

## STRONG INTERACTION DYNAMICS

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INTRODUCTION

In this review I shall concentrate mainly on high energy dynamics. The main sections are as follows:

- A) Exclusive Channels and the S-channel unitarity equation.
- B) The Pomeron and its consistency.
- C) Rising total cross-sections.
- D) Field theoretic Reggeisation.
- E) Nuclear Collisions.

A. Exclusive Channels and the S-channel unitarity equation

In this section I would like to describe the efforts being made to understand the following classical problem. Can we write down an explicit realistic amplitude for 2 particles  $\rightarrow$  N particles such that when we write

$$\text{Im } T_{22}(s,t) = \sum_N \int d\Omega_N T_{2N}^* T_{2N} \propto \sum \sigma_{2 \rightarrow N} \quad (\text{A.1.})$$

the right hand side gives a satisfactory imaginary part for the two body amplitude?

More or less stringent requirements may be imposed on this exercise. These have been taken to include

- a)  $\sigma_{2 \rightarrow N}$  correctly calculated together with exclusive mass and momentum distributions.
- b)  $T_{22}(s,t) \approx s^{\alpha(t)}$   $s \rightarrow \infty$  with  $\alpha(0) \approx 1$

In other words we obtain a Pomeron like object on calculating  $\sigma_{\text{TOT}}$

- c) The slope of the forward elastic amplitude in  $t$  is near the physical value.
- d) One particle inclusive couplings qualitatively correct.
- e) Multiplicity distributions should look like the real world as well as the two particle correlations.

Before starting a description of the progress in these problems let me start by describing the theorist's dream world of weak coupling so that we can understand why these problems are hard.

Consider the process  $AB \rightarrow 1, 2, \dots, N$  with  $N$  fixed  $S$  large. Now write the momenta of the particles in Fig. 1 using rapidity ( $y_i$ ) and transverse momentum ( $p_{\perp}^i$ ) in the A,B centre of mass system. Then, if the particles are ordered in rapidity i.e.  $y_i > y_{i+1}$  and all the sub-energies  $S_{i,i+1}$  are large<sup>(1)</sup>,

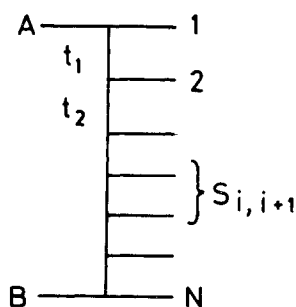


Fig. 1. Multiperipheral kinematics.

$$\begin{aligned}
S_{i,i+1} &\approx e^{(y_i - y_{i+1})} |\mu_{i+1}^2| \\
t_i &\approx - \left| \sum_i^i p_{\perp}^i \right|^2 \\
\mu_i^2 &= m^2 + |p_{\perp}^i|^2 \\
\prod_{i=1}^{N-1} S_{i,i+1} &= \prod_{i=2}^{N-1} \mu_i^2 \quad (A.2.)
\end{aligned}$$

Now experimentally the  $p_{\perp}$ 's are  $\leq 350$  MeV/c so  $t_i$  is apparently very small. This is the motivation for the original ABFST<sup>(2)</sup> model. Since the  $t_i$  are small then, where quantum numbers allow, we may hope for pion pole dominance. The model is then shown in Fig. 2 with off-mass-shell  $\pi\pi \rightarrow \pi\pi$  amplitudes for the bubbles.

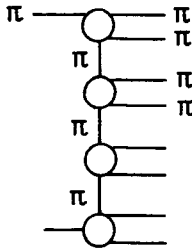


Fig. 2. The ABFST model.

As is well known this model gives reasonable fits to the exclusive, unnormalised single particle distributions. However, it has a lot of trouble in giving  $\sigma_{2,n}$ . Typically it gives  $\sigma_{2,n}$  much too small<sup>(3)</sup>.

However in calculating  $\sigma_{TOT}$  we require  $\sigma_{2,n}$  for  $n \geq \langle n \rangle$  the mean multiplicity. In this region it is clear the  $S_{i,i+1} \sim \text{const}$  as  $s \rightarrow \infty$  because of the multiplicative behaviour of (A2). If this constant is taken the same for all  $S_{i,i+1}$  and it is small there is a dramatic change in the expression for  $t_i$ <sup>(4)</sup>

$$t_i \approx - \left| \sum_i^i p_{\perp}^i \right|^2 - \langle \mu^2 \rangle - \frac{e^{-d}}{(1-e^{-d})} \quad (A.3.)$$

Here  $d$  is the average gap in rapidity between  $i, i+1$ , and  $\mu^2$  is an average transverse mass.

For multiplicities near the mean this second term is much larger than the first. Typically one may find  $t_i \geq 1 \text{ GeV}^2$  while  $S_{i,i+1} \leq 0.5 \text{ GeV}^2$ . Notice that the situation is made worse if we do not order the particles in rapidity first.

In fig. 3 we show some typical results of an experiment with  $n \sim \langle n \rangle$ . These are plots from the 16 GeV/c data of the ABCLV collaboration plotted in a theoretical manner by P Dornan and B Pollock. The final state particles are ordered in rapidity and the momentum transfers plotted as shown. If we do not order in rapidity we obtain even larger  $t_i$ . The mean charged multiplicity at 16 GeV/c is 4.0 and the total multiplicity 5-6. Thus at  $n \sim \langle n \rangle$  we see  $t_i$  up to  $1 \text{ GeV}^2$ . Notice also that nearest neighbour pions in the rapidity plot typically have a mass well below the  $\rho$ -meson. Next to nearest neighbours have a clear  $\rho$ -signal.

