

Review of Parton Recombination Models

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Abstract. Parton recombination models have been very successful in explaining data taken at RHIC on hadron spectra and emission patterns in Au+Au collisions at transverse momenta above 2 GeV/c, which have exhibited features which could not be understood in the framework of basic perturbative QCD. In this article I will review the current status on recombination models and outline which future challenges need to be addressed by this class of models.

1. Introduction

Collisions of heavy nuclei at relativistic energies are expected to lead to the formation of a deconfined phase of strongly interacting nuclear matter, often referred to as a Quark-Gluon-Plasma (QGP). Recent data from the Relativistic Heavy-Ion Collider (RHIC) at Brookhaven Lab have provided strong evidence for the existence of a transient QGP – among the most exciting findings are strong (hydrodynamic) collective flow [1, 2, 3, 4, 5, 6], the suppression of high- p_T particles [7, 8, 9, 10] and evidence for parton recombination as hadronization mechanism at intermediate transverse momenta [11, 12, 13, 14, 15].

The development of parton recombination as a key hadronization mechanism for hadrons with transverse momenta up to a couple of GeV/c was triggered by a series of experimental observations which could not be understood in a straightforward manner either in the framework of perturbative QCD or via regular soft physics, i.e. hydrodynamic scaling.

These observations, which in a broader context have often been referred to as the *baryon puzzle at RHIC* are in particular:

- **baryon/meson particle ratios:** It has been found that the ratio of protons over positively charged pions is equal or above one for $p_T > 1.5$ GeV/c and remains approximately constant up to 4 GeV/c [16, 17, 18, 19]. The rise of this ratio as a function of p_T can be well understood for transverse momenta between zero and roughly 1.5 GeV/c as a result of a hydrodynamic expansion with a strong radial flow component (which affects baryons more strongly than light mesons). However, for $p_T > 1.5$ GeV/c one would expect hadron productions to be described by perturbative QCD and the fragmentation of hard partons – here the proton/pion ratio would be given by the ratio of the respective fragmentation functions and be on the order of 0.1 – an order of magnitude below the observed value.
- **flavor dependence of the nuclear modification factor:** The strong nuclear suppression of the pion yield at transverse momenta larger than 2 GeV/c in central Au + Au collisions, compared to $p + p$ interactions is widely seen as the experimental confirmation of jet

quenching, the phenomenon that high energy partons lose energy when they travel through the hot medium created in a heavy ion collision [20, 21, 22], entailing a suppression of intermediate and high P_T hadrons. However, the amount of suppression seems to depend on the hadron species [24, 23]. In fact, in the production of protons and antiprotons between 2 and 4 GeV/c the suppression seems to be completely absent. Generally, mesons appear to suffer from a strong energy loss while baryons and antibaryons do not. This behavior is surprising, since the fragmentation of a hard parton into a hadron should first occur after it has traversed the deconfined medium – jet-quenching should therefore be flavor-blind and not exhibit a meson/baryon asymmetry.

- **constituent quark number scaling of the elliptic flow:** Measurement of the elliptic flow v_2 as a function of p_T shows a surprising species dependence above $p_T \approx 1$ GeV/c. For transverse momenta below 1 GeV/c the elliptic flow coefficient v_2 shows a monotonous increase as a function of p_T , which is well described in the framework of hydrodynamics and exhibits the expected mass-scaling. Above 1 GeV/c, however, the elliptic flow saturates in a species dependent fashion which cannot be described by hydrodynamics. A rescaling of the v_2 vs. p_T curve by the number of valence quarks contained in the respective hadron on both axes (i.e. plotting v_2/n vs. p_T/n with n the number of valence quarks) yields a *universal* curve for all hadron species [25, 14], strongly indicative of parton recombination.
- **two component shape of single particle spectra:** Transverse momentum spectra of identified hadrons exhibit a unique two-component shape as a function of p_T : exponential (i.e. thermal) for low and intermediate transverse momenta (up to 2-4 GeV/c, depending on the hadron species) and a power-law shape for higher transverse momenta. The power-law shape is indicative of pQCD driven particle production, the exponential hints at a thermalized system, possibly with hydrodynamic features. While the low and high p_T limits of the spectral shape do not come as surprise, the details of the transition from one regime to the other can neither be directly addressed in a thermal model nor by a pQCD mini-jet calculation.

While attempts have been made to describe the above observations with a variety of different physics concepts, e.g. gluon junctions in the case of the high p_T baryon excess [26], or a hydrodynamic plus pQCD jet-production approach for the spectral shape [27], only models based on parton recombination at intermediate p_T combined with hadron production via fragmentation at high p_T have been able to describe all of the above phenomena within one consistent framework.

