

Preheating Mechanism in F-term SUSY-Hybrid Inflation

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Abstract. Supersymmetric F-term hybrid inflation is one of the most popular models of inflation. Preheating process occurs in this model via two different mechanism. Firstly the standard parametric resonance and secondly, the tachyonic preheating. Generally tachyonic preheating dominates the parametric resonance for this type of models. For different values of the parameters of the theory dominance of tachyonic preheating can vary.

1. Introduction

Inflationary paradigm has been greatly successful in solving the horizon problem and flatness problem and describing the origin of observed cosmic microwave background (CMB) anisotropy. While shorting out the best model for inflation is still a very active area of modern cosmology, theory for describing matter formation from the energy of inflation is still not well developed. Since an exponential expansion at the time of inflation extremely decreases the temperature of the universe a process of reheating is necessary to get present observed CMB temperature. But a standard reheating process dominated by the elementary decay of inflaton field to the other matter fields is not sufficient enough to produce proper reheating temperature. In 90's a nonlinear and non-perturbative theory of rapid production of matter field from the inflaton field got developed, which helped to reduce the time of long matter dominated era just after the end of inflation[1]. This rapid growth of matter fields expected to happen prior to the elementary reheating, that is why this process is known as *preheating*.

In this particular note we will be concentrating on the F-term SUSY Hybrid model of inflation. From WMAP data it became prominent that scalar spectral index is less than one. So standard hybrid inflation was replaced by its supersymmetric version to match with the observation. Specially F-term SUSY Hybrid model is tailor-made to fit with Higgs mechanism.

Preheating mechanism in this model is of two types, tachyonic preheating and parametric resonance. In general tachyonic preheating is more fast and effective than parametric resonance. But depending on the parameters of the model and comoving wavenumber of the matter field one process may be favored over other.

2. F-term SUSY Hybrid Model

In general F-term SUSY-Hybrid inflation assumes a superpotential of the following form[2].

$$W = \kappa\Phi(\Psi\bar{\Psi} - M^2) \quad (1)$$

Here $\Psi, \bar{\Psi}$ are chiral scalar multiplets conjugate to each other. Φ is a gauge singlet scalar and contains inflaton (ϕ). F-term scalar potential coming from this super-potential is

$$V_F = \kappa^2(|\psi|^2 - M^2)^2 + 2\kappa^2|\phi|^2|\psi|^2. \quad (2)$$

where ψ and $\bar{\psi}$ can be any scalar multiplet conjugate to each other. Inflation occurs in flat direction $\psi = \bar{\psi} = 0$ which has a constant potential of $\kappa^2 M^4$. Total effective potential by driving the inflation is the sum of this constant part and one loop correction in this direction[3, 4]:

$$V(\phi) = \kappa^2 M^4 + \frac{\kappa^4 \mathcal{N}}{32\pi^2} \left((\phi^2 + M^2)^2 \log \frac{\kappa^2(\phi^2 + M^2)}{\Lambda^2} \right. \\ \left. + (\phi^2 - M^2)^2 \log \frac{\kappa^2(\phi^2 - M^2)}{\Lambda^2} - 2\phi^4 \log \frac{\kappa^2 \phi^2}{\Lambda^2} \right) \quad (3)$$

where \mathcal{N} is the dimensionality of the representation of $\psi, \bar{\psi}$ and Λ is the renormalization scale. This potential has two minima, one is the local minima at $\phi = 0, \psi = 0$ and another is the global minima at $\phi = 0, \psi = M$. After the end of inflation another susy-breaking term, say $m^2 \phi^2$ is required to drive inflaton to $\phi = 0$. Value of m^2 is taken much smaller than $\kappa^4 M^2$.

So, generally these types of potentials can be written as

$$V(\phi) = V_0 + v(\phi). \quad (4)$$

where V_0 is the constant vacuum energy density and $v(\phi)$ is the loop correction term.

Scalar spectral index(n_s) and amplitude of density perturbation ($\Delta_{\mathcal{R}}$) of CMB are related to the inflation potential through the slowroll parameters(ϵ, η).

$$n_s = 1 - 6\epsilon + 2\eta \quad (5)$$

$$\Delta_{\mathcal{R}}^2 = \frac{8}{m_{Pl}^4} \left. \frac{V}{\epsilon} \right|_{k=aH} \quad (6)$$

where,

$$\epsilon = \frac{m_{Pl}^2}{16\pi} \left(\frac{V_\phi}{V} \right)^2 = \frac{m_{Pl}^2}{16\pi} \left(\frac{v_\phi}{V_0 + v} \right)^2, \quad (7)$$

$$\eta = \frac{m_{Pl}^2}{8\pi} \left(\frac{V_{\phi\phi}}{V} \right) = \frac{m_{Pl}^2}{8\pi} \left(\frac{v_{\phi\phi}}{V_0 + v} \right). \quad (8)$$

Here ϕ in subscript means derivative with respect to ϕ and $m_{Pl} = 1/\sqrt{G}$.

