DETECTING ADDITIONAL POLARIZATION MODES WITH LISA

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Within the frame of General Relativity, gravitational waves possess two tensorial polarizations (h_+ and $h_\times$), whereas more general metric theories of gravity predict the existence of additional modes: up to 2 vector and 2 scalar modes. An arbitrary signal such a stochastic GW background is thus expected to contain a mixture of up to 6 polarizations with no dominant mode, and the analysis of its content could give us constraints on GR or its extensions. We addresse the question of whether a given LISA configuration can provide a sufficient sensitivity to detect additional polarization modes and then allow the extraction of the latter in order to determine the GW spectrum for each mode.

1 Introduction

The recent detections of gravitational waves (GW) announced by the LIGO Collaboration [1,2] mark the beginning of a new chapter in GW research. Furthermore, the future ESA L3 mission, the space-borne detector "Laser Interferometer Space Antenna" (LISA) [3], is expected to open new perspectives in the low-frequency domain from 0.1 mHz to 1Hz, giving access to the observation of e.g. supermassive black holes at cosmological distances or binary systems of close white dwarves, or even a stochastic GW background (SGWB).

The dynamics of those mergers can be affected by alterations of GR. The proposed design of the LISA mission [4] makes it possible to reach an unprecedented signal sensitivity and therefore measure the inprints of alternative theories, such as $f(R)$ or scalar-tensor theories. We consider the detection of a SGWB which would in general contain a mixture of all polarization modes. Depending if additional modes are effectively detected or not, this will allow in any case to put constraints and discriminate between alternative gravitation theories.

2 Mode sensitivity and mode extraction

Within the frame of GR, GW possess two tensorial polarizations, the so-called h_+ and $h_\times$ modes, but one can expect additional modes when considering more general metric theories of gravity: up to 2 vector and 2 scalar modes. The perturbed metric of a propagating GW can be expressed as:

$$h_{ij}(\omega t - \mathbf{k}\cdot\mathbf{x}) = \sum_A h_A(\omega t - \mathbf{k}\cdot\mathbf{x}) e^A_{ij} \tag{1}$$

with $A = +, \times, x, y, b, l$ the six possible polarization modes, h_A the GW amplitude of the mode A, and e^A_{ij} the following polarization tensors (tensor ($+$, $\times$), vector (x, y) and scalar (b, l) modes):

$$e_{ij}^{+} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad e_{ij}^{x} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad e_{ij}^{b} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$e_{ij}^{\times} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad e_{ij}^{y} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad e_{ij}^{l} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

From now on, we will not assume a particular gravitation theory, which means that a GW is in general expected to contain a mixture of up to 6 polarizations. We will focus on the determination of the detection threshold for each polarization modes present in a SGWB signal and look for a way of extracting this polarization content.

2.1 Network of detectors

The determination of the sensitivity to additional polarization modes was discussed for an earlier version of the LISA mission [5,6,7,8,9,10]. It is therefore necessary to investigate this question for the new design beforehand and establish the sensitivity curves for several time-delay interferometric (TDI) combinations (i.e. different combinations of the signals depending on the number of laser links between the satellites). We already found that the noise spectrum of the future detector presents large similarities with the older project, especially in the frequency range $f \in [10^{-2}, 1]$ Hz, and thus expect the sensitivity spectrum to be quite similar in that frequency domain where, in particular, the sensitivity to the vector and longitudinal-scalar modes is higher.

In its current proposed design (3 satellites with 3 arms, i.e. 3 times 2 laser links between the satellites), LISA is actually equivalent to two single detectors; we will describe this arrangement of three satellites as a cluster. The SGWB signal measured by a single detector can be written as

$$h(t, \mathbf{x}) = \sum_A \int_{S^2} d\hat{\mathbf{\Omega}} \int_{-\infty}^{\infty} df \; \tilde{h}_A(f, \hat{\mathbf{\Omega}}) e^{2\pi i f (t - \hat{\mathbf{\Omega}}\mathbf{x}/c)} F_A(\hat{\mathbf{\Omega}}), \tag{2}$$

with F_A the antenna pattern function of that single detector (which describes its geometry), and h_A the GW amplitude of the mode A. In a single detector, the vectors $\hat{u}$ and $\hat{v}$ give the direction of each arm and one can define the detector tensor D as

$$D_{ij} = \frac{1}{2}(\hat{u}_i \hat{u}_j - \hat{v}_i \hat{v}_j), \tag{3}$$

which describe the response of that detector to a signal.

If one now considers a network of several identical clusters [11,12,13], the first step consists in determining the so-called overlap reduction functions (ORFs), defined for a pair of detectors I and J separated by $\Delta\mathbf{x}$ as

$$\begin{aligned} \gamma_{IJ}^{M}(f) \;=\; & \frac{1}{\sin^2(\chi)} \Big(\rho_1^M(\alpha) D_I^{ij} D_{ij}^J + \rho_2^M(\alpha) D_{I,k}^{i} D_J^{kj} \hat{d}_i \hat{d}_j \\ & + \rho_3^M(\alpha) D_I^{ij} D_J^{kl} \hat{d}_i \hat{d}_j \hat{d}_k \hat{d}_l \Big), \end{aligned} \tag{4}$$

with M denoting the tensor (T), vector (V) or scalar (S) polarization modes, $\sin^2(\chi) = 1 - (\hat{u} \cdot \hat{v})^2$ a geometry factor (which is $\frac{3}{4}$ for triangular cluster like LISA), $\rho_i^M = f(j_0(\alpha), j_2(\alpha), j_4(\alpha))$ a linear combination of spherical Bessel functions, D_I^{ij} the detector tensor of the interferometer I, $\hat{d}_i = \dfrac{\Delta\mathbf{x}}{|\Delta\mathbf{x}|}$, $\alpha = \dfrac{2\pi f \, |\Delta\mathbf{x}|}{c}$. An ORF tells how much degree of correlation is preserved when one correlates the output of two detectors, according to their relative orientation.