2. Formalism of the Recombination plus Fragmentation approach

Inclusive hadron production at sufficiently large momentum transfer can be described by perturbative quantum chromodynamics (pQCD). The invariant cross section for a hadron h with momentum P can be given in factorized form [28]

$$E \frac{d\sigma_h}{d^3 P} = \sum_a \int_0^1 \frac{dz}{z^2} D_{a \rightarrow h}(z) E_a \frac{d\sigma_a}{d^3 P_a}. \quad (1)$$

The sum runs over all parton species a and σ_a is the cross section for the production of parton a with momentum $P_a = P/z$. Thus the parton production cross section has to be convoluted with the probability that parton a fragments into hadron h . The probabilities $D_{a \rightarrow h}(z)$ are called fragmentation functions [29]. Like parton distributions they are non-perturbative quantities. However, they are universal and once measured, e.g. in e^+e^- annihilations, they can be used to describe hadron production in other hard QCD processes.

Using (1) we can estimate the ratio of protons and pions. Taking, e.g., the common parametrization of Kniehl, Kramer and Pötter (KKP) [30], the ratio $D_{a \rightarrow p}/D_{a \rightarrow \pi^0}$ is always smaller than 0.2 for each parton a . This reflects the well known experimental fact that pions are much more abundant than protons in the domain where pQCD is applicable. The excess of pions over protons even holds down to very low P_T , smaller than 1 GeV, where perturbative calculations are no longer reliable. In that domain one can argue that the difference in mass, $M_p \gg M_\pi$, lays a huge penalty on proton production. The small value of the p/π^0 ratio predicted by these calculations over the entire range of P_T is the reason why the ratio $p/\pi^0 \sim 1$ measured at RHIC is so surprising.

It has been suggested that the fragmentation functions $D_{a \rightarrow h}(z)$ can be altered by the environment [31, 32]. The energy loss of the propagating parton in the surrounding medium leads, in first approximation, to a rescaling of the variable z . This would affect all produced hadrons in the same way, and thus cannot explain the observations at RHIC. In a picture with perturbative hadron production and jet quenching alone, the different behavior of hadrons can not be described by one consistent set of energy loss parameters. To save the validity of the purely perturbative approach, species dependent non-perturbative contributions to the fragmentation functions have to be introduced *ad hoc* to explain the data [33].

Perturbative hadron production consists of three steps: production of a parton in a hard scattering, propagation and interaction with a medium, and finally hadronization of the parton. Only modifications in our understanding of hadronization are able to provide an explanation of the experimental observations, since the other steps are blind to the hadron species that will eventually be created.

Recombination can be interpreted as the most “exclusive” form of hadronization, the endpoint of a hypothetical resummation of fragmentation processes to arbitrary twist. Comparing a simple model for recombination with single parton fragmentation, one finds that these two mechanisms of hadron production compete differently depending on the phase space density of partons. The competition between fragmentation and recombination is dominated by the slope and the absolute value of the phase space distribution of partons. Recombination always wins over fragmentation for an exponentially falling parton spectrum, but fragmentation takes over if the spectrum has the form of a power law, e.g. as provided by pQCD. This insight can be applied to hadron production in relativistic heavy ion collisions at midrapidity and transverse momenta of a few GeV/c where a densely populated phase space is expected. For the recombination of three quarks into a baryon the momenta of three partons have to be added up, but only two momenta in the case of a meson. Assuming an exponential parton spectrum this implies for a baryon a distribution $\sim [\exp(-P_T/3)]^3$ and for mesons $\sim [\exp(-P_T/2)]^2$, predicting e.g. a constant p/π^+ ratio where the value is determined by simple counting of quantum numbers [11].

The idea of quark recombination was proposed long ago to describe hadron production in the forward region of $p + p$ collisions [34]. This was later justified by the discovery of the leading particle effect, the phenomenon that, in the forward direction of a beam of hadrons colliding with a target, the production of hadrons sharing valence quarks with the beam hadrons are favored. E.g. in the Fermilab E791 fixed target experiment with a 500 GeV π^- beam [35] the asymmetry

$$\alpha(x_F) = \frac{d\sigma_{D^-}/dx_F - d\sigma_{D^+}/dx_F}{d\sigma_{D^-}/dx_F + d\sigma_{D^+}/dx_F} \quad (2)$$

between D^- and D^+ mesons grows nearly to unity when the Feynman variable x_F , measuring the longitudinal momentum relative to the beam momentum, approaches 1. Fragmentation would predict this asymmetry to be very close to zero. However, recombination of the \bar{c} quark from a $c\bar{c}$ pair produced in a hard interaction with a d valence quark from the π^- , propagating in forward direction with large momentum, is highly favored compared to the recombination of the c with a \bar{d} which is only a sea quark of the π^- . This leads to the enhancement of D^- over

D^+ mesons in the forward region.

The leading particle effect is a clear signature for the existence of recombination as a hadronization mechanism and has been addressed in several publications recently [36, 37]. In this case recombination is favored over fragmentation only in a certain kinematic situation (the very forward direction), which is only a small fraction of phase space.