We therefore only expect the ABFST model to work for small multiplicities or high energies. In an interesting series of papers Dash and collaborators<sup>(5)</sup> have in fact put in an enhancement for large  $t_i$  in the double off-shell amplitude. They get reasonable fits to  $\pi^+ p \rightarrow 3\pi^+ 2\pi^- p$ ,  $\pi^- p \rightarrow 2\pi^- 2\pi^+ n$  with cross-sections as shown. Notice that at 16 GeV/c the value of  $\langle n_{TOT} \rangle \approx 6.5 \pm 0.5$ . This strong enhancement of the  $\langle n \rangle$  cross sections also results in pushing the output Pomeron up to  $\alpha(0) \approx 0.85$ . The enhancement is however rather arbitrary. Clearly this effect needs further study.

Notice that if  $n \gg \log s$  as  $s \rightarrow \infty$  then  $d \rightarrow 0$  and  $t_i \rightarrow \infty$ . This means that the canonical multiperipheral diagrams are rather unbelievable. Then one must presumably have production as in Fig. 5. Whether this is an important mechanism at energies  $\leq 30 \text{ GeV}$  is clearly an open question.

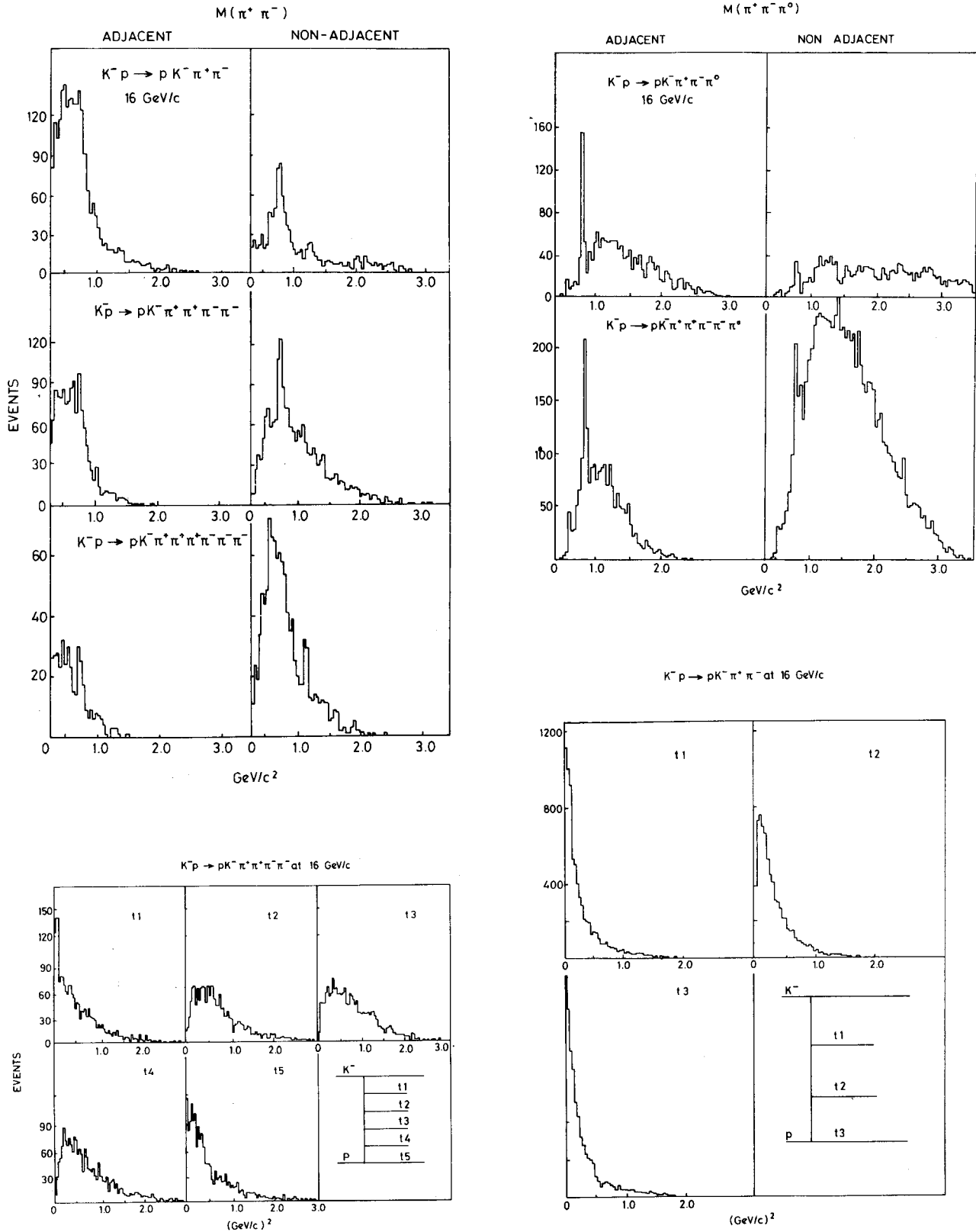


Fig. 3. 16 GeV/c  $K^-p$   $t_{i,m}^2$  histograms.

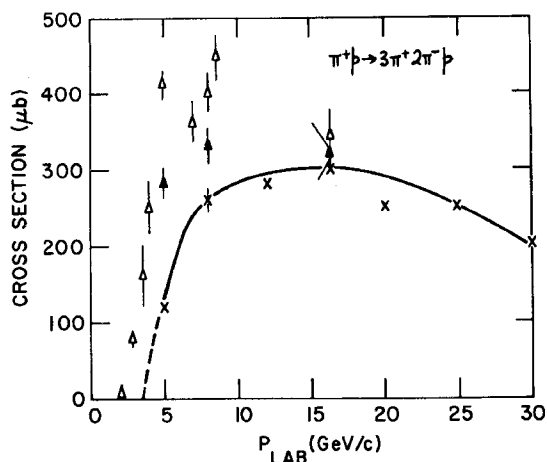


Fig. 4. The ABFST cross-sections of Dash et al.  
Experimental  $\sigma_6$  and ABFST (x) model  
predictions.

