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ACCURATE BEAM SPOT FITTING ALGORITHM USING GENERALIZED AND SKEWED GAUSSIAN TYPE DISTRIBUTIONS*

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Abstract

To address non-standard Gaussian beam spot profiles in injectors, this paper proposes a fitting algorithm based on Gaussian, the newly introduced Generalized Gaussian Type and Skewed Gaussian Type distributions. These distributions are specifically designed to better fit non-Gaussian and asymmetric beam spots by automatically selecting the most suitable model. Validation using beam spot images from the YAG screen after the electron gun in the Hefei Light Source II (HLS-II) injector demonstrates that the Generalized Gaussian Type is effective for fitting sharp or broad profiles, while the Skewed Gaussian Type is well-suited for handling asymmetry. Compared to traditional methods, the proposed algorithm improves fitting accuracy and adaptability, providing a practical solution for complex beam measurement challenges.

INTRODUCTION

Traditional transverse beam size measurement methods often rely on symmetric Gaussian fitting, which is adequate for many conditions. However, in practical applications, especially in linear accelerators (LINACs) based injectors, beam spots typically show non-Gaussian distributions [1], including asymmetry, skewness, or multiple peaks. These non-Gaussian features limit the effectiveness of traditional Gaussian fitting, making it insufficient for accurate transverse beam distribution measurements.

To overcome these limitations, this paper proposes an improved fitting algorithm that introduces the newly Generalized Gaussian Type and Skewed Gaussian Type distributions to more accurately fit the transverse beam spot shapes. Compared to traditional Gaussian fitting, these models can flexibly adjust symmetry, peak position, width, and skewness, offering higher fitting accuracy and better adaptability to complex beam spot shapes. The goal of this study is to determine the optimal fitting method by comparing the performance of different algorithms, thereby improving the accuracy of beam size measurements in LINACs-driven injectors of synchrotron radiation source [2].

FITTING THEORY

To accurately measure and describe the transverse beam size in synchrotron radiation sources, the fitting algorithm must be capable of flexibly handling various complex beam spot shapes. The improved algorithm proposed in this paper is based on three fitting models: the traditional Gaussian distribution, and the newly introduced Generalized Gaussian Type and Skewed Gaussian Type distributions, both of which were developed in this study. The following sections provide a detailed introduction to these three fitting models and their theoretical foundations.

Gaussian Distribution

The probability density function of the Gaussian distribution is given by:

$$f(x; a, x_0, \sigma) = a \cdot exp\left(-\frac{(x-x_0)^2}{2\sigma^2}\right), \qquad (1)$$

where a is the amplitude, representing the peak height; x_0 is the center position, indicating the peak location of the beam spot; and σ is the standard deviation, corresponding to the beam spot size. A schematic of the Gaussian distribution is shown in Fig. 1, illustrating the symmetry of the distribution and the range of the standard deviation. The standard deviation σ determines the width of the curve, reflecting the size of the transverse beam.

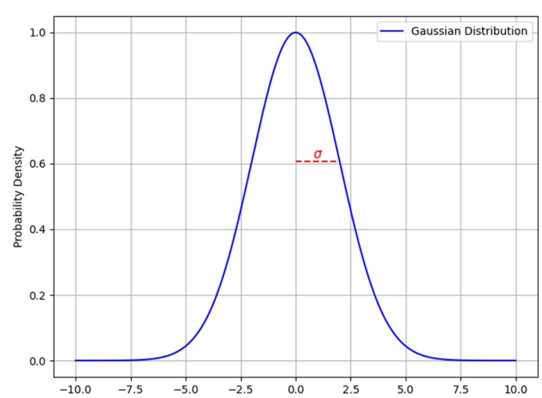

Figure 1: Schematic of the Gaussian distribution.

The Gaussian distribution assumes that the beam spot is symmetric and unimodal, which is reasonable in many cases. However, when the beam spot exhibits asymmetry, skewness, or multiple peaks, Gaussian fitting often fails to accurately represent the actual shape of the beam spot.

Generalized Gaussian Type Distribution

To better accommodate the diverse shapes of beam spots, this study introduces a distribution similar to the Generalized Gaussian, called the Generalized Gaussian

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Type. This model includes an additional parameter p, allowing for greater flexibility in controlling the shape of the distribution. This makes it particularly effective for fitting non-standard Gaussian beam spots, such as those that are unusually sharp or broad. The formula for this distribution is given by:

$$f(x; a, x_0, \beta, p) = a \cdot \exp\left(-\left(\frac{|x - x_0|}{\beta}\right)^p\right), \quad (2)$$

where β is the scale parameter, similar to the standard deviation, and it controls the width of the distribution. The parameter p is the shape parameter that determines the distribution's form. When $p = 2$, the Generalized Gaussian Type distribution simplifies to resemble the standard Gaussian distribution; When $p < 2$, the distribution becomes sharper; and when $p > 2$, it becomes flatter. A schematic of the Generalized Gaussian Type distribution is shown in Fig. 2, illustrating how the distribution transitions from a sharp peak to a broad flat top as p varies.

