action. The transformations that are allowed are transformations from the symmetry group of the theory, these are the only one that preserve the action.

A conformal transformation will not change the action for a theory with conformal symmetry, and in the following I will show how to use a conformal transformation to remove a non-minimal coupling to gravity in a inflationary model. A conformal transformation is in general an angle preserving mapping from one metric to another. Here I will work with the conformal transformation of scaling. In the flat space scenario this is nothing else than decide whether to use miles or meters when describe physics.

Starting from a general single field inflationary model with a non-minimal coupling to gravity the action reads

$$\mathcal{S} = -\frac{M_P^2}{2} \int d^4x \sqrt{-g} (f(\phi)R + 2P(\phi, X)) \quad (4.5)$$

where $f(\phi)$ is some function that both contain the normal 1 from the Einstein Hilbert action and something else from the non-minimal coupling, $P(\phi, X)$ is given by

$$P(\phi, X) = \frac{1}{M_P^2} (V(\phi) + K(\phi)X + \mathcal{O}(X^2)),$$

where $X = \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$ is the normal canonical kinetic term.

The idea is now to make a conformal transformation of the metric, and see how the shape of the action integral changes. The hope is that it can transform the action from the Jordan frame, with the non-minimal coupling, into the Einstein frame, without the non-minimal coupling.

A general scaling is given by the transformation $g_{\mu\nu} \rightarrow \tilde{g}_{\mu\nu} = T^2 g_{\mu\nu}$, where T^2 is some factor. This will change both X and R and the prefactor $\sqrt{-g}$ of (4.5).

In general this transformation will give us the following action [16]

$$\mathcal{S} = -\frac{M_P^2}{2} \int d^4x T^{-4} \sqrt{-\tilde{g}} \left[T^2 f \left(\tilde{R} + 6\tilde{\square} \ln(T) - 6\tilde{g}_{\mu\nu} (\partial_\mu \ln(T)) (\partial_\nu \ln(T)) \right) + 2P(\phi, X) \right].$$

The factor in front of \tilde{R} is $T^{-2}f$, so if one choose a specific transformation, namely $T = \sqrt{f}$ the action will land in the Einstein frame. Doing this one get

$$\mathcal{S} = -\frac{M_P^2}{2} \int d^4x \sqrt{-\tilde{g}} \left(\tilde{R} + 2\tilde{P}(\phi, \tilde{X}) \right), \quad (4.6)$$

where $\tilde{X} = T^{-2}X$ and

$$\tilde{P}(\phi, \tilde{X}) = T^{-4}P(\phi, X) - \frac{6\tilde{X}}{T^2} \left(\frac{dT}{d\phi} \right)^2.$$

This is now in some new frame where all quantities have a tilde, and this is the new frame for computations where there is no non-minimal coupling to gravity. The full action now reads

$$\mathcal{S} = \int d^4x \sqrt{-\tilde{g}} \left[-\frac{M_P^2}{2} \tilde{R} - \frac{1}{T^4} \tilde{V}(\phi) - \frac{1}{T^2} K(\phi) \tilde{X} + \frac{6M_P^2}{T^2} \tilde{X} \left(\frac{dT}{d\phi} \right)^2 \right].$$

Compared to [16] there are some changes in sign, both due to definition of the kinetic term and from direction of the time axis.

4.2.3 MCI Action in The Einstein Frame

Returning to MCI, equation (4.4) can be transformed into the Einstein frame with the method of section 4.2.2. I use the following definitions

$$\begin{aligned} \phi = \sigma, \quad T = \Omega, \quad K(\phi) = -1 & \quad f(\sigma) = \Omega^2 = \frac{M_P^2 + \xi \sigma^2}{M_P^2} \\ V(\sigma) = -\frac{m^2}{2} \sigma^2 + \frac{\kappa}{4} \sigma^4 & \quad P(\sigma, X) = \frac{1}{M_P^2} (V(\sigma) - X). \end{aligned}$$

With this equation (4.4) will transform into

$$\begin{aligned} \mathcal{S}_{MCI,E} = & -\frac{M_P^2}{2} \int d^4x \sqrt{\tilde{g}} \left[\tilde{R} + \frac{2}{M_P^2} \frac{1}{\Omega^4} \tilde{V}(\sigma) \right. \\ & \left. - \frac{2}{M_P^2} \frac{1}{\Omega^4} X - \frac{2}{M_P^2} \frac{6\tilde{X}}{\Omega^2} \left(\frac{d\Omega}{d\sigma} \right)^2 \right]. \end{aligned} \quad (4.7)$$

That the potential here has a tilde comes from the fact that everything is in the new frame. The potential is invariant under the conformal transformation, so it is the same with or without the tilde. What I need to do now is to replace the X with \tilde{X} and calculate the derivative in equation (4.7), this yields:

$$\mathcal{S}_{MCI,E} = \int d^4x \sqrt{\tilde{g}} \left(\frac{-M_P^2}{2} \tilde{R} + \left(\frac{\Omega^2 M_P^2 + 6\xi^2 \sigma^2}{\Omega^4 M_P^2} \right) \tilde{X} - \frac{1}{\Omega^4} \tilde{V}(\sigma) \right). \quad (4.8)$$

Now I have landed in the full "tilde"-frame where all quantities have tildes, further calculations will be made without tildes, as I adopt this new frame. More over the frame is the Einstein frame as desired, and it can be seen that the non-minimal coupling have been transformed into a non-canonical kinetic term. In the following section I will redefined the inflaton to remove this non-canonical kinetic term.

4.2.4 Slow-Roll Minimal Composite Inflation

First thing to do with the newly obtained action, $\mathcal{S}_{E,MCI}$ (4.8), is to put the kinetic term into canonical form. Doing this will standardize my theory to a general scalar inflaton (3.5) and make calculations of slow-roll parameters more direct.

Taking the kinetic term from (4.8) the requirement for a canonical kinetic term leads to following definition of a redefined inflaton field:

$$\frac{1}{2} g^{\mu\nu} \partial_\mu \chi(\sigma) \partial_\nu \chi(\sigma) = \frac{1}{2} \left(\frac{d\chi}{d\sigma} \right)^2 g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma.$$

This equation links σ and χ , and to get a canonical kinetic term I have

$$\chi' \equiv \frac{d\chi}{d\sigma} = \sqrt{\frac{\Omega^2 M_P^2 + 6\xi^2 \sigma^2}{\Omega^4 M_P^2}}. \quad (4.9)$$

This redefining from σ to χ will not only change the kinetic term, but also the potential of (4.8). How the potential will change can be calculated by solving (4.9) for σ and inserting the result in (4.8), but this is not straight forward since (4.9) is a non-linear differential equation. It can however be solved by separation of variables with a complicated result.

As one so often do in physics, I can make an approximation to simplify (4.9). As in section 3.2, one way of insuring inflation was that the potential of the inflaton should dominate the kinetic term. This can be applied to (4.8) to see what type of approximation could be valid.

Counting powers of σ I see that the kinetic term, without the derivatives, goes roughly as σ^{-2} and the first term in the potential also goes as σ^{-2} . What gives us a clue is the second term of the potential. Ω^{-4} is to highest order proportional to σ^{-4} which matches with σ^4 in the potential. So if σ increases, both the kinetic term and the first part of the potential will decrease, but the second term in the potential will roughly be constant. If the goal is to make the

potential dominate the kinetic term I can make the assumption that σ is large, this will help me obtain inflation.

Taking a closer look at Ω^{-4} I see that if this should go as σ^{-4} the following approximation should be made

$$\sigma > \frac{M_P}{\sqrt{\xi}}.$$

This approximation will of course have consequences for the other quantities;

$$\Omega^2 \approx \frac{\xi \sigma^2}{M_P^2}, \quad \frac{d\chi}{d\sigma} \approx \sqrt{6} \frac{M_P}{\sigma},$$

where I for the last also need to assume the ξ is large. This new differential equation is easy to solve. Taking appropriate factors I get

$$\chi = \sqrt{6} M_P \ln \left(\frac{\sqrt{\xi} \sigma}{M_P} \right).$$

and equivalent

$$\sigma = \frac{M_P}{\sqrt{\xi}} \exp \left(\frac{\chi}{\sqrt{6} M_P} \right).$$

To rewrite the potential of (4.8) in terms of χ , it is convenient to simplify the expression first. Using the above approximation, the potential can be written as

$$\begin{aligned} U(\sigma) &= \frac{1}{\Omega^4} \tilde{V}(\sigma) = \left(\frac{M_P^2}{M_P^2 + \xi \sigma^2} \right)^2 \left(-\frac{m^2}{2} \sigma^2 + \frac{\kappa}{4} \sigma^4 \right) \\ &= \left(\frac{M_P^2}{\xi \sigma^2 \left(\frac{M_P^2}{\xi \sigma^2} + 1 \right)} \right)^2 \left(-\frac{m^2}{2} \sigma^2 + \frac{\kappa}{4} \sigma^4 \right) \\ &= \frac{M_P^4}{\xi^2} \frac{\kappa}{4} \left(1 + \frac{M_P^2}{\xi \sigma^2} \right)^{-2} \left(1 - \frac{2m^2}{\kappa} \sigma^{-2} \right) \\ &= \frac{M_P^4}{\xi^2} \frac{\kappa}{4} \left(1 + \frac{M_P^2}{\xi \sigma^2} \right)^{-2}, \end{aligned} \tag{4.10}$$

where there for the last step is used that I am working in the large field approximation, so σ is far from its minimum $\sqrt{\frac{m^2}{\kappa}}$. Now it is easy to calculate the potential in χ

$$U(\chi) = \frac{M_P^4}{\xi^2} \frac{\kappa}{4} \left(1 + \exp \left(\frac{-2\chi}{\sqrt{6} M_P} \right) \right)^{-2}.$$

A plot of $U(\sigma)$ can be seen in figure 8. It can be seen that the conformal transformation have changed the potential from the form $\sigma^2 + \sigma^4$ to a new more inflation friendly potential. It is more inflation friendly in the sense that it

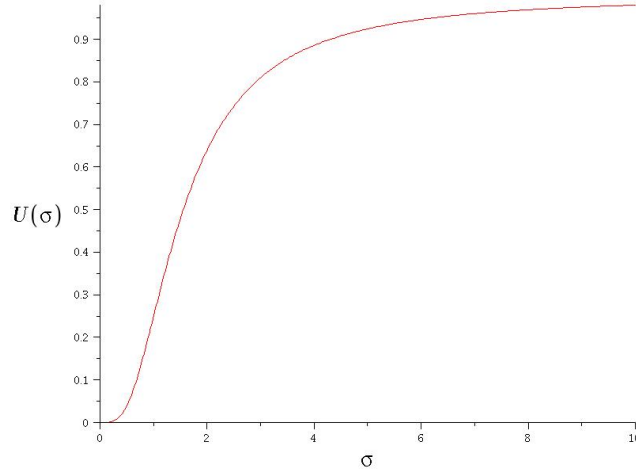


Figure 8: Plot of the potential in MCI. All constant factors have been dropped as they will only scale the potential, but not change its overall shape. The potential is seen to flatten as we move to larger field values, this is an indication of a regime where slow-roll inflation might occur.

flattens in the regime where inflation takes place, whereas the original potential had no such feature.

To ensure that this model will provide an inflationary scenario I can use the slow-roll parameters to check conditions on the different couplings. Using the slow-roll parameters from (3.10) I have

$$\varepsilon = \frac{M_P^2}{2} \left(\frac{dU(\chi)/d\chi}{U(\chi)} \right)^2, \\ \eta = M_P^2 \frac{d^2 U(\chi)/d\chi^2}{U(\chi)}.$$

Using the following calculus calculations I can obtain an expression for the slow-roll parameters in terms of σ instead of χ . As will be shown, however, the use of (4.9) can not be omitted.

