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SLED:

A Method for Doubling SLAC's Energy

This note gives the theory of a method for increasing SLAC's energy using passive microwave components. The overall system will be called SLED (SLAC Energy Doubler) for convenience. In SLED, microwave networks are inserted in the output waveguide line of each klystron. These networks, which contain high Q resonant cavities, store energy in the klystron power output pulse over a large fraction of the pulse length and then deliver it to the accelerating sections during a much shorter period. By this means, peak power is enhanced at the expense of pulse width. Triggered phase shifters which can flip the input phase to the klystrons by 180° during the drive pulse are also required. In addition, it will be advantageous to lengthen the klystron modulator pulse in order to increase the power enhancement factor that can be obtained from SLED. Second-order effects, such as pulse jitter, ripple on the pulse amplitude, finite pulse rise-time, and inperfections in the microwave components of the power enhancement network are not taken into account in this note. Engineering design problems such as the temperature stability requirements on the high Q cavities, although important to the practical realization of an operating system, also will not be discussed here. Initial considerations indicate, however, that no unusual problems should be encountered in the engineering design of SLED.

Figure 1 shows a sketch of the microwave components in a typical power enhancement network (PEN) inserted between a klystron and its associated accelerating sections. We will first calculate the fields at the output of the PEN as a function of time. Next, the electric field $E(z,t)$ in the accelerating sections will be computed as a function of time t and length z along the structure, taking into account the variation in group velocity with z. By integrating the field with respect to z, the accelerating voltage is then obtained as a function of time. Examples of the accelerating field along the structure, and the energy gain as a function of time, will be given for a particular choice of practical system parameters which leads to an increase in the maximum SLAC energy by a factor of 1.84. Beam loading is investigated, assuming that the accelerating voltage waveforms are identical for each klystron and that they follow the theoretical variation. It is calculated that an average current of 13 μA can be accelerated at a loaded energy of 43 MeV (unloaded energy = 48 MeV), within an energy spectrum width of about one-half percent.

Field at the Output of the PEN

The variation as a function of time of the fields in the waveguide at the output of the klystron is shown at the top of Fig. 2. This will also be the input waveform to the PEN. At time t_1 an instantaneous phase reversal of exactly 180° is assumed. At time t_2 the klystron output pulse ends. The rise and fall times of the pulse are assumed to be negligible. The two cavities are also assumed to be tuned exactly to resonance. If the phase reversal is not exactly 180°, or if the cavities are not tuned precisely to resonance, the voltages must be treated by means of a vector diagram and the analysis is then considerably more complex. Assuming that the cavities in the PEN are identical, there will be no power reflected back to the klystron. This will also be true even if the cavities are off resonance, assuming that they are both detuned by the same amount. It will prove convenient to consider the field E_L at the output of the PEN to be the algebraic sum of an incident wave E_g from the generator and an emitted wave E_e from the cavities. We will calculate the fields at the output of the PEN during three time intervals, denoted as A $(0 < t < t_1)$; B $(t_1 < t < t_2)$; and C $(t > t_2)$. During time interval A, the PEN cavity fields and hence the emitted field vary as

$$E_e(A) = -\alpha\left(1 - e^{-t/T_C}\right) ,$$

where $\alpha \equiv 2\beta/(1 + \beta)$, β is the cavity coupling coefficient, and T_C is the cavity filling time, $T_C = 2Q_L/\omega$. At time $t = t_1$, the emitted field is

$$E_{e1} = -\alpha\left(1 - e^{-t_1/T_C}\right) .$$

The minus sign is used because the emitted wave has a phase which tends to cancel the direct incident field from the generator, which has been assumed positive ($E_g = +1$). For $t_1 \rightarrow \infty$ the cancellation is exact for critical coupling ($\alpha = 1$, $E_e \rightarrow -1$), and the load field would then be zero. The emitted field waveform during time interval A is sketched in Fig. 2.

