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# Tensor force and delta excitation for the structure of light nuclei

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**Abstract.** We treat explicitly  $\Delta(1232)$  isobar degrees of freedom using a bare nucleon-nucleon interaction for few-body systems, where  $\Delta$  excitations can be the origin of the three-body force via the pion exchange. We adopt the Argonne two-body potential including  $\Delta$ , named as AV28 potential, and study the role of  $\Delta$  explicitly in two-body and three-body systems. It was found that the additional  $\Delta$  states generate strong tensor correlations caused by the transitions between  $N$  and  $\Delta$  states, and change tensor matrix elements largely from the results with only nucleons. We studied the effects of three-body force in the triton and obtained 0.8 MeV attraction due to the intermediate  $\Delta$  excitation. Due to the lack of the total binding energy for the triton in the delta model, we further studied carefully the effects of the delta excitation in various two body channels and compared with the nucleon only model in the AV14 potential. We modified slightly the AV28 potential in the singlet S channel so that we could reproduce the triton binding energy due to the appropriate amount of the three-body force effects.

## 1. Introduction

For a quantitative description of nuclei, it is important to include three-body force [1]. The contributions of the three-body force turned out to be 0.85 MeV for the triton and 4.26 MeV for  $^4\text{He}$  in the GFMC simulations with the AV18/UIX Hamiltonian. The origin of the three body force is considered to come from the delta intermediate excitation and the relativistic effect [1]. Particularly for the delta intermediate excitation one delta is excited by two nucleon interaction and the excited delta decays back to a nucleon by interacting with another nucleon, which is named as the Fujita-Miyazawa mechanism [2]. Usually this three-body process is introduced in the calculation of nuclear many-body systems as a phenomenological three-body force expressed in the nucleon space [1, 3].

In the literature, there were a few attempts to include delta degrees of freedom explicitly for the discussion of the three-body force due to the delta excitation. Sauer and his collaborators studied the effect of single delta excitation with the knowledge of the pion exchange based on the Bonn potential [4]. Pikelesimer and his collaborators studied the three body system using the Argonne AV28 potential, where the excitation of one and two delta's was included for the construction of the nucleon-nucleon potential [5]. They used the Faddeev equation to solve for the triton using both the single delta and double delta amplitudes. Their results were quite reasonable as compared with the three body contribution of the Argonne group. The Los Alamos group, however, found that the net binding energy for the triton turned out to be



7.29 MeV short of the experimental binding energy of 8.48 MeV. Hence, it was concluded that the delta excitation model was not able to explain the triton binding energy.

Recently, there was a renewed interest on the nucleon-nucleon interaction in the singlet S channel. In the neutron-neutron quasi-free scattering cross section in the neutron-deuteron break-up reaction process, Witala and Glockle showed the the quasi-free scattering cross section was underestimated, when they used the standard two-body nucleon-nucleon interaction in the Faddeev calculation [6]. They suggested that the neutron-neutron interaction in the singlet S channel should be increased by about 10% so that the quasi-free cross section was reproduced. It is therefore very important to revisit the effect of the delta excitation in order to understand the vast amount of the three-body scattering data accumulated until now [7]. The increase of the neutron-neutron attraction in the singlet S channel by 10% might remove the discrepancy found in the calculation of the Los Alamos group for the triton.

The inclusion of the delta isobar excitation needs a large amount of model space. Recently, we have developed the tensor optimized few-body model (TOFM) for few body systems in order to take into account the strong tensor force in the nucleon-nucleon interaction. In the TOFM, we include only a single  $Y_2$  component in the few body wave function, which has a direct connection to the major  $Y_0$  configuration by the tensor interaction [8]. The TOFM is able to truncate the model space largely and the computational effort becomes much lighter than the rigorous method [9]. Hence, we should be able to reduce the computational effort for few body calculations in the case of explicit introduction of the delta degrees of freedom. Particularly, when we like to calculate heavier systems, the success of TOFM in few body systems encourage the use of the tensor optimized shell model (TOSM) including the delta degrees of freedom. We are then able to analyze the wave functions and the dynamics easier than other sophisticated methods.