Using the Chain rule I get.

$$\frac{dU}{d\sigma} = \frac{dU}{d\chi} \frac{d\chi}{d\sigma} \Leftrightarrow \\ \frac{dU}{d\chi} = \frac{dU}{d\sigma} \bigg/ \frac{d\chi}{d\sigma}$$

There is no need to specify whether U depends on σ or χ since these are in one to one correspondence, which can make calculations of the slow-roll parameters

easier. The second derivative is a bit more complicated to derive, but the idea is the same.

$$\begin{aligned}
\frac{d^2 U}{d\sigma^2} &= \frac{d^2 U}{d\chi^2} \left(\frac{d\chi}{d\sigma} \right)^2 + \frac{dU}{d\chi} \frac{d^2 \chi}{d\sigma^2} \Rightarrow \\
\frac{d^2 U}{d\sigma^2} - \frac{dU}{d\chi} \frac{d^2 \chi}{d\sigma^2} &= \frac{d^2 U}{d\chi^2} \left(\frac{d\chi}{d\sigma} \right)^2 \Rightarrow \\
\frac{d^2 U}{d\sigma^2} - \frac{dU}{d\sigma} \frac{d^2 \chi}{d\sigma^2} / \frac{d\chi}{d\sigma} &= \frac{d^2 U}{d\chi^2} \left(\frac{d\chi}{d\sigma} \right)^2 \Rightarrow \\
\frac{d^2 U}{d\chi^2} &= \frac{\frac{d^2 U}{d\sigma^2} \frac{d\chi}{d\sigma} - \frac{dU}{d\sigma} \frac{d^2 \chi}{d\sigma^2}}{\left(\frac{d\chi}{d\sigma} \right)^3}.
\end{aligned}$$

For simplicity let me denote derivatives with respect to σ with a prime.

$$\frac{d^2 U}{d\chi^2} = \frac{U'' \chi' - U' \chi''}{(\chi')^3}$$

Even if all derivatives are calculated with respect to σ still there is the need for the derivative of χ . Luckily this comes from (4.9) without the need to solve it, however calculations will of course be easier with a more simple expression for (4.9).

I am now able to calculate the slow-roll parameters for this composite inflationary model. Working from (4.10) I have the following form of the potential and its derivatives when working in the large field approximation:

$$\begin{aligned}
U &\simeq \frac{M_P^4}{\xi^2} \frac{\kappa}{4} \\
U' &\simeq \frac{M_P^4}{\xi^2} \frac{\kappa}{4} \left(4 \frac{M_P^2}{\xi} \sigma^{-3} \right) \\
U'' &\simeq \frac{M_P^4}{\xi^2} \frac{\kappa}{4} \left(-12 \frac{M_P^2}{\xi} \sigma^{-4} \right).
\end{aligned} \tag{4.11}$$

The above is consistent with calculations where the approximation is not taken until after differentiation. The slow-roll parameters are then

$$\begin{aligned}
\varepsilon &= \frac{M_P^2}{2} \left(\frac{4 M_P^2 \sigma}{\xi \sigma^3 \sqrt{6} M_P} \right)^2 \\
&= \frac{4}{3} \frac{M_P^4}{\xi^2} \frac{1}{\sigma^4} \\
\eta &= M_P^2 \frac{-12 \frac{M_P^2}{\xi} \sigma^{-4} \sigma^{-1} - 4 \frac{M_P^2}{\xi} \sigma^{-3} (-1) \sigma^{-2}}{6 M_P^2 \sigma^{-3}} \\
&= -\frac{4}{3} \frac{M_P^2}{\xi} \frac{1}{\sigma^2}.
\end{aligned} \tag{4.12}$$

A similar calculation can be done with χ , but it will give the same result as a simple insertion of $\sigma(\chi)$, namely

$$\begin{aligned}
\varepsilon &= \frac{4}{3} \exp \left(\frac{-4\chi}{\sqrt{6} M_P} \right) \\
\eta &= -\frac{4}{3} \exp \left(\frac{-2\chi}{\sqrt{6} M_P} \right).
\end{aligned} \tag{4.13}$$

Now the theory have been formulated in terms of the general parameters of inflationary models, and from here I am able to set further constraints on field values and coupling strength for the non-minimal coupling to gravity.

4.2.5 Constrains on Minimal Composite Inflation

Given the slow-roll parameters, (4.12), I can now move to constraints of the MCI model. This will give an idea of how the scalar field evolve with time, and if it is reasonable to assume that this theory could drive inflation. Moreover it is possible to calculate the effective scale, e.i. at what scale it is no longer valid to assume the effective Lagrangian. This can in turn associate a composite scale, the scale at which the composite theory breaks down and more fundamental particles must be taken into consideration.

This composite scale will give some clues to what particles could drive inflation. A low composite scale raises the hope that some low energy bound state could do the job, but more reasonable would be if the composite scale was of the order of the TeV scale, since no useable bound state have been seen. This would imply that LHC was within reach of not only the Higgs particle from the electroweak symmetry breaking, but that also the inflaton could emerge from this experiment. These two particles might even be one and the same.

An even higher composite scale would also be good, at least for this model. I am working with an effective description of the model, making it valid up to some energy scale. However inflation occurred at very high energy density, so if the effective description should be valid, the initial argument would be that the composite scale should be higher than the inflationary one. Until I have investigated what this scale is, there is no reason why it should be the TeV scale or some other.

First I can calculate a what value the field no longer will expand the universe further. I know that any inflationary model will ultimately stop when ε reaches 1, this will give me σ_{end} , the value of σ when inflation ends. With (4.12) I get:

$$\sigma_{end} \simeq \left(\frac{4}{3}\right)^{\frac{1}{4}} \frac{M_P}{\sqrt{\xi}} = 1.075 \frac{M_P}{\sqrt{\xi}}. \quad (4.14)$$

With one endpoint of inflation calculated, I can calculate the other by knowing how long inflation lasted. Since time is a relatively hard quantity to calculate in this setup, I turn to the relative size of the universe, e.i. the scaling factor. With the number of e-foldings I can estimate the duration of inflation in terms of the scalar field, using equation (3.11) I get

$$\mathcal{N} = \frac{1}{M_P^2} \int_{\chi_{end}}^{\chi_{ini}} \frac{U}{dU/d\chi} d\chi = \frac{1}{M_P^2} \int_{\sigma_{end}}^{\sigma_{ini}} \frac{U}{\frac{dU}{d\sigma} \frac{d\sigma}{d\chi}} \frac{d\chi}{d\sigma} d\sigma.$$

By the inverse function theorem from calculus, or a simple calculation of the derivatives $\frac{d\sigma}{d\chi}$ and $\frac{d\chi}{d\sigma}$, I have

$$\mathcal{N} = \frac{1}{M_P^2} \int_{\sigma_{end}}^{\sigma_{ini}} \frac{U}{dU/d\sigma} \left(\frac{d\chi}{d\sigma}\right)^2 d\sigma. \quad (4.15)$$

Using the approximations from (4.11) I get

$$\mathcal{N} = \frac{1}{M_P^2} \int_{\sigma_{end}}^{\sigma_{ini}} \frac{\xi \sigma^3}{4M_P^2} \left(\frac{\sqrt{6}M_P}{\sigma} \right)^2 d\sigma = \frac{6\xi}{8M_P^2} (\sigma_{ini}^2 - \sigma_{end}^2).$$

Solving this for σ_{ini} and substituting (4.14) I arrive at an initial value for composite inflaton field;

$$\sigma_{ini} = \sqrt{\frac{8M_P^2}{6\xi} \mathcal{N} + \sigma_{end}^2} = \sqrt{\frac{8}{6} \mathcal{N} + (1.075)^2} \frac{M_P}{\sqrt{\xi}}. \quad (4.16)$$

Different estimates can be given on the size of \mathcal{N} [2, 5], but to take the lowest value capable of producing the observables seen today, both in the CMB spectra and for the horizon and flatness problem, I obtain $\mathcal{N} \sim 60$. This implies that

$$\sigma_{ini} = 9.01 \frac{M_P}{\sqrt{\xi}} \quad (4.17)$$

I have now estimated by how much the composite scalar field must change for inflation to run successfully. There is however more constraints I can put on MCI. With the normalization condition [14] from WMAP it is possible to calculate the strength of the strong coupling to gravity, ξ . This can be done by estimating the inflaton field value at the WMAP measurements, σ_{WMAP} to be of the same size as the initial inflaton value, σ_{ini} . The normalization condition is roughly a requirement of the inflaton oscillations to match current matter densities, so that the decay of the inflaton does not produce too much energy:

$$\frac{U_{ini}}{\varepsilon_{ini}} = (0.0276 M_P)^4. \quad (4.18)$$

With this condition and equation (4.11), (4.12) and (4.17) I get

$$\xi \simeq 46043 \sqrt{\kappa}. \quad (4.19)$$

Remember from section 4.1 that κ was the quartic self coupling of the inflaton field. Since MCI originates from a strongly coupled gauge theory, MWT, I can assume that κ is of order unity, this in turn implies that the strong coupling to gravity, ξ , is of the order 46000. To compare, the strong coupling from the Higgs Inflation scenario was $49000\sqrt{\lambda}$, where λ is the " ϕ^4 "-self coupling of the SM Higgs boson.

There is however a difference with the analogy to the Higgs Inflation scenario, the MCI inflaton is a composite state and is thereby formulated in an effective theory. This effective theory is not valid up to an arbitrary scale, but has some cutoff scale at which its formulations no longer make sense, or it produces cross sections that break unitarity.

An analogy to the standard model can be found here [17], where it is seen how unitarity is broken when reaching the scale at which the effective Lagrangian

does not work. In the higgsless standard model this happens when the total energy of the system reaches $\Lambda = 4\pi v \simeq 3TeV$, the v is then associated with the scale where there is the need to change the Lagrangian. For the standard model this change is to add the Higgs boson [17].

For MCI there is similar a breakdown of the effective Lagrangian, this happens at $\Lambda_{MCI} \equiv 4\pi v$. The change needed for this model is however not "simply" to add a new boson, but rather to change from the effective Lagrangian to the fundamental Lagrangian. This will make calculations more complicated, and the use of the scalar field σ impossible, since it no longer would be a bound state due to the high energy. This new scale, v , is therefore what I will call the composite scale.

In the previous sections all calculations have been done in the effective theory, so if there should be any hope that MCI is a phenomenological valid theory, the scales of inflation must be below this cutoff. This highest scale we have in the model is the initial value of the composite inflaton, $\sigma_{ini} = 9 \frac{M_P}{\sqrt{\xi}}$, the model is therefore valid if:

$$\begin{aligned} \sigma_{ini} < 4\pi v &\Rightarrow \\ v &> \frac{9M_P}{4\pi\sqrt{\xi}}. \end{aligned}$$

Plugging in numbers I end up with the following composite scale[14]:

$$v > 4.07 \cdot 10^{16} GeV.$$

A lower bound can be found with the reduced Planck mass, actually this might be the real estimate due to lost of a $8\pi G$ factor throughout calculations. With the reduced Planck mass the composite scale is of the order

$$v > (0.813 - 4.07) \cdot 10^{16} GeV. \quad (4.20)$$

This scale is around the GUT scale. Since inflation probably runs at energies lower than the GUT scale I can trust the effective description of my inflationary model through out the inflationary period.

4.3 Preliminary Findings for Minimal Composite Inflation

A standard inflationary theory obtains inflation via a fundamental scalar field. There has however only been observed one possible fundamental scalars in nature. To explore the possibility of a non-fundamental scalar, an effective theory for a composite scalar field have been set up in the form of the MCI theory. The main difference from MCI to a standard inflationary theory is the compositeness of the scalar field, and the presence of a non-minimal coupling between the scalar field and gravity. With the effective description of MCI it has been possible to formulate a theory that will inflate the universe via the slow-roll paradigm. In this setup a reasonable change of the inflaton field was obtained, and furthermore an estimate of the non-minimal coupling to gravity was given.

From these estimates and the cutoff scale of the effective theory a composite scale was found. This is the scale at which it can be expected that the composite field will decompose into more fundamental particles.

With a composite scale of the size found, (4.20), there are several things I can conclude. The hope that this theory could be solved with a composite Higgs field has to be dismissed. If the inflaton and the Higgs scalar were the same, the two theories must agree, and there for the scale of the theories must be the same. This analysis shows that a composite inflaton decompose around the Grand Unification scale, $10^{16} GeV$, whereas work on electroweak symmetry breaking results in a decomposition of the composite Higgs at much lower scales[17]. With the composite scale of the order of the GUT scale it is safe to assume that the inflaton is composite through out the period of inflation in this model.

Assuming that the inflaton is a bound state of some higher gauge theory MCI serves as an example that composite inflation can work. In its current form it is however only a template for further investigations of compositeness in the inflationary sector. Realistic candidates for the composite inflaton will have to be examined together with reheating properties of the specific theory.