Amplitude of tensor perturbation is related to inflation potential as

$$\Delta_h^2 = \frac{128}{3m_{Pl}^4} V(\phi) \Big|_{k=aH} \quad (9)$$

So tensor-to-scalar ratio takes the form $r = \frac{\Delta_h^2}{\Delta_{\mathcal{R}}^2} = 16\epsilon \approx \frac{128}{3m_{Pl}^4 \Delta_{\mathcal{R}}^2} \kappa^2 M^4$. SUSY hybrid model is a model with negative η , which gives negligibly small tensor-to-scalar ratio. If we take number of *e-foldings*, N , corresponding to the WMAP pivot scale 0.002 Mpc^{-1} , to be 60 and \mathcal{N} is 8 then for $\Delta_{\mathcal{R}}^2 = (2.41 \pm 0.11) \times 10^{-9}$ and $n_s = 0.983$ [5] we should have $M = 9.55 \times 10^{15} \text{ GeV}$ and $\kappa = 0.008 - 0.01$.

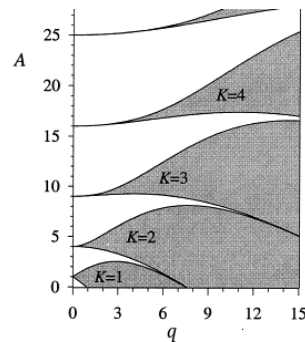


Figure 1. Mathieu chart showing stable(white) and unstable(dark) region in A, q space.[6]

3. Parametric Resonance Mechanism

We Fourier decompose the spatial part of ψ and derive the equations of motion from equation(2). After taking mean-field approximation we get

$$\ddot{\psi}_k + 3H\dot{\psi}_k + \frac{k^2}{a^2}\psi_k + 4\kappa^2\langle\psi^2\rangle\psi_k + 4\kappa^2\phi^2\psi_k - 4\kappa^2M^2\psi_k = 0 \quad (10)$$

$$\ddot{\phi} + 3H\dot{\phi} + 2\kappa^2\langle\psi^2\rangle\phi + m^2\phi = 0 \quad (11)$$

As usual inflaton ϕ is regarded as spatially homogeneous. $\langle\psi^2\rangle$ is the spatially averaged value of ψ^2 and k is the comoving wavenumber. By taking $X_k = a^{3/2}\psi_k$ we re-write equation(10) as:

$$\ddot{X}_k + \left[\frac{k^2}{a^2} + 4\kappa^2\phi^2 + 4\kappa^2\frac{\langle X^2 \rangle}{a^3} - 4\kappa^2M^2 \right] X_k = 0 \quad (12)$$

Here pressure term has been neglected since at the time of free oscillation it is vanishingly small. For oscillatory case of ϕ we assume $\phi = \phi_0 \cos \omega_\phi t$ and change the variable t to $\tau = \omega_\phi t$. In this way we arrive at the familiar form of Mathieu equation[7] from (10)

$$\frac{d^2 X_k}{d\tau^2} + [A_k + 2q \cos(2\tau)]X_k = 0 \quad (13)$$

where

$$A_k = \left(k^2/a^2 + 4\kappa^2\frac{\langle X^2 \rangle}{a^3} - 4\kappa^2M^2 \right) / \omega_\phi^2 + 2q \quad (14)$$

$$q = \kappa^2\phi_0^2/\omega_\phi^2 \quad (15)$$

Solution of Mathieu equation takes the form like $X_k \sim e^{\nu_k \tau}$ where ν_k is the critical exponent and depends on A_k and q . In an unstable band of Mathieu chart (fig. 1) ν gets a real value which gives exponential growth in X_k . Occupation number of ψ_k is proportional to $|X_k^2| \sim e^{2\nu_k \tau}$ [8]. From numerical solution of eqn.(10) occupation number can also be estimated using the the well known formula[7]

$$n_k = \frac{\omega_k}{2} \left(\frac{|\dot{\psi}_k|^2}{\omega_k^2} + |\psi_k|^2 \right) - \frac{1}{2} \quad (16)$$

where $\omega_k^2 = \frac{k^2}{a^2} - 4\kappa^2M^2$.

In an expanding universe parametric resonance is always expected to be in broad resonance regime. A_k and q both change with time and parametric resonance is not constrained in a single unstable band of Mathieu chart (fig. 1). By observing the growth of n_k we can find the average critical exponent[9].

