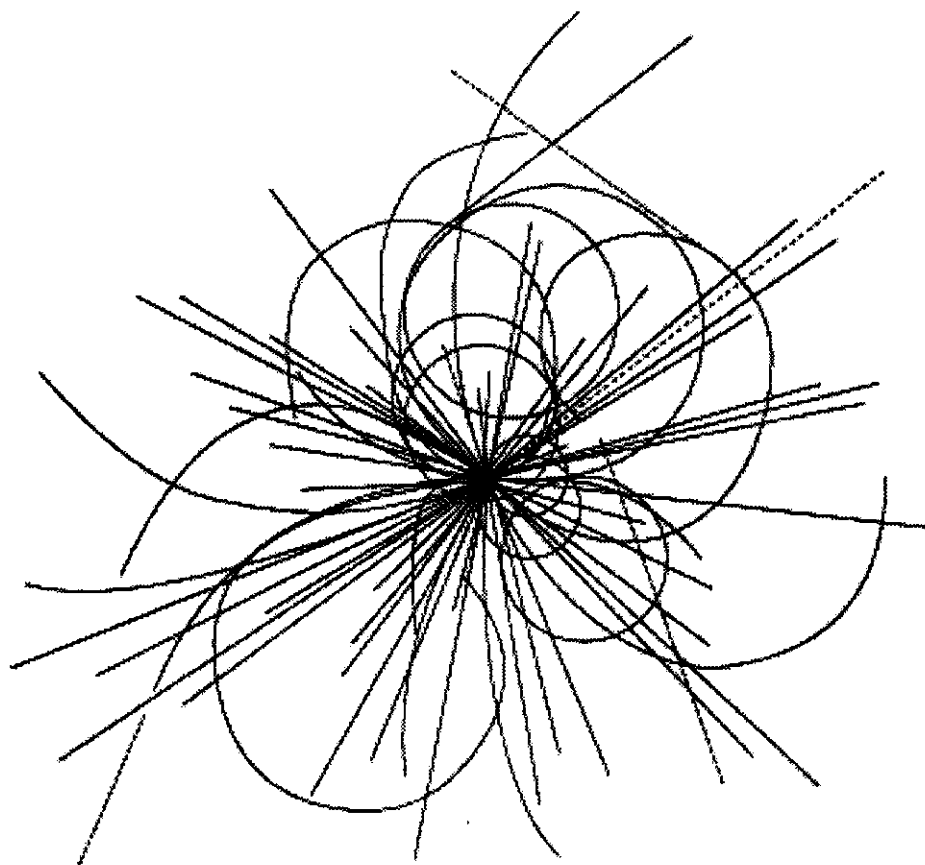


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**Superconducting Super Collider
Laboratory**

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Amplitude Function Mismatching and Correction of LEB-MEB Transfer Line

N. Mao and R. Gerig

Abstract

The LEB-MEB transfer line at the Superconducting Super Collider Laboratory plays an important role in the amplitude function matching between the Low Energy Booster (LEB) and the Medium Energy Booster (MEB). Amplitude function mismatching due to LEB and MEB amplitude function variations and transfer line quadrupole gradient errors is analyzed statistically. The correction for the amplitude function mismatching by using the β -matching section in the transfer line is studied.

1.0 INTRODUCTION

The LEB-MEB transfer line¹ at the Superconducting Super Collider Laboratory transports 12-GeV/c proton beam from the Low Energy Booster (LEB) to the Medium Energy Booster (MEB), as shown in Figures 1 and 2. The transfer line plays an important role in beam position matching, amplitude function matching, and dispersion function matching between these two boosters. The design amplitude functions at the LEB extraction point and MEB injection point are listed in Table 1. But the amplitude functions in the LEB and MEB rings may vary because of field errors and magnet misalignment in the rings. And the transfer matrix of the transfer line may also vary, because of quadrupole gradient errors. All these variations may cause amplitude function mismatching at the MEB injection point. The corresponding emittance dilution factor is

$$\begin{aligned} F_{\beta} &= 1 + 0.5(\beta_2/\beta_1) ((1 - \beta_1/\beta_2)^2 + ((\beta_1/\beta_2)\alpha_2 - \alpha_1)^2) \\ &= 0.5 (\beta_1\gamma_2 + \beta_2\gamma_1 - 2\alpha_1\alpha_2), \end{aligned}$$

where

$$\gamma = (1 + \alpha^2)/\beta$$

and subscripts 1 and 2 denote the quantities without and with error effect, respectively. If emittance growth of less than 1% is required, $|\alpha_2 - \alpha_1|$ should be less than 0.141 when $\beta_1 = \beta_2$, and β_1/β_2 should be larger than 1/1.063 and less than 1.063 when $\alpha_1 = \alpha_2$ in the horizontal plane. As we can see later, these matching requirements are not easy to meet. Therefore, a correction for β -mismatching is necessary. The effect of these errors on the amplitude function mismatching and its correction is studied using the codes TRANSPORT² and EAC (specially for beam line error analysis with a statistical method).³

Table 1. Design Amplitude Functions.

	LEB Extraction Point (tune E)	MEB Injection Point
β_x (m)	4.722	51.0927
α_x	-0.0895	-2.095 88
β_y (m)	22.223	14.4093
α_y	0.3698	0.731 054

2.0 LEB AMPLITUDE FUNCTION VARIATIONS

If β -function variations at the extraction point of the LEB ring are $\Delta\beta_0/\beta_0 = \pm 20\%$, the analysis shows that the maximum α - and β -mismatching at the MEB injection point are 0.3 m and 4 m, corresponding to an emittance dilution of 2-3% (the upper part of Table 2). The β -functions along the transfer line for $\Delta\beta_{x0}/\beta_{x0} = \Delta\beta_{y0}/\beta_{y0} = -20\%$ are shown in Figure 3 (solid line for β_x and dashed line for β_y). The MEB injection septum magnet (SEP3) has the smallest horizontal acceptance in the transfer line. The β_x value within the septum magnet increases by 7.07 m due to $\Delta\beta_{x0}/\beta_{x0} = \Delta\beta_{y0}/\beta_{y0} = -20\%$ errors. It corresponds to a horizontal beam size increase of 1.6 mm if the normalized emittance of the beam is 36π -mm-mrad (3σ test beam).

In the transfer line there is a β -matching section, consisting of four independent quadrupole components (dual quadrupoles QFM1, 2, 3 and single quadrupole QFM4, Figures 1 and 2). Readjusting the gradients of these quadrupoles may correct the β -mismatching at the MEB injection point, as shown in the lower part of Table 2. The corresponding maximum gradient change needed for correction in the β -matching section is about 13%, and the maximum gradient is 13.67 T/m, less than the nominal value of 15 T/m. The beam envelopes after correction for $\Delta\beta_{x0}/\beta_{x0} = \Delta\beta_{y0}/\beta_{y0} = -20\%$ are given in Figure 4; the beam size within the MEB injection septum magnet decreases to its normal value.

3.0 MEB AMPLITUDE FUNCTION VARIATIONS

The variations of MEB lattice functions due to field errors and misalignment have been simulated.^{4,5} The results show that the β - and α -errors at MEB injection point may reach up to 10%. If the LEB and LEB-MEB transfer line operate in ideal conditions, the maximum emittance dilution factor will be 1.10 for different combinations of 10% β - and α -errors in MEB, as shown in Table 3.

By using the β -matching section in the transfer line, the β -function mismatching due to MEB errors can be entirely corrected. Table 4 lists the maximum gradients ($G_{\max}$) in the β -matching section required for the correction. Obviously, all the gradients for different error combinations are lower than the nominal value of 15.0 T/m.