## 2. Three body system in the tensor optimized few body model

We discuss here the structure of the triton including  $\Delta$  degrees of freedom. We calculate the ground state energy for the triton using the few-body method [8]. Our model space allows single and double  $\Delta$  excitations, and the wave function is described as,

$$|\Psi\rangle = |\Psi_{NNN}\rangle + |\Psi_{N\Delta N}\rangle + |\Psi_{\Delta\Delta N}\rangle . \quad (1)$$

Using the Jacobi coordinate, we treat the strong short-range repulsion and the strong tensor correlation. We consider all the nucleons excite to the  $\Delta$  states. Hence, we include all the possible anti-symmetrization. In this study, we perform variational calculations to obtain the triton state by using Stochastic Variational Method which is developed by the Niigata group [9].

The wave function is expressed in terms of the basis function of the correlated Gaussian basis,  $\Psi_{JM} = \sum_k c_k \psi_k$ , where  $\psi_k$  is given by

$$\psi_k = \mathcal{A}[\psi_L(x, A_k, u_k)\chi_s]_{JM}\eta_t . \quad (2)$$

The spin wave function is  $\chi_s = [[\chi_1\chi_2]_{s_{12}}\chi_3]_s$  with  $\chi_1, \chi_2, \chi_3$  are the spin wave functions of nucleon or  $\Delta$ . The isospin wave function is  $\eta_t = [[\eta_1\eta_2]_{t_{12}}\eta_3]_t$ , where  $\eta_1, \eta_2, \eta_3$  are the isospin wave functions of nucleon or  $\Delta$ . As for the spatial part of  $\psi_L$ , the basis functions are described by using the Correlated Gaussian basis with the global vector  $\psi_L = e^{-\frac{1}{2}\tilde{x}Ax}|\tilde{u}x|^L Y_L(\tilde{u}x)$ . In the present calculation, we include also  $[Y_1 \times Y_1]_1$  components so that we can compare with the rigorous calculations of Niigata group [9].

## 3. AV28 potential and two-body systems

### 3.1. Two nucleon interaction: AV28

Wiringa *et al.* constructed phenomenological two-body  $NN$  interactions with and without  $\Delta(1232)$  degrees of freedom, Argonne v14 (AV14) and v28 (AV28) models, respectively [10]. We

**Table 1.** Various quantities of the deuteron with the AV14 and AV28 potentials.

${}^2\text{H}$	Energy	Kinetic	Central	Tensor	LS	$L^2$	$(\text{LS})^2$
AV14	-2.2	19.1	-1.9	-18.8	0.4	3.1	-4.0
AV28	-2.2	23.9	8.8	-35.3	0.8	3.5	-4.1

review briefly the content of the  $NN$  interactions of the AV28 potential together with the AV14 potential. These potentials have three radial components: long-range one-pion exchange part  $v_\pi$ , and phenomenological intermediate-range and short-range parts  $v_I$  and  $v_S$ :

$$V_{ij}(r_{ij}) = \sum_p [v_\pi^p(r_{ij}) + v_I^p(r_{ij}) + v_S^p(r_{ij})] \mathcal{O}_{ij}^p, \quad (3)$$

where operators  $\mathcal{O}^p$  represent operators of spin, isospin, tensor and other forms. For the AV14 potential, it is described by the sum of 14 operators ( $p = 1, \dots, 14$ ). On the other hand, the AV28 potential includes the delta excitation, and contains 14 components in the  $NN$  channel and have 6 operators in the  $N\Delta$  channel and 8 operators in the  $\Delta\Delta$  channels to represent all the possible processes due to  $\pi N\Delta$  and  $\pi\Delta\Delta$  vertices.

The  $NN - NN$  interaction is expressed by the first 14 operators with the same form as the AV14 potential. The AV28 potential including the transition between  $N$  and  $\Delta$  appearing in  $NN - N\Delta, NN - \Delta\Delta, N\Delta - \Delta\Delta, N\Delta - \Delta N$  processes are assumed to be made only through the pion exchange ( $v_\pi$ ) along with the corresponding spin-isospin and tensor-isospin operators. The ratio of coupling constants are taken as  $f_{\pi N\Delta}^2/f_{\pi NN}^2=4$  and  $f_{\pi\Delta\Delta}^2/f_{\pi NN}^2=1/25$  [10]. The diagonal  $N\Delta - N\Delta$  and  $\Delta\Delta - \Delta\Delta$  processes include not only the pion-exchange part but also the short-range repulsion ( $v_I$  and  $v_S$ ). These parameters in the short-range and intermediate-range potentials for  $N\Delta$  and  $\Delta\Delta$  channels are taken as the same values as those in the  $NN$  channel. Thus, the total number of parameters does not change for the AV28 potential compared with the AV14 potential. All the phenomenological parameters should be determined to reproduce the experimental data of  $NN$  scattering phase shifts and deuteron properties for both AV14 and AV28 potentials.