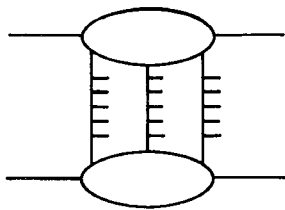


Fig. 5. A large multiplicity mechanism. Each chain  
has small  $t_i$ .

The value of the  $t$ -slope of the forward amplitude has given rise to a prolific literature and the dust has not settled yet.

From the numerical calculations of Michejda and collaborators<sup>(6)</sup> the slope at  $t=0$  is typically much too small ( $\sim 1/10$  experimental value) for the Chew-Pignotti and CLA ( $n$  phase) models. However, several authors using the large sub-energy kinematics for  $n \sim \langle n \rangle$  obtained too large a slope. Thus in calculations at  $n \sim \langle n \rangle$  one should be extremely careful in using the separated form of (A2). The random walk picture in impact parameter space proposed by several authors<sup>(7)</sup> should therefore be treated with caution<sup>(8)</sup>. There is, of course,

always the option that the cut-off is not in the  $t_i$  but in  $(p_{\perp}^i)^2$ .

Teper<sup>(8)</sup> has argued that in multiperipheral models it is impossible to get the slope and its  $s$ -dependence up to ISR energies correct. He considers the introduction of phases and of clusters.

In a paper submitted to this conference Dash and Jones have stressed the importance of spin and in a simplified ABFST model where the  $\pi\pi \rightarrow \pi\pi$  model is  $\rho$ -dominated they get reasonable agreement with experiment for  $E \lesssim 30$  GeV.

This includes the difference between  $\pi p$  and  $pp$  elastic slopes.

The possible large effects, in the single diffractive contribution to  $d\sigma/dt$ , due to spin have been studied by Sakai and White<sup>(10)</sup>.

They show how  $t$ -channel helicity conservation for the diffracted resonant states is crucial in obtaining a reasonable slope.

We should mention in this context the study by Chan Hong-Mo and Paton<sup>(11)</sup> of the slopes due to the Dual model. Here they study both Pomeron and isospin exchanges. This is qualitatively correct.

If the  $p_{\perp}$  cut-offs are due to the second term in (A.3.) then of course we expect a dependence of the  $p_{\perp}$  distribution on the gap size in rapidity  $d$ . Thus at increasing multiplicity we expect sharper cut offs in  $p_{\perp}^2$ . Some experiments have found a small effect of this kind<sup>(12)</sup>. Alternatively one can study strictly exclusive events where the particle whose  $p_{\perp}$  is measured has or has not large gaps on either side of it. Henyey<sup>(8)</sup> has carried out such a study with null results in semi-inclusive events at 200 GeV/c. It is important that such studies should be carried further.

One of the most elegant sets of results from the ISR is the Pisa-Stony-Brook measurements<sup>(13)</sup> on two

particle correlations. Several simplified models on the lines of the multi-peripheral model have been put forward to fit this data. These are the cluster emission models.

For example Berger and Fox<sup>(14)</sup> write

$$|M_{2 \rightarrow n_c}|^2 = \frac{\lambda^{n_c}}{n_c!} \frac{n_c!}{1} e^{-p_{\perp}^2/2\sigma^2}$$

for the non-diffractive  $2 \rightarrow n_c$  clusters matrix element. This is supplemented with the experimental proton x-distribution near  $x=1$ . The clusters decay isotropically in their rest frame.

This then gives a fit to a) Multiplicity distribution b)  $p_{\perp}$  distribution, c) Single particle inclusive and d) Correlations.

Notice here the use of  $e^{-p_{\perp}^2/2\sigma^2}$  rather than  $e^{-t/2\sigma}$ . It is found that there are  $\sim 4$  particles per cluster, with  $m_{\text{cluster}} \sim 1.6$  GeV. Whether these clusters are meaningful is an open question. Thus we have a rather arbitrary matrix element, spin is ignored and there is little contact with known resonance structure.

One of the other reasons put forward for cluster formation of the existence of  $(++)$  and  $(--)$  short range rapidity correlations comparable with the  $(+-)$  correlations<sup>(13,15)</sup>. The naivest reason for  $(+-)$  correlations is resonance, say  $\rho$ , production. No such comparable contribution is available for the  $(++)$  or  $(--)$  correlations. Thus perhaps the clusters can contain  $(++)$  or  $(--)$  combinations of pions giving these apparently non resonant contributions to the correlation functions<sup>(16)</sup>.

Clearly these peripheral models have a great deal of uncertainty due to the experimental large  $\langle n \rangle$  with its subsequent implication of small rapidity gaps. These lead to momentum transfers  $t_i$  which rather undermine the multiperipheral assumptions.

### B. The Pomeron and its consistency

This section adopts the philosophy that we succeeded in A in satisfying our constraints and obtained  $\alpha(0) \sim 1$ . The question that now arises is the consistency of this object. Let me assume for ease that it was a Regge pole. It has been known for 10 years now that the unitarity equation leads to the iteration of this object.

This means that we have production processes of the form of Fig. 6. When these are put together they give contributions to  $\text{Im } T_{22}$ , by the optical theorem, or inclusive cross-sections as shown in Fig. 7.