The β -functions along the transfer line after the correction for a combination of 10% α - and -10% β -errors are shown in Figure 5. The maximum β_x function downstream of the β -matching section reaches up to 118 m, 15 m larger than the design value of 103 m. It is unacceptable because of the limited magnet aperture. Therefore, the β_x and α_x errors at MEB injection point have to be reduced. Simulations show that the maximum β_x will decrease to 103 m and 94 m, respectively, if β_x and α_x errors are reduced to 7% and 5% (Figures 6 and 7, Table 4). In order to leave some margin, 5% β_x and α_x tolerance is suggested. As for β_y and α_y errors, 10% errors are allowed in view of β_y values along the transfer line, as shown in Table 4.

The β -function within the MEB injection septum magnet (SEP3) will increase by about 3 m, due to 5% β_x and α_x errors and 10% β_y and α_y errors. It corresponds to an increase of about 1 mm in the beam size (36 π -mm-mrad beam).

Table 2. Amplitude Function Mismatching and Emittance Dilution Due to LEB Amplitude Function Variations, and Corrections for the Mismatching.

LEB635E**, MISMATCH & PHASE SPACE DILUTION CAUSED BY LEB EXTRACTION ERRORS, SEP. 7, 93

AT THE END OF THE BEAM LINE

RUN	ALPHAX1	BETAX1 (m)	ALPHAY1	BETAY1 (m)	Fx	Fy	
LEB630E	-2.09590	51.09230	0.73110	14.40940			
RUN	CORRECTION	ALPHAX2	BETAX2 (m)	ALPHAY2	BETAY2 (m)	Fx	Fy
LEB635E01	BETAX0 +20%	-2.43380	55.10210	0.73110	14.40940	1.01680	1.00000
LEB635E02	-20%	-1.78390	48.70930	0.73110	14.40940	1.02521	1.00000
LEB635E03	+20%	-2.09590	51.09230	0.51920	13.95910	1.00000	1.01895
LEB635E04	-20%	-2.09590	51.09230	1.02010	15.45500	1.00000	1.02841
LEB635E13	-20%	-1.78390	48.70930	1.02010	15.45500	1.02521	1.02841
LEB635E09	-40%	-1.52400	49.58010	1.46350	17.69090	1.13440	1.15154
LEB635E10	-50%	-1.43300	52.45570	1.80100	19.70170	1.25199	1.28418
LEB635E05	+20%	-2.09590	51.09230	0.73110	14.40940	1.00000	1.00000
LEB635E06	-20%	-2.09590	51.09250	0.73110	14.40940	1.00000	1.00000
LEB635E07	+20%	-2.09590	51.09230	0.73110	14.40940	1.00000	1.00000
LEB635E08	-20%	-2.09590	51.09230	0.73110	14.40940	1.00000	1.00000
LEB635E14	-20%	-2.09590	51.09250	0.73110	14.40930	1.00000	1.00000
LEB635E11	-40%	-2.09590	51.09270	0.73110	14.40940	1.00000	1.00000
LEB635E12	-50%	-2.09590	51.09260	0.73110	14.40930	1.00000	1.00000

Table 3. Amplitude Function Mismatching and Emittance Dilution Due to MEB Amplitude Function Variations.

LEB636E**, MISMATCH & PHASE SPACE DILUTION CAUSED BY MEB LATTICE FUNCTION ERRORS, SEP. 8, 93

AT THE END OF THE BEAM LINE

RUN	MEB LATTICE ERRORS						ALPHAX1	BETAX1 (m)	ALPHAY1	BETAY1 (m)	Fx	Fy
	ALPHAX2	BETAX2	ALPHAY2	BETAY2	ALPHAX1	BETAX1 (m)						
LEB636E					-2.09590	51.09230			0.73110	14.40940		
RUN	MEB LATTICE ERRORS						ALPHAX2	BETAX2 (m)	ALPHAY2	BETAY2 (m)	Fx	Fy
	ALPHAX2	BETAX2	ALPHAY2	BETAY2	ALPHAX1	BETAX1 (m)						
LEB636E06	-10%	+10%			-1.88630	56.20200			0.73110	14.40930	1.08443	1.00000
LEB636E08	-10%	-10%			-1.88630	45.98340			0.73110	14.40930	1.00555	1.00000
LEB636E05	+10%	+10%			-2.30550	56.20200			0.73110	14.40930	1.00455	1.00000
LEB636E07	+10%	-10%			-2.30550	45.98340			0.73110	14.40930	1.10317	1.00000
LEB636E18	+7%	-7%			-2.24260	47.51620			0.73110	14.40930	1.04891	1.00000
LEB636E17	+5%	-5%			-2.20070	48.53810			0.73110	14.40930	1.02443	1.00000
LEB636E10		-10%	+10%		-2.09590	51.09270			0.65800	15.85020	1.00000	1.01426
LEB636E12		-10%	-10%		-2.09590	51.09270			0.65800	12.96840	1.00000	1.00556
LEB636E09		+10%	+10%		-2.09590	51.09270			0.80420	15.85020	1.00000	1.00454
LEB636E11		+10%	-10%		-2.09590	51.09270			0.80420	12.96840	1.00000	1.01743
LEB636E21	+10%	-10%	+10%		-2.30550	45.98340			0.80420	12.96840	1.10317	1.01743
LEB636E19	+7%	-7%	+10%		-2.24260	47.51620			0.80420	12.96840	1.04891	1.01743
LEB636E20	+5%	-5%	+10%		-2.20070	48.53810			0.80420	12.96840	1.02443	1.01743

Table 4. Corrections for Amplitude Function Mismatching Due to MEB Amplitude Function Variations.

MAXIMUM GRADIENTS IN MATCHING SECTION, MAXIMUM BETA FUNCTIONS IN THE FODO ARRAY,
AND BETA FUNCTIONS IN THE MEB INJECTION SEPTUM MAGNET

RUN	MEB LATTICE ERRORS				G(MAX) (T/m)	BETAX(MAX) (m)	BETAY(MAX) (m)	BETAX(SEP3) (m)	BETAY(SEP3) (m)
	ALPHAX2	BETAX2	ALPHAY2	BETAY2					
LEB630E					13.34	75.66	57.60	51.10	17.77
RUN	MEB LATTICE ERRORS				G(MAX) (T/m)	BETAX(MAX) (m)	BETAY(MAX) (m)	BETAX(SEP3) (m)	BETAY(SEP3) (m)
	ALPHAX2	BETAX2	ALPHAY2	BETAY2					
LEB636E06	-10%	+10%			13.18	68.16	57.60	58.91	17.77
LEB636E08	-10%	-10%			13.33	80.88	57.60	56.67	17.77
LEB636E05	+10%	+10%			13.35	71.67	57.60	46.55	17.77
LEB636E07	+10%	-10%			13.43	118.06	57.60	62.26	17.77
LEB636E18	+7%	-7%			13.41	102.51	57.60	56.56	17.77
LEB636E17	+5%	-5%			13.40	93.57	57.60	53.93	17.77
LEB636E10		-10%	+10%		13.42	75.66	48.70	51.10	16.71
LEB636E12		-10%	-10%		13.43	75.66	60.54	51.10	19.74
LEB636E09		+10%	+10%		13.26	75.66	59.02	51.10	16.16
LEB636E11		+10%	-10%		13.33	75.66	69.12	51.10	19.96
LEB636E21	+10%	-10%	+10%	-10%	13.42	118.05	69.12	62.26	19.96
LEB636E19	+7%	-7%	+10%	-10%	13.40	102.51	69.12	56.56	19.96
LEB636E20	+5%	-5%	+10%	-10%	13.39	93.57	69.12	53.93	19.96

4.0 TRANSFER LINE QUADRUPOLE GRADIENT ERRORS

The rms gradient tolerance of the transfer line quadrupoles is required to be 0.1%.⁶ According to the statistical simulation (Code EAC, Run LEB645E02), the emittance dilution in the MEB due to gradient errors is about 0.4% (99% events, Table 5 and Figure 8); it meets the emittance budget requirement. In the simulation, the gradient errors of all quadrupoles are randomly chosen with an rms value of 0.1% (1000 seeds), and the dilution factors reflect 99% random events.

