

α_s Measured at DELPHI

Hermann Fürstenau*

Institut für Experimentelle Kernphysik,
Universität Karlsruhe, D-7500 Karlsruhe



Abstract

The determination of the strong coupling constant and the extracting of informations about possible unification of the fundamental interactions has recently attracted our attention. Therefore different ways of determining α_s at LEP with the DELPHI detector are presented. The measurement of the hadronic and leptonic branching fraction of the τ -lepton, applying a recently proposed method leads to $\alpha_s(M_Z) = 0.114^{+0.009}_{-0.020}$. The hadronic jet and shape variables analysis of DELPHI gives $\alpha_s(M_Z) = 0.113 \pm 0.007$, adding all systematically and statistical errors in quadrature. The recommended recently published Next to Leading Logarithm Approximation is studied and tested for a significant reduction in the scale error. Using the proposed method one gets $\alpha_s(M_Z) = 0.122 \pm 0.006$.

1 α_s from R_τ

The hadronic and leptonic branching fraction R_τ of the τ lepton is experimentally determined by

$$R_\tau = \frac{B(\tau \rightarrow \text{hadrons} + \nu_\tau)}{B(\tau \rightarrow e\nu_e\nu_\tau)} . \quad (1)$$

The theoretical expressions for R_τ is[1]

$$R_\tau = 3(|V_{ud}|^2 + |V_{us}|^2) \cdot (1 + \delta_{\text{pert}} + \delta_{\text{non-pert}} + \delta_{\text{EW}}), \quad (2)$$

*Bitnet: FURSTENA@CERNVM

Postal address: Inst. für Experimentelle Kernphysik
Universität Karlsruhe, Postfach 6980, D-7500 Karlsruhe 1

where V_{ud} and V_{us} are the Kobayashi-Maskawa matrix elements. δ_{pert} and $\delta_{non-pert}$ are the perturbative and non-perturbative QCD corrections and δ_{EW} is the electroweak correction. δ_{EW} is of the order of 2% and $\delta_{non-pert}$ around -1% while δ_{pert} varies between 10% and 40% depending on α_s . It is remarkable that the perturbative QCD correction is at least 5 times larger than the non-perturbative corrections and this at a scale below 2 GeV where perturbative QCD is just one order of magnitude above the confinement scale Λ .

However, the perturbative correction in third order can be written as[1]

$$\delta_{pert} = \frac{\alpha_s}{\pi} + 5.202 \left(\frac{\alpha_s}{\pi} \right)^2 + 26.37 \left(\frac{\alpha_s}{\pi} \right)^3 \quad (3)$$

for three active flavors. Assuming lepton universality, DELPHI measured the branching ratio $R_\tau = 3.51 \pm 0.23$ [2] which corresponds to

$$\alpha_s(M_\tau) = 0.293^{+0.080}_{-0.106} \quad (4)$$

or

$$\alpha_s(M_Z) = 0.114^{+0.009}_{-0.020} . \quad (5)$$

I would like to point out three facts which can be deduced from Fig. 1 which shows R_τ as a function of α_s . Firstly, the renormalization of α_s from M_τ to M_Z shrinks the error on $\alpha_s(M_Z)$ in our case the relative error decreases by a factor 2.5. Secondly, the error on $\alpha_s(M_Z)$ depends on the mean value of R_τ . The smaller R_τ , the larger the error on $\alpha_s(M_Z)$, so that a precise measurement on R_τ does not necessarily lead to a small uncertainty in the strong coupling at M_Z . And finally one cannot get values of $\alpha_s(M_Z)$ larger than 0.14. The calculations have been performed up to third order in α_s . Nevertheless one still worries about the higher orders, since for $\alpha_s(\tau) = 0.4$ (or $\alpha_s(M_Z) = 0.125$) the third (second) order contribution relative to the first is 43% (66%). One might ask for higher order corrections before being convinced of the correct treatment of the series expansion at large α_s values.