In central heavy ion collisions many more partons are produced than in collisions of single hadrons. The idea that recombination may then be important for a wide range of rapidities – and at least up to moderate transverse momenta – was advocated before [39, 40, 41]. However, it was only recently that RHIC data indicated that recombination could indeed be a valid approach up to surprisingly high transverse momenta of a few GeV/c .

Charm and heavy hadron production have the advantage that the heavy quark mass provides a large scale that permits a more rigorous treatment of the recombination process [37]. The description of recombination into pions and protons seems to be theoretically less rigorous. However, a simple counting of quantum numbers in a picture where the structure of hadrons is dominated by their valence quarks often provides surprisingly good results. This has been pointed out for particle spectra and ratios [38, 11, 42] and for elliptic flow [47, 14, 48]. Recombination of D mesons in heavy ion collisions has been investigated in [43].

The recombination approach can be formulated in terms of Wigner functions. The yield of mesons M recombining from two partons a, b is given by [12]

$$\frac{dN_M}{d^3P} = \sum_{a,b} \int \frac{d^3R}{(2\pi)^3} \frac{d^3qd^3r}{(2\pi)^3} W_{ab} \left(R - \frac{r}{2}, \frac{P}{2} - q; R + \frac{r}{2}, \frac{P}{2} + q \right) \Phi_M(r, q). \quad (3)$$

W_{ab} and Φ_M are the Wigner functions of the partons and the meson respectively, P and R are the momentum and spatial coordinate of the meson and the sum runs over all possible combinations of quantum numbers, essentially leading to a degeneracy factor C_M . The generalization of this formula for baryons is straightforward [12]. The Wigner function for the partons is usually factorized into classical one-particle phase space distributions, $W_{ab}(r_a, p_a; r_b, p_b) = w_a(r_a, p_a)w_b(r_b, p_b)$. This assumes that the partons are completely uncorrelated before hadronization.

One of the interesting conclusions of this ansatz is that if the parton spectrum is exponential in transverse momentum p_T , $w = Ae^{-p_T/T}$, with a slope parameter $1/T$, then fragmentation and recombination would provide meson spectra

$$dN_{\text{frag}}/d^2P_T D \propto Ae^{-p_T/\langle z \rangle T} \langle D \rangle, \quad dN_{\text{reco}}/d^2P_T \propto A^2 e^{-p_T/T} \quad (4)$$

respectively. Here $\langle D \rangle$ and $\langle z \rangle$ are average values of the fragmentation function and the scaling variable z . Since $\langle z \rangle < 1$, fragmentation is less effective than recombination on an exponential spectrum, as long as the normalization A is not too small. On the other hand, if the parton spectrum has power law form, $w = Bp_T^{-\alpha}$, the yields are $dN_{\text{frag}}/d^2P_T \sim P_T^{-\alpha}$ and $dN_{\text{reco}}/d^2P_T \sim P_T^{-2\alpha}$ for mesons. This implies that fragmentation will dominate at large P_T for power law spectra (which are predicted by perturbative QCD).

3. Implementations

Four groups have contributed to the development of the recombination approach in the past years in a major fashion. These groups are:

- Duke/Minnesota/Osaka (Duke) [11, 44, 12, 45, 46]
- Texas A& M/Budapest (TAM) [13, 42, 52, 53]
- Oregon [38, 51]

- Ohio State/Wayne State (Ohio) [48, 50, 49]

While utilizing the same basic principles of the recombination approach, the groups differ in their particular implementation of the model. The Duke and Oregon groups chose a semi-analytic approach and try to evaluate Equation (3) using certain assumptions that essentially result in a convolution of one-particle distributions and hadron wave functions in longitudinal momentum space (longitudinal with respect to the hadron momentum). For example, the Duke group writes the meson spectrum from recombination as [12]

$$E \frac{dN_M}{d^3P} = C_M \int_{\Sigma} d\sigma \frac{P \cdot u}{(2\pi)^3} \int dx w_a(\sigma, xP^+) w_b(\sigma, (1-x)P^+) |\phi_M(x)|^2 \quad (5)$$

where $d\sigma$ integrates over the hadronization hypersurface Σ , u is the four vector orthogonal to Σ , x is the momentum fraction of parton a in the meson, P^+ is the light cone momentum and ϕ_M the wave function of the meson. The parton phase that undergoes hadronization is assumed to have an exponential part at low p_T (soft partons) and a power law tail at high p_T (hard partons). For central Au+Au collisions at RHIC, the parameters chosen by the Duke group are those of a thermal distribution with temperature $T = 175$ MeV and average radial flow velocity $v = 0.55c$ for the soft partons, while the hard partons are taken from a minijet calculation [54] including energy loss [12]. The temperature is being kept fixed in these calculations, whereas the radial flow velocity and the energy loss coefficient have been fitted to data [12].