Table 5. Quadrupole Gradient Error and Emittance Dilution.

$\Delta G/G$ (rms)	ΔF_x (99% Events)	ΔF_y	Run (EAC)
1×10^{-3}	0.37%	0.40%	LEB645E02
2×10^{-3}	1.5%	1.6%	LEB645E03
1×10^{-2}	39%	41%	LEB645E01

The quadrupole gradient error mainly consists of setting error, regulation error, and ripple. The setting error may be as large as 1%; therefore, an evaluation of the effect of 1% error on the emittance dilution in the MEB is made. The statistical simulation results show that the dilution due to 1% error is about 40% (Run LEB645E01, Table 5 and Figure 9), far beyond the budget. A few measures are being considered to solve the possible problem, such as:

1. quadrupoles in the transfer line FODO array (downstream of the β -matching section, 12 quadrupoles in total) powered in series;
2. fine calibration for the quadrupole excitation curve;
3. use of beam position monitors to measure transfer matrix elements (x/θ) and (y/ϕ), and then correcting the quadrupole gradients;
4. use of β -matching section in the transfer line to correct the β -mismatching due to the gradient errors.

The last measure is studied here, and it is pointed out that the β -mismatching due to the gradient error of 1% at the MEB injection point can be corrected by using the β -matching section of the transfer line. The details of the simulation are as follows:

1. *Quadrupoles with random rms gradient error of 1%*

Some seeds are selected for correction simulation of 1% rms gradient error (Runs LEB640E24–35). The results show that without correction, the β -mismatching at the MEB injection point reaches up to 16 m, corresponding to an emittance dilution of about 40% (the upper part of Table 6). Figures 10 and 11 are the β -functions along the transfer line for gradient error seeds 358084 and 321454. The results also show that the mismatching of this magnitude can be entirely corrected by the β -matching section (the lower part of Table 6). The maximum quadrupole gradient required for the correction is 13.9 T/m.

2. *All the quadrupoles with the same gradient errors of $\pm 1.0\%$*

If all the quadrupoles in the transfer line have the same gradient errors of $\pm 1\%$, the simulation (Runs LEB640E04, 05, 10, and 11) shows that without correction the maximum β -mismatching is about 12 m, and the maximum emittance dilution in the MEB is about 22% (Table 6). The β -functions along the transfer line for -1% gradient error are shown in Figure 12. The β -mismatching caused by the gradient errors can be entirely corrected with the β -matching section, and there will be no dilution in the MEB, as shown in Table 6 and Figure 12. The maximum gradient in the β -matching section needed for correction is 13.8 T/m, lower than the nominal value of 15 T/m.

5.0 SUMMARY

Using the β -matching section in the transfer line, the following β -mismatching corrections are possible:

1. LEB 20% β_x and β_y variations;
2. MEB 5% β_x and α_x variations, 10% β_y and α_y variations;
3. β -mismatching caused by 1% beam line quadrupole gradient errors.

The maximum gradient required for corrections is lower than the nominal value of 15 T/m. The beam size along the transfer line after correction is smaller than the dimension of the magnet nominal good-field region, except within the MEB injection septum magnet, where the beam size is 1 mm larger (36 π -mm-mrad beam).

Table 6. Amplitude Function Mismatching and Emittance Dilution Due to Transfer Line Quadrupole Gradient Errors, and Corrections for the Mismatching.

LEB640E**, BETA, ETA MISMATCH AND PHASE SPACE DILUTION, CAUSED BY QUADRUPOLE ERRORS, JULY 14, 93

AT THE END OF THE BEAM LINE

RUN	QUAD ERROR	RANDOM NUMBER	CORREC- TION	ALPHAX1 (rad)	BETAX1 (m)	ALPHAY1 (rad)	BETAY1 (m)	FX	FY
LEB630E				-2.09590	51.09230	0.73110	14.40940		
RUN	QUAD ERROR	RANDOM NUMBER	CORREC- TION	ALPHAX2 (rad)	BETAX2 (m)	ALPHAY2 (rad)	BETAY2 (m)	FX	FY
LEB640E24	1.0%	358084	NO	-1.79600	67.10290	0.69530	17.97720	1.38582	1.04341
LEB640E25	1.0%	285616	NO	-1.19660	49.91900	0.64700	18.52330	1.37103	1.06506
LEB640E26	1.0%	267301	NO	-0.96450	41.90090	0.78970	17.02780	1.36667	1.01630
LEB640E27	1.0%	321454	NO	-2.13560	58.03290	2.14660	23.03530	1.03455	1.41114
LEB640E28	1.0%	297991	NO	-1.45750	40.75360	0.86540	30.22590	1.05445	1.39361
LEB640E29	1.0%	265915	NO	-1.32800	35.09380	0.78380	28.97170	1.08044	1.37107
LEB640E04	+1.0%		NO	-2.75090	62.89470	0.62000	22.61290	1.03353	1.19187
LEB640E05	-1.0%		NO	-1.53400	43.64140	1.72620	20.73210	1.05089	1.22492
LEB640E30	1.0%	358084	YES	-2.09590	51.09250	0.73110	14.40940	1.00000	1.00000
LEB640E31	1.0%	285616	YES	-2.09590	51.09250	0.73110	14.40940	1.00000	1.00000
LEB640E32	1.0%	267301	YES	-2.09590	51.09270	0.73110	14.40940	1.00000	1.00000
LEB640E33	1.0%	321454	YES	-2.09590	51.09240	0.73110	14.40940	1.00000	1.00000
LEB640E34	1.0%	297991	YES	-2.09590	51.09220	0.73110	14.40940	1.00000	1.00000
LEB640E35	1.0%	265915	YES	-2.09590	51.09230	0.73110	14.40940	1.00000	1.00000
LEB640E10	+1.0%		YES	-2.09590	51.09240	0.73110	14.40940	1.00000	1.00000
LEB640E11	-1.0%		YES	-2.09590	51.09220	0.73110	14.40940	1.00000	1.00000