2 α_s from Shape and Jet Variables

In the recent DELPHI analysis[3] the following hadronic variables were considered for determining $\alpha_s(M_Z)$: Thrust (T), Oblateness, C-parameter, Heavy Jet Mass (M_H^2/E_{vis}^2), Difference between heavy and light Jet Mass, Energy Energy Correlation (EEC), the Asymmetry of EEC and the Differential two Jet Rate in the so called JADE scheme. Two different methods were used to determine α_s in 2nd order, the Parton Shower method (PS) and the Matrix Element method (ME). In the PS method one corrects the data to the parton level with a parton shower Monte Carlo. The correction was done bin by bin. A second order QCD[4] fit to the corrected data was performed to extract α_s . In case of the ME method the data were corrected with a second order matrix element Monte Carlo generator using string fragmentation. A matrix was used to correct to the parton level. The ERT-p generator[5] implemented in JETSET 7.3[6] gave the second order expressions which include also a cut against infrared and ultraviolet divergences. The advantage of using these two different methods is twofold. Firstly, both methods have different hadronisation corrections. Secondly, in case of the PS one corrects with a $O(\alpha_s) + \text{LLA}$ calculation

to the parton level, so if the two methods differ one has to conclude that hadronisation corrections are important. Several other systematic errors sources have been studied which are described in detail in Ref. [3]. The major systematic error arises from the scale uncertainty. The gluon radiation occurs at a particular scale Q which is the physical scale of the process. One has to distinguish this scale from the so called renormalization scale μ which is not determined by QCD; one usually parametrizes μ as a function of the center of mass energy s and defines a scale parameter

$$f = \mu^2/s \quad . \quad (6)$$

If one identifies the physical scale Q with the renormalization scale μ , which is not obvious, higher order terms of the type $\ln Q^2/\mu^2$ vanish. Therefore one possible choice of μ could be the center of mass energy of the process. This is quite a large scale because the typical energy of a gluon is 20 GeV at LEP. On the other hand, PS Monte Carlos provide the best description of the hadronic events at scales which are typically of the order of the transverse momentum of the gluon relative to its mother particle. They are of the order of several GeV which would correspond to a scale parameter of about $f = 0.002$. Further below this small scale the perturbation series of QCD collapses. In Fig. 2 one can see the fit results of $\alpha_s(M_Z)$ for the ME and PS method as a function of the scale parameter f for the different variables as indicated. The $\alpha_s(M_Z)$ values are spread over a large range and the obvious problem is the averaging of the numbers to a single value. We have chosen a method in which at each scale the root mean square and the mean value are calculated. The final value was determined by taking the weighted averaged over all α_s values at each scale. The resulting α_s of this procedure is

$$\alpha_s(M_Z) = 0.113 \pm 0.002(\text{exp.}) \pm 0.003(\text{had.}) \pm 0.006(\text{scale}) \quad . \quad (7)$$

A detail description of the determination of the experimental(exp.) and hadronisation(had.) error can be found in Ref. [3]. If one adds the systematic errors in quadrature one obtains a total error of 0.007.

3 Comments on NLLA

The dominating error in the determination of α_s is the scale error. To avoid these theoretical uncertainties, the Next to Leading Logarithm Approximation (NLLA) [7] has been proposed. Leading Logarithm Approximations (LLA) are well known from the PS Monte Carlos, while in the NLLA one sums up one order of logarithms more. This has the advantage of a well defined renormalization scheme in which it is possible to extract $\Lambda_{\overline{MS}}^{(5)}$. However the quantities have to exponentiate. Therefore not all shape and jet variables can be calculated in the NLLA. The preferred field of application should be the range of multiple gluon emission which is indicated in the example of T in Fig. 3b. To fit second order calculations to the data, fit ranges sensitive to hard gluon radiation are chosen. A typical fit range is indicated for T in Fig. 3a. The combination of the second order matrix elements with the NLL approximation is also possible and is included in Fig. 3 as well. The resulting $\alpha_s(M_Z)$ as a function of the scale parameter f together with resulting χ^2 distribution can be seen in Fig 3c-f. In the following the χ^2 -estimator method is applied.

This exercise considers a scale parameter variation of more than four orders of magnitude and one has to distinguish four different cases.

- 1.) 2nd order calculation with fit range of Fig. 3a (full line). The minimum of the χ^2 distribution in Fig. 3e is around the scale $f = 0.005$ and $\Delta\chi^2 = 1$ would lead to $\alpha_s(M_Z) = 0.113^{+0.030}_{-0.002}$ (Fig. 3c). The smaller lower error is caused by the breakdown of fixed order perturbative QCD calculations at scales below $f = 0.001$.
- 2.) 2nd order + NLLA with fit range as above (dashed line). The minimum of the χ^2 distribution in Fig. 3e is around the scale $f = 19$ and $\Delta\chi^2 = 1$ would lead to $\alpha_s(M_Z) = 0.139^{+0.010}_{-0.020}$ (Fig. 3c). Note that the upper error is just limited by the restriction of the scales to be below $f = 100$.
- 3.) 2nd order + NLLA with fit range of Fig. 3b (dashed line). The minimum of the χ^2 distribution in Fig. 3f lies at scales larger than $f = 100$. The $\chi^2 = 3$ per degree of freedom is quite high and the fit should be considered as bad. $\alpha_s(M_Z) = 0.150$ would correspond to the minimum χ^2 in the considered scale interval (Fig. 3d).
- 4.) NLLA only with fit range as above (dotted line). The minimum of the χ^2 distribution in Fig. 3f lies at scales larger than $f = 100$ and leads to $\alpha_s(M_Z) = 0.150^{+0.0}_{-0.040}$ (Fig. 3d).