5 Inflatonic Glueballs

*Nature, it seems, is the popular name
for millions and millions and millions
of particles playing their infinite game
of billiards and billiards and billiards*
Piet Hein

When I now have shown that composite inflation in general can be made to work, I will in this section give another possible origin of the composite state, which does not require a Technicolor model. In section 4 I used a composite state from Minimal Walking Technicolor, an extension of the current Standard Model, since all the complicated work of constructing an effective Lagrangian was already done in [15]. In this section I will, motivated strongly by the particle content of the standard model, show that a simple $SU(N)$ gauge theory described through the Yang-Mills Lagrangian can produce inflation.

5.1 What are Glueballs?

Before stating on the inflationary theory I will give a short recap of what a glueball actually is. For this I will start on what a gluon is, since glueballs are condensates of gluons.

5.1.1 Gauge Theories

When one says that a theory has a $SU(N)$ gauge symmetry it means that if the field content is transformed by a local transformation from this gauge group, the theory must be invariant. An example is the mass term for a fermion, $m\bar{\psi}\psi$. It is invariant under a $U(1)$ gauge symmetry, which means that when the fermion field transforms with a $U(1)$ transformation phase like:

$$\psi(x) \rightarrow e^{i\alpha(x)}\psi(x),$$

the mass term is invariant, which easily can be seen by insertion, just remember that the transformation angle changes sign when there is a bar over ψ .

But particle theories contain not only mass terms, it should include kinetic terms as well. Since the derivative is defined as a limiting process, namely the derivative in direction n^μ is :

$$n^\mu \partial_\mu \psi = \lim_{\epsilon \rightarrow 0} \frac{1}{\epsilon} (\psi(x + \epsilon n) - \psi(x)),$$

one ends up having problems when naively including standard kinetic terms, $\bar{\psi}\not{\partial}\psi$, since the transformation of the derivative includes a local transformation.

A way to solve this is to introduce the covariant derivative, D_μ . It is constructed very similar to the ordinary derivative, but it includes a scalar $U(x, y)$ that "moves" the two fields in the derivative together, so that the over all derivative transforms nicely, see chapter 15 of [1]. For an $SU(N)$ gauge theory the covariant derivative is given by

$$D_\mu \psi(x) = \partial_\mu \psi(x) - ig A_\mu^a t^a \psi(x),$$

with summation over a , where t^a is the generators of the gauge group, A_μ^a is the gauge connection and g is the gauge coupling.

In Quantum Electrodynamics, QED, the gauge connection, A_μ can be identified with the photon. So in an atom, where the nuclei is interpreted as a static charge, the force carrier, responsible for keeping the electron in orbit, is the photon. A continuous exchange of photons between the electron and the nuclei acts like a force, and binds the atom together.

Likewise is it with any other gauge theory, the gauge connection can be seen as a particle. Take a look at Quantum Chromodynamics, QCD, which has a $SU(3)$ gauge group, the force carries comes from the gauge connection, A_μ^a , unlike in the QED case this field has an extra index, a . The force carrier in QCD is called the gluon, and the extra index has a huge impact on the way the gluon behaves. For one thing photons do not interact directly with each other, gluons do. The extra index can be associated to the color charge of QCD so that gluons them self carry charge, this makes them interact with each other, photons can not do this since they do not carry any charge.

The self-interaction of gluons is what brings us to glueballs. The strong nuclear force is, not surprisingly, strong, and therefore the interaction between gluons is also strong. An invariant kinetic term for the gluons can be formed from the covariant derivative, it is simplified in the form $(F_{\mu\nu}^a)^2$ where $F_{\mu\nu}^a$ is defined by

$$[D_\mu, D_\nu] = -ig F_{\mu\nu}^a t^a,$$

again with summation over a . This new quantity is called the field strength tensor. With the ingredients presented above a theory with a given gauge symmetry can be formed via the Yang-Mills Lagrangian into the following theory of fermions:

$$\mathcal{L}_{YM} = \bar{\psi}(iD_\mu)\psi - \frac{1}{4} (F_{\mu\nu}^a)^2 - m\bar{\psi}\psi.$$

The Yang- Mills Lagrangian is the term with $F_{\mu\nu}^a$.

Taking only the Yang-Mills Lagrangian, this is called the Yang-Mills theory, it describes a non-trivial interacting theory due to the cubic and quartic term within $(F_{\mu\nu}^a)^2$ [1]. Its particle content is only gluons, or a generalization of these in the case of non-QCD models, which interact in different ways. For us, one important interaction is the condensation due to the strong coupling of the theory. Two gluons can attract each other and form a condensation called a glueball.

5.1.2 Effective Glueballs

Francesco Sannino and Joseph Schechter have derived an effective theory for glueballs [18], in which individual gluons can be omitted and only the glueballs are modeled. From this I can take the effective description of the glueball field and its potential which I later will use for an inflationary model driven by glueballs. With the only change being notation the effective glueball field, in an $SU(N)$ gauge theory, was derived to be

$$H = \frac{11N}{3} \frac{g^2}{32\pi^2} F^{\mu\nu,a} F_{\mu\nu}^a,$$

with summation over all repeated indices. Geoffrey B. West finds the same form of the glueball field in [19], from here the glueball field is

$$G = \frac{\beta(g)}{g} F^{\mu\nu,a} F_{\mu\nu}^a,$$

here β is the full beta function, and g is the coupling constant of the gauge theory. I will use this formulation of the glueball field, which I will denote Φ

$$\Phi = \frac{\beta(g)}{g} F^{\mu\nu,a} F_{\mu\nu}^a. \quad (5.1)$$

It should be noted that the glueball field is not just two single gluons with gauge charge $a = 1$, it is a summation of all possible condensations of gluons with $a = 1..N^2 - 1$ in the $SU(N)$ theory. Moreover the glueball field has dimension 4, since $F_{\mu\nu}$ is of dimension 2¹⁰.

With an effective description of the glueball field I can now move the potential of the glueball. Again Sannino and Schechter have an effective potential ready to use. Whether I use the notation of Sannino and Schechter or the one by West does not matter. The potential reads

$$V(H) = \frac{H}{4} \ln \left(\frac{H}{8e\Lambda_{YM}^4} \right),$$

where e is Euler's number and Λ_{YM} is the scale of the $SU(N)$ Yang-Mills theory. To make calculations look a bit simpler I absorb the $8e$ into Λ_{YM}^4 and call it Λ^4 , furthermore I use the glueball field notation of West so that the potential will take the form

$$V(\Phi) = \frac{\Phi}{4} \ln \left(\frac{\Phi}{\Lambda^4} \right). \quad (5.2)$$

Now that I have outlined how glueballs emerge from a pure gauge theory, I will move to coupling glueballs to gravity.

¹⁰This is easy to see in the case of global $U(1)$ symmetry of QED, where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and A_μ is of dimension 1.

5.2 Glueballs In Spacetime

In this section I will show how the glueballs, condensates from a pure Yang-Mills theory, can produce an inflationary state of the universe in a theory called Glueball Inflation, GI [20]. The recipe for inflatonic glueballs is more or less the same as for Minimal Composite Inflation, so for a start I will outline the plan for a generic non-minimal coupled inflationary theory. The plan is as follows:

- Write the effective action in the Jordan frame with a non-minimal coupling to gravity
- Rewrite the action via a conformal transformation, so that there is only a minimal coupling to gravity.
- Define a new field so that the non-canonical kinetic term is made canonical
- Calculate the slow-roll parameters using the definition of this new field.

In this section I will do the first 3 steps, moving from the first action to an action in the form where the slow-roll parameters can be calculated.

The action for the glueball field Φ is given by

$$\mathcal{S}_J = \int d^4x \sqrt{-g} \left[-\frac{M^2 + \xi \Phi^{\frac{1}{2}}}{2} R + \Phi^{-\frac{3}{2}} \partial_\mu \Phi \partial^\mu \Phi - \frac{\Phi}{2} \ln \left(\frac{\Phi}{\Lambda^4} \right) \right]. \quad (5.3)$$

There are a few things to note about (5.3). First thing that might be odd is that non-analytic powers in both the non-minimal coupling to gravity and in the non-canonical kinetic term. The reason is the same for both of them, to match dimensions. The glueball field, Φ , is a dimension 4 operator and therefore, if I would like to have a dimensionless coupling ξ , must form a non-analytic coupling to the scalar curvature R since this is a dimension 2 operator. The non-canonical kinetic term is dimensionally constrained as well, counting dimensions in a canonical kinetic term will the field of dimension 4 leads to the need to correct for 6 powers of mass in the Lagrangian density, this can be done with the field itself to the power of $-\frac{3}{2}$. That I here have M instead of M_P , as in MCI, is due to the note at the end of section 4.2.5 that there at some point might be missing some constant factors, so that M either is the Planck mass or the reduced Planck mass. Further more, the exact form of graviton interactions are not known, so in a non-minimally coupled theory the constant in the Hilbert-Einstein action need not be neither the Planck mass nor the reduced Planck mass, so using M opens for a more general setup.

With the dimension 4 operator, Φ , it could make calculations easier if I introduced a rewriting of the field into a dimension 1 operator, this is simply done with $\Phi = \phi^4$. This field will be referred to as the one dimensional glueball field or just glueball field, but ϕ will always be of dimension 1 and not 4 as the actual glueball field, Φ . With this the action (5.3) now reads:

$$\mathcal{S}_J = \int d^4x \sqrt{-g} \left[-\frac{M^2 + \xi \phi^2}{2} R + 16 \partial_\mu \phi \partial^\mu \phi - 2 \phi^4 \ln \left(\frac{\phi}{\Lambda} \right) \right]. \quad (5.4)$$

Now I only need to set this up for easier use of the general conformal transformation in section 4.2.2, (5.4) then takes the form:

$$\mathcal{S}_J = -\frac{M_P^2}{2} \int d^4x \sqrt{-g} \left[\frac{M^2 + \xi\phi^2}{M_P^2} R - \frac{32}{M_P^2} \partial_\mu \phi \partial^\mu \phi + \frac{4\phi^4}{M_P^2} \ln\left(\frac{\phi}{\Lambda}\right) \right].$$

Using the notation of section 4.2.2 I have

$$f(\phi) = \frac{M^2 + \xi\phi^2}{M_P^2}, T = \sqrt{f}$$

$$V(\phi) = 2\phi^4 \ln\left(\frac{\phi}{\Lambda}\right), K(\phi) = -32,$$

Making the conformal transformation I end up with the following action in the Einstein frame

$$\begin{aligned} \mathcal{S}_E &= \int d^4x \sqrt{-g} \left[-\frac{M_P^2}{2} R + \left(\frac{6M_P^2}{T^2} \left(\frac{dT}{d\phi} \right)^2 + \frac{32}{T^2} \right) \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{2\phi^4}{T^4} \ln\left(\frac{\phi}{\Lambda}\right) \right] \\ &= \int d^4x \sqrt{-g} \left[-\frac{M_P^2}{2} R + \left(\frac{6\xi^2\phi^2 + 32T^2M_P^2}{T^4M_P^2} \right) \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{2\phi^4}{T^4} \ln\left(\frac{\phi}{\Lambda}\right) \right]. \end{aligned} \quad (5.5)$$

Now that I have landed in the Einstein frame, again with the cost of a non-canonical kinetic term, I am able to redefine the inflaton field into one with a canonical kinetic term, just like I did in section 4.2.4.

This new field, χ , will be linked to the inflaton field, ϕ , via the non-canonical kinetic term of (5.5):

$$\frac{d\chi}{d\phi} \equiv \chi' = \sqrt{\frac{6\xi^2\phi^2 + 32T^2M_P^2}{T^4M_P^2}}. \quad (5.6)$$

With this redefinition I can now make (5.5) into canonical form in the Einstein frame, where our slow-roll parameters are defined.