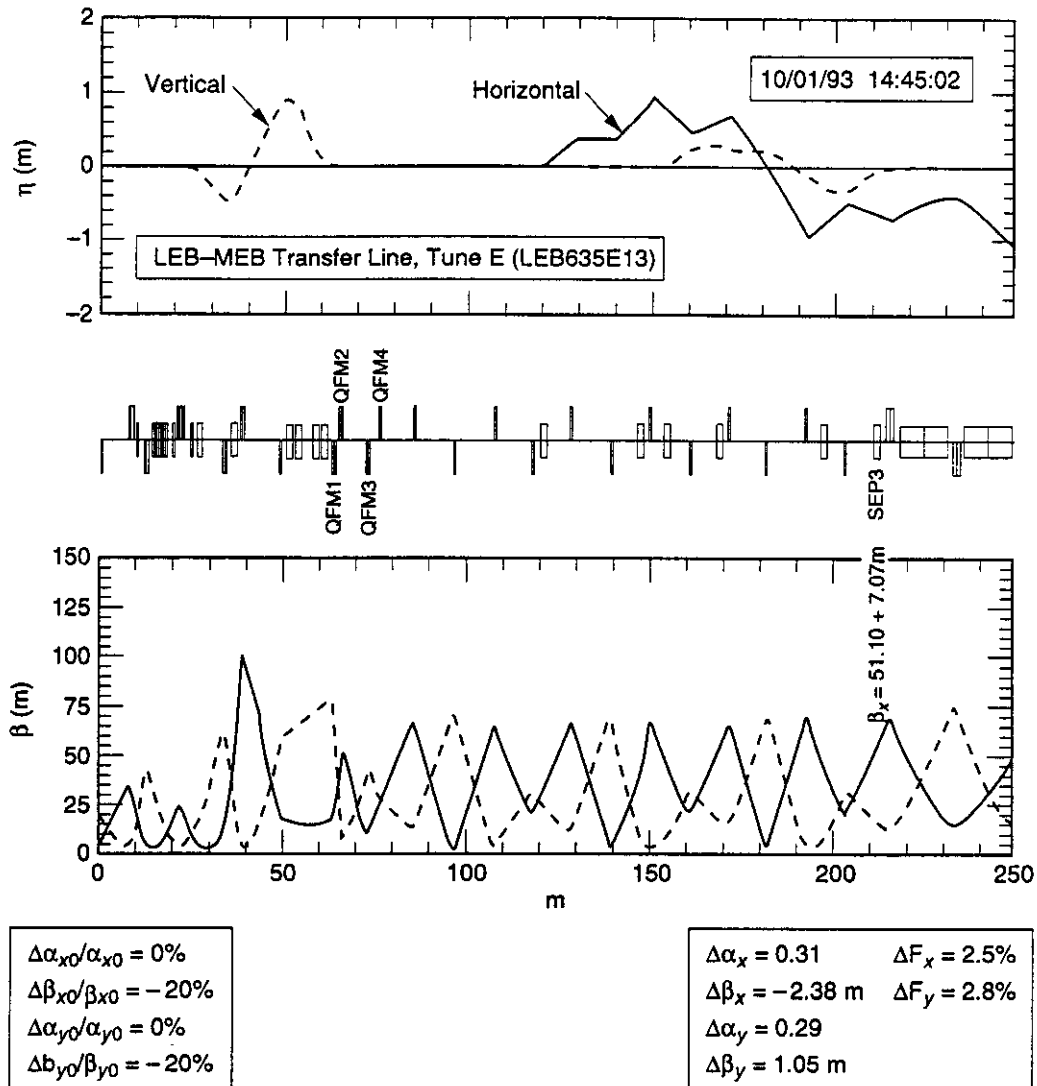
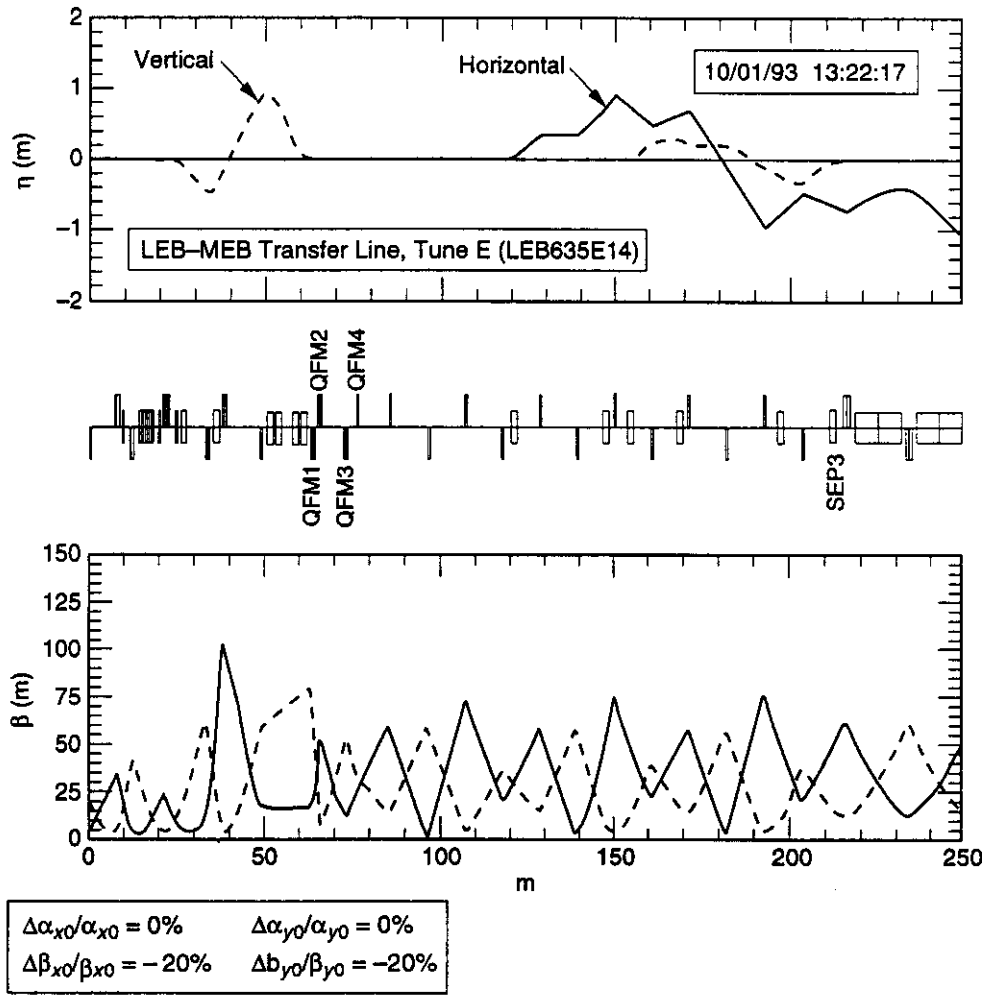


Figure 3. β -functions Along the Transfer Line for LEB $\Delta\beta_{x0}/\beta_{x0} = \Delta\beta_{y0}/\beta_{y0} = -20\%$ Variations (without correction).



TIP-05766

Figure 4. β -functions Along the Transfer Line for LEB $\Delta\beta_{x0}/\beta_{x0} = \Delta\beta_{y0}/\beta_{y0} = -20\%$ Variations (with correction).