The upper limit of the scale parameter $f \leq 100$ is justified by the fact that $f = 100$ already leads to a scale of almost 1TeV and terms of the type $\ln Q^2/\mu^2$ become large. A physical argument for using such large scales is maybe beyond our naive picture and therefore the renormalization scale is in this case a more technical parameter. Following strictly the χ^2 -estimator method, a clear reduction of the scale uncertainty with the help of the NLL approximation cannot be observed. Reasons for restricting the range of the scale parameter have to be well justified. Therefore the desirable effect of having a steep χ^2 -distribution for the fit and a flat scale dependence on α_s cannot be achieved with the NLLA. This was of course just shown for the Thrust variable T but seems to be also true for the Heavy Jet Mass M_H^2/E_{v1s}^2 . The calculation of the differential 3-jet cross section in the so called Durham scheme[8] has not yet been calculated in all orders of NLLA and therefore the scale dependence is not complete. Ignoring all suspicious questions and following closely the ideas of Refs. [7], the second order plus NLLA analysis for T and M_H^2/E_{v1s}^2 gives the combined value of

$$\alpha_s(M_Z) = 0.1218 \pm 0.0008_{stat.} \pm 0.0010_{exp.} \pm 0.0018_{had.} \pm 0.0052_{scale} . \quad (8)$$

The scale was varied between $0.3 \leq f \leq 1$, which is twice the range suggested in Ref. [7]. The small variation of the scale around the so called 'natural scale' [7] of $f = 1$ which is orders of magnitude smaller than with the χ^2 estimator method, leads to a smaller theoretical error. Some people may feel more confident in using larger f ranges which are not excluded by data. But ad hoc there is no physical argument why the χ^2 estimator method should give the only correct range of scale parameters.

In conclusion, the value of $\alpha_s(M_Z)$ from R_r fits into the picture of the strong coupling constant obtained by the shape variable analysis in second order and in the NLL approximation.

Acknowledgments

I would like to thank the organizers of this conference for creating a pleasant and creative atmosphere and S.Catani, A.Pich, T. Sjöstrand and B. Webber for many useful discussions. A special thanks also to the DELPHI collaboration and the LEP machine people without whom this work would not be possible.

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Figure captions

- Fig. 1. a) $\alpha_s(M_\tau)$ as a function of R_τ in third order(full line) and in second order (dashed line). Fig. 2b shows the extrapolated $\alpha_s(M_Z)$ as a function of R_τ in third order(full line) and in second order (dashed line).
- Fig. 2. $\alpha_s(M_Z)$ as a function of the scale parameter f for a) the PS method b) for the ME method.
- Fig. 3. a) Thrust distribution with the best fit of the second order calculation (full line), 2nd order plus NLLA (dashed line) and NLLA only (dashed dotted line). Fig. 3a and 3b show the best fit of the particular model for the fit ranges indicated. Fig. 3c (3d) shows $\alpha_s(M_Z)$ as a function of the scale for the fit range indicated in Fig. 3a (3b). While Fig. 3e (3f) shows the χ^2 of the fit as a function of the scale for the fit range indicated in Fig. 3a (3b).