The load field during pulse interval A is $E_L(A) = E_e(A) + E_g(A) = E_e(A) + 1$, or

$$E_L(A) = 1 - \alpha\left(1 - e^{-t/T_C}\right) = \alpha e^{-t/T_C} - (\alpha - 1) . \quad (1)$$

At time $t = t_1 - \delta t$, the load field is

$$E_{L1}^{-} = 1 - \alpha\left(1 - e^{-t_1/T_C}\right) .$$

At time $t = t_1 + \delta t$, the load field is obtained as $E_{L1}^{+} = E_{e1} + E_g(B) = E_{e1} - 1$, or

$$E_{L1}^{+} = -\left[1 + \alpha\left(1 - e^{-t_1/T_C}\right)\right]. \quad (2)$$

Note that $\Delta E_{L1} = E_{L1}^{-} - E_{L1}^{+} = -2$. This discontinuity in the load field waveform at time t_1 is shown in Fig. 2.

During time interval B, the fields in the PEN cavities and hence the emitted field vary exponentially between $E_e = E_{e1}$ and $E_e = +\alpha$, which is the level the field would eventually reach were the pulse $E_g(B) = -1$ to last indefinately. That is,

$$E_e(B) = (E_{e1} - \alpha) \; e^{-(t - t_1)/T_C} + \alpha \quad .$$

At time t_2 the emitted field is

$$E_{e2} = (E_{e1} - \alpha) \; e^{-(t_2 - t_1)/T_C} + \alpha .$$

Since $E_g(B) = -1$, the load field is given by $E_L(B) = E_e(B) - 1$, or

$$E_L(B) = (E_{e1} - \alpha) \; e^{-(t - t_1)/T_C} + (\alpha - 1) .$$

Expressing the field in terms of $E_{L1}^{+} = E_{e1} - 1$,

$$E_L(B) = (E_{L1}^{+} - \alpha + 1) \; e^{-(t - t_1)/T_C} + (\alpha - 1) . \tag{3}$$

At time $t = t_2 - \delta t$, $E_L(B) = E_{L2}^{-}$ where

$$E_{L2}^{-} = (E_{L1}^{+} - \alpha + 1) \; e^{-(t_2 - t_1)/T_C} + (\alpha - 1) . \tag{4}$$

During time interval C, the generator field is zero, the load field is then equal to the emitted field, and both vary as

$$E_L(C) = E_e(C) = E_{e2} \; e^{-(t - t_2)/T_C} .$$

Since $E_{e2} = E_{L2}^{+} = E_{L2}^{-} + 1$,

$$E_L(C) = (E_{L2}^{-} + 1) \; e^{-(t - t_2)/T_C} . \tag{5}$$

Note again that the discontinuity in the load field waveform at $t = t_2$ is equal to the discontinuity in the incident waveform, ΔE_g. This is shown in Fig. 2.

Equations (1), (3) and (5) show how the output field from the PEN varies during time intervals A, B and C in terms of the basic cavity and pulse parameters and the fields E_{L1}^{+} and E_{L2}^{-}. In turn, the fields E_{L1}^{+} and E_{L2}^{-} are given by Eq. (2) and (4). The field E_{L1}^{+} is important because it is the highest field reached at any time, while the field E_{L2}^{-} is important because it is a measure of the droop in the output field of the PEN during the time interval $t_2 - t_1$, which will usually be set equal to the accelerator filling time T_A.

In Fig. 3 the PEN input and output waveforms are shown to scale for the following system parameters:

$$t_2 = 5.4 \quad \mu\text{sec}$$
$$T_A = 0.83 \quad \mu\text{sec}$$
$$\beta = 5$$
$$\alpha = 1.67$$
$$Q_0 = 1.15 \times 10^5$$
$$T_C = 2Q_0/\omega(1+\beta) = 2.13 \ \mu\text{sec}$$
$$\omega/2\pi = 2856 \quad \text{MHz}$$

where Q_0 is the unloaded Q for the PEN cavities. This Q_0 can be obtained using room-temperature cavities of reasonable dimensions operating in the TE_{023} mode. The value shown for β has been chosen to optimize the enhancement in energy gain. Note also that the assumed pulse length is is twice the present SLAC rf pulse length.