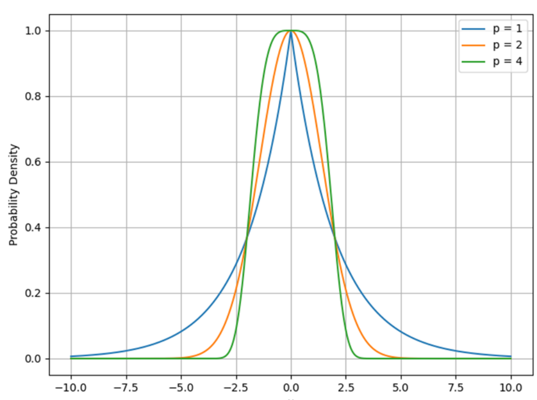

Figure 2: Schematic of the Generalized Gaussian Type distribution.

Skewed Gaussian Type Distribution

To address asymmetric beam spot shapes, this study introduces a distribution similar to the Skewed Gaussian, called the Skewed Gaussian Type distribution. This distribution extends the standard Gaussian by incorporating a skewness parameter α, allowing it to effectively describe the asymmetry in the beam spots. The functional for this distribution is given by:

$$f(x; a, x_0, \sigma, \alpha) = a \cdot \frac{2}{\sigma\sqrt{2\pi}} \cdot exp\left(-\frac{(x - x_0)^2}{2\sigma^2}\right) \cdot \Phi\left(\alpha \frac{x - x_0}{\sigma}\right), \quad (4)$$

where α is the skewness parameter, which controls the direction and degree of the distribution's skew. When $\alpha > 0$, the distribution skews to the right; when $\alpha < 0$, it skews to the left. Φ represents the cumulative distribution function (CDF) of the standard normal distribution, which adjusts the asymmetry of the distribution. A schematic of the Skewed Gaussian Type distribution is shown in Fig. 3, illustrating the shapes of the distribution under different skewness values.

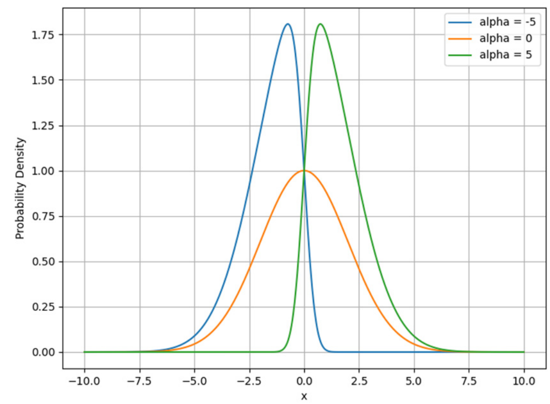

Figure 3: Schematic of the Skewed Gaussian Type distribution.

Fitting Algorithm

The fitting algorithm is based on the least squares method, which adjusts model parameters to minimize the error between the actual data and the fitted curve. The specific steps are as follows.

(1) Initial Parameter Estimation: Select appropriate initial parameters (such as amplitude, center position, and width) to ensure rapid convergence of the fitting process.

(2) Error Minimization: Utilize the least squares method to continuously adjust the parameters, optimizing the fitted curve so that it closely matches the actual data points.

(3) Model Selection: Evaluate the fitting models using the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) [3]. AIC and BIC consider not only the goodness of fit but also introduce penalties for model complexity, ensuring the selection of models that accurately fit the data without overfitting.

$$AIC = 2k - 2\ln(L) \quad (5)$$

$$BIC = k\ln(n) - 2\ln(L), \quad (6)$$

where k is the number of parameters in the model, L is the likelihood function of the fit, and n is the number of data points.

EXPERIMENTAL VALIDATION

Data Source

The experimental data were obtained from beam spot images measured at the YAG screen located closest to the electron gun in the injector of the Hefei Light Source II (HLS-II) [4]. Positioned at a critical location after the electron gun, this YAG screen accurately reflects the initial transverse size distribution of the electron beam.

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Algorithm Workflow

The specific workflow of the algorithm includes:

Data Preprocessing: The collected beam spot images undergo background subtraction and edge cropping to remove background noise and non-beam light interference at the image edges. These preprocessing steps are intended to improve data quality, ensuring that the fitting process focuses on the relevant beam spot information, thereby enhancing the accuracy and precision of the measurements.