If  $\alpha(0) < 1$  as is the case in the ABFST case then one can in a rather straightforward manner calculate the effect of these diffractive processes. It is found that  $\sigma_{\text{TOT}}(\infty) = 0$  although we may have many bumps and oscillations on the way there. There is therefore no real problem with consistency. In particular the famous triple Pomeron coupling  $g_{\text{ppp}}$

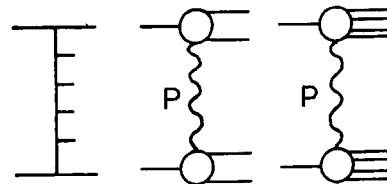


Fig. 6. Diffractive Production mechanisms.

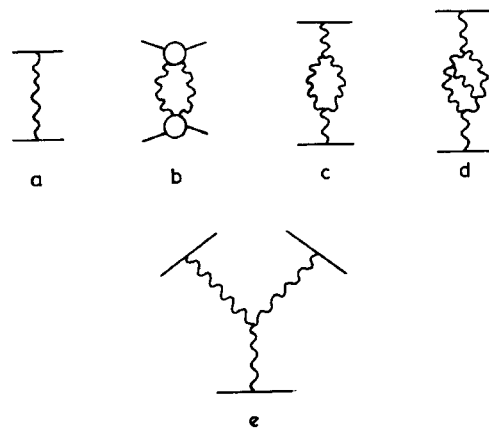


Fig. 7. Contributions to elastic and inclusive cross-sections.

(t) is not zero at  $t=0$ . In an interesting contribution to this conference Shankar<sup>(17)</sup> studies the triple Regge couplings in an ABFST model. He finds, in agreement with fits to ISR data, that lower trajectories have much larger couplings.

If  $\alpha(0) = 1$  then the contributions of the graphs of Fig. 7 c,d are in general bigger than graph a and on summing the series we lose our dominant pole at  $\alpha(0) = 1$  unless  $g_{ppp}(t=0) = 0$ . Then we obtain Gribov and Migdal's quasi-stable Pomeron. In other words the summation of the above series has a dominant pole plus small cut corrections. Then  $\sigma_{TOT} \rightarrow \text{const.}$  as  $s \rightarrow \infty$ . The data of inclusive reactions appears not to give  $g_{ppp}(0) = 0$ <sup>(19)</sup>.

Unless one has rather clever dominant cuts in two particle inclusive matrix elements, this zero acts as a cancer centre and eventually also forces the Pomeron to decouple from total cross-sections<sup>(20)</sup>.

In two intriguing papers Migdal, Polyakov and Ter-Martirosyan and Abarbanel and Bronzan<sup>(21)</sup> show how the problem of summing the above series for  $g_{ppp}(t=0) \neq 0$  can be tackled by the renormalization, group techniques of field theory.

Let me outline how this comes about. Since we are dealing with Pomerons we can assume large subenergies and hence the kinematics of (A2). Then, in two space dimensions ( $p_1$ ), if we write the non-relativistic perturbation theory for the above diagrams with  $E = 1 - j$  and  $\omega(p) = 1 - \alpha(0) + \alpha'(0)p^2$  we find for Figs 5 a,b respectively

$$\frac{1}{E - \omega(p)} \sim \frac{1}{j - \alpha(p)}$$

$$\int \frac{d^2 q N^2}{(E - \omega(q) - \omega(p - q))} \approx \int \frac{d^2 q N^2}{(j - \alpha(q) - \alpha(p - q) + 1)}$$

Thus if we interpret  $j$  as the complex angular momentum in the  $t$ -channel the first term is the usual Regge pole formula and the second the usual

Regge cut term. The general expressions can be derived in two ways. Either, following Gribov<sup>(22)</sup>, we can derive the rules for the high energy limits of Feynman diagrams or, as emphasized by Cardy and White<sup>(20)</sup>, we may look upon them as the solution of the continued  $t$ -channel unitarity equations.

This set of propagator rules is then equivalent to a field theory in 2 space dimensions

$$\begin{aligned} &= \frac{i}{2} \left( \psi^+ \frac{\overleftrightarrow{\partial}}{\partial t} \psi \right) - \alpha'_0 \nabla \psi^+ \nabla \psi - \Delta_0 \psi^+ \psi \\ &- \frac{\lambda_0}{2} (\psi^+ \psi^2 + \psi^+ \psi^2 \psi) \quad \Delta = 1 - \alpha(0) \end{aligned}$$

plus possible higher order or derivative couplings.

We pretend now that we are in  $D$  dimensions. Notice that we are interested in infrared behaviour since  $j \sim 1$  is  $E \rightarrow 0$  and  $\omega \sim 1 - \alpha(p) \sim 0$  at  $p \sim 0$ .

Now derive the standard renormalization group equations. Thus we renormalize the coupling constant at the point  $E = -E_n$ ,  $p^2 = 0$ , to define  $\lambda(E_n)$  and then introduce the dimensionless quantities

$$y(E_n) = \frac{\lambda(E_n) E_n^{D/4-1}}{\{\alpha'(E_n)\}^{D/4}}$$

$$\beta(y) = E_n \frac{\partial}{\partial E_n} y(E_n)$$

A zero of  $\beta$  at  $y = 0$  when  $\frac{\partial \beta}{\partial y} > 0$  would give the infrared behaviour. To lowest order in perturbation theory

$$\beta(y) = - \left( \frac{4-D}{4} \right) y - (\tilde{K} + D/4 K) y^3$$

$\tilde{K}, K(D)$  are computable positive constants for  $2 \leq D \leq 4$ . This gives a zero at  $\beta = 0$  but  $\frac{\partial \beta}{\partial y} < 0$  and so the zero governs the ultraviolet behaviour. However, in the Gribov calculus our cut term is negative i.e.  $\lambda_0$  is pure imaginary. This turns the result the way we want it. The graph for the coupling constant now looks like Fig. 8 and we have a zero ( $y = ig$ )

$$g_1 = \frac{4 - D}{4K + DK}$$

with positive slope.