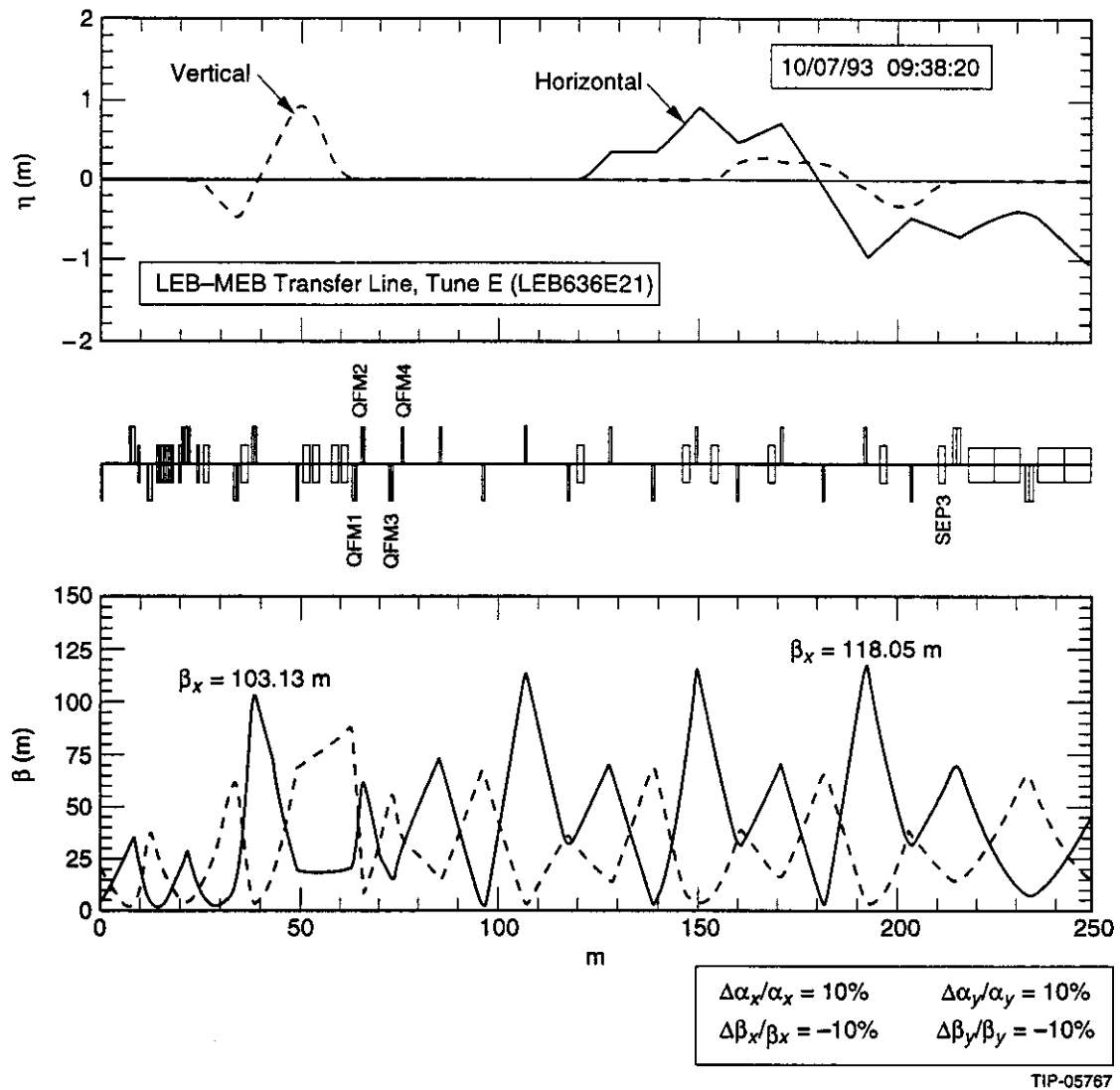
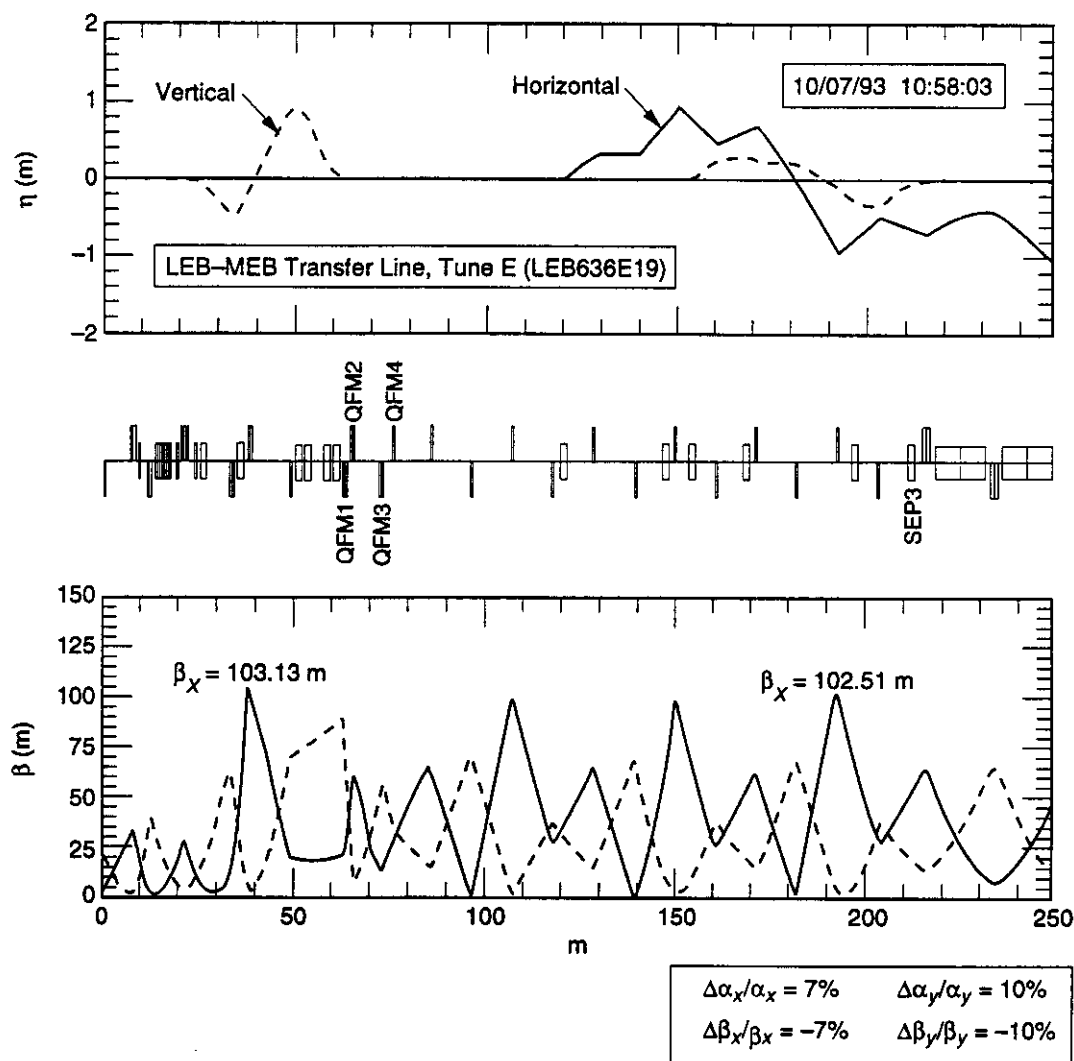
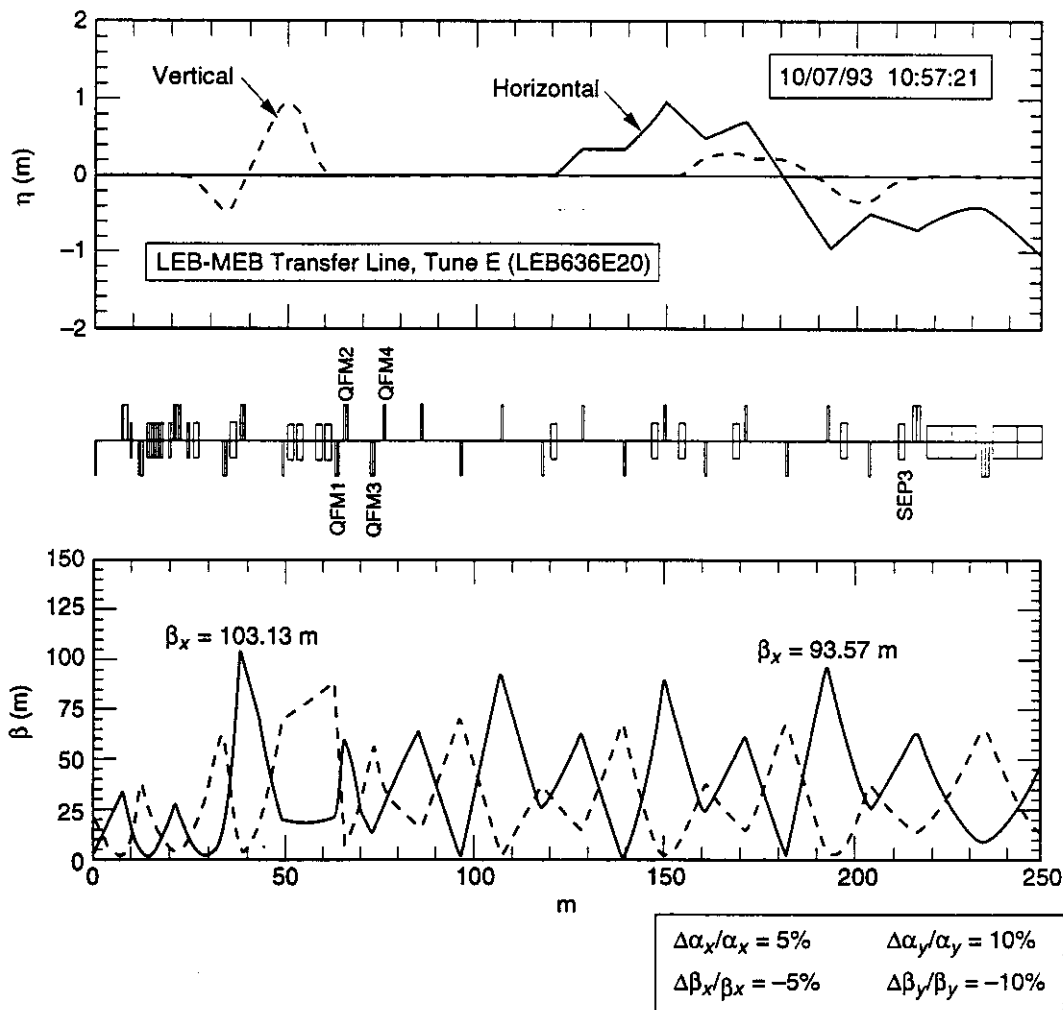