The solid curves in Fig. 3 show waveforms for the case when the phase reversal at t_1 takes place one accelerator filling time before the end of the pulse; that is, $t_2 - t_1 = T_A$. For this case, $E_{L1}^{+} = -2.47$ and $E_{L2}^{-} = -1.46$. The dashed waveform shows the effect of flipping the phase two filling times before the end of the pulse. For this case, $E_{L1}^{+} = -2.38$ and $E_{L2}^{-} = -0.73$.

Field in the Accelerating Structure

We next calculate, as a function of time, the field along a SLAC constant gradient disk-loaded structure. The group velocity in a structure of length L varies as

$$v_g(z) = v_{g0}\left[1 - g(z/L)\right],$$

where, for the SLAC structure, $v_{g0} = 0.0204c$ and the constant g is equal to 0.681. The time for a wavefront to travel to position z along the structure is

$$\Delta t = \frac{L}{v_{g0}} \int_0^{z'} \frac{dz'}{(1 - gz')} = \frac{L}{v_{g0}}\left[\frac{1}{g}\ \ell n\left(\frac{1}{(1 - gz')}\right)\right]$$

where $z' = z/L$. The accelerator filling time is the value of Δt for $z' = 1$, or

$$T_A = \frac{L}{v_{g0}}\left[\frac{1}{g}\ \ell n\left(\frac{1}{1 - g}\right)\right].$$

Therefore,

$$\frac{\Delta t}{T_A} \equiv \Delta t' = \frac{\ell n\left[1/(1 - gz')\right]}{\ell n\left[1/(1 - g)\right]} = \frac{1}{b}\ \ell n\left(\frac{1}{1 - gz'}\right), \tag{6}$$

where $b \equiv \ln\left[1/(1-g)\right] = 1.144$ for the SLAC structure. Solving the preceding relation for z',

$$z' = \frac{1}{g}\left(1 - e^{-b\Delta t'}\right). \tag{7}$$

The field at any point along the structure is now obtained from

$$E_S(z,t) = E_S(0,\ t - \Delta t)\ ,$$

where Δt is given by Eq. (6) and where $E_S(0,t)$ is obtained from Eqs. (1), (3) and (5). Thus

$$E_S(A) = \alpha e^{-ut'}(1 - gz')^{-v} - (\alpha - 1)\ , \tag{8a}$$

$$E_S(B) = (E_{L1}^{+} - \alpha + 1)\ e^{-u(t'-t_1')}(1 - gz')^{-v} + (\alpha - 1)\ , \tag{8b}$$

$$E_S(C) = (E_{L2}^{-} + 1)\ e^{-u(t'-t_2)}(1 - gz')^{-v}\ , \tag{8c}$$

where the additional constants $u = T_A/T_C$ and $v = u/b$ have been introduced. In using the preceding relations, the position that the waveform discontinuities (which occur at $t = 0$, t_1 and t_2) have propagated to must be taken into account. The location of a field discontinuity on the structure produced by a waveform discontinuity at time t_d' is given, using Eq. (7), by

$$z_d' = \frac{1}{g}\left[1 - e^{-b(t'-t_d')}\right]. \tag{9}$$

Using, for example, $t'_d = t_1$ in Eq. (9), there will be a discontinuity on the structure at $z'_d \equiv z'_1$ for $t' - t'_1 < 1$. The field is given by Eq. (8b) for $0 < z' < z'_1$, while for $z'_1 < z' < 1$, the field on the structure is given by Eq. (8a). Figure 4 shows the development with time of the field on the structure for the parameters listed previously on p. 6. In addition, the following structure constants have been used: $g = 0.681$, $b = 1.144$, $u = 0.390$ and $v = 0.341$.