Projection Transformation: The preprocessed image data are projected onto the horizontal and vertical directions, generating probability density curves that represent the transverse distribution of the beam.

Fitting Models: Three fitting models—Gaussian, Generalized Gaussian Type, and Skewed Gaussian Type distributions—are sequentially applied to fit the projection curves, estimating the beam size and shape characteristics for each model.

Model Evaluation and Selection: The fitting results are evaluated using the AIC and BIC. By comparing AIC and BIC values, the algorithm comprehensively considers fitting accuracy and model complexity to select the optimal model and provide the corresponding fitting results.

Fitting Results

The actual beam size is related to pixel size and optical magnification; therefore, this discussion focuses solely on the effectiveness of the fitting algorithm without involving specific units.

Figure 4 shows the original beam spot image on the left and the preprocessed image on the right, where the preprocessing successfully retains the relevant beam spot information.

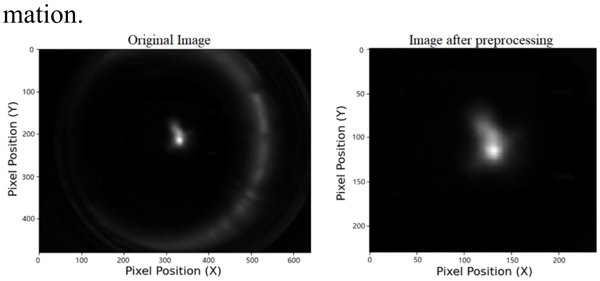

Figure 4: Original beam spot image (left) and preprocessed image (right).

Figure 5 shows the beam spot fitting results, with the horizontal direction on the left and the vertical direction on the right, comparing the performance of the three fitting models. The black solid line represents the original transverse beam distribution, while the blue, green, and orange dashed lines correspond to the fitting results of the Gaussian, Generalized Gaussian Type, and Skewed Gaussian Type distributions, respectively.

To quantitatively compare the fitting performance of each model, we calculated the AIC and BIC values for the fitted curves. In the horizontal direction, the AIC and BIC values for the Gaussian distribution are 2878.68 and 2889.12, respectively; for the Generalized Gaussian Type distribution, they are 2703.16 and 2717.09; and for the Skewed Gaussian Type distribution, they are 2878.22 and 2892.14. In the vertical direction, the AIC and BIC values for the Gaussian distribution are 2669.57 and 2679.89, respectively; for the Generalized Gaussian Type distribution, they are 2647.43 and 2661.18; and for the Skewed Gaussian Type distribution, they are 2583.57 and 2597.32.

The results show that in the horizontal direction, the Generalized Gaussian Type distribution has the lowest AIC and BIC values, indicating the best fitting performance. In the vertical direction, the Skewed Gaussian Type distribution has the lowest AIC and BIC values, also indicating the best fitting performance. Since our focus is on the transverse beam size and fitting effectiveness, the amplitude and center position fitting results are not provided in the following.

In the horizontal direction, the parameter results for the Generalized Gaussian Type fitting curve are as follows: the scale parameter $\beta = 16.06$ pixels, and the shape parameter $p = 1.08 < 2$, indicating a distribution sharper than the standard Gaussian; in the vertical direction, the Skewed Gaussian Type fitting results show a skewness parameter $\alpha = -2.47 < 0$, indicating a left-skewed distribution, with $\sigma = 33$ pixels.

In transverse beam size measurements, the Gaussian distribution is commonly used for fitting, and its standard deviation σ is typically used to represent the beam size. However, the scale parameter β in the Generalized Gaussian Type distribution and the σ in the Skewed Gaussian distribution. Therefore, to provide a consistent measure across different distributions, we choose to represent the transverse beam size using the Root Mean Square (RMS) of the fitted curve. The RMS of the fitted curves is initially in pixel units. To obtain the true RMS, these values need to be multiplied by the camera scaling factor of $0.1224\ \mathrm{mm/pixels}$.

The transverse beam sizes represented by the RMS of the fitted curves from the three models are summarized in Table 1. In this table, RMS_h denotes the RMS in the horizontal direction, while RMS_v represents the RMS in the vertical direction.