Figure 5. β -functions Along the Transfer Line with Correction for MEB $\Delta\alpha_x/\alpha_x = 10\%$, $\Delta\beta_x/\beta_x = -10\%$, $\Delta\alpha_y/\alpha_y = 10\%$, and $\Delta\beta_y/\beta_y = -10\%$ Variations.



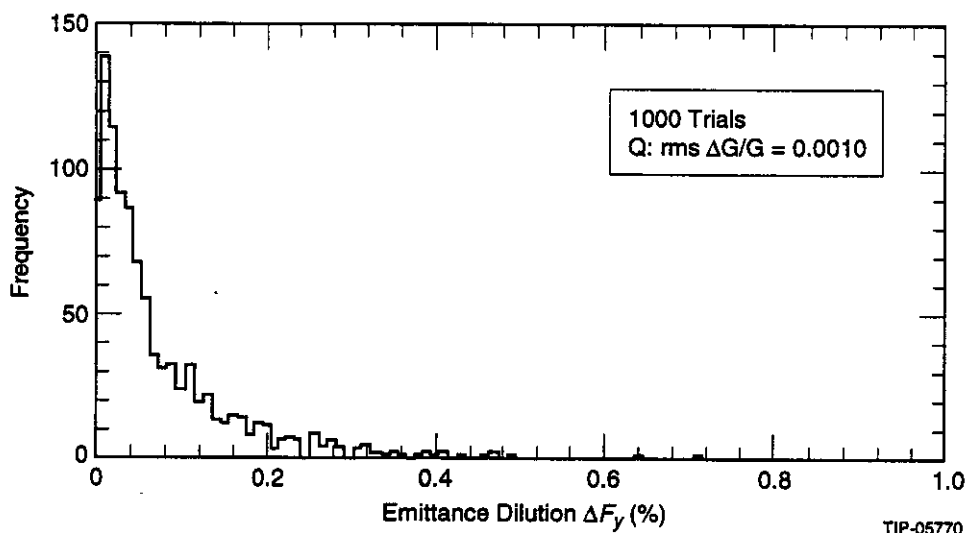
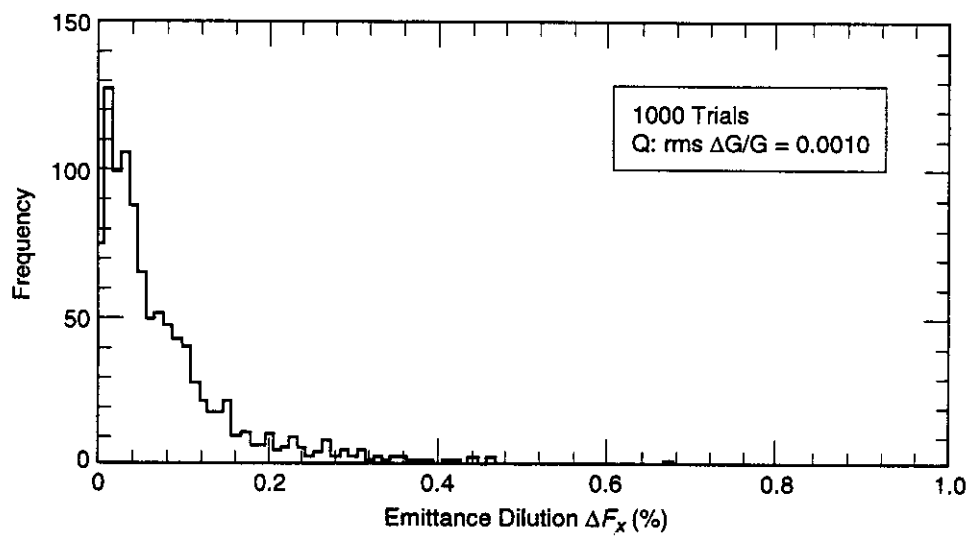
TIP-05768

Figure 6. β -functions Along the Transfer Line with Correction for MEB $\Delta\alpha_x/\alpha_x = 7\%$, $\Delta\beta_x/\beta_x = -7\%$, $\Delta\alpha_y/\alpha_y = 10\%$, and $\Delta\beta_y/\beta_y = -10\%$ Variations.



TIP-05769

Figure 7. β -functions Along the Transfer Line with Correction for MEB $\Delta\alpha_x/\alpha_x = 5\%$, $\Delta\beta_x/\beta_x = -5\%$, $\Delta\alpha_y/\alpha_y = 10\%$, and $\Delta\beta_y/\beta_y = -10\%$ Variations.



TIP-05770

Figure 8. Emittance Dilution Due to Transfer Line Quadrupole RMS Gradient Error of 0.1%.