If the transient response of the disk-loaded structures to step changes in field were to be taken into account, we would expect the field profiles shown in Fig. 4 to develop ripples with an amplitude on the order of ±10% at the end of the 3-meter long structure.[1] Ripples of this order are, in fact, observed experimentally at the end of the structure for an input waveform as shown at the bottom of Fig. 2. The effect on the energy gain should be considerably less. In the case of SLAC sections for a step input pulse, the ripples on the energy gain waveform have an applitude of ±0.5%, which is only 5% of the amplitude of the ripples on the field profile.[2]

Calculation of the Energy Gain

The accelerating voltage is obtained by integrating Eqs. (8a), (8b) and (8c) with respect to z'. A complicating factor is that the integration must sometimes be carried out in two parts, using one relation for the

[1] For example, see R. B. Neal, ed., The Stanford Two-Mile Accelerator, W. A. Benjamin, Inc., New York, 1968), p. 123.

[2] Ibid, p. 125.

field up to a wavefront discontinuity as given by Eq. (9), and another relation for the field following the discontinuity. For example, in pulse interval A the following partial energy gain expressions are required:

$$V_A(0\rightarrow z_d') = \frac{e^{-ut'}}{g(1 - v)}\left[1 - (1 - gz_d')^{1-v}\right] - (\alpha - 1)z_d' , \tag{10a}$$

$$V_A(0\rightarrow L) = \frac{\alpha e^{-ut'}}{g(1 - v)}\left[1 - (1 - g)^{1-v}\right] - (\alpha - 1) , \tag{10b}$$

$$V_A(z_d'\rightarrow L) = \frac{\alpha e^{-ut'}}{g(1 - v)}\left[(1 - gz_d')^{1-v} - (1 - g)^{1-v}\right] - (\alpha - 1)(1 - z_d') , \tag{10c}$$

where

$$V_A(0\rightarrow z_d') = \int_0^{z_d'} E_S(A)dz'$$

and so forth. In these expressions, z_d' is given by Eq. (9) with $t_d' = 0$. By substituting for z_d' using Eq. (9), the three above voltage gain expressions can, if desired, be written as functions of t' only. The partial energy gain expressions for pulse intervals B and C can be written in a similar manner:

$$V_B(0\rightarrow z_d') = \frac{(E_{L1}^{+} - \alpha + 1)}{g(1 - v)} e^{-u(t'-t_1')}\left[1 - (1 - gz_d')^{1-v}\right] + (\alpha - 1)z_d' , \tag{11a}$$

$$V_B(0 \rightarrow L) = \frac{(E_{L1}^{+} - \alpha + 1)}{g(1 - v)} e^{-u(t' - t_1')} \left[1 - (1 - g)^{1-v}\right] + (\alpha - 1) \tag{11b}$$

$$V_B(z_d' \rightarrow L) = \frac{(E_{L1}^{+} - \alpha + 1)}{g(1 - v)} e^{-u(t' - t_1')} \left[(1 - gz_d')^{1-v} - (1 - g)^{1-v}\right] + (\alpha - 1)(1 - z_d') \tag{11c}$$

$$V_C(0 \rightarrow z_d') = \frac{(E_{L2}^{-} + 1)}{g(1 - v)} e^{-u(t' - t_2')} \left[1 - (1 - gz_d')^{1-v}\right] \tag{12a}$$

$$V_C(0 \rightarrow L) = \frac{(E_{L2}^{-} + 1)}{g(1 - v)} e^{-u(t' - t_2')} \left[1 - (1 - g)^{1-v}\right] \tag{12b}$$

The total accelerating voltage is now obtained as

$$0 < t' < 1 \ : \ V = V_A(0 \rightarrow z_d') \ ; \ t_d' = 0$$

$$1 < t' < t_1' \ : \ V = V_A(0 \rightarrow L)$$

$$t_1' < t' < (t_1' + 1) \ : \ V = V_B(0 \rightarrow z_d') + V_A(z_d' \rightarrow L) \ ; \ t_d' = t_1$$

$$(t_1' + 1) < t' < t_2' \ : \ V = V_B(0 \rightarrow L)$$

$$t_2' < t' < (t_2' + 1) \ : \ V = V_C(0 \rightarrow z_d') + V_B(z_d' \rightarrow L) \ ; \ t' = t_2$$

$$(t_2' + 1) < t' \ : \ V = V_C(0 \rightarrow L)$$

Figure 3 gives the energy gain V as a function of time for the PEN output wavefore, E_L, as shown. The peak energy gain for the case