Table 1: Transverse Beam Size Represented by RMS of Fitted Curves

Item	**Gaussian**	**Generalized Gaussian Type**	**Skewed Gaussian Type**
RMS_h	1.624 mm	2.445 mm	1.632 mm
RMS_v	2.575 mm	2.982 mm	2.718 mm

Discussion

The experimental results demonstrate that the Generalized Gaussian Type and Skewed Gaussian Type distributions perform exceptionally well in handling beam spots with non-standard Gaussian characteristics. The Generalized Gaussian Type distribution, by adjusting the shape parameter p, can flexibly accommodate shapes that are sharper or flatter than the standard Gaussian, thereby providing more precise fitting. The Skewed Gaussian Type

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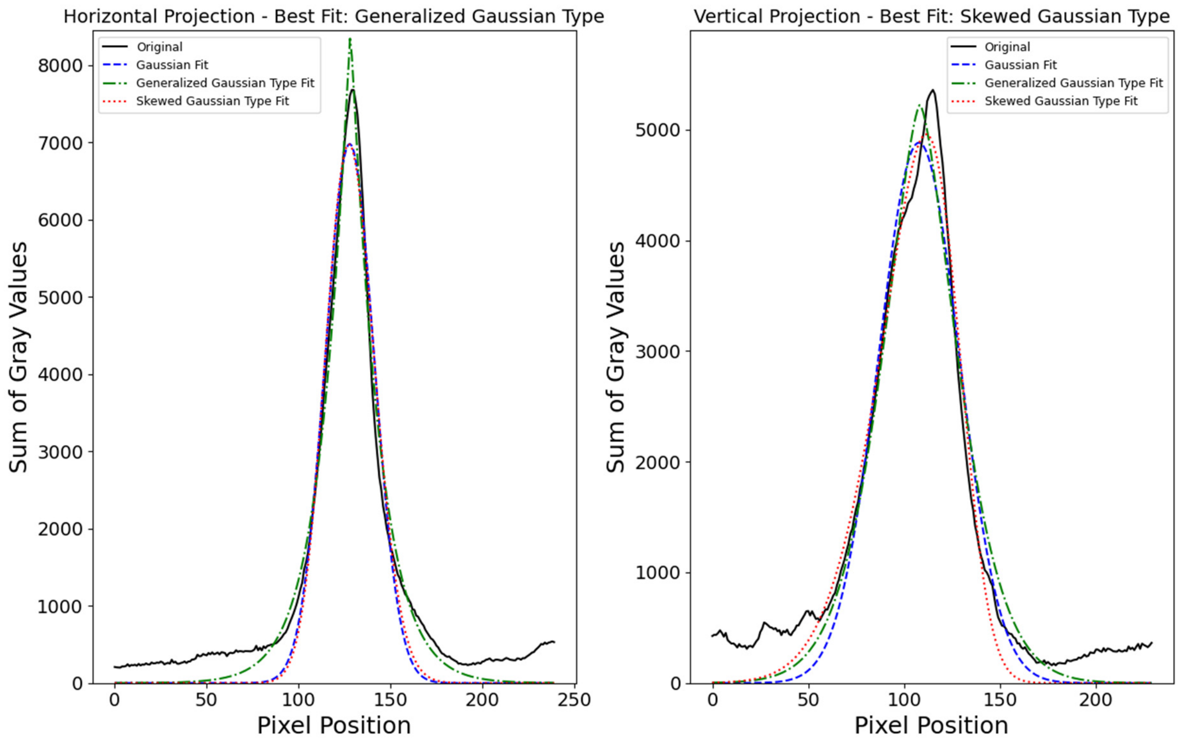

Figure 5: Beam spot fitting results in the horizontal and vertical directions.

distribution, through the introduction of the skewness parameter α, effectively captures the asymmetry of the beam spot, making it particularly advantageous for fitting asymmetric transverse beam distributions.

These results validate the applicability and flexibility of the proposed fitting algorithm across different distribution shapes, highlighting its potential in high-precision beam measurements. Compared to traditional Gaussian fitting methods [5], the Generalized Gaussian Type and Skewed Gaussian Type distributions can more accurately depict the actual distribution characteristics of the beam. Additionally, the algorithm presented in this paper can automatically select the optimal model among the Gaussian, Generalized Gaussian Type, and Skewed Gaussian Type distributions based on fitting performance. This selection mechanism not only improves fitting accuracy but also enhances the robustness of the algorithm under complex beam conditions, providing critical support for further improving the beam measurement accuracy in synchrotron radiation sources.

CONCLUSION

This paper proposes a fitting algorithm based on Gaussian, Generalized Gaussian Type, and Skewed Gaussian Type distributions for measuring the transverse beam size in synchrotron radiation sources. The experimental results indicate that the Generalized Gaussian Type distribution excels in handling non-standard Gaussian distributions, while the Skewed Gaussian Type distribution effectively fits asymmetric beam spots. Compared to traditional Gaussian fitting methods, the Generalized Gaussian Type and Skewed Gaussian Type distributions more accurately reflect the actual characteristics of the beam.

The algorithm enhances fitting accuracy and adaptability by automatically selecting the optimal fitting model among the three distributions, providing reliable support for high-precision beam measurements. Future work will explore incorporating models that combine multiple Gaussian distributions into the algorithm to further improve fitting accuracy and robustness for complex beam distributions.

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