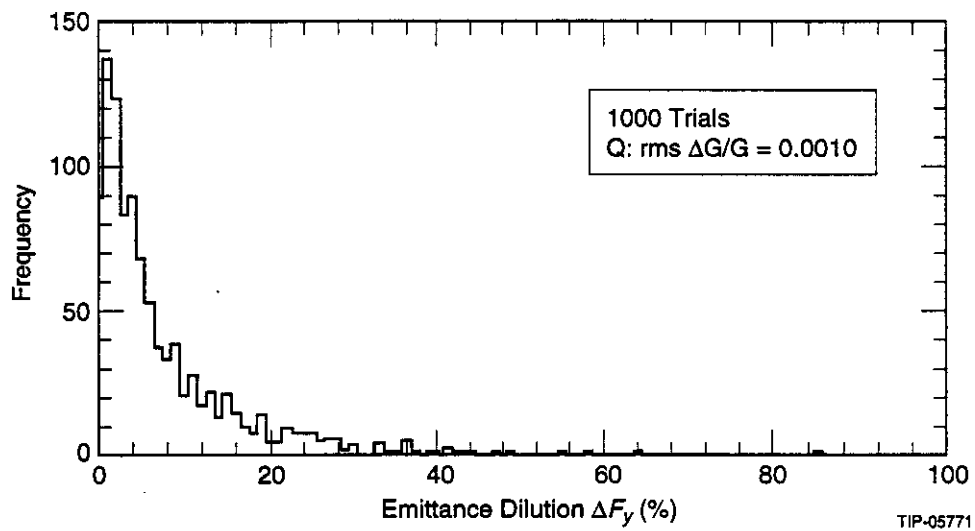
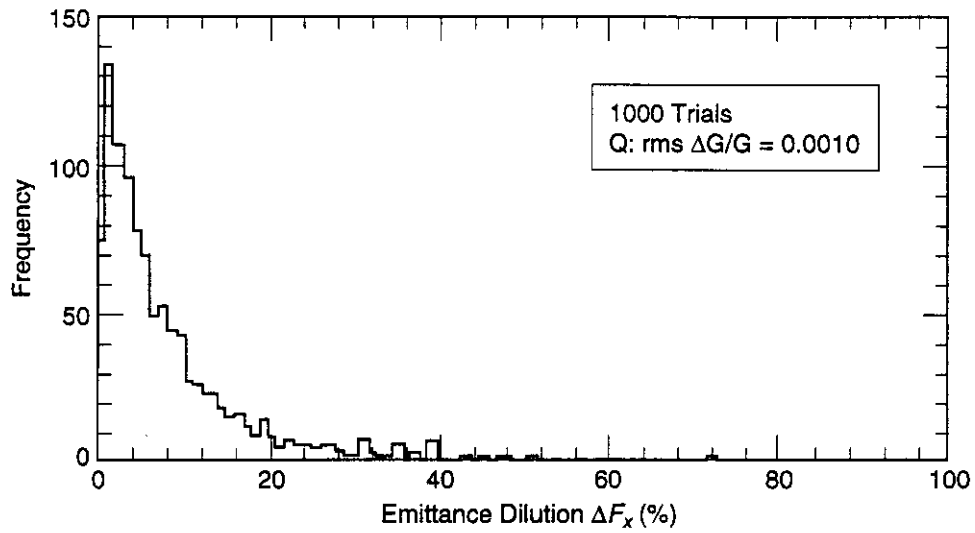
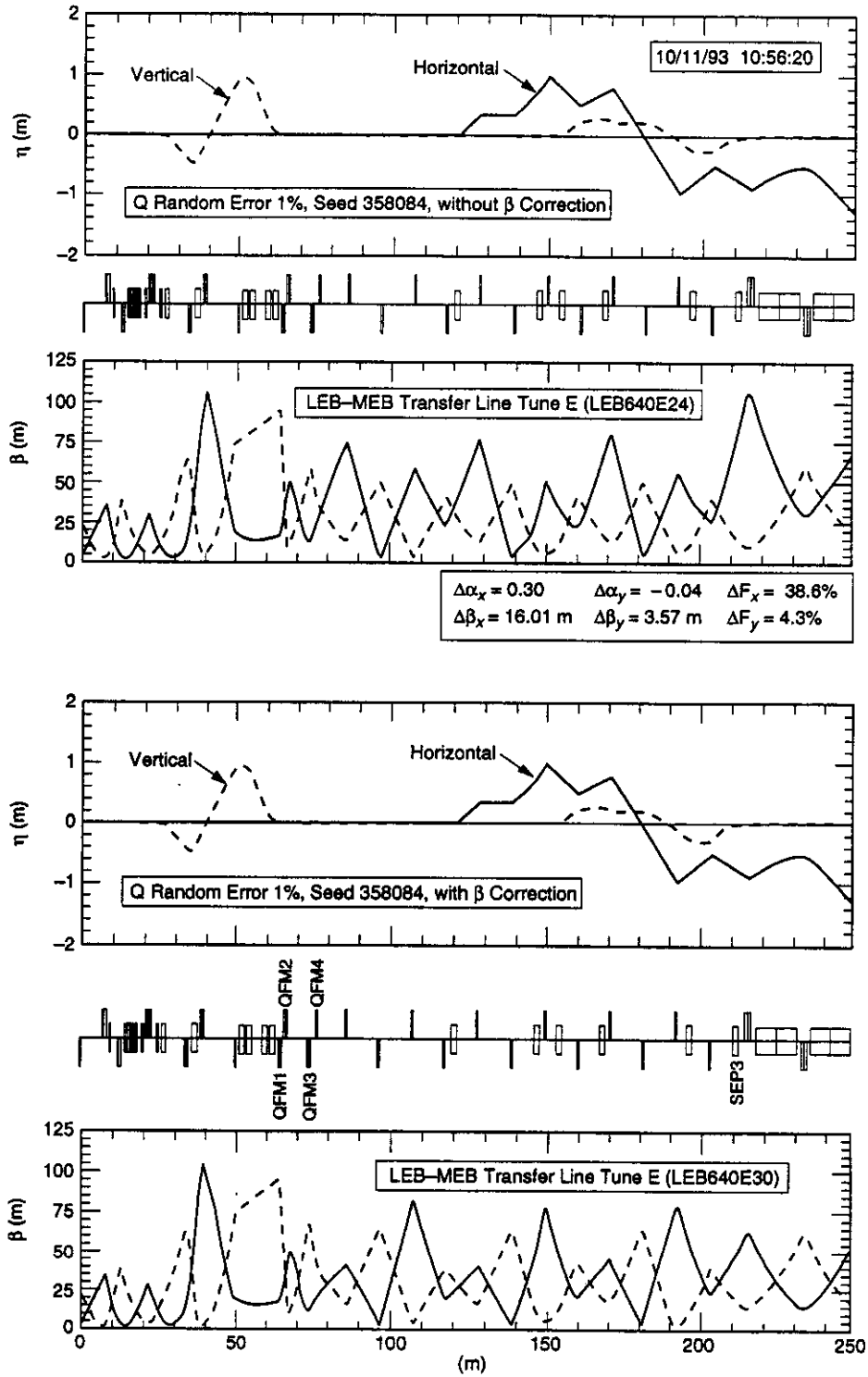


Figure 9. Emittance Dilution Due to Transfer Line Quadrupole RMS Gradient Error of 1%.



TIP-05772

Figure 10. β -functions Along the Transfer Line Without (top) and With (bottom) Corrections for the Mismatching Due to Quadrupole RMS Gradient Error of 1% (error seed 358084).