The Ohio and TAM groups utilize Monte Carlo implementations of the recombination process. These can be connected to microscopic transport models that prepare the parton state before hadronization. One of the main differences lies in the treatment of the connection between soft and hard partons. While the Duke group strictly separates soft and hard physics, allowing only the soft partons to recombine and only the hard partons to fragment, the TAM group includes additional coalescence of soft and hard partons [13, 42]. The Oregon group carries the approach even a step further and replaces fragmentation functions by a scenario where minijet partons develop a shower which subsequently recombines. In this approach, one is able to describe fragmentation functions reasonably well. Applied to heavy ion collisions, this allows for the recombination of soft partons with shower partons [51].

4. Current status

All four model implementations describe hadron data from RHIC at intermediate and large transverse momenta very well. The calculations for spectra reproduce the two-component shape of the spectra with the transition around $P_T = 4$ GeV/c for mesons and 6 GeV/c for baryons from soft (recombination) to hard (perturbative) particle production. Soft-hard coalescence can improve the fit to data points in the transition region, which is then extended to even higher P_T . The baryon/meson ratios in the recombination approach are essentially given by the ratio of degeneracy factors C_B/C_M , leading naturally to an enhanced proton/pion ratio. The most recent results can be found e.g. in [12, 42, 51].

It should be noted, however, that recombination plus fragmentation models are limited to providing a microscopic description of hadronization. They do not attempt to describe the full dynamics of a relativistic heavy-ion collision, but require the phase space distribution of partons at the time of hadronization as input (Duke and TAM approach) or can be used as a reverse filter to extract this distribution from the experimental data (Oregon approach).

The dependence on the parton distribution at hadronization limits the predictive power of the recombination models to such observables which exhibit a flavor-dependency (baryons vs. mesons) or allow for a comparison between mass scaling and constituent quark number scaling. Among the predictions of the recombination plus fragmentation approach, which were subsequently confirmed by experiments, were the nuclear suppression factor of the phi meson

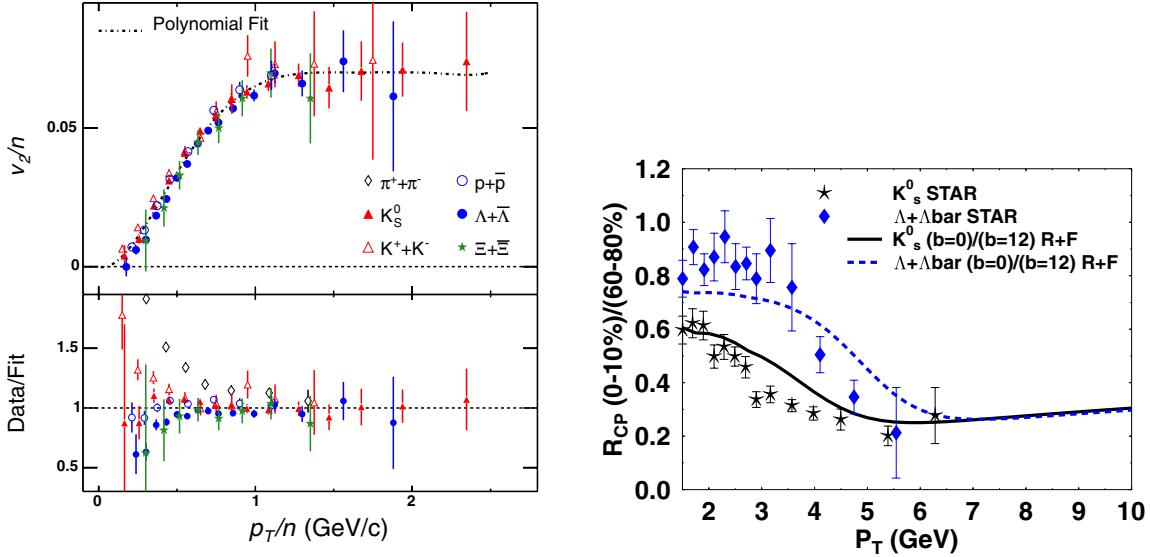


Figure 1. Left: scaled elliptic flow v_2/n vs. scaled transverse momentum p_t/n , with n the number of valence quarks in the respective hadron. The scaling law is strongly indicative of parton recombination (figure taken from [55]). Right: nuclear modification factor R_{CP} as a function of transverse momentum for Λ and K_S^0 in the recombination+fragmentation approach compared to STAR data (figure taken from [12]).

[56, 57] (exhibiting the same behavior vs p_T as for kaons and pions) as well as the quark number scaling of the elliptic flow of the cascade and the omega baryon [58].