$t_2 - t_1 = T_A$, which occurs at time t_2, is 1.84 for the PEN parameters and rf pulse length (5.4 μsec) chosen for this example. Note that a direct integration of the E_L waveform over one accelerator filling time would give an energy of 1.95. The effect of the variation in group velocity along the constant gradient structure causes, in this case, a 6% energy loss as compared to the case for a constant impedance, lossless structure. The reason for this energy decrease is that the early, high field, output from the PEN is compressed at the low group velocity end of the structure relative to the low field portion of the pulse in the high group velocity region at the front of the structure. The high field portion of the pulse therefore contributes relatively less, and the low field portion relatively more, to the energy integral. A similar effect would of course take place for a constant impedance structure, where the early, high field, portion of the pulse is at the end of the structure after one filling time and hence suffers the greatest attenuation.

The dashed waveform in Fig. 3 shows the incident generator field, the PEN output waveform and the energy gain for the case $t_2 - t_1 = 2T_A$. Although the total time that the energy gain is greater than unity is substantially increased as compared to the case $t_2 - t_1 = T_A$, the maximum absolute value of the energy gain at time $t_1' + 1$ is decreased somewhat from 1.84 to 1.76.

Beam Loading and Energy Spectrum

It is obvious from the plot of energy vs. time in Fig. 3 that the pulse length of a beam accelerated near peak energy will necessarily be short compared to the structure filling time. From Fig. 3 it is seen, in addition, that the energy gain is rising as a function of time as peak energy is approached. It seems reasonable, therefore, to expect that by turning on the beam prior to reaching peak energy, the transient energy droop due to beam loading might roughly compensate the rising unloaded energy gain characteristic, resulting in a reasonably tight energy spectrum.

Table I gives the energy gain multiplication factor M_E as a function of time for the time interval immediately preceding maximum energy for the case $t_2' - t_1' = T_A$. Assuming that the unloaded energy of the present SLAC accelerator would be 26.0 GeV with 30 MW klystrons, V_0 in Table I gives the unloaded energy to be expected from SLED as a function of the time interval before maximum energy, $\Delta t' = t_2' - t'$. The next column gives the difference between the energy at time $t' = t_2' - \Delta t'$ and the maximum energy V_m at time t_2'. The induced energy change due to beam loading at time $\Delta t'$ after turning on a beam of peak current i_p is given by

$$V_b = ki_p(2\Delta t' - \Delta t'^2)$$

where k = 35 MV/mA for the total SLAC length. Setting $V_b = V_m - V_0$, we can solve for the peak current i_p as a function of $\Delta t' = (t_2' - t')$. For each peak current obtained in this way, the energy is the same at the beginning and end of the pulse and is equal to V_0. Values for the peak current are given in Table I together with the beam pulse width $T_b = T_A \Delta t'$, with $T_A = 0.83$ μsec. The average current is then obtained as $i_a = rT_b i_p$, where r is the pulse repetition rate. Values for average current are given in the Table for r = 180 pps.

We consider, finally, the energy spectrum to be expected. Although the loaded energy is equal to V_0 and is the same at the beginning and end of the beam pulse, it will deviate from V_0 during the pulse. The deviation is a maximum at a time roughly at the mid-point of the pulse. The maximum relative values, $\Delta V/V_0$, for this deviation are shown in the final column in Table I.

By shaping the pulse current as a function of time, the energy spectrum width can be reduced below the values shown in Table I. Consider the case for $t_2' - t' = 0.4$ (or $T_b = 0.4T_A$), which gives an energy spectrum width of 1.8% at a constant peak pulse current of 219 mA.* By pulsing the current to 280 mA for the first 0.2 T_A and then to 170 mA for the final 0.2 T_A, we calculate that the maximum total energy deviation at any time during the pulse is less than 0.5%. By more complex pulse shaping, the spectrum width could in principle be reduced even further. In a real machine, the effects of trigger timing errors and jitter will at some point limit the energy resolution that can be achieved.