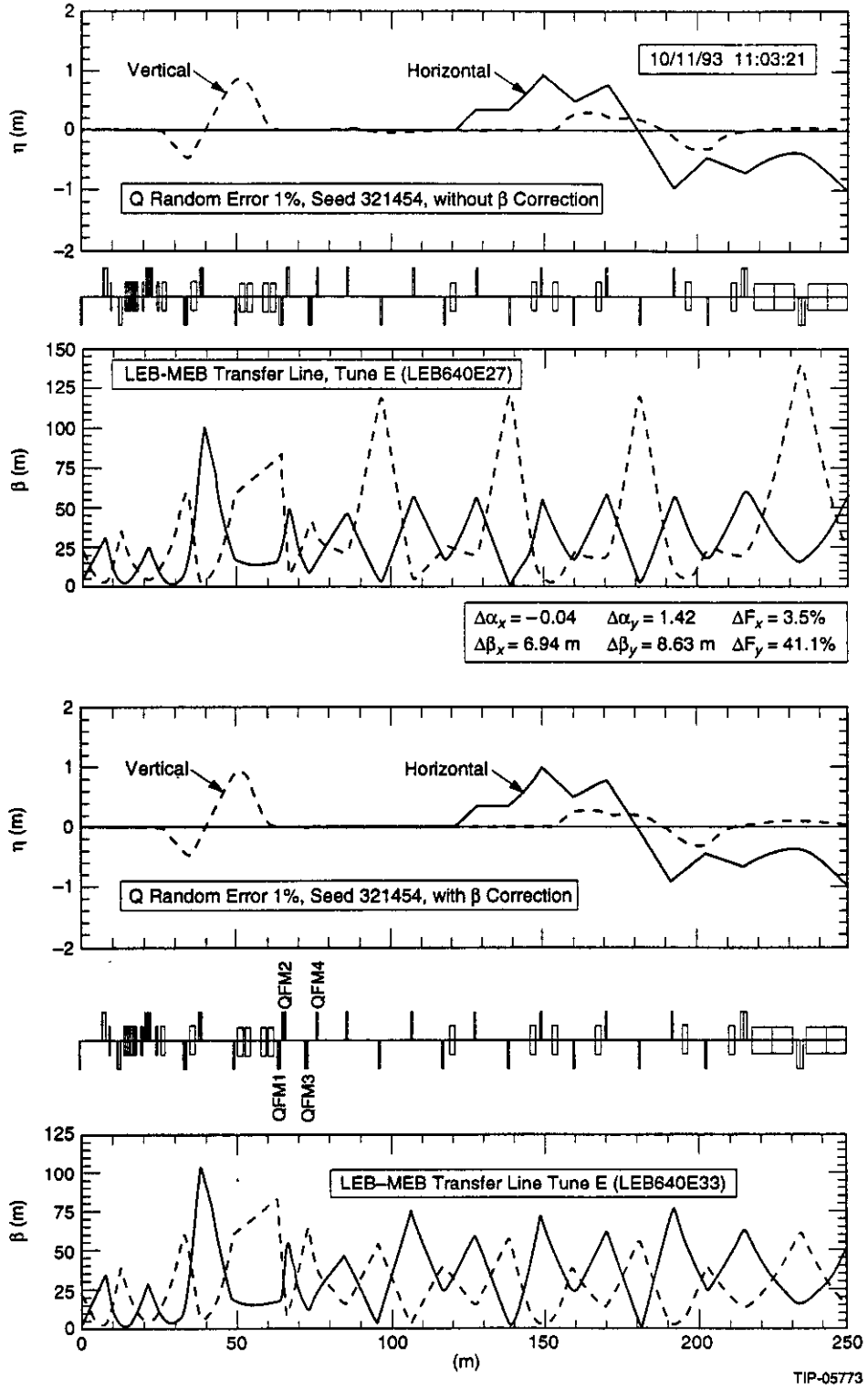
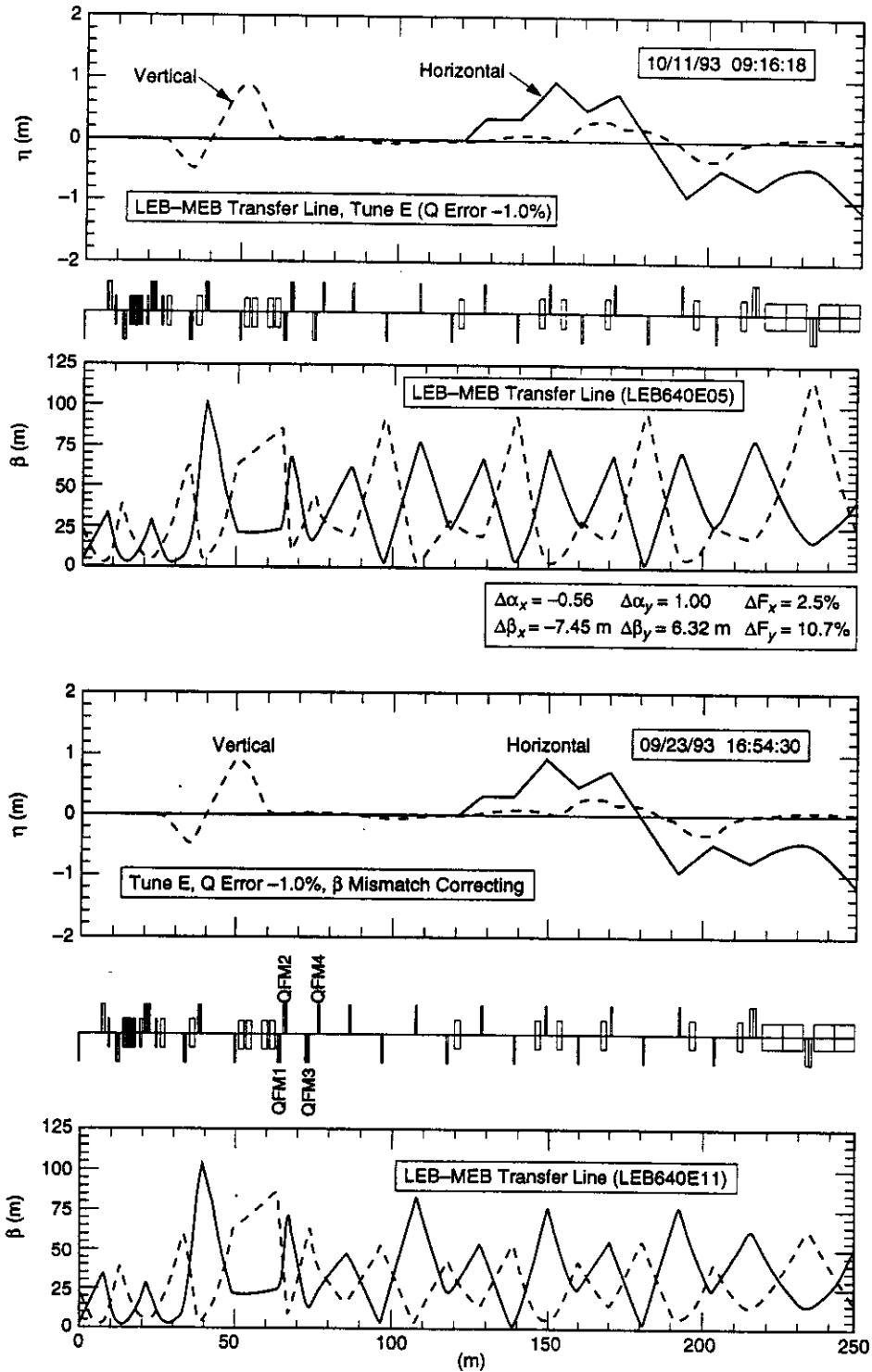


Figure 11. β -functions Along the Transfer Line Without (top) and With (bottom) Corrections for the Mismatching Due to Quadrupole RMS Gradient Error of 1% (error seed 321454).



TIP-05774

Figure 12. β -functions Along the Transfer Line Without (top) and With (bottom) Corrections for the Mismatching Due to Quadrupole RMS Gradient Error of -1%.

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