5. Challenges:

The recombination plus fragmentation model has been extremely successful in describing single particle observables such as spectra, ratios and elliptic flow. Recently, however, more sophisticated dynamical correlation observables have become available and questions have been posed concerning the range of applicability of the recombination approach as standard model of hadronization:

5.1. Jet-like two particle correlations

One of the biggest challenge for the recombination models has been the measurement of dynamic two-particle correlations. The picture of quarks recombining from a collectively flowing, deconfined thermal quark plasma appears to be at odds with the observation of “jet-like” correlations of hadrons observed in the same transverse momentum range of 2 to 5 GeV/c [59, 60, 61]. Triggering on a hadron, e.g., with transverse momentum $2.5 \text{ GeV}/c < p_T < 4 \text{ GeV}/c$, the data shows an enhancement of hadron emission in a narrow angular cone around the direction of the trigger hadron in a momentum window below $2.5 \text{ GeV}/c$. Can such correlations be reconciled with the claim that hadrons in this momentum range are mostly created by recombination of quarks?

Obviously, the observation is incompatible with any model which assumes that no correlations exist among the quarks before recombination. Such correlations require deviations from a global thermal equilibrium in the quark phase. One mechanism for correlations is already well established: elliptic flow.

In [62] it was shown that correlations among partons in a quark-gluon plasma naturally

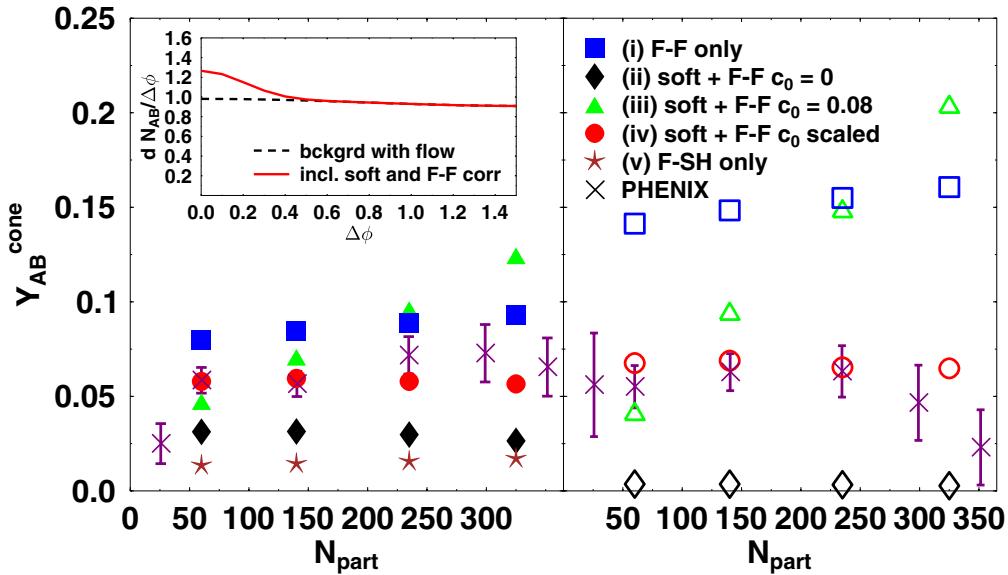


Figure 2. Associated particle yield per trigger hadron A , for meson (left panel) and baryon triggers (right panel) as a function of centrality. Different correlation scenarios in the recombination approach are compared to PHENIX data [60] (figure taken from [62]).

translate into correlations between hadrons formed by recombination of quarks. Correlations are even enhanced by an amplification factor $Q = n_A n_B$ similar to the scaling of elliptic flow. The interaction of hard partons with the medium has been discussed as one plausible mechanism for the existence of such parton correlations, even though other scenarios for the creation of parton-parton correlations in the deconfined phase are possible. A numerical example that shows that two-parton correlations of order $\approx 10\%$ will be sufficient to explain hadron correlations as measured by the PHENIX collaboration. One may conclude that the existence of localized angular correlations among hadrons are not in contradiction with the recombination scenario.

5.2. Charged particle fluctuations:

Fluctuations of the net electric charge of all particles emitted into a specified rapidity window have been proposed as a possible signal for the formation of deconfined quark matter in relativistic heavy ion collisions [63, 64]. The argument at the basis of this proposal is that charge fluctuations in a quark-gluon plasma are expected to be significantly smaller (by a factor 3 – 4) than in a hadronic gas. Because the net charge contained in a given volume is locally conserved and can only be changed by particle diffusion, thermal fluctuations generated within the deconfined phase could survive hadronization and final state interactions.