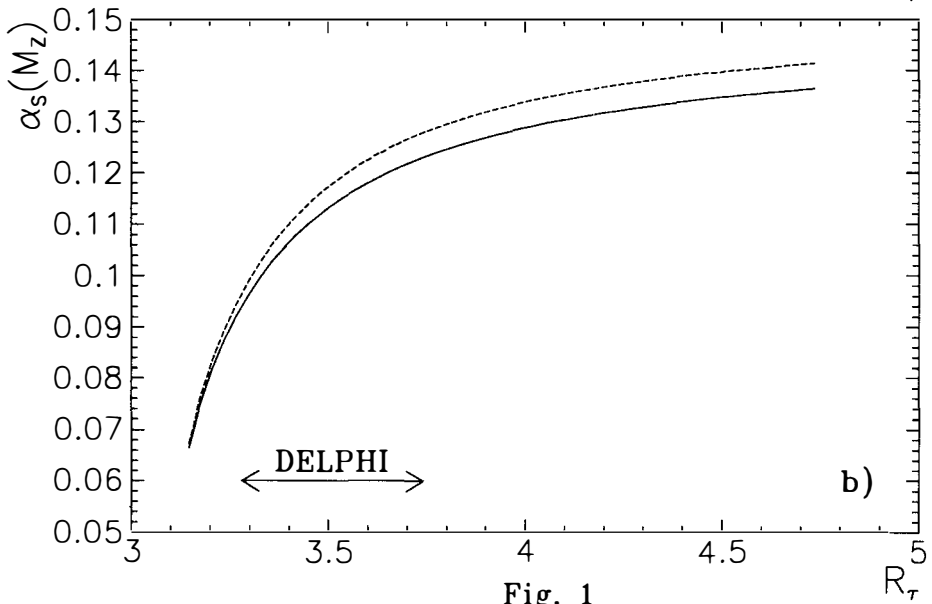
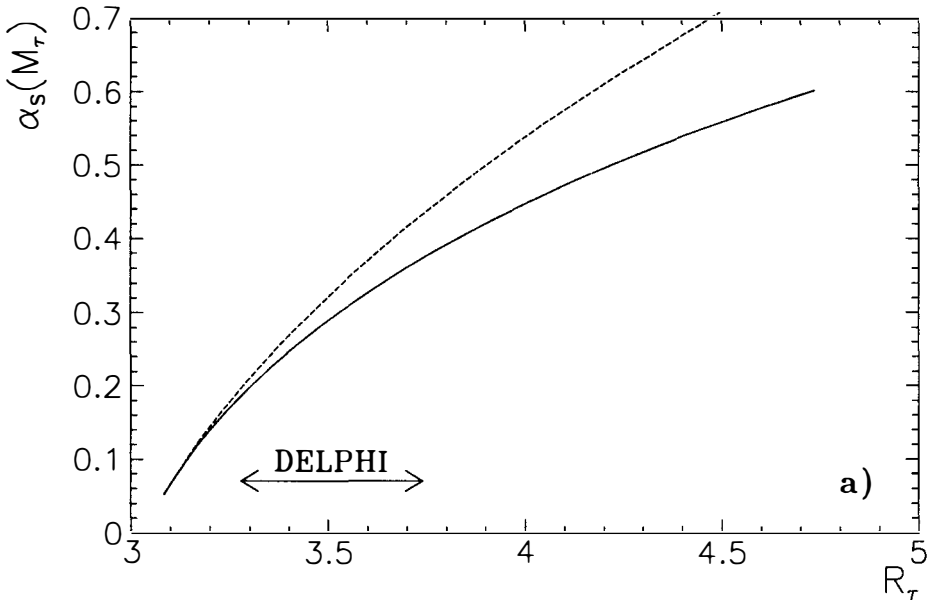