For  $D = 4$  the coupling is small and hopefully may be computed in perturbation theory. Even more hopefully this is still true at  $D = 2$  where the physics lies.

In two papers Sugar and White<sup>(21)</sup> have presented a careful study of the pole model. This care is required for the renormalization of an infrared divergent theory.

As stressed by Migdal, Polyakov and Ter-Martirosyan the solutions to these equations now avoid all the Pomeron decoupling arguments coming from inclusive sum rules etc. If we now work back to the total cross-section we find

$$\sigma_{\text{tot}}(a,b) = f_a f_b (\log s)^{1/6} (1 + O(\frac{1}{\log s}))$$

There is a pole at  $\alpha(t) \approx 1 + Bt^{(1/(1 + \epsilon/24))}$

moreover the "triple Pomeron" vertex has a non-analytic zero at  $t = 0$ . However  $\langle n \rangle \approx (\log s)^{1+x}$  which we have seen to be dangerous.

Clearly there is a great deal of arbitrariness in this procedure. There are many Lagrangians allowed. Thus Jengo<sup>(23)</sup> in a paper to the conference and Brower and Ellis have shown how the quasi-stable Pomeron comes from a gradient coupling term.

Dash and Bronzan<sup>(24)</sup> have also raised doubts about the convergence of the expansion in  $(4-D) = \epsilon$ .

Abarbanel and Sugar<sup>(25)</sup> have studied how lower trajectories may be modified. There are solutions where a linear input trajectory is modified either

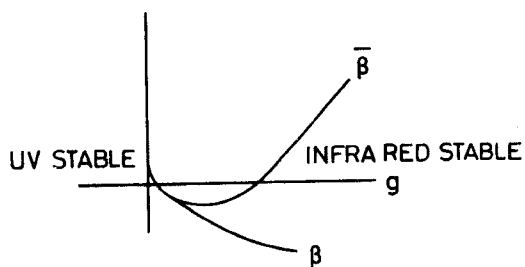


Fig. 8. The fixed points.

into a linear trajectory or one with an infinite slope at  $t = 0$ .

In an extremely interesting paper Bartels and Savit<sup>(26)</sup> have shown how this renormalization group technique may get rid of the wrong parity nucleons for fermion trajectories.

Another field theory has also been studied. If, as in the Cheng and Wu or Chang and Yan<sup>(27)</sup> models we obtain a Reggeon above 1 then eikonalise in the  $s$ -channel we obtain a "Reggeon" propagator

$$\frac{1}{((j-1)^2 - a^2 t)^p}$$

Again the renormalization group<sup>(28)</sup> may be applied to give an infra-red stable point if  $p = \frac{1}{2}$ . This gives  $\sigma_{\text{tot}} \approx \text{const}$ ,  $s \rightarrow \infty$ .

This is clearly an exciting theoretical development which is already being pushed very hard.

At a fundamental level one would clearly like to have a rigorous derivation of the Reggeon field theory. Thus we remind readers that the complex angular momentum unitarity condition is still not proven<sup>(29)</sup>.

We should also mention a very different approach by Ciafaloni and Marchesini<sup>(30)</sup>. One of the standard "proofs" of  $g_{\text{ppp}}(0) = 0$  runs as follows. We know

$$\begin{aligned} \frac{\sqrt{s}}{2} \sigma_{\text{tot}} &= \int \frac{d^3 p}{E} \cdot E \cdot \frac{d\sigma_{\text{incl}}}{d^3 p/E} \\ &\geq g_{\text{ppp}}(0) \ln \ln s \end{aligned} \quad (\text{B.1.})$$

If we evaluate the integral of the right hand side over the triple Regge region with the inclusive cross-section given by Fig. 9 we find the right hand side is greater than  $g_{\text{ppp}}(0) \ln \ln s$ . Ciafaloni and Marchesini show that all the other terms in Fig. 9 give a series in  $(\ln \ln s)^n$  which can be summed and

gives a constant total cross-section. Since  $\ln \ln (M_0^2/m_p^2) = 5.5$  where  $M_0 =$  mass of the sun it is not clear whether  $\ln \ln s$  is large or small.

However, in an intriguing paper submitted to the conference Bronzan and Jones<sup>(31)</sup> have raised doubts about one's naive assumptions. If  $0 < 1 - \alpha \leq 0.1$  then following the standard path to the decoupling theorems the find at present energies  $\sigma_{TOT}^{\pi\pi} \leq .03$  mb.

Thus in diagram language we compute a lower bound on  $\sigma_{tot} \propto g_{\pi\pi p}^2$  from the above energy momentum sum rule. The triple Pomeron coupling is then given a lower bound from the energy sum rule relating the two particle inclusive to are particle inclusive cross-sections. This is schematically shown in Fig. 10. If we pull out a pion then is small and we insert the pion pole. Thus it is found that

$$g_{ppp} \geq C \cdot \frac{2}{m_\pi} g_{\pi\pi p}^3$$

Finally we get the above result for  $\sigma_{tot}^{\pi\pi}$

For  $\alpha(0) > 1$  as occurs in massive Quantum Electrodynamics we must apply absorption in the s-channel to pull the cross-section below the Froissart bound. The famous historical calculations of Cheng and Wu, and Chang and Yan<sup>(27)</sup> may have technical flaws. These are concerned with the imposition of energy momentum conservation. Cardy<sup>(27)</sup> has given a heuristic treatment which shows how some, at least, of these calculational problems may be avoided in the Gribov calculus.