5.3 Slow-rolling Glueballs

In this section I will calculate the slow-roll parameters in our glueball inflationary model [20]. Let me to begin with restate the slow-roll parameters in this setup where I redefine the inflaton field to remove a non-canonical kinetic term. The two slow-roll parameters are given by

$$\varepsilon = \frac{M_P^2}{2} \left(\frac{U'}{U} \frac{1}{\chi'} \right)^2, \quad \eta = M_P^2 \frac{U''\chi' - U'\chi''}{U(\chi')^3}, \quad (5.7)$$

where primes denote derivatives with respect to ϕ , and $U = T^{-4}V(\phi)$. Although the euqations (5.7) looks fairly simple, with (5.6) they quickly becomes

very messy, so like in section 4.2.4 I need an approximation to simplify things. Remembering that in an inflationary model, the inflaton potential dominates the kinetic term, I can count powers of ϕ . This leads to the conclusion that I must assume ϕ to be large. To simplify things the assumption I will make is

$$\phi > \frac{M}{\sqrt{\xi}},$$

since this reduces both T and χ' into the following

$$\begin{aligned} T^2 &\simeq \frac{\xi \phi^2}{M_P^2} \Rightarrow \\ \chi' &= M_P \sqrt{6} \phi^{-1}, \end{aligned} \quad (5.8)$$

where I also have assumed that ξ is large. With the approximation the transformed potential, U , becomes

$$U = T^{-4} 2\phi^4 \ln\left(\frac{\phi}{\Lambda}\right) \simeq \frac{2M_P^4}{\xi^2} \ln\left(\frac{\phi}{\Lambda}\right). \quad (5.9)$$

A sketch of $U(\phi)$ can be seen in figure 9, where the general form of the potential can be seen. All constant factors have been dropped in the plot these will only scale the graph.

To calculate the slow-roll parameters I need just to evaluate the derivatives of (5.9) and (5.8):

$$\begin{aligned} U' &= \frac{2M_P^4}{\xi^2 \phi} & U'' &= -\frac{2M_P^4}{\xi^2 \phi^2} \\ \chi' &= \frac{\sqrt{6}M_P}{\phi} & \chi'' &= -\frac{\sqrt{6}M_P}{\phi^2}. \end{aligned}$$

With this and (5.9) I get the slow-roll parameters from (5.7) for glueball driven inflation to be

$$\begin{aligned} \varepsilon &= \frac{M_P^2}{2} \left(\frac{\phi}{\sqrt{6}M_P \phi \ln(\frac{\phi}{\Lambda})} \right)^2 = \frac{1}{12(\ln(\frac{\phi}{\Lambda}))^2} \\ \eta &= M_P^2 \frac{-\phi^{-2} \phi^{-1} - \phi^{-1}(-\phi^{-2})}{\ln(\frac{\phi}{\Lambda})(\sqrt{6}M_P)^2 \phi^{-3}} = 0. \end{aligned} \quad (5.10)$$

It is a bit surprising that the second slow-roll parameter is identical zero. But a quick check by counting powers of χ indicates that U is linear in χ , taking the large field for granted. If U is linear in χ the second derivative with respect to χ must of course vanish. Then simplicity of η is due only to the large field approximation, and not the to fact that I also assumed ξ to be large. This assumption only simplifies the prefactor of the slow-roll parameters, and hence can not change the fact that η is zero. However it could be an indication for calculation of higher order is needed. I must stress that this will not move the final result away from zero by any significant amount, but it might give some further insight.

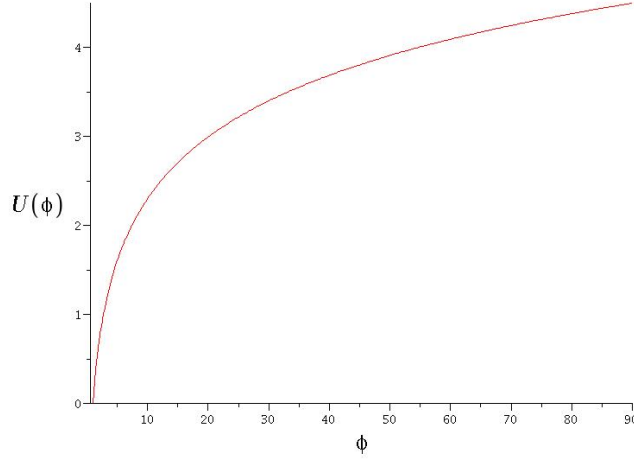


Figure 9: A plot of the potential in Glueball Inflation, $U(\phi)$, where ϕ is the one dimensional glueball field. The general form of the potential can be seen in the plot, all constant factors have been dropped as they will only scale the potential. Note that $\Lambda = 1$. This sets the scale of the first axis. What further should be emphasized is that the potential flattens when moving towards higher field values, which might indicate a working slow-roll regime.

5.4 The Strong Coupling and its Implications

As it was the case with Minimal Composite Inflation, I will now move to calculations of the strong coupling constant, ξ . This can, as it did with MCI, give us some clues to the scale of the theory, and hence an estimate of the validity of this model.

The layout is fairly simple, calculate final and initial value of the inflaton field, and use this to derive ξ from the WMAP normalization condition (4.18). Inflation ends when $\varepsilon = 1$, this gives a final value of the dimension one glueball field of size

$$\phi_{end} = 1.335\Lambda. \quad (5.11)$$

It should be noted that for now I can only calculate with reference to the scale of the theory, since this for now is a free parameter. Using the number of e-foldings in the form of equation (4.15) I can calculate the initial value assumed by the

glueball field.

$$\begin{aligned}
\mathcal{N} &= \frac{1}{M_P^2} \int_{\phi_{end}}^{\phi_{ini}} \frac{U}{U'} (\chi')^2 d\phi \\
&= 6 \int_{\phi_{end}}^{\phi_{ini}} \frac{\ln\left(\frac{\phi}{\Lambda}\right)}{\phi} d\phi \\
&= 3 \left(\ln^2\left(\frac{\phi_{ini}}{\Lambda}\right) - \ln^2\left(\frac{\phi_{end}}{\Lambda}\right) \right) \\
&= 3 \ln^2\left(\frac{\phi_{ini}}{\Lambda}\right) - \frac{1}{4}.
\end{aligned}$$

Here $\ln^2(x) = (\ln(x))^2$ adopted from $\sin^2(x)$ and $\cos^2(x)$. Again assuming that inflation at least lasted for 60 e-foldings I can get an estimate of the initial value of the glueball field:

$$\begin{aligned}
\phi_{ini} &= \Lambda \exp \sqrt{\frac{\mathcal{N} + \frac{1}{4}}{3}} \\
&= 88.36\Lambda.
\end{aligned} \tag{5.12}$$

This high value of the initial field might be a problem. The theory is only an effective theory, and therefor with scales this high compaired to the scale of the theory the effective description may be insufficient. I will not adress this problem further.

What I aim to do is to evaluate the scale of the theory, Λ . Since I am working in the large field approximation I have assumed that

$$\phi > \frac{M}{\sqrt{\xi}}.$$

This implies that also the smallest value assumed by ϕ must be above this energy. The smallest value of the one dimensional glueball field is Λ by construction. Hence

$$\Lambda > \frac{M}{\sqrt{\xi}},$$

all I need here is the size of ξ , this is however within reach. Using the WMAP normalization condition (4.18) I can derive ξ .

$$\begin{aligned}
\frac{U_{ini}}{\varepsilon_{ini}} &= (0.0276 M_P)^4 \Rightarrow \\
0.0276^4 &= \frac{24}{\xi^2} \ln^3\left(\frac{\phi_{ini}}{\Lambda}\right) \Rightarrow \\
\xi &= \sqrt{\frac{24 \ln^3(88.36)}{0.0276^4}} \simeq 61011.
\end{aligned}$$

With this I get the scale of the theory to be

$$\Lambda > 4.94 \cdot 10^{16} GeV,$$

where M was identified with the Planck mass.

As I found in the Minimal Composite setup a typical scale is in the Grand Unification region. Like for MCI there is a meaning to this scale of the theory. The description of glueball I have used is an effective description at relative low energies. What this lower bound on Λ means is that I can trust the effective description up to scales of this order. That it is of the order of GUT is a good thing. If glueballs couple strongly to gravitons at the GUT scale, and they would do so all the way to the Planck scale, then the physics of the GUT scale would impose huge constraints on for instance unitarity at Planck scales. But since the scale of Glueball Inflation is "only" the GUT scale, then gravitons and glueball could easily decouple at higher scales, before the Planck scale, removing any strong constraints from a Grand Unification Theory. Another aspect is that at the GUT scale, which is defined as the unification energy of the 3 standard model forces, the standard model coupling¹¹ is weak. Our new $SU(N)$ Yang-Mills theory will, with this lower bound on Λ , be a strongly coupled theory at the Grand Unification scale and hence the standard model decouples from the inflation theory. This justifies that the action describing inflation do not include any aspects from the standard model, but only the new Yang-Mills theory. This idea can be visualized in figure 10 where the couplings of the 3 standard model forces are plotted together with the new $SU(N)$ gauge coupling, assuming an extension of the standard model which is unification friendly¹².

5.5 Preliminary Findings for Glueball Infaltion

Working of the basis of Minimal Composite Inflation I have constructed a new composite inflationary theory called Glueball Inflation[20]. In here the inflaton is a condensate from a pure $SU(N)$ Yang - Mills Theory featuring only gluons. An effective description of the condensate of these gluons led the glueball and its effective potential. Like with MCI, I coupled GI to gravity non-minimally, making it a working inflationary theory. It has been shown that GI is an inflationary theory in the energy region supposedly governed by a era of inflation, thereby making it a reasonably theory of this era.

However the effective description of Glueball Inflation breaks down at some energy scale, this was found to be the GUT scale, like with MCI. This opens for studies of only the inflationary theory in the region, since the Yang - Mills Theory is strongly coupled and the Standard Model at this energy might be weakly coupled. Further more it could be interesting to see whether or not GI and the Standard Model unify at some point which suggests a theory explaining both inflation and the Standard model.

¹¹Remember that there at the GUT scale only is one coupling for the standard model

¹²The standard model as it is does not unify, but it shows indications to unify[1].

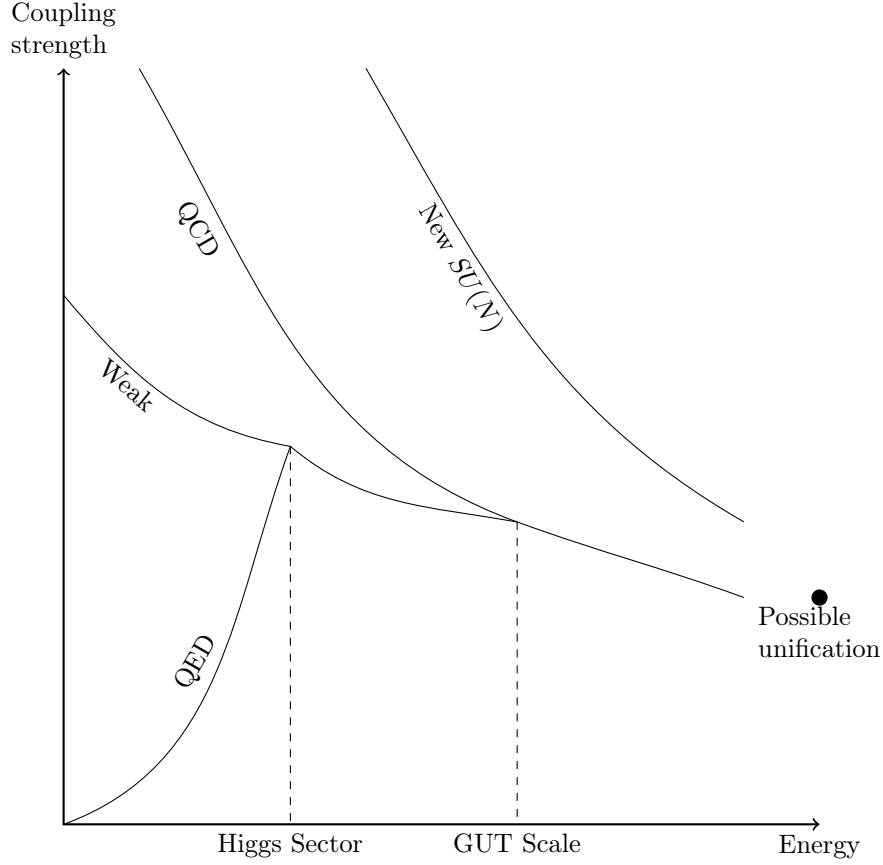


Figure 10: Qualitative visualization of the gauge couplings relevant for discussion of inflation. Both axis are far off linear scale. With this setup the standard model gauge couplings unify at different scales. First QED and the weak force unify to the electroweak theory, and later (in a setup allowing for this) the electroweak theory unifies with the strong interactions. At this point the single standard model gauge coupling is weak where as the new $SU(N)$ gauge coupling still strong. At some higher scale these two couplings might unify, this unification does not need to be at the Planck scale.