### 3.2. Triplet-even ( ${}^3E$ ) channel

In order to understand the roles of  $\Delta$  in nuclei, we start with the deuteron including explicitly  $\Delta$  degrees of freedom. In Ref. [10], some deuteron properties have been presented such as the binding energy and the probabilities of the wave function components. The wave function of the deuteron consisting of  $NN$  and  $\Delta\Delta$  states with the isospin  $T = 0$  channel is given as,  $|\Psi\rangle = |\Psi_{NN}\rangle + |\Psi_{\Delta\Delta}\rangle$ . The  $NN$  wave function is written by the  ${}^3S_1$  and  ${}^3D_1$  channels and  $\Delta\Delta$  state is written in four channels:  ${}^3S_1, {}^3D_1, {}^7D_1$  and  ${}^7G_1$ . We present the energy and various Hamiltonian matrix elements using the AV28 potential [10] in Table 1 and also in Fig. 1. We also compare the results with those of the AV14 potential [10]. Both results provide the same binding energies of 2.2 MeV, but the matrix elements are largely different. Especially the tensor force component becomes large with the AV28 potential as  $-35.3$  MeV in comparison with the value  $-18.8$  MeV of the AV14 potential. The enhancement of tensor force contribution increases the kinetic energy, where the contributions of the mass difference between  $NN$  and  $\Delta\Delta$  is obtained as 3.1 MeV. This mass contribution is much smaller than the original value of the mass difference of about 600 MeV, which is due to the small probabilities of about 0.5% for the total  $\Delta\Delta$  state. Most of the kinetic and central energy components come from the  $NN$  wave functions. However as for the tensor energy components, even the small mixing of  $\Delta$  generates the large tensor matrix elements in the deuteron. We find that the additional  $\Delta$  comprises about  $-10$  MeV of

**Table 2.** Various energy components for a two-body system in the  $^1E$  channel with the AV28 and AV14 potentials using the radius constraint. The upper part shows various energy components in unit of MeV, and the lower one for the probabilities of the wave function in %.

$^1E$	Energy	Kinetic	Central	Tensor
AV14	1.6	8.3	-6.7	0.0
AV28	1.5	14.0	2.2	-14.7
P [%]	$^1S(NN)$	$^1S(\Delta\Delta)$	$^5S(\Delta\Delta)$	$^5D(N\Delta)$
AV28	98.86	0.02	0.13	0.99

the tensor matrix elements, and the transitions between  $^3S_1$  of  $NN$  state and  $^7D_1$  of  $\Delta\Delta$  state play the dominant role. The probability of the  $^7D$ -state is 0.42%, which is particularly larger than other  $\Delta\Delta$  states. In comparison to the same  $D$ -state between  $^7D$  with spin 3 and  $^3D$  with the total spin 1, the tensor correlations prefer the state which has spin alignment. The central force components provide the repulsive effect to cancel out the large attraction of the tensor force, which is the opposite sign to the result of the AV14 potential.

### 3.3. Singlet-even ( $^1E$ ) channel

In two-body system,  $T = 1$  channels are constructed by not only  $\Delta\Delta$  state but also  $N\Delta$  state. Specifically, it is important to investigate the role of  $N\Delta$  state with  $T = 1$ , because single  $\Delta$  excitation components in three-body system would contribute to the three-body process such as the Fujita-Miyazawa model. In order to examine the properties of  $N\Delta$  state, we discuss another two-body system of the singlet-even ( $^1E$ ) channel. The wave function of  $^1E$  channel with the total spin  $J=0$  consists of  $NN$ ,  $N\Delta$  and  $\Delta\Delta$  states,  $|\Psi\rangle = |\Psi_{NN}\rangle + |\Psi_{N\Delta}\rangle + |\Psi_{\Delta\Delta}\rangle$ , where the components of each wave function are  $^1S_0$  state for the  $NN$  channel,  $^5D_0$  state for the  $N\Delta$  channel and  $^1S_0$  and  $^5D_0$  states for the  $\Delta\Delta$  channel.