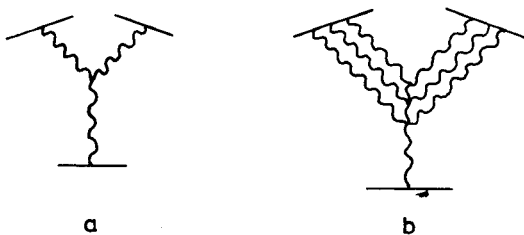


Fig. 9. The Ciafaloni-Marchesini graphs.

It has been stressed by Blankenbecler and collaborators<sup>(32)</sup> that absorption may be more visible in  $p_\perp$  distributions. Thus in two body scattering absorption suppresses low impact parameter collisions. Thus in the random walk in impact parameter space (cf section A) the path should not wind up on itself. This is a famous problem in the theory of polymers. The correlation imposed between the impact parameters then shows up in correlated  $p_\perp$  distributions between the particles.

We should stress that these eikonal or absorptive models with trajectories cannot be faulted in the s-channel. The only doubts arise from their t-channel structure.

At this point we should perhaps mention the evidence in favour of the existence of multiple Pomeron exchange. There are of course the ISR triple-Regge<sup>(19)</sup> fits which fit into the usual scheme rather easily. There is also the NAL data on the exclusive channel  $pp \rightarrow pp \pi^+ \pi^-$  in the region shown in Fig. 11<sup>(33)</sup>. This cross section is surprisingly small.

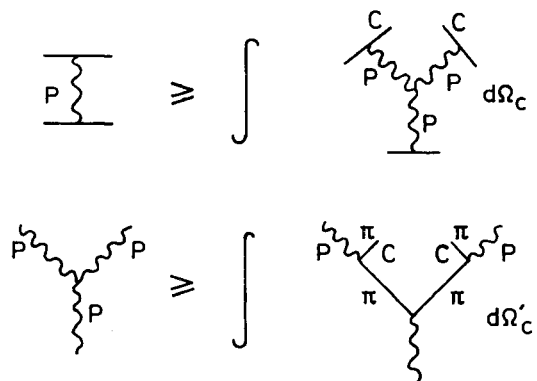


Fig. 10. The Bronzan-Jones upper bound graphs.

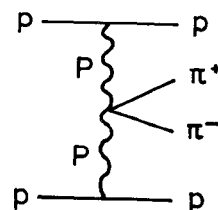


Fig. 11. Double Pomeron exchange.



### C. Rising Total cross-sections

The mechanisms which have been suggested for this rise were admirably reviewed by Caneschi and Ciafaloni at the Aix-en-Provence conference. I shall merely run through them rapidly with reference to work done since then.

$\sigma_{TOT}^{(\infty)} = \infty$  These are models based on some mechanism for saturating the Froissart bound as in massive Q.E.D. In a paper presented at this conference Cheng, Walker and Wu<sup>(34)</sup> presented their latest fits with the inclusion of lower Regge trajectories. These are quite impressive. Goldman and Sivers<sup>(34)</sup> have pointed out that if one calculates the multiplicity coming from the dominant quantum electrodynamic diagrams then the ISR total average multiplicity should be  $\leq 4$ .

Similar fits are given by Collins, Gault and Martin<sup>(34)</sup>. It is however not clear that, at the ISR, the saturation of Froissart mechanism is really relevant. Thus we are really a long way from the Froissart bound of

$$\sigma_{tot} \leq \frac{\pi}{m_\pi^2} \log^2(s/s_0)$$

$$= 3800 \text{ mb for } s_0 = 1 \text{ GeV}^2$$

There are also models with a Regge dipole. The latest of these, giving rather good fits, is due to R.J.N. Phillips<sup>(35)</sup>.

$\sigma_{tot}^{(\infty)} = \text{constant}$  This is the quasi-stable pomeron prediction. The growth in  $\sigma_{tot}$  is due to the decrease of the negative two pomeron cuts ( $\sim 1/\log s$ ). However, this decrease is caused by exactly the same mechanism as should cause  $(\frac{d\sigma}{dt})_{el}$  to shrink. Moreover we require  $g_{ppp}(0) = 0$ .

Capella and Kaplan<sup>(36)</sup> have studied this in the Gribov calculus. By insisting that the cut is

calculated from the inclusive data they find the rise over the ISR range is too small at  $\leq 1 \text{ mb}$ .

### Threshold Effects

Many authors<sup>(37)</sup> have suggested that the rise is due to the opening of the large mass diffractive region. This positive growth of the form of (B.1.) is however rather small and Blankenbecler<sup>(38)</sup> has given strong grounds for believing that this growth is more than cancelled by a consequent decrease in the pionisation region.

Another possible mechanism is due to the  $B\bar{B}$ <sup>(39)</sup> channels finally opening up. From A1 for Fig. 17  $S_1 S_2 S_3 = S_1 u_2^2 u_3^2$ . If we take  $S_1 \sim 4 \text{ GeV}^2$ , then  $S \geq 64 \text{ GeV}^2$ . This rough estimate is numerically too low. Typically only above  $s = 200 \text{ GeV}^2$  is  $B\bar{B}$  production important.

However, in  $\pi\pi$  collisions the rise threshold should at first sight be much lower since  $s_1$  may be missing (cf Fig. 12). This does not seem to be the case.

This should be tempered with the knowledge that some of this process is diffractive. This is possibly cancelled by the Blankenbecler mechanism.