The most widely used measure for the entropy normalized net charge fluctuations is the D measure [64]:

$$D = 4\langle(\Delta Q)^2\rangle/N_{\text{ch}}, \quad (6)$$

where $\langle(\Delta Q)^2\rangle$ denotes the event-by-event net charge fluctuation within a given rapidity window Δy , and N_{ch} is the total number of charged particles emitted in this window. For a free plasma of quarks and gluons $D \approx 1$, while for a free pion gas $D \approx 4$. Several experiments have measured net charge fluctuations in heavy ion collisions at the CERN Super-Proton Synchrotron (SPS) and at the Relativistic Heavy Ion Collider (RHIC) in Brookhaven [65, 66, 67, 68]. The results for D are generally somewhat smaller than 4, but much larger than the value predicted for a free quark-gluon gas. Applying the formalism of parton recombination to the charged particle fluctuation

observables [69, 70] one finds, however, that within the present systematic uncertainties, parton recombination is compatible with the measured charged particle fluctuations. This finding is of significant importance, since the incompatibility of the charged particle fluctuation measurements with the deconfinement hypothesis has posed a serious problem to the emerging picture of Quark-Gluon-Plasma formation at RHIC.

5.3. Entropy balance between the two phases:

The recombination model has been developed for hadron production at intermediate transverse momenta of a few GeV/c . In that domain, one does not have to be concerned about the apparent violation of entropy conservation due to recombining two or three partons respectively into one meson or baryon, since the bulk matter at low transverse momenta can act as a heat-bath and compensate for the perceived reduction in entropy. It is tempting, however, to attempt the extension of the recombination concept to low P_T : once recombination is recognized to be the dominant hadronization mechanism at intermediate P_T from 2 to 5 GeV/c , it is quite reasonable to expect that this mechanism extends down to very small transverse momenta, where quarks are even more abundant. However, the theoretical description will only be on solid ground once the problems of energy and entropy conservation have been addressed properly.

One proposal to deal with the entropy balance in the recombination picture has been the explicit treatment of resonances [53]. While the decay of resonances will certainly decrease the mismatch in particle number between the deconfined and the confined phase, this line of argument alone might be insufficient to fully balance the entropies in both phases. A comprehensive analysis of the total entropy created at RHIC was performed in [70] and contrasted it with lattice-gauge predictions on the entropy-density of a deconfined system of quarks and gluons approaching T_c . The results of that analysis indicate that

- (i) The entropy per particle in the hadronic gas, and therefore the entropy content of the hadronic phase at chemical freezeout, is considerably larger than often assumed (mostly due to large masses of hadronic resonances in the system).
- (ii) The entropy density of the quark phase is significantly suppressed near T_c , most likely due to correlations among the quasiparticles caused by their strong interactions.

These two conclusions make the recombination picture of hadronization far more compatible with the entropy constraint. If the quark-gluon plasma at hadronization consists of strongly interacting quasi-particles (e.g. constituent quarks) with strong correlations, and if many of the hadrons created at hadronization are heavy, quark recombination and the concomitant particle number decrease could be reconciled with the second law of thermodynamics. At present, however, a more rigorous statement concerning the entropy balance cannot be made, since the state of the art does not allow a direct comparison of the entropy content of both phases, due to the volume at hadronization not being unambiguously known. Besides, lattice calculations do not provide the number of (quasi-)particles, because there is no lattice definition of particle density. Therefore, a more detailed comparison of the entropy content of the hadronic phase and the quark phase at hadronization remains as a problem for future investigations.

5.4. Resonances and elliptic flow scaling law violations:

The v_2 scaling law has been impressively confirmed by measurements at RHIC. Pions and kaons ($n = 2$), protons, Λ and Ξ ($n = 3$) [58] all fall on one universal curve, if v_2 and P_T are divided by the number n of valence quarks [25]. Slight deviations can be observed and are largest for pions. This can be due to its nature as a Goldstone boson – its mass is much smaller than the sum of its constituent quark masses – or because a large fraction of all pions are not created at hadronization but by the subsequent decay of hadron resonances [53]. This poses the interesting question how the existence of a hadronic phase generally affects hadron production

at intermediate and large momenta. In general the v_2 scaling law implies that the elliptic flow is *additive* for any type of composite particle with respect to the v_2 of its constituents. However, this implies that e.g. the v_2 of a K_0^* meson in the recombination domain will therefore be the sum of the v_2 contributions of the d and \bar{s} quarks if it has been formed from a hadronizing QGP, or the sum of the contributions of the K^+ and π^- if it has been formed through the coalescence of a kaon and a pion in the hadron phase. Naively, if the kaon and pion themselves are formed by quark recombination, one would expect a scaling of the K_0^* with $n = 4$ in the hadronic phase, while it scales with $n = 2$, like the stable mesons, if it is created at the phase transition. The scaling could further be altered by the possibility that one or both of the coalescing hadrons could come from fragmentation, which breaks the scaling law. The interplay of these production mechanism may therefore result in scaling law violations for hadronic resonances. These scaling law violations expected for hadronic resonances in turn can be utilized to estimate the amount of hadronic rescattering [46].