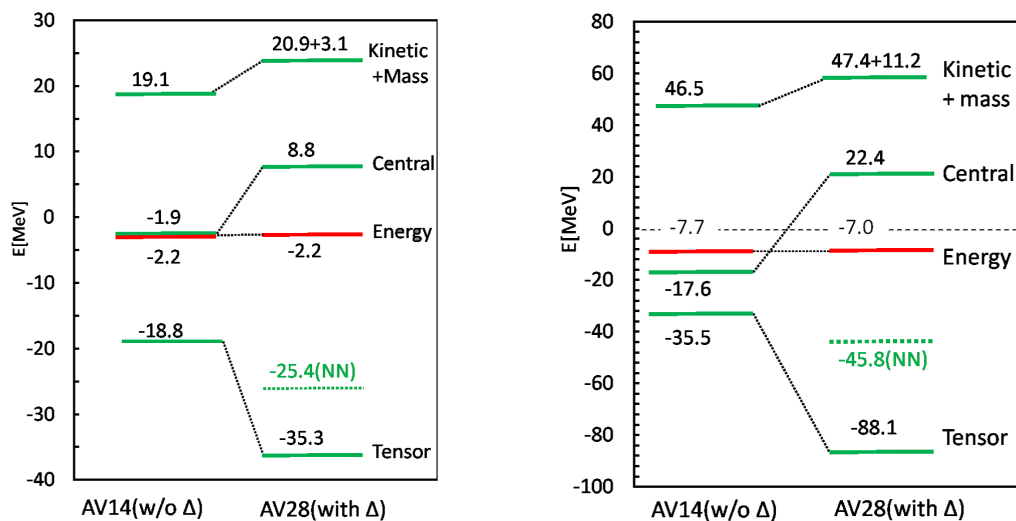
Since the  $^1E$  channel is unbound, we use a method to deal with unbound systems as bound states by using a Hamiltonian with radius constraint. It enables us to find out the energies and the matrix elements for the unbound two-body systems. The Hamiltonian with the constraint for the radius is given as,  $\tilde{H} = H + \lambda\hat{r}^2$ , where  $\hat{r}^2$  is the operator of the radius, and we can obtain the energy by subtracting the effect of the radius operator. We constrain the two-body system with the same radius as the deuteron 2.0 fm and show the results in Table 2.

As a result we obtain a large tensor energy of  $-14.7$  MeV, where the total energy is 1.5 MeV with the kinetic energy 14.0 MeV and central energy 2.2 MeV. It is very important to point out that the energy gained by adding the delta component is about 15 MeV, while the probability is only about 1%. The  $NN$  wave function consisting of only  $S$ -wave does not make the tensor correlations by itself, hence this large tensor components come from the transitions between  $N$  and  $\Delta$ . Particularly we estimate the couplings between  $^1S_0$  of  $NN$  state and  $^5D_0$  of  $N\Delta$  state are dominant in this tensor component,  $-13.2$  MeV.

## 4. The three body force in the triton

We show here the results for the triton together with the deuteron with both the AV28 and AV14 potentials in Fig. 1. It is very interesting to see that the results of AV28 are largely different from those of AV14 for both the two and three nucleon systems. As for the binding energy of the triton, the result with the AV28 potential was less than that of the AV14 potential. We calculated the triton system by dropping the three body process and found the effects of three-body force due to the  $\Delta$  excitations as 0.8 MeV attraction. Hence, we concluded that the  $\Delta$  excitations gave a reasonable amount of the three-body effect. This means that the two body

interaction terms of AV28 gave a smaller binding energy as compared to AV14 by about 1 MeV. We have checked the two body systems where the AV28 potential gave almost the same amount of attraction as compared to the AV14 potential. Hence, we concluded that the attraction in the singlet S channel in AV28 was decreased due to the blocking effect in the three body system. We multiplied then the central interaction in the singlet S channel by 0.93 and calculated the triton binding energy with the result of -7.8 MeV as compared with the value of -7.7 MeV for the case of the AV14 potential. In this case, the energy of the singlet S channel becomes 0.5 MeV as compared with the value of 1.6 MeV as shown in Table 2.



**Figure 1.** Calculated results for the binding energies and various matrix elements in the deuteron and the triton with AV28 and AV14.

## 5. Discussion and Summary

We have studied the two body and three body systems using the AV28 and AV14 potentials. We have found that the three body force effect in fact came from the delta excitation by the right amount of 0.8 MeV. We have found at the same time that the attractive effect in the singlet S channel in two body interaction was decreased by about 1 MeV in the triton system. This amount corresponds to the necessary strength for the reproduction of the neutron-neutron quasi-elastic process in the pd reactions [6]. It is very interesting to see that the two body interaction changes in the triton due to the Pauli blocking effect for the case of the AV28 potential.

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