In the  $pp$  case the rise is estimated from the sum rule

$$\langle N \rangle \sigma(N\bar{N}) = \int \frac{d^3 p}{2E} \left( \frac{d\sigma_{p}^{incl.}}{d^3 p/2E} + \frac{d\sigma_{n}^{incl.}}{d^3 p/2E} \right)$$

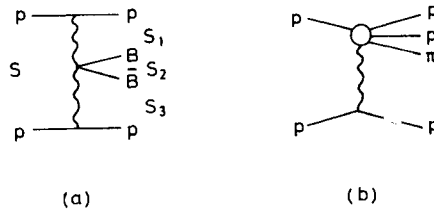


Fig. 12.  $B\bar{B}$  production mechanism.

where  $\langle N \rangle$  is the average number of  $B\bar{B}$  pairs  $\sim 1$  in our range. Then  $\sigma(NN)$  increases by 5-6 mb over ISR range.

In an interesting paper presented to this conference Chew and Koplik<sup>(40)</sup> have pointed out that  $\bar{p}p$  interactions may be anomalous. This is due to their strong long range attraction due to pion exchange.

The actual final state may have a high percentage of pion pairs. This makes the above calculation slightly shaky. Thus a large rise in  $\sigma_{el}$  and  $\sigma_{tot}$  ( $\pi\pi$ ) is expected near  $s = 4 m_p^2$ . An estimate based on an ABFST calculation for the  $B\bar{B}$  production cross section gives  $\langle \sigma(\pi\pi \rightarrow B\bar{B}) \rangle \geq 7\text{mb}$  for  $4m_p^2 < s < 30 \text{ GeV}^2$ .

This cross section is for all  $B\bar{B}$  pair production e.g.  $\Delta\Delta$  as well as  $pp$ .

These results may tie in rather nicely with a study by Sakai<sup>(41)</sup> who estimates that in the energy momentum sum rule the proton total contribution is zero the  $|x| > 0.94$  protons effectively cancelling the  $0.6 \leq |x| \leq 0.94$  protons with others negligible. The central pion rise could then be the cause, in the energy sum rule, for the rise within large experimental uncertainties.

In the large transverse momentum session Jarlskog<sup>(42)</sup> also claims that the rise in the total cross-section is due to the production of pions in the pionization region. From the equation

$$\langle n \rangle \sigma_{TOT} = \int \frac{d\sigma_{incl.}}{d^3q/2E} d^3q/2E$$

if  $\frac{d\sigma_{incl.}}{d^3q/2E}$  rises at  $X \rightarrow 0$  then, unless  $\sigma_{TOT}$  rises,  $\langle n \rangle$  increases faster than  $\sim \log s$ . The rise is therefore interpreted as a rise in  $\sigma_{TOT}$ . However these pions may well come from the high mass diffraction.

At a purely phenomenological level the simplest fits are due to a model with  $\alpha(0) = 1.06$  with high mass diffraction giving the major rise in  $\sigma_{TOT}$ . Eventually the Froissart bound must be obeyed and the absorptive corrections become important. At present energies they may be safely ignored<sup>(43)</sup>.

#### D. Field Theoretic Reggeisation

One of the long term problems in high energy theory has been the problem of Reggeisation. Thus if we start with a Lagrangian field theory and compute its high energy behaviour we often find Regge poles in the result. The question then arises of whether out original "elementary" particles lie on these trajectories.

Until recently the only particle for which this happened was the spin  $\frac{1}{2}$  particle in a theory with spin  $\frac{1}{2}$  coupled to a conserved vector current<sup>(44)</sup> (Massive quantum electrodynamics).

The recent startling result is that, in spontaneously broken, non-abelian gauge theories, using the sufficiency arguments of Mandelstam<sup>(45)</sup> Grisaru, Schnitzer and Tsao<sup>(46)</sup> have proved for  $SU(2)$  Yang-Mills that the spin  $\frac{1}{2}$ , 1 mesons do Reggeise but in general the scalars do not.

The explicit high energy behaviour in spinor-spinor scattering has been studied by Nieh and Yao<sup>(47)</sup> in a recent paper. This is very odd looking

$$A(s,t) \approx iCs \left( 1 - \frac{4g^2}{16\pi} \ln^2 s + \frac{5}{2} \left( \frac{g^2}{16\pi} \right)^2 \ln^4 s \right)$$

The rather unexpected sign changes and the fact that to get the leading terms they need integrations over large  $p_\perp^2$  need further exploration. If cross sections are dominated by large  $p_\perp^2$  in these models they would be severely weakened.

### E. Nuclear Scattering

Here I would like to mention some results of high energy scattering on nuclei. This has the rather astonishing property that the pion multiplicity on emulsion ( $\langle A \rangle \approx 69$  is only  $\sim 1.7 \pm 0.2 \times pp$  (multiplicity) from  $E_{lab} = 67$  to  $8000$  GeV<sup>(48)</sup>. Why is this surprising? If we assume a high energy incident proton hits a nuclear proton then we expect a large shower of downstream particles. These will typically rescatter and give an enormous final multiplicity. This is true even if only the fast forward proton is allowed to cascade. Numerical calculations have been performed by many authors confirming the above picture.

Notice that we are assuming that the time of particle production say in the peripheral model is small compared to the average time of flight between collisions in a nuclei.

However, hydro-dynamical models as recently revived to fit ISR inclusive data give a very different picture<sup>(49)</sup>. Here it is imagined that in the original impact a superdense fluid, in which pions, protons, etc. are not identifiable, is formed. This then expands until typically each pion mass has  $\sim (1/m_\pi^3)$  of space available. They then become identifiable as pions. Thus in nuclear collisions this particle formation only takes place outside the nucleus. In cascade type collisions rather than  $\langle n_{pp} \rangle$  particles cascading downstream we have a flux of undifferentiated energy density.