6 Modified Glueball Inflation

*Not only is the universe stranger than we think,
it is stranger than we can think.*
Werner Heisenberg

In this section I will explore a modified version of the glueball inflation. This is motivated mainly for the reason that the coupling $\xi\sqrt{\Phi}R$ seems unnatural, since square roots do not appear often in fundamental particle physics.

6.1 Producing Non-Minimal Couplings

Until this point the coupling to gravity has only been postulated, but it is not without reasoning. A short recap of [21] should provide some insight on how these non-minimal couplings are naturally formed. In [21] a framework is set up. In this framework the non-minimal coupling of the composite scalar bosons and gravity can be calculated. What I will use from this is not the value the coupling takes, but rather the reasoning on how the non-minimal coupling can arise in nature.

The field in [21] that is non-minimally coupled to gravity is a chiral condensate from the Nambu - Jona-Lasinio model called H . It is coupled to gravity via a coupling term very similar to the one from section 4, namely $\xi H^\dagger H R$. The claim of [21] is then that this coupling is generated in nature via the scattering process of Figure 11a or in the case of broken symmetry, where the coupling is $\sqrt{2}\xi v\phi R$, via Figure 11b. Both scatterings go via a fermion loop.

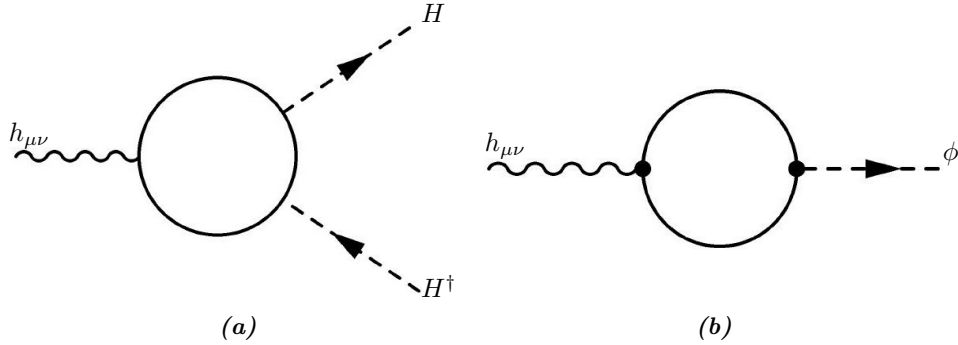


Figure 11: *a* Generation of non-minimal coupling in the NJL setup. *b* Generation of non-minimal coupling in the NJL setup with broken symmetry by the VEV of H

I would like to follow this idea and use it for the models already presented. Take a look at Minimal Composite Inflation once again, the non-minimal cou-

pling was $\frac{1}{2}\xi_0\sigma^2 R$, where ξ_0 is the dimensionless coupling, σ is the inflaton field of the theory and R is the scalar curvature.

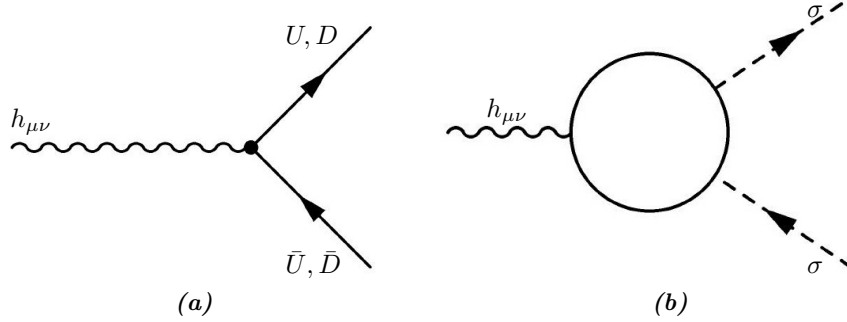


Figure 12: *a* Possible tree-level scattering process with a graviton into a pair of techni-fermions in a theory with a non-minimal coupling to gravity. *b* The graviton decays into techni-fermions which then condensates to a scalar particle, which in turn are the inflaton from MCI.

Remember, from Minimal Walking Technicolor, that the scalar I used was a condensate of the two new fermions from the extension of the standard model, I can say that the coupling fundamentally is not between the graviton and the inflaton, but rather between the graviton and the two new fermions. In this case a tree-level process of scattering gravitons and techni-fermions could be the one in Figure 12a, where a graviton decays into two techni-fermions. With the two new fermions produced, I can imagine that these condensate at some lower energy. This is when I can start to use the effective descriptions used in section 4.

From section 4.1 the condensate in question is of the form $\langle U\bar{U} + D\bar{D} \rangle$, this condensation forms the scalar particle I called σ . This production of an inflaton can be pictured as in Figure 12b where the graviton decays into techni-fermions and these fermions then condensate into the scalar particle from MCI, that I used for the inflaton in section 4. With this reasoning the non-minimal coupling to gravity seems rather natural and further studies could be made to investigate if similar calculations to the ones in [21] can be done in the MCI setup. Now let me move to the Glueball Inflation from section 5, can I here make the same argument of the origin of the non-minimal coupling?

At this level I can not. The reason being that the scattering of type $\xi_0\sqrt{\Phi}R$ does not follow standard Feynman diagrams, since it is not possible to form a $\sqrt{\Phi}$ particle, so to speak. This is what brings me to the next section, where I present a modified version of glueball inflation.

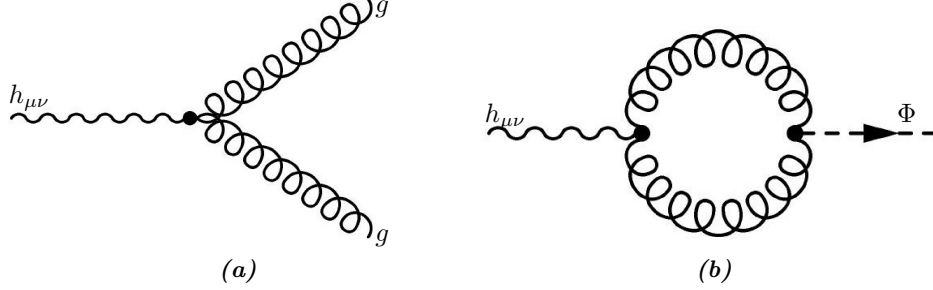


Figure 13: First look at how the graviton, $h_{\mu\nu}$, could decay into a glueball, Φ . *a* Naive production of gluons, g , from the graviton. *b* Production of gluons, which then condensates into a glueball, Φ .

6.1.1 Producing Glueballs

The production of inflatons in the MCI model came from the condensation of techni-fermions, see Figure 12b, and I will here try and explain how I can form inflatonic glueballs in the same manner. Glueballs are a condensation of gluons, so I need some way to produce gluons from the graviton, thereby creating a process like the one in Figure 13a. This will then lead to a condensation of the gluons into a glueball, see Figure 13b.

I will pause for a moment at the implications of Figure 13b, since these considerations will help in understanding the possibilities of this way to visualize the non-minimal coupling in glueball inflation. The coupling associated with Figure 13b is $\xi_2 \Phi R$, where ξ_2 is the coupling constant, Φ the 4 dimensional glueball field from section 5 and R the scalar curvature. The original argument for the $\sqrt{\Phi}R$ coupling in the glueball inflationary model was to match dimensions and keeping ξ_0 dimensionless. With this new coupling I have relaxed the constrain on the dimension of ξ and by counting it is now of dimension -2 . The origin of this can come from the way to produce gluons from gravitons. The production of Figure 13b could be true, but there is another relevant process as well.

If gravity couples non-minimally to fermions, then the underlying process of Figure 13a could in fact be via fermions which couple both to gravity and the gauge bosons. A process that then could produce the effects of Figure 13a could be the one in Figure 14a. Here the graviton decays to fermions which then produce the desired gluons.

Once this idea that the gluons in fact come from some intermediate state and not the graviton directly, there is the need to consider Figure 14b. That there is an intermediate massive state will affect the coupling term in the Lagrangian.

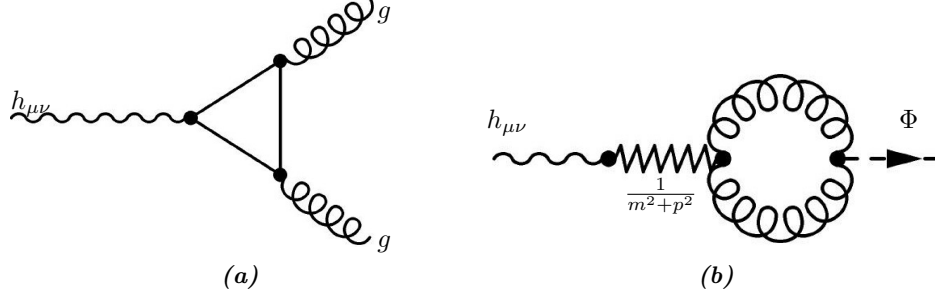


Figure 14: *a* Production of gluon from the graviton via a massless fermion loop. *b* Condensation of gluons to a glueball, Φ , where the gluons are produced, via some heavy intermediate state, from the graviton. This leads to a suppression with a factor $\frac{1}{m^2 + p^2}$ compared to Figure 13b.

With this way of coupling glueballs to gravitons the new coupling term is

$$\mathcal{L} \supset \frac{\xi_0 \Phi}{2m^2} R.$$

In this way I have obtained a term with dimension 4 as needed and still kept ξ_0 dimensionless. This is however not directly the term I will use since ξ_0 and $\frac{1}{m^2}$ anyway always will appear together, so I might as well collect them in one factor, call that ξ_2 , which is of dimension -2 . It should be kept in mind that I have absorbed on of the scales of the theory into the non-minimal coupling.

The coupling term I will use is

$$\mathcal{L} \supset \frac{1}{2} \xi_2 \Phi R,$$

which is the exact same thing. A more complete calculation of factors can be done in a specific $SU(N)$ model, but for now this light weight version will be sufficient.

My argument of using an intermediate fermion state could be casted into not a suppression by m^{-2} but simply by a new scale of this kind of interactions, say Λ'^{-2} . This is more general, since it is not a specific fermion state, but takes into account the whole dynamic of that scale.

6.2 Modified Potential for Modified Glueball Inflation

I will now start the work on calculating slow-roll parameters in a new modified glueball inflationary scenario, Modified Glueball Inflation, MGI. Again it is the same tactic of deriving the parameters, so I will not comment to much on calculations. One thing should be noted, I will not make any approximations on the transformation factor T^2 , the redefined field, χ , or on the new potential, $U(\phi)$ as I did in the previous sections.

With the newly derived coupling term to gravity the Lagrangian and action describing this modified version is

$$\mathcal{S} = \int d^4x \sqrt{-g} \left[-\frac{M^2}{2} R - \frac{1}{2} \xi_2 \Phi R + \Phi^{-\frac{3}{2}} \partial_\mu \Phi \partial^\mu \Phi - \frac{\Phi}{2} \ln \left(\frac{\Phi}{\Lambda^4} \right) \right].$$

Setting this up to make the conformal transformation of section 4.2.2 and replacing Φ with ϕ^4 , where ϕ is of dimension 1, I get

$$\mathcal{S} = -\frac{M_P^2}{2} \int d^4x \sqrt{-g} \left[\frac{M^2 + \xi_2 \phi^4}{M_P^2} R - \frac{64}{M_P^2} X + \frac{4\phi^4}{M_P^2} \ln \left(\frac{\phi}{\Lambda} \right) \right].$$

Using section 4.2.2 where the transformation factor, T^2 , is given by

$$T^2 = \frac{M^2 + \xi_2 \phi^4}{M_P^2},$$

I get the action in the following form

$$\mathcal{S} = \int d^4x \sqrt{-g} \left[-\frac{M_P^2}{2} R + \left(\frac{32}{T^2} + \frac{6M_P^2}{T^2} \left(\frac{dT}{d\phi} \right)^2 \right) X - 2 \frac{\phi^4}{T^4} \ln \left(\frac{\phi}{\Lambda} \right) \right]. \quad (6.1)$$

Defining the field χ will make the kinetic term canonical, χ is related to ϕ via

$$\chi' = \sqrt{\frac{32}{T^2} + \frac{6M_P^2}{T^2} \left(\frac{dT}{d\phi} \right)^2},$$

and evaluating the derivative of T yields

$$\chi' = \frac{\sqrt{4M^2 + 4\xi_2 \phi^4 + 3\xi_2^2 \phi^6}}{M^2 + \xi_2 \phi^4} \sqrt{8} M_P, \quad (6.2)$$

where primes denote derivatives with respect to ϕ . From equation (6.1) I read off the potential:

$$U(\phi) = \frac{2M_P^4 \phi^4}{(M^2 + \xi_2 \phi^4)^2} \ln \left(\frac{\phi}{\Lambda} \right), \quad (6.3)$$

note that $U(\phi)$ is of dimension 4, as it should be, since ξ_2 is dimension -2 .