Fig. 1

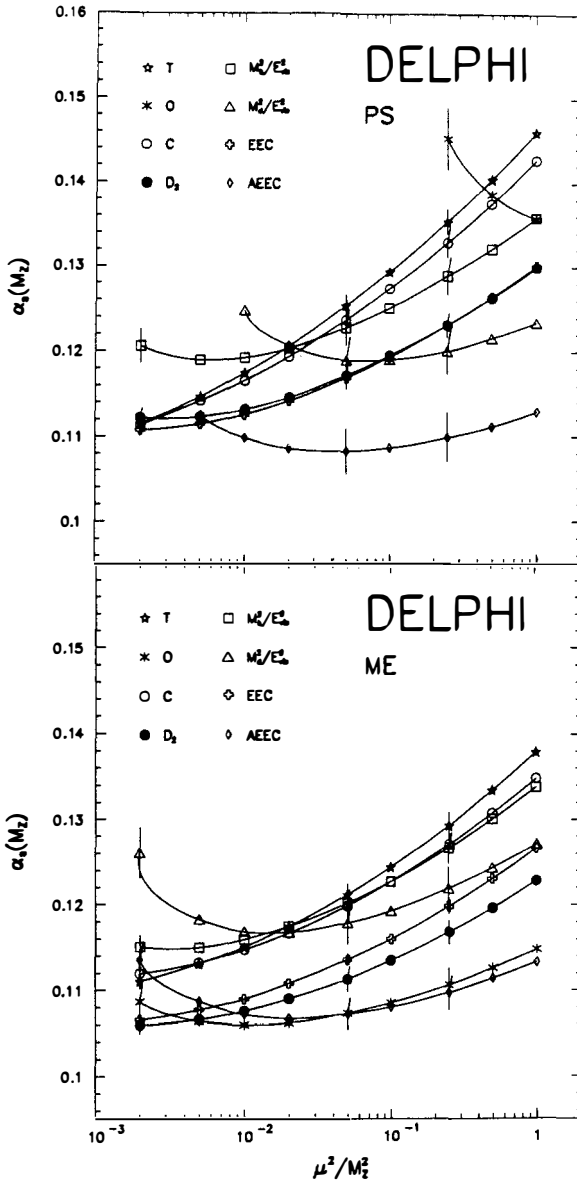


Fig. 2

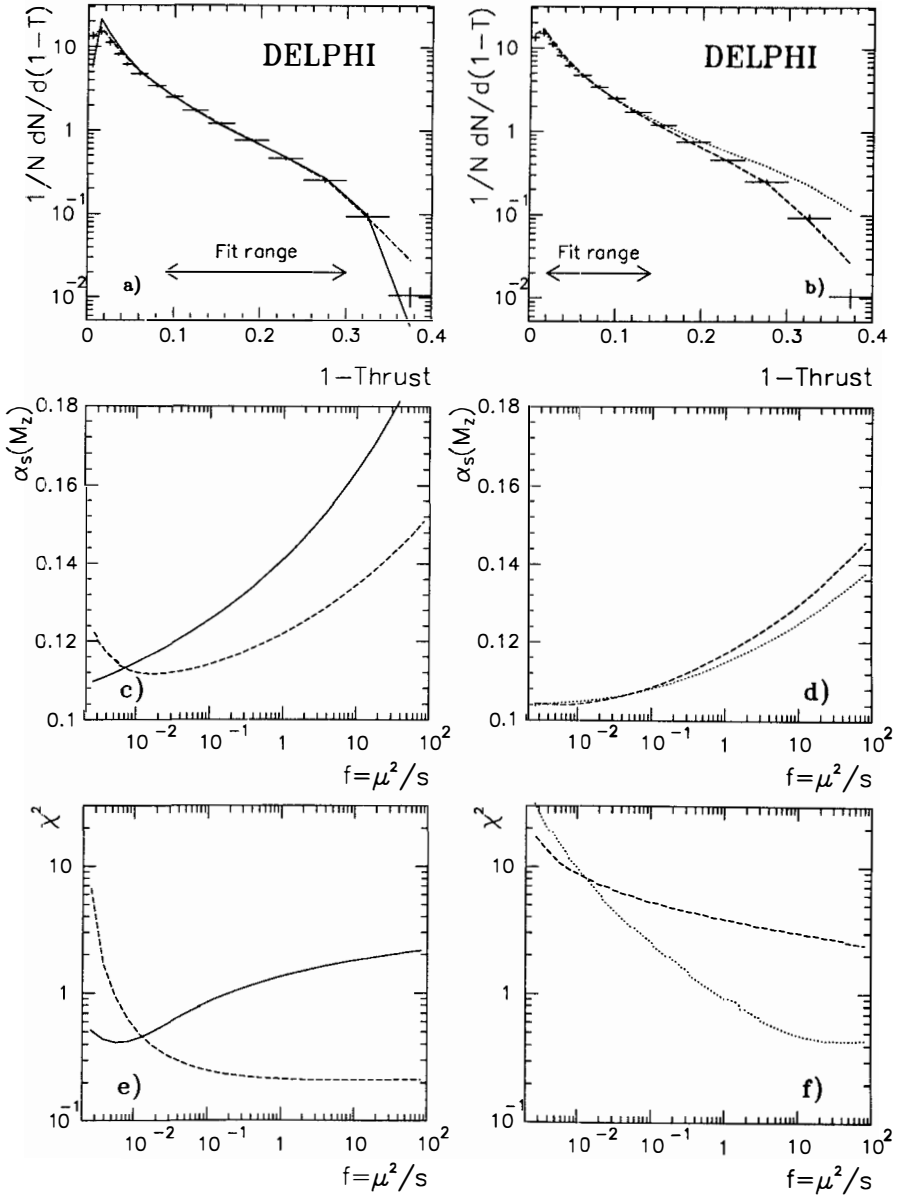


Fig. 3