Gottfried<sup>(48)</sup> has proposed a model on these lines which gives reasonable agreement with data. However, these studies deserve to be pushed further.

The actual time dependence of the multiperipheral model has been studied many times<sup>(7)</sup> and is very different from the above instantaneous picture. The

effect on nuclear collision multiplicities has been studied by Lehman and Winbow and by Kancheli<sup>(50)</sup>. We shall only consider the pionization region in this review. For further details see the original papers.

In diagram language the simplest peripheral pion production mechanisms are shown in Fig. 13. If these were incoherent then we would have cascading.

To simplify our discussion now turn to the diagram of Fig. 14 which is a simple example typical of the problem. There are three important s-channel cuts. The first does not contribute to the logs term in the multiplicity. The second corresponds to  $\beta \times \beta$ . Now it is a remarkable fact that the contribution to the multiplicity cancels exactly between these diagrams. The multiplicities are  $\langle n \rangle$  for a single ladder cut and  $2 \langle n \rangle$  for the double ladder cut. The contributions to the square of the total matrix element  $-4A$  and  $+2A$  respectively<sup>(51)</sup>. Similar cancellations hold for all further diagrams and cascading does not happen.

In Fig. 15 we show the experimental results and the predictions by Lehman and Winbow. A is the result of a cascading calculation. B, C, D are results of typical hydro-dynamic like models.

In the same context we should mention the apparent "experimental" result that the low mass non-resonating  $3\pi$ ,  $5\pi$  systems which apparently have  $\sim 25$  mbs. of cross-section in nuclei<sup>(52)</sup>.

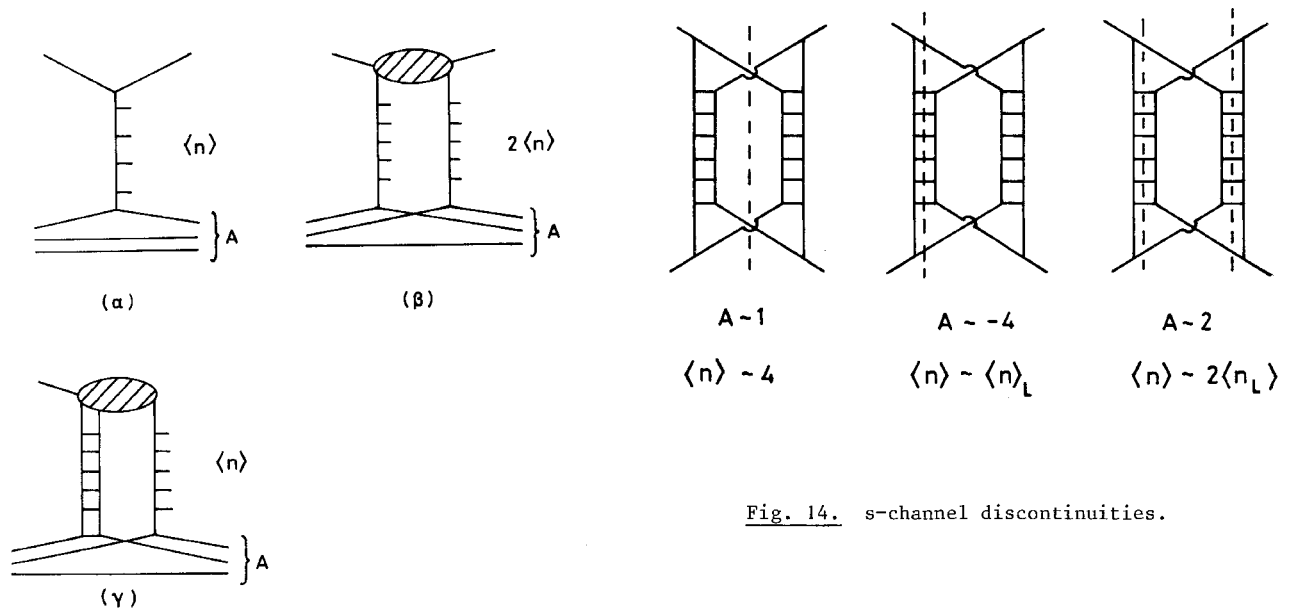


Fig. 14. s-channel discontinuities.

Fig. 13. Cascade graphs.

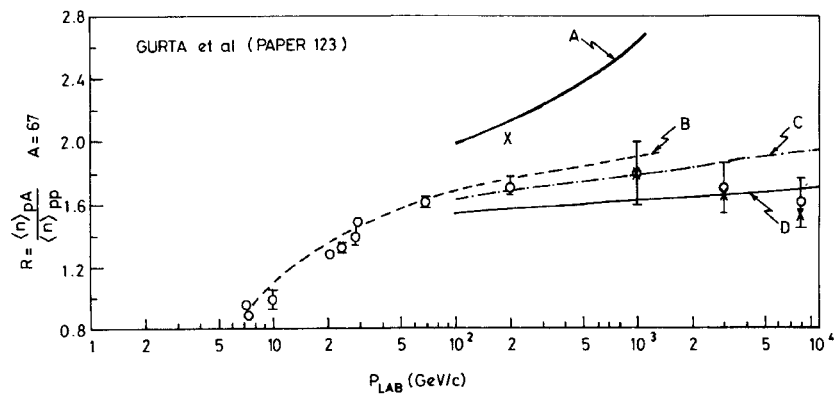


Fig. 15. The ratio of nuclear to proton multiplicities as a function of  $p_{lab}$ . The Lehman Winbow calculations are shown by X.

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