Figure 5 shows the peak and average beam current as a function of loaded energy. At 10% beam loading (a 10% reduction in energy below the

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*The SLED pulse current will in practice be limited by beam break-up. For the present SLAC accelerator, the beam break-up limit on pulse current is 160 mA for a beam pulse width of 0.3 μsec and a final energy of 20 GeV. For a final energy of 40 GeV, a pulse current on the order of 250 mA could in principle be accelerated at this pulse width, although the focusing along the accelerator would have to be increased.

unloaded energy), the loaded energy is 43 GeV, the average current is 13 μA and the beam power is 560 kW. For the present SLAC machine (again assuming 30 MW klystrons), the average beam current at 10% beam loading is 40 μA and the beam power is 950 kW. Thus, from the standpoint of the conversion of average rf power into average beam power, SLED is not substantially less efficient than the present machine.

It is worth noting that conversion to SLED can take place with only a minimum amount of interference with accelerator operation during the changeover period. After a PEN is installed, operation of that station can be restored to the normal, pre-SLED mode of operation simply by detuning the two high Q cavities and deactivating the trigger that produces the 180° phase reversal. Under these conditions the rf pulse length will be 5.4 μsec at a maximum repetition rate of 180 pps, and the beam pulse length can be increased to about 4.3 μsec. Since the beam pulse is 2.7 times as long as the present 1.6 μsec, the post-SLED duty cycle in this mode of operation will be higher by 35% than at present. The energy spectrum can also be expected to be somewhat tighter, since the ratio of the steady-state portion of the beam pulse to the transient portion is increased by more than a factor of four.

A summary of parameters for SLED and for the present SLAC accelerator is given in Table II. Because the modulator high-voltage pulse length has been doubled and the maximum repetition rate has been cut in half, the energy increase provided by SLED is achieved without an increase in the average input power to the accelerator.

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Computations by Dave Farkas (SLAC-TN-73-8) have been instrumental in the development of the SLED concept. In particular, his idea of reversing the klystron phase during the rf pulse has made possible a viable energy enhancement factor for SLED. In addition, he suggested the design shown in Fig. 1 for the power enhancement network, based on the use of a 3-db hybrid and two identical resonant cavities. Harry Hogg has also contributed substantially to the microwave design of the PEN and to the engineering associated with its installation in the existing high-power waveguide system.

TABLE I

SLED Beam Loading Characteristics

$t_2' - t'$	M_E	V_0 (GeV)	$V_m - V_0$ (GeV)	i_p (mA)	T_b (μsec)	i_a (μA)	$\Delta V/V_0$ (%)
0.5	1.545	40.17	7.72	294	0.42	22.0	2.9
0.4	1.653	42.98	4.91	219	0.33	13.1	1.8
0.3	1.732	45.03	2.86	160	0.25	7.2	0.9
0.2	1.789	46.51	1.38	109	0.17	3.3	0.4
0.1	1.824	47.42	0.47	71	0.08	1.0	0.2
0	1.842	47.89	0	0	0	0	0

TABLE II

Comparison of SLED and Present SLAC Parameters

(Computed assuming 30 MW klystrons)

		Present SLAC	SLED
Unloaded Energy	(GeV)	26	48
Loaded Energy	(GeV)	23.5	43
Repetition Rate	(pps)	360	180
Rf Pulse Width	(μsec)	2.7	5.4
Beam Pulse Width	(μsec)	1.6	0.33
Average Current	(μA)	40	13
Peak Current	(mA)	70	220
Duty Cycle		6×10^{-5}	6×10^{-6}
Energy Spread	(%)	1.0	0.5*
Average Beam Power	(kW)	940	560

*Assumes 280 mA pulse for the first 0.16 μsec, then 170 mA for the next 0.16 μsec. For constant 220 mA peak current, estimated energy spread is 1.8%.