Note that, schematically, the signal $h(t)+n(t)$ measured by a detector is composed of the GW signal $h(t)$ as well as the noise $n(t)$. Next, we can consider the one-sided power spectral density S_h^A:

$$\langle \tilde{h}_A^*(f,\hat{\mathbf{\Omega}})\tilde{h}_{A'}(f',\hat{\mathbf{\Omega}}')\rangle = \delta(f-f')\frac{1}{4\pi}\delta^2(\hat{\mathbf{\Omega}},\hat{\mathbf{\Omega}}')\delta_{AA'}\cdot\frac{1}{2}S_h^A(|f|), \tag{5}$$

as well as the noise spectrum $P_I(f)$:

$$\langle \tilde{n}_I(f)\tilde{n}_J(f')\rangle = \frac{1}{2}\delta(f-f')\delta_{IJ}\cdot P_I(|f|), \tag{6}$$

and the GW background energy density

$$\Omega_{\rm gw}^M(f)\propto f^3 S_h^A(f), \tag{7}$$

with e.g. $\Omega_{\rm gw}^T=\Omega_{\rm gw}^+ + \Omega_{\rm gw}^\times$ (and similarly for $M=V,S$). It is then possible to find the optimal signal-to-noise ratio (SNR) necessary to separately detect the modes:

$$SNR^M \propto \int_0^\infty df\left[\frac{(\Omega_{\rm gw}^M(f))^2\det\mathbf{F}(f)}{f^6\mathcal{F}_M(f)}\right]^{(1/2)}, \tag{8}$$

where $\mathbf{F}$ is a (3×3)-matrix whose elements are given by

$$F_{MM'} = \sum_{\text{det. pairs}(I,J)}\int_0^{T_{\rm obs}}\mathrm{d}t\frac{\gamma_{IJ}^M(t,f)\gamma_{IJ}^{M'}(t,f)}{P_I(f)P_J(f)}, \tag{9}$$

($T_{\rm obs}$ is the mission duration) and $\mathcal{F}_M(f)$ is the determinant of the matrix obtained by removing all the M-elements from the matrix $\mathbf{F}$.

The separation of the modes can then be achieved as follows [11]: we first define the statistics

$$\begin{aligned} Z_{IJ} &\propto \left|f^3\right|\tilde{s}_I^*(f)\tilde{s}_J(f) \\ &= \sum_M \Omega_{\rm GW}^M\gamma_{IJ}^M(f) + \text{ noise }. \end{aligned} \tag{10}$$

By averaging it, one gets the matrix equation

$$\langle Z_{IJ}\rangle = \sum_M \Omega_{\rm GW}^M\gamma_{IJ}^M(f) \tag{11}$$

or shortly $\mathbf{Z}=\mathbf{\Pi}\,\mathbf{\Omega}$. By inverting so-called correlation matrix $\mathbf{\Pi}$ containing the ORF (as long as $\det(\mathbf{\Pi})\neq 0$), one can finally find the mode densities $\mathbf{\Omega}$.

This method thus gives the detection threshold for each mode and is valid for a network of independant detectors in space, i.e. several clusters (for instance 2 independant LISA-like clusters, or 4 clusters such as in the DECIGO project [15]), in the low frequency limit and for a full polarized GW background. Moreover, it allows to extract from a signal the energy density of each mode.

2.2 Single detector

A study of the sensitivity of LISA (i.e. a cluster with 3 arms) to additional polarization modes has been performed [9] for an earlier version of the project, and the sensitivity curves for each mode as well as various TDI simply need to be updated according to the new proposed design. However, it is worth investigating a minimal version consisting of a cluster with only 2 arms (this situation could arise in case of a technical problem). Such an alternative solution requires the use of the so-called autocorrelation method [14]. As previously, we can write the output data of the detector as $h(t) + n(t)$, with $n(t)$ the noise and $h(t)$ the GW signal. In that case, the autocorrelation of the signal reads

$$\langle \tilde{h}(f)\tilde{h}^*(f')\rangle = \frac{1}{2}\delta(f - f')S_h(|f|), \tag{12}$$

and similarly for the noise density P_n:

$$\langle \tilde{n}(f)\tilde{n}^*(f')\rangle = \frac{1}{2}\delta(f - f')P_n(|f|), \tag{13}$$

and in that case, it is possible to find the optimal SNR to detect a signal as

$$SNR = \left(\frac{T_{\rm obs}}{2}\int_{-\infty}^{\infty} df \; \frac{S_h(|f|)^2}{[S_h(|f|) + P_n(|f|)]^2}\right)^{1/2}. \tag{14}$$

This method applies to a single-detector and is valid in the high-frequency limit. Since this analysis did not assume a particular polarization content, it only gives a detection threshold for a GW signal, but it will be necessary to generalize it in order to take into account all the possible modes, similarly to the network analysis.

3 Conclusion

So far, the current analysis method for a network of space-borne detectors requires the use of multiple LISA-like clusters and presents the advantage of considering a general polarization of the signal. A study was already performed for the use of a 3-arm detector and simply needs to be adapated to the most recent LISA design. One can also focus on a single 2-arm detector, but this method does not yet address the polarization of the signal and needs to be generalized in order to fully treat the polarization content; one especially needs to take the correlation of the noise into account.

With the proposed LISA design as well as future possible projects of space-borne inteferometers, it is therefore necessary to investigate both methods in order to set limits on the detectability of each polarization mode potentially present in a SGWB.

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