6.2.1 Slow-Roll Relevant Derivatives

Next thing to do is to evaluate the slow-roll parameters, but for that I need the first and second derivative of the potential as well as the field χ . This can of course be made with pen and paper, but I have used Maple for further calculations. A way to avoid this is to make similar approximations as I did in section 4 and 5. This will not change things alot, but as will be clear in the end, I need the precision of a non-approximated calculation. I did the approximated case first, which agreed with the full calculations done in Maple made later on, up to the approximation.

The derivatives of the potential is

$$U'(\phi) = \frac{8M_P^4\phi^3 \ln\left(\frac{\phi}{\Lambda}\right)}{(M^2 + \xi_2\phi^4)^2} + \frac{2M_P^4\phi^3}{(M^2 + \xi_2\phi^4)^2} - \frac{16M_P^4\xi_2\phi^7 \ln\left(\frac{\phi}{\Lambda}\right)}{(M^2 + \xi_2\phi^4)^3} \quad (6.4)$$

and

$$U''(\phi) = \frac{24M_P^4\phi^2 \ln\left(\frac{\phi}{\Lambda}\right)}{(M^2 + \xi_2\phi^4)^2} + \frac{14M_P^4\phi^2}{(M^2 + \xi_2\phi^4)^2} - \frac{176M_P^4\phi^6 \ln\left(\frac{\phi}{\Lambda}\right)}{(M^2 + \xi_2\phi^4)^3} \\ - \frac{32M_P^4\xi_2\phi^6}{(M^2 + \xi_2\phi^4)^3} + \frac{192M_P^4\xi_2^2\phi^{10} \ln\left(\frac{\phi}{\Lambda}\right)}{(M^2 + \xi_2\phi^4)^4}. \quad (6.5)$$

The second derivative of χ is needed for the second slow-roll parameter η ;

$$\chi'' = \frac{\sqrt{8}M_P}{(M^2 + \xi_2\phi^4)^2} \left[\frac{(M^2 + \xi_2\phi^4)(16\xi_2\phi^3 + 18\xi_2\phi^5)}{2\sqrt{4M^2 + 4\xi_2\phi^4 + 3\phi^6\xi_2^2}} \right. \\ \left. - 4\xi_2\phi^3\sqrt{4M^2 + 4\xi_2\phi^4 + 3\phi^6\xi_2^2} \right]. \quad (6.6)$$

These equations are far from appealing, but as Einstein is quoted; "If you are out to describe the truth, leave elegance to the tailor." Let us hope that there is some lesson to learn from this.

6.3 Slow Roll MGI

With the complexity of the equations going into the slow-roll parameters there is no hope that these will be any simpler. From equation (5.7) I have the slow-roll parameters, inserting the already calculated factors I get for the first slow-roll parameter

$$\varepsilon = \frac{1}{16} \frac{\left(4 \ln\left(\frac{\phi}{\Lambda}\right) M^2 - 4\phi^4\xi_2 \ln\left(\frac{\phi}{\Lambda}\right) + M^2 + \xi_2\phi^4\right)^2}{\phi^2 \ln^2\left(\frac{\phi}{\Lambda}\right) (4M^2 + 4\xi_2\phi^4 + 3\xi_2^2\phi^6)}$$

Like with MCI and Glueball Inflation I can now calculate the end value of the field from ε . Setting ε equal one, and keeping Λ as the scale, the end value of ϕ is

$$\phi_{end} = 1.1146\Lambda$$

I did this calculates nummerically with $\Lambda = 1$, $M_P = 100 = M$ and $\xi_2 = 50000$. These values were picked from the experience of section 5 since there was no better found choice.

When I now have a starting point of where inflation might end it could be of help to plot the potential, see Figure 15a. I know that inflation ends at ϕ_{end} , which is at the far left of Figure 15a, and ε have same zero point as $U'(\phi)$ e.i. at

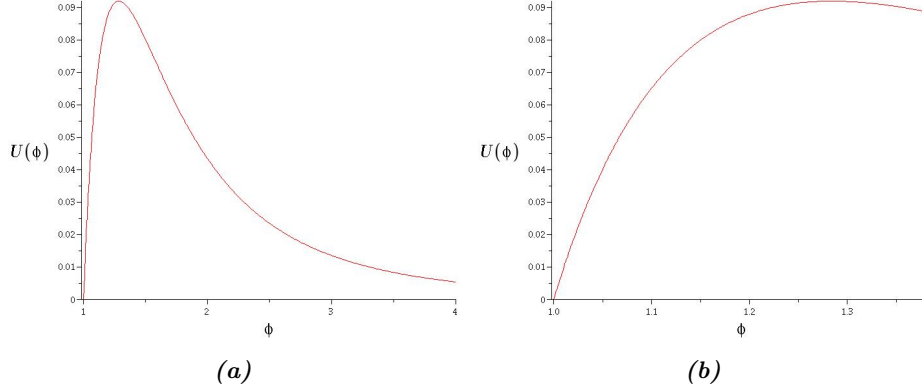


Figure 15: *a* The potential of MGI, where ϕ is in units of Λ , M_P and M is set to 100 and $\xi_2 = 50000$. Inflation ends almost at the left end of the plot and starts at the top of the potential. *b* Zoom of the relevant part of the potential, scales are as in *a*

the top of the potential. I could then try and see if inflation could work starting at the top and then rolling down to ϕ_{end} .

There is however another field value for which ε is 1, but this is below the one I have picked. If inflation runs from the top of the potential it will come to ϕ_{end} first and inflation will end there, never reaching the other value. Since these two are the only values for which inflations can end it is safe to say that inflation could run from the top of the potential at $\phi_{ini} \simeq 1.3288\Lambda$ to $\phi_{end} \simeq 1.1146\Lambda$. These values are the reason why I need to do the full calculations. Making a large field approximations at the start would have been wrong, since I am working around one Λ , the same goes for a small field approximation. A qualitative value can however be extracted from approximated calculations, this is what got me started on the full calculations.

6.3.1 Duration of MGI

With this I have constructed a new inflationary theory, which makes the universe expand. But how large will it make the universe? This can be calculated using the number of e-folds, the standard requirements, as used before, is that \mathcal{N} is at least sixty to explain observations on CMB. The number of e-folds are in this setup defined as

$$\mathcal{N} = \frac{1}{M_P^2} \int_{end}^{ini} \frac{U}{U'} (\chi')^2 d\phi.$$

From this there is a problem, inflation starts where U' is zero so there is a divergence in the integral. A way to solve this is simply to set the start of inflation a bit lower than ϕ_{ini} . In the approximated case the first calculations

led me to believe that moving closer and closer to ϕ_{ini} could increase \mathcal{N} to any order, but this is not the case.

As can be seen in Table 1, \mathcal{N} does increase with ϕ_{ini} moving closer and closer to the top value of the potential at around 1.3288, but it does not increase forever. There is what looks like a maximum value for \mathcal{N} at 28.9 e-foldings, this does not change if I use the reduced Planck mass for M instead of just the Planck mass, neither if I change ξ_2 .

Even though this modified glueball inflation model did produce inflation it does not produce enough. There is a way to solve this problem and the outline will follow, since I have not had the time to go through the calculations.

ϕ_{ini}	\mathcal{N}
1.3	2.26
1.32	3.95
1.328	7.51
1.3287	10.6
1.32879	14.7
1.328796	18.6
1.3287964	21.8
1.32879645	25.2
1.328796455	28.3
1.3287964552	28.8
1.32879645521	28.9

Table 1: Number of e-fold compared to the precision on the initial inflaton field in the MGI model.

6.3.2 Possible Extensions of MGI

The most common way to explain inflation is with one inflaton field, but as I briefly mentioned in section 3.2.5, one could have inflationary models with several inflatonic fields, making a collection of inflatons. This idea have in some special cases been worked out in *Assisted Inflation* [13] and *N-flation* [12]. The main idea is that with more than one inflaton, each field need not expand the universe all sixty e-folds, but only a fraction of this. With 3 similar inflaton fields, each need only inflate around 20 e-folds.

The dependance of the number of inflaton fields arises in both the number of e-folds and in the slow-roll parameters for the models investigated in [12]. If I now would like to invoke the N-flation idea on Modified Glueball Inflation it might be done in several ways. The glueballs originate from a $SU(N)$ gauge theory so at first glance I could imagine that two gluons with one color form one glueball, say a red, and two other gluons of some other color form another glueball, say a blue. Then I could tune the number of inflaton fields with the size of the gauge group by changing N .

This however is not possible because of the way the effective glueball field is derived. The form was $Tr [G_{\mu\nu}^a G^{\mu\nu a}]$, which takes into account the different colors of the glueballs by summing over them. This seems reasonable since if all different colors of glueballs are of the same mass, there is no reason why one should not "decay" into another. Furthermore, one can never know the color of a gluon and thereby neither of a glueball, so I have no means to differentiate one from another.

A more direct approach would be just to invoke a larger gauge group from the beginning like $SU(N) \times SU(N) \times SU(N)$ making 3 sets of glueballs. This might solve the problem of the duration of MGI, since the result from [12] was that if one glueball produces 25 e-folds, then 3 would make 75 e-folds. This would, combined with the standard model, give us a huge gauge group, that might not be as simple as I would like to. However for the low energies of the standard model there is still the need for a product of 3 gauge groups, so I can not rule out that at higher energies an even larger gauge group will arise for inflation, even besides the $SU(5)$ grand unification proposal.

One last idea comes from the fact that in the $SU(N)$ gauge theory there are more massive glueball states. As with [12] if I have states of different masses it opens up for the possibility for the inflationary era to be a stepwise process. As a heavy glueball rolls down its potential it produces inflation. At the bottom, or near it, it could then decay into a lighter state, which then starts to roll down its potential again producing inflation. This could go on for a number of steps until the last and lightest state would produce reheating at the end of its inflationary era. I will call this series inflation. Another model could be parallel inflation where there from the start was a mixture of both heavy and light states, which then, side by side, roll down their potential and at the end produce reheating.

These two models might solve the shortcoming of the MGI duration, but at first glance they could not solve the following problem.

6.3.3 Fine-Tuning in MGI

With tabel 1 it is seen that the number of e-folding during inflation is highly fine-tuned, meaning that there is a high dependence on the exact value of the field ϕ at the start of inflation. At first glance it might not look fine-tuned but remember that the size of the universe is exponential in the number of e-foldings, making a change in \mathcal{N} of just 1 dramatic for the size. Assume that this model was able to produce sixty e-folds of inflation. With this order of fine-tuning a relative fluctuation in the initial field value of just $5 \cdot 10^{-7}$ would change the number of e-foldings by 3 – 4, implying that the universe was only about a 1/10 as big as it should have been or 10 times bigger. Remember from section 3 that all large structures in the universe could be explained by inflation, I can argue that there is an upper limit on the number of e-foldings. If there is too many e-folds, today's structures must come from some extremely small fluctuations, in fact with \mathcal{N} being too large the fluctuations must be of Planck length. This imposes problems with our theories, since they do not work all the way to the Planck scale. In [22] an upper bound on \mathcal{N} is given, not from my argument, but on the entropy during inflation, \mathcal{N} can not be any larger than 85. At the moment this does not have any implications on the model presented, but it does set a constrain if the fine-tuning region of the model was somehow moved to around 60 e-folds, thereby making 85 within reach of change in the initial field value. Of course this is not true a priori for any change, but it should be considered if one tries to change the duration of this inflationary model.

6.4 Preliminary Findings of Modified Glueball Inflation

Even though Glueball Inflation was a well working theory it does have a problem. The non-minimal coupling to gravity seems unnatural. This is due to the square root form which does not come up often in particle physics. One way to overcome this is to make a theory which includes these kinds of interactions, or, as I did, avoid couplings of this form.

Arguing from Feynmann diagrams led to a coupling of the form $\frac{1}{2}\xi_2\Phi R$, where a graviton decays into a single glueball. From calculations point of view this is a more complicated theory, since some of the cancellations in GI did not happen in MGI and in the end precision in the calculations was needed.