6. Outlook

The recombination plus fragmentation approach is able to describe most available RHIC data on spectra, ratios, nuclear suppression and elliptic flow of hadrons, including their impact parameter dependence, for transverse momenta above 1–2 GeV/ c – for v_2 even down to very low P_T – consistently with a very small number of globally adjusted parameters. The challenges described in section 5 are being addressed and improved model implementations are likely to succeed in describing the forthcoming data in those sectors. The biggest challenge in the long run will be to determine whether the recombination approach can be extended to the bulk of the matter at low transverse momenta – for this a far more detailed knowledge of the non-perturbative dynamics of QCD seems necessary. To date, the success of the recombination approach (and in particular the elliptic flow scaling law) is among the most compelling and direct pieces of evidence for the formation of a deconfined phase of partons at RHIC.

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References

- [1] P. Huovinen, P. F. Kolb, U. W. Heinz, P. V. Ruuskanen and S. A. Voloshin, Phys. Lett. B **503**, 58 (2001).
- [2] D. Teaney, J. Lauret and E. V. Shuryak, [arXiv:nucl-th/0110037].
- [3] S. S. Adler *et al.* [PHENIX Collaboration], Phys. Rev. Lett. **91**, 182301 (2003).
- [4] S. Esumi (for the PHENIX collaboration), Nucl. Phys. A **715**, 599 (2003).
- [5] C. Adler *et al.* [STAR Collaboration], Phys. Rev. Lett. **90**, 032301 (2003); *ibid.* **89** 132301 (2002); *ibid.* **87** 182301 (2001).
- [6] J. Adams *et al.* [STAR Collaboration], Phys. Rev. Lett. **92**, 052302 (2004).
- [7] X. N. Wang, Phys. Rev. C **63**, 054902 (2001).
- [8] M. Gyulassy, I. Vitev, X. N. Wang, Phys. Rev. Lett. **86**, 2537 (2001).
- [9] K. Adcox *et al.* [PHENIX Collaboration], Phys. Rev. Lett. **88**, 022301 (2002).
- [10] J. Adams *et al.* [STAR Collaboration], Phys. Rev. Lett. **91**, 172302 (2003).
- [11] R. J. Fries, B. Müller, C. Nonaka and S. A. Bass, Phys. Rev. Lett. **90**, 202303 (2003).
- [12] R. J. Fries, B. Müller, C. Nonaka and S. A. Bass, Phys. Rev. C **68**, 044902 (2003).
- [13] V. Greco, C. M. Ko, and P. Lévai, Phys. Rev. Lett. **90**, 202303 (2003).
- [14] S. A. Voloshin, Nucl. Phys. A **715** (2003) 379.
- [15] C. Nonaka, R. J. Fries and S. A. Bass, Phys. Lett. B **583**, 73 (2004).
- [16] K. Adcox *et al.* [PHENIX Collaboration], Phys. Rev. Lett. **88**, 242301 (2002).