4. Tachyoninc Preheating Mechanism

Tachyonic preheating is a process in which if a field has tachyonic instability around some extrema, its amplitude can increase explosively and preheating gets complete within a very short time. In our model as the inflaton ϕ crosses the critical value M , potential for ψ becomes tachyonic. Amplitude of ψ_k keeps increasing as long as $k^2 + 4\kappa^2\psi_k^2 + 4\kappa^2\phi^2 - 4\kappa^2M^2$ remains negative. Occupation number of the ψ_k can be defined as equation(16) using $\omega_k^2 = \frac{k^2}{a^2}$ without any loss of generality [10]. If initially $\langle\psi^2\rangle$ is small compared ϕ and M , we can have

$$\psi_k \sim \exp(t\sqrt{4\kappa^2M^2 - k^2 - 4\kappa^2\phi^2}) \quad (17)$$

At the time of the start of preheating a has been taken to be 1. Damping term is neglected for the time being. ϕ decreases with time and $\langle\psi^2\rangle$ increases very fast. Therefore, within a short time growth of ψ_k and $\langle\psi^2\rangle$ gets complete. So initial form of $\langle\psi^2\rangle$ is

$$\langle\psi^2\rangle = \int_0^{\sqrt{4\kappa^2\phi^2 - 4\kappa^2M^2}} \frac{dk^2}{8\pi^2} \exp(2t\sqrt{4\kappa^2M^2 - k^2 - 4\kappa^2\phi^2}) \quad (18)$$

$$= \frac{1}{16\pi^2t^2} \left(1 + (2t\sqrt{4\kappa^2\phi^2 - 4\kappa^2M^2} - 1)e^{2t\sqrt{4\kappa^2\phi^2 - 4\kappa^2M^2}} \right) \quad (19)$$

Then after some time when $\langle\psi^2\rangle$ reaches M^2 and it settles down at that value. Growth of occupation number can also be estimated to be

$$n_k \sim |\psi_k^2| \sim \exp(2t\sqrt{4\kappa^2M^2 - k^2/a^2 - 4\kappa^2\phi^2}) \quad (20)$$

5. Discussion

For different values of κ and M different modes of ψ_k undergoes different processes. For large M like 10^{17} GeV and $\kappa = 0.01$ modes with initial physical wavenumber $k^2/a^2 = 0.3(4\kappa^2M^2)$ and $k^2/a^2 = 0.1(4\kappa^2M^2)$ undergo tachyonic growth. But for small M like 10^{15} GeV and $\kappa = 0.001$ first one of these mode undergoes parametric resonance and the second one gets tachyonic growth. A detailed discussion on this parameter dependence of the preheating process has been done in Ref.[11]. For experimental tuned values of κ and M , i.e. $M = 9.55 \times 10^{15}$ GeV and $\kappa = 0.008 - 0.01$ first of the above modes shows parametric resonance and second mode grows due to tachyonic instability.

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References

- [1] L. Kofman, A. D. Linde and A. A. Starobinsky, Phys. Rev. Lett. **73** (1994) 3195 [hep-th/9405187].
- [2] G. R. Dvali, Q. Shafi and R. K. Schaefer, Phys. Rev. Lett. **73**, 1886 (1994) [hep-ph/9406319].
- [3] S. R. Coleman and E. J. Weinberg, Phys. Rev. D **7** (1973) 1888.
- [4] M. Huq, Phys. Rev. D **14**, 3548 (1976).
- [5] D. Larson, J. Dunkley, G. Hinshaw, E. Komatsu, M. R.olta, C. L. Bennett, B. Gold and M. Halpern *et al.*, Astrophys. J. Suppl. **192**, 16 (2011) [arXiv:1001.4635 [astro-ph.CO]].
- [6] N. W. McLachlan "Theory and Application of Mathieu Functions", Oxford Univ. Press, Clarendon (1947)
- [7] J. Garcia-Bellido and A. D. Linde, Phys. Rev. D **57**, 6075 (1998) [hep-ph/9711360].
- [8] I. Zlatev, G. Huey and P. J. Steinhardt, Phys. Rev. D **57**, 2152 (1998) [astro-ph/9709006].
- [9] M. Bastero-Gil, S. F. King and J. Sanderson, Phys. Rev. D **60**, 103517 (1999) [hep-ph/9904315].
- [10] G. N. Felder, L. Kofman and A. D. Linde, Phys. Rev. D **64**, 123517 (2001) [hep-th/0106179].
- [11] A. Mazumdar, arXiv:1204.4113 [astro-ph.CO].