Modified Glueball Inflation does produce inflation, however in insufficient amounts. Furthermore I showed that even if MGI produced sixty e-fold of inflation, it would still be highly fine-tuned in the initial region of inflation, making it an unattractive theory to solve fine-tuning problems in the first place. Included in the work still needed for this model are calculations if N-flation could work in this setup, and a more precise calculations on the dependence of M and ξ_2 . More over relevant scales of the theory might be within reach of calculations.

7 Excluded by Reality?

*Experts have their expert fun
ex cathedra telling one
just how nothing can be done.*
Piet Hein

Until now this project has developed 2 working models of inflation, Minimal Composite Inflation and Glueball Inflation. Just taking a part of the models briefly described in section 3.2.5 there is already over 10 different models of inflation. As it is with science not only a theoretical model is needed, but verification from experiments as well. In precision cosmology there is one experiment not to be overlooked; the Wilkinson Microwave Anisotropy Probe, and as will be clear afterwards this experiment more or less put the precision in precision cosmology. I will start with a short description of the experiment and its impact, followed by an explanation of the technical details for the observed parameters and their implications for our models.

7.1 Wilkinson Microwave Anisotropy Probe

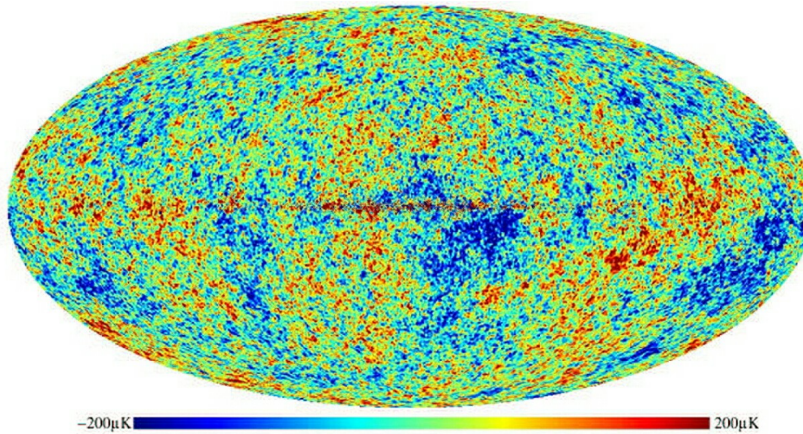


Figure 16: Picture of the WMAP full sky map of the CMB temperature. The picture is a projection of the 3 dimensional space unto a 2 dimensional plot, just like an atlas. The color indicates the temperature fluctuations in μK around the mean temperature [23].

In cosmology one of the observables there is of the early universe is the CMB, and even with the huge impact it has had on the verification of a Hot

Big Bang scenario there is still a lot to learn from CMB [6]. The Wilkinson Microwave Anisotropy Probe, WMAP, is a satellite with instruments designed to measure the temperature of the CMB. The temperature of the CMB is defined by assuming a Planck distribution of the photons and from that use the statistical temperature coming from the density function.

What WMAP does, is to point its detectors in some direction and then sample frequencies of photons, effectively measuring the CMB temperature in that direction. But is there enough statistical material to make such an assumption? Yes; the average number density of CMB photons is around $4 \cdot 10^8 m^{-3}$ [6], so even if WMAP only detects a small part of these the temperature is still calculated based on a lot of photons. Following this procedure, WMAP was able to make a full sky map of the CMB temperature, see figure 16, from which a number of cosmological parameters can be calculated.

The satellite WMAP is funded by NASA in collaboration with Princeton University and was initially planned for a 2 years observation period. This however was extended to a total of 9 years ending in 2010. WMAP was launched to improve the findings of a previous mission, the COBE experiment, from which it became clear that CMB was almost isotropic. Like WMAP, COBE measured CMB temperatures and found that its mean was $2.725K$ with fluctuations of order 10^{-5} , or around $30\mu K$ [6]. It is these fluctuations that gave rise to the primary successes for WMAP, since WMAP had the resolution to observe the structure of these fluctuations. Moreover WMAP has other detectors, designed to measure other aspects of the CMB radiation, for instance the polarization of the photons, which can be linked to what kind of matter fluctuations caused the CMB fluctuations. This makes the WMAP observations ideal for testing inflationary theories since inflation should produce measurable implications on spacetime and the universe which in turn affects the CMB.

With the accuracy of WMAP, cosmologists have moved closer to understanding the universe by further constraining the cosmological standard model; Λ CDM. The current measurements suggest a flat universe with 4.5% baryonic matter, 22.6% cold dark matter and 73.4% dark energy [7]. This is not all of the parameters WMAP extracted from their CMB measurements. In table 2 is a list of inflationary relevant parameters measured by the WMAP group.

Besides this table the WMAP group also produced different kinds of plots, where correlated quantities are shown. One of these is the scalar spectral index and the tensor-to-scalar ratio plot, see figure 17, what these are will come later. From an inflationary model one can calculate some of the quantities from table 2 and see what that model predicts, one way to check one's model is by calculating the scalar spectral index and the tensor-to-scalar ratio, and then insert it into the plot on figure 17 to see if it is within the experimental bounds.

7.2 Linking Theory to Observations

When working with inflationary theories, most of the expressions are formulated in actions or slow-roll parameters, but neither of them are observed directly in nature, so how do one test inflationary theories? This section will try to build

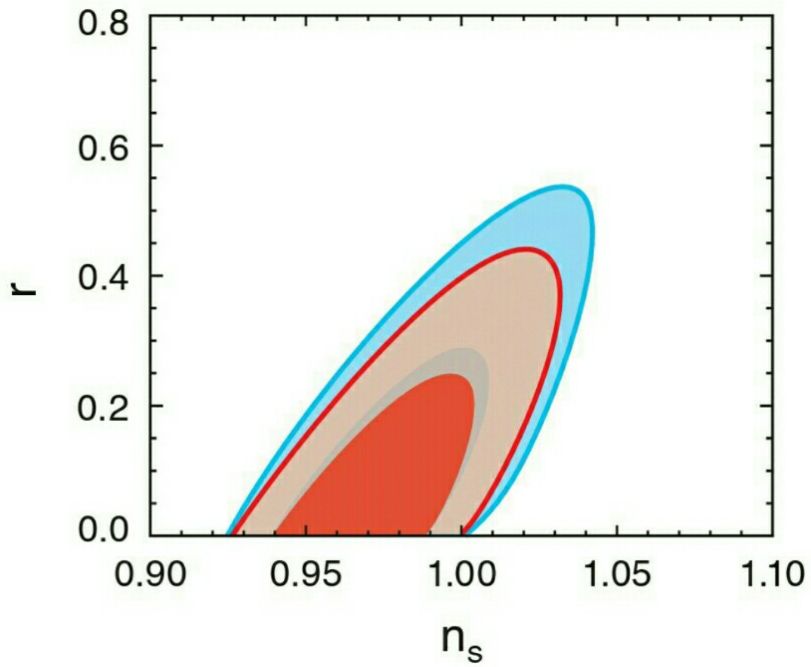


Figure 17: Plot of the scalar spectral index, n_s , vs tensor-to-scalar ratio, r . Starting from outside and going in, the first line (blue) marks the 68% confidence level for 5yr WMAP measurements, next (red) is the 68% CL for 7 yr WMAP, then (blue) 95% CL for 5yr and last (red) the 95% CL for 7yr WMAP measurements[24]. A inflation model can be set in the plot by a point corresponding to the quantities predicted by that model.

Baryonic density	Ω_b	0.0449 ± 0.0028
Cold dark matter density	Ω_c	0.222 ± 0.0028
Dark energy density	Ω_Λ	0.734 ± 0.029
Total density	Ω_{tot}	$1.080^{+0.093}_{-0.071}$
Curvature fluctuations amplitude	$\Delta_{\mathcal{R}}^2$	$(2, 43 \pm 0.11) \cdot 10^{-9}$
Scalar spectral index	n_s	0.963 ± 0.014
Running of spectral index	$\frac{dn_s}{d \ln k}$	-0.034 ± 0.0266
Equation of state	ω	$-1.12^{+0.42}_{-0.43}$
Tensor-to-scalar ratio	r	< 0.36 at 95% CL

Table 2: Results from the 7yr WMAP experiment[7], all measurements at calculated from the full sky map measured by WMAP with local distortions removed.

a bridge between theory and experiments mainly based on the so-called scalar spectral index and the tensor-to-scalar ratio. What these are, will hopefully be clear after reading this section.

When measuring the CMB spectrum, WMAP found that the mean temperature was $2.725K$, but also that there was a perturbation around this of the order 10^{-5} . This motivates cosmological perturbation theory, meaning that this perturbation of the CMB temperature could be seen as some perturbations of the spacetime. But where does these perturbation of spacetime then have their origin? For now the hope is that inflation could solve exactly this problem, and as will be shown, this is actually a natural way to explain the CMB perturbations, or at least a credible explanation.

Assume that the metric is somehow altered with some small change in the following way:

$$g_{\mu\nu}(t, \mathbf{x}) = \bar{g}_{\mu\nu}(t) + \delta g_{\mu\nu}(t, \mathbf{x}),$$

where $\bar{g}_{\mu\nu}$ is our background, the FRW metric. There are 3 different types of perturbations of the metric; scalar, vector, and tensor perturbations. I will work with scalar and tensor perturbations only, since vector perturbation are not produced during inflation, and even if they where they would decay as the universe expanded [2].

To explain the origin of metric perturbations via the inflaton field the Einstein field equation is needed. This is a long and complicated calculation which I will not go through here, it can be found in [2, 5]. I will however list some of the results relevant for this project.

If the metric is perturbed it is not unreasonable to assume that there is some curvature change associated to this perturbation. Assuming that the inflaton field is responsible for the perturbation a gauge invariant quantity can be formed. This quantity is linked to the curvature and is called the comoving curvature perturbation;

$$\mathcal{R} = \Psi - \frac{H}{\dot{\phi}} \delta\phi,$$

here Ψ is the spatial diagonal perturbation of the metric [2], and $\bar{\phi}$ is the background of the inflaton field. One nice property of the comoving curvature perturbation is that it is frozen outside the horizon. This means that once the curvature perturbations are formed during inflation, the rapid expansion of inflation will blow these perturbations outside the Hubble horizon, where the curvature is kept constant. At some later time after inflation the curvature perturbations will reenter the horizon, due to expansion of the Hubble sphere, making it possible to measure them. It is however not directly the comoving curvature perturbation that is calculated from the CMB measurements. It is the curvature fluctuation amplitude, $\Delta_{\mathcal{R}}^2$. A statistical measure of the comoving curvature perturbation is the power spectrum of \mathcal{R} , $P_{\mathcal{R}}(\mathbf{k})$, defined by

$$\langle \mathcal{R}_{\mathbf{k}} \mathcal{R}_{\mathbf{k}'} \rangle = (2\pi)^3 \delta(\mathbf{k} + \mathbf{k}') P_{\mathcal{R}}(k),$$

where $\mathcal{R}_{\mathbf{k}}$ is the Fourier transform of \mathcal{R} . From this the curvature fluctuation amplitude can be defined:

$$\Delta_{\mathcal{R}}^2 = \frac{k^3}{2\pi^2} P_{\mathcal{R}}(k).$$

Once the curvature fluctuation are in place, calculations of the scalar spectral index, which measures the scale dependence of the curvature fluctuations, can be started;

$$n_s - 1 = \frac{d \ln \Delta_{\mathcal{R}}^2}{d \ln k}. \quad (7.1)$$

The scalar spectral index is one of the main quantities I will use for testing inflationary theories.