- [17] C. Adler *et al.* [STAR Collaboration], Phys. Rev. Lett. **86**, 4778 (2001).
- [18] K. Adcox *et al.* [PHENIX Collaboration], Phys. Rev. C **69**, 024904 (2004).
- [19] J. Adams *et al.* [STAR Collaboration], Phys. Rev. Lett. **92**, 112301 (2004).
- [20] M. Gyulassy and X. Wang, Nucl. Phys. B **420**, 583 (1994); R. Baier, Y. L. Dokshitzer, A. H. Mueller, S. Peigne and D. Schiff, Nucl. Phys. B **484**, 265 (1997).
- [21] R. Baier, Y. L. Dokshitzer, A. H. Mueller and D. Schiff, JHEP **0109**, 033 (2001);
- [22] B. Müller, Phys. Rev. C *in print* (2003), nucl-th/0208038.
- [23] S. S. Adler *et al.* [PHENIX Collaboration], Phys. Rev. Lett. **91**, 172301 (2003).
- [24] J. Adams *et al.* [STAR Collaboration], Phys. Rev. Lett. **92**, 052302 (2004).
- [25] P. Sorensen [STAR Collaboration], J. Phys. G **30**, S217 (2004).
- [26] I. Vitev and M. Gyulassy, Nucl. Phys. A **715**, 779 (2003).
- [27] T. Hirano and Y. Nara, Phys. Rev. C **69**, 034908 (2004).
- [28] J. F. Owens, Rev. Mod. Phys. **59**, 465 (1987).
- [29] J. C. Collins and D. E. Soper, Nucl. Phys. B **194**, 445 (1982).
- [30] B. A. Kniehl, G. Kramer and B. Pötter, Nucl. Phys. B **582**, 514 (2000).
- [31] X. F. Guo and X. N. Wang, Phys. Rev. Lett. **85**, 3591 (2000); Nucl. Phys. A **696**, 788 (2001).
- [32] E. Wang and X. N. Wang, Phys. Rev. Lett. **89**, 162301 (2002).
- [33] X. N. Wang, nucl-th/0305010.
- [34] K. P. Das and R. C. Hwa, Phys. Lett. B **68**, 459 (1977); Erratum *ibid.* **73**, 504 (1978); R. G. Roberts, R. C. Hwa and S. Matsuda, J. Phys. G **5**, 1043 (1979).
- [35] E. M. Aitala *et al.* [E791 Collaboration], Phys. Lett. B **371**, 157 (1996).
- [36] J. C. Anjos, J. Magnin and G. Herrera, Phys. Lett. B **523**, 29 (2001).
- [37] E. Braaten, Y. Jia and T. Mehen, Phys. Rev. Lett. **89**, 122002 (2002);
- [38] R. C. Hwa and C. B. Yang, Phys. Rev. C **67**, 034902 (2003) [arXiv:nucl-th/0211010].
- [39] C. Gupt, R. K. Shivpuri, N. S. Verma and A. P. Sharma, Nuovo Cim. A **75**, 408 (1983)
- [40] T. Ochiai, Prog. Theor. Phys. **75**, 1184 (1986).
- [41] T. S. Biro, P. Levai and J. Zimanyi, Phys. Lett. B **347**, 6 (1995); T. S. Biro, P. Levai and J. Zimanyi, J. Phys. G **28**, 1561 (2002).
- [42] V. Greco, C. M. Ko and P. Levai, Phys. Rev. C **68**, 034904 (2003).
- [43] R. Rapp and E. V. Shuryak, hep-ph/0301245.
- [44] R. J. Fries, B. Muller, C. Nonaka and S. A. Bass, J. Phys. G **30**, S223 (2004).
- [45] C. Nonaka, R. J. Fries and S. A. Bass, Phys. Lett. B **583**, 73 (2004).
- [46] C. Nonaka, B. Muller, M. Asakawa, S. A. Bass and R. J. Fries, Phys. Rev. C **69**, 031902 (2004).
- [47] Z. W. Lin and C. M. Ko, Phys. Rev. Lett. **89**, 202302 (2002).
- [48] D. Molnár and S. A. Voloshin, Phys. Rev. Lett. **91**, 092301 (2003).
- [49] D. Molnár, J. Phys. G **30**, S235 (2004).
- [50] Z. w. Lin and D. Molnar, Phys. Rev. C **68**, 044901 (2003).
- [51] R. C. Hwa and C. B. Yang, Phys. Rev. C **67**, 034902 (2003).
- [52] V. Greco, C. M. Ko and R. Rapp, Phys. Lett. B **595**, 202 (2004).
- [53] V. Greco and C. M. Ko, Phys. Rev. C **70**, 024901 (2004).
- [54] R. J. Fries, B. Muller and D. K. Srivastava, Phys. Rev. Lett. **90**, 132301 (2003).
- [55] M. D. Oldenburg [STAR Collaboration], J. Phys. G **31**, S437 (2005).
- [56] J. Adams *et al.* [STAR Collaboration], arXiv:nucl-ex/0406003.
- [57] S. S. Adler *et al.* [PHENIX Collaboration], arXiv:nucl-ex/0410012.
- [58] J. Castillo [STAR Collaboration], J. Phys. G **30**, S1207 (2004).
- [59] C. Adler *et al.* [STAR Collaboration], Phys. Rev. Lett. **90**, 082302 (2003)
- [60] A. Sickles [PHENIX collaboration], J. Phys. G **30**, S1291 (2004).
- [61] S. S. Adler *et al.* [PHENIX Collaboration], arXiv:nucl-ex/0408007.
- [62] R. J. Fries, S. A. Bass and B. Muller, Phys. Rev. Lett. *in print*, arXiv:nucl-th/0407102.
- [63] M. Asakawa, U. W. Heinz, and B. Müller, Phys. Rev. Lett. **85**, 2072 (2000).
- [64] S. Jeon and V. Koch, Phys. Rev. Lett. **85**, 2076 (2000).
- [65] J. Adams *et al.* [STAR Collaboration], Phys. Rev. C **68**, 044905 (2003); G. D. Westfall [STAR collaboration], J. Phys. G **30**, S1389 (2004).
- [66] K. Adcox *et al.* [PHENIX Collaboration], Phys. Rev. Lett. **89**, 082301 (2002); J. Nystrand [PHENIX Collaboration], Nucl. Phys. A **715**, 603 (2003).
- [67] H. Sako and H. Appelshäuser [CERES/NA45 Collaboration], J. Phys. G **30**, S1371 (2004).
- [68] C. Alt *et al.* [NA49 Collaboration], [arXiv:nucl-ex/0406013].
- [69] A. Biallas, Phys. Lett. B **532**, 249 (2002).
- [70] C. Nonaka, B. Muller, S. A. Bass and M. Asakawa, arXiv:nucl-th/0501028.