Now that the short treatment of the scalar perturbations relevant for inflation are done I can move to tensor perturbations which also can become relevant for testing inflation. The perturbations on the metric might have a change in the spatial components, lets call this change h_{ij} . This looks like the notation of gravitational waves, and in fact tensor perturbations are also called gravitational wave perturbations. Just like the scalar perturbations, tensor perturbations are not directly measured, but one can derive quantities that are possible to measure. First it should be noted that the tensor perturbations can be formulated via two orthogonal polarization modes, h^+ and h^\times . To each of these, the power spectrum can be associated:

$$\langle h^a(\mathbf{k}) h^a(\mathbf{k}') \rangle = (2\pi)^3 \delta(\mathbf{k} + \mathbf{k}') P_{h^a}(k),$$

here a is either $+$ or \times . Then the total tensor fluctuation amplitude is given by the sum of the two polarizations:

$$\Delta_t^2 = 2\Delta_{h^a}^2.$$

And as I did for equation (7.1) I can now form the tensor spectral index,

$$n_t = \frac{d \ln \Delta_t^2}{d \ln k}.$$

The last quantity I need is the tensor-to-scalar ratio, r , not surprisingly this is the ratio between the tensor and scalar fluctuations:

$$r = \frac{\Delta_t^2}{\Delta_{\mathcal{R}}^2}.$$

The definitions above are aimed for experimentalists and are not very useful from a theoretical point of view. However assuming the slow-roll paradigm both the scalar spectral index and the tensor-to-scalar ratio take very simple forms in the slow-roll parameters [2]. One thing must be emphasized; when calculating the following quantities all factors must be evaluated at the time where the relevant scale leave the Hubble sphere. The reason being that the comoving curvature perturbation freezes outside the horizon. More over there is no way to have any clues on what happens outside the horizon, so the fact that perturbations are fixed is both a necessary and a lucky coincidence. From now on quantities that are evaluated when they leave the horizon are marked with a star, *. It need not, however, be at the same time, or number of e-foldings, for all quantities, since different scales exits the horizon at different moments. Since I mostly will be comparing theory to WMAP observations of CMB, the relevant scale is when CMB exits. This happens 60 e-foldings before the end of inflation [2].

For the slow-roll paradigm I have from [2] that for the scalar spectral index

$$n_s - 1 = 2\eta^* - 6\varepsilon^* \quad (7.2)$$

and for the tensor-to-scalar ratio

$$r = 16\varepsilon^*. \quad (7.3)$$

Here then, slow-roll parameters are the potential slow-roll parameters, as my convention from section 3.2.3 implies. With a link between observables and theory, I am now ready to check if WMAP have excluded Minimal Composite Inflation or Glueball Inflation.

7.3 Theoretical Observables

In the following I will calculate both the scalar spectral index and the tensor-to-scalar ratio in both the Minimal Composite Inflation model and in Glueball Inflation. As already mentioned I have to compute them at horizon exit for the relevant scale. The relevant scale here is the CMB scale which is 60 e-foldings before inflation ends

7.3.1 Minimal Composite Inflation

First let me restate the slow-roll parameters from MCI

$$\varepsilon = \frac{4}{3} \frac{M_P^4}{\xi^2 \sigma^4}, \quad \eta = \frac{4}{3} \frac{M_P^2}{\xi \sigma^2}.$$

Next thing to do is to find at what value of σ , where the CMB exits the horizon. From section 4.2.5 remember that the requirement for the observed CMB spectra was for $\mathcal{N} \simeq 60$. Moreover this was used to obtain the lowest initial value for the composite inflaton capable of producing such a spectra. This implies that for the CMB relevant scale I can use what is called the initial value of the composite inflaton field in section 4.2.5, this was $\sigma_{ini} = 9 \frac{M_P}{\sqrt{\xi}}$. It is now trivial to calculate the following

$$n_s^{MCI} = 1 + 2 \frac{4}{3} \frac{1}{9^2} - 6 \frac{4}{3} \frac{1}{9^4} = 1.0317 \quad (7.4)$$

$$r^{MCI} = 16 \frac{4}{3} \frac{1}{9^4} = 0.00325. \quad (7.5)$$

Now that I have some measurable numbers for MCI I can compare with table 2. It is seen that the tensor-to-scalar ratio is safely within 95% CL, so with only this parameter I can not exclude MCI. However by taking the scalar spectral index into account I must, from only table 2, say that the MCI model is excluded. Table 2 was a recap of the findings from [7]. There is however a more detailed article regarding the CMB spectrum [24], it is from here figure 17 is taken. The MCI model can of course be found in this plot as well. This puts it outside both the 95% and the 68% confidence level for the 5 year WMAP measurements and for the improved 7 year WMAP measurements. So it is safe to say that WMAP has excluded Minimal Composite Inflation. Even though it would be exciting with MCI as a fully working and plausible theory it is excluded. The MCI model is mainly a toy model, build to show that inflation could be running under a strongly coupled theory with a composite scalar as the inflaton. The concepts from MCI can be used as building blocks for other models, as it is with the Glueball model.

7.3.2 Glueball Inflation

What I now will do is the same analysis as I did with Minimal Composite Inflation, just in the case of Glueball Inflation. One might remember that the second slow-roll parameter for Glueball Inflation was identical zero, so for this analysis I have to worry only about the first slow-roll parameter, ε (5.10). Evaluating this at the initial value of the glueball field gives

$$\varepsilon^* = \frac{1}{12 \ln^2 \left(\frac{\phi_{ini}}{\Lambda} \right)} = \frac{1}{241}.$$

The reason for using the initial value of the glueball field is the same as it was with the initial value of the composite field in MCI, I have already assumed inflation to start, so that in at least explains observations from CMB i.e. inflation start from 60 e-foldings. The model could be extended to last for a longer period, but for this analysis there is no need to do so.

With the slow-roll parameter calculated I can now move to calculate the two

observables. from (7.2) and (7.3) i get for Glueball Inflation

$$n_s = -\frac{6}{241} + 1 \simeq 0.9751$$

$$r = \frac{16}{241} \simeq 0.06639$$

Comparing to table 2 it can be seen that both the scalar spectral index and the tensor-to-scalar ratio is within the bounds set by WMAP, hence Glueball Inflation in the form presented in [20] can not be excluded by current measurements, and hence is still a possible explanation for inflation. Using the n_s vs r plot, figure 17, Glueball Inflation is in the lower center of the 95 % confidence level, this means that it is not excluded. A naive estimate from only the plot shows that the shortest distance from the Glueball Inflation point to outside the 95 % CL region, is by having a lower bound on the tensor-to-scalar ratio. This can be obtained by observation of tensor modes in the CMB spectrum, one of the goals of the Planck satellite. This might exclude Glueball Inflation in its current form.

8 Conclusions

For more than the last 60 years cosmologists have been working with the big bang theory. In its first layout, the universe started of as a hot dense state and expanded from here, forming the universe as it is today 13.7 billion years after. There has been many successes of the standard big bang theory, where the biggest might be the production of the CMB and the natural formation of primordial hydrogen and helium. There is, however, also a lot of problems with the standard big bang theory. Some are The Flatness Problem, The Horizon Problem and The Monopole Problem. The first two require extreme fine-tuning of the initial state of the universe to explain todays observables, and the third is a problem of unobserved exotic particles.

A way of combining the good sides of a big bang theory with a theory that solves the problems of the standard big bang is via a period called inflation. Inflation is a era of rapid expansion of the universe. It stretches spacetime, solving the Flatness problem, makes it possible for a small universe before inflation solving The Horizon Problem and enlarges the universe by many factors explaining why exotic particles have not been observed. Furthermore at the end of inflation is a period called reheating, which heats up the universe once again ensuring the formations of primordial elements and later the CMB. Every era in the history of the universe is explained by some type of particle being dominant, so what governs inflation?

The initial proposal in 1981 by A. Guth and A. Linde in 1982 was that inflation was driven by a scalar particle, which later was named the inflaton. In most models the inflaton is a fundamental scalar, which via a dominant potential, produces a period of nearly exponential expansion. As inflation solves the problems of the standard big bang it imposes its own problems; only one, for the moment, fundamental scalar have been observed. Evenmore a fundamental scalar particle on its own, produces huge problems in our theory of nature: quantum field theory. The most well know scalar particle is the Higgs boson of the standard model. When trying to renormalize a theory including a fundamental scalar is produces untamed divergences in, for instance, the mass operator of the Higgs boson. A lot of the problems with the Higgs scalar particle applies to a fundamental scalar inflaton as well, but why not use some of the same tools to solve the problems with the inflaton as one does for the Higgs.

One of the type of models that solve the problems of the Higgs sector is Technicolor models, where the scalar is not fundamental, but a composite state. This setup had until 2011 not been used for inflation, but Phongpichit Channuie, Jakob Jark Jørgensen and Francesco Sannino in their article "*Minimal Composite Inflation*" [14] worked out how inflation could come to work in a composite setup.

With the work of the project several things have become clear in the composite inflation setup. I have worked with 3 different models of composite inflation, of which 2 of them are working models of inflation and 1 is still not excluded by precision cosmology. That a composite setup can be made to work should at this point be clear, but at first glance it might have seemed that any com-

posite model could work since it includes extra parameters to be tuned, thereby making it more flexible. This is however not true.

The first model, Minimal Composite Inflation, was the first setup for composite inflation. As an extension of the standard model I assumed a Technicolor model similar to Minimal Walking Technicolor [15] to be responsible for inflation. This new model included two new fermions and the condensations of these was used as the inflaton of Minimal Composite Inflation. The main feature of the composite inflation models I have worked on is that the inflaton is coupled non-minimally to gravity, and the same goes for Minimal Composite Inflation. With this coupling to gravity I was able to derive the slow-roll parameters of the model, thereby checking if inflation was working in this framework. Furthermore the new coupling to gravity includes a coupling constant. With constraints from the WMAP experiment I was able to derive the composite scale from this coupling. The composite scale is the energy at which the composite inflaton decomposes into its building blocks, the fundamental fermions. This scale was derived to be the GUT scale, imposing that the composite inflaton could not be the composite Higgs particle from former electroweak Technicolor models.

Minimal Composite Inflation however turned out to be merely a toy model for compositeness at the inflationary scale, since measurements from WMAP excluded the model. It did serve as a foundation for another model, Glueball Inflation, also by Phongpichit Channuie, Jakob Jark Jørgensen and Francesco Sannino as well as Fedor Bezrukov [20].

Inflation is not at this point included in the standard model of physics, so there is the need for an extension to explain inflation. In Minimal Composite Inflation the extension was a Technicolor model, but in Glueball Inflation it is a pure Yang - Mills theory. In this model the inflaton condensates from a new $SU(N)$ gauge symmetry forming glueballs. Again, what made the model work was a non-minimal coupling to gravity, from which the coupling constant again implied the GUT scale as the scale of the theory. This neatly decoupled the inflaton from the standard model and opened the opportunity that the inflaton could decompose before the Planck scale, removing any divergences that might be in the scalar description.

Glueball Inflation as a model of composite inflation is a simple looking model, it includes only a Yang - Mills Theory on top of the standard model to work. Besides that it, in this context, looks simple, Glueball Inflation is not yet excluded by WMAP at the level of my investigation. Maybe in the near future, when the results of the Planck experiment arrive, there will come more clear answer to whether or not Glueball Inflation could be the right way to explain inflation.

Even though Glueball Inflation seems to be the best model so far for composite inflation it suffers from a significant problem; the non-minimal coupling to gravity does not look like any ordinary particle interaction. It includes a square root of the glueball field coupled to the graviton. This type of scattering have not been seen. Maybe this is due to the fact that physicist have not yet made gravity renormalizable and that this would fix the problem, but another way is to remove this type of coupling, imposing a new, more straightforward

coupling, where one glueball couples to one graviton.

This led to Modified Glueball Inflation, which invokes just this change. By arguing from Feynmann diagrams I came up with this new coupling, and worked out a model that produces inflation of the universe. This, however, is not a working model of inflation, since it does not inflate the universe enough, thereby not solving the initial problems of the Standard Big Bang. Further more the model suffers from fine-tuning at the start of inflation, so unless these problems are solved Modified Glueball Inflation is not a theory that could explain inflation.

Overall I have argued that using a composite scalar to drive inflation can be made to work, and it seems like a general tendency that the scale of these types of inflationary models is the GUT scale, making the effective description of the condensate valid for inflationary era. Further calculations is needed in all models to ensure precision and to work out reheating properties of that specific model, opening for even more tests.

Acknowledgments

I would like to thank Francesco Sannino for his guiding comments and discussions together with his help and support for my project. Thanks goes to Martin Sloth as well for discussions on the physics of CMB and inflation in general. Jakob Jørgensen and Phongpichit Channuie are to be thanked for help and support in general on composite inflation. During the process and in the final stages Jens Krog provided helpfull discussions on formulations and clarity. Furthermore I would like to thank Isabella Masina for taking her time as referee. Special gratitude goes to my girlfriend Heidi Hansen for patiently listening to progress on the project every day without complaining.

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