

Weak charges in $SU(5)_L \times U(1)_Y$ gauge models

Adrian Palcu*

Aurel Vlaicu University of Arad, 2 Elena Drăgoi Street, 310330 - Arad, Romania

*E-mail: a_palcu@yahoo.com

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Within the framework of a renormalizable $SU(5)_L \times U(1)_Y$ electro-weak gauge model with no exotic electric charges, we obtain all the neutral weak charge operators and their quantization, once the diagonalization of the neutral boson mass matrix is properly performed. Our results open up the path to a rich and promising phenomenological outcome. All the Standard Model phenomenology is recovered by simply decoupling the latter's scale ($v_{\text{SM}} = 246$ GeV) from the higher scale ($V \sim 10$ TeV) specific to our new electro-weak unification.

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Subject Index B00, B40, B59

1. Introduction

In a recent paper [1] the author proposed an original $SU(5)_L \times U(1)_Y$ gauge symmetry for the electro-weak unification, assuming that the Standard Model (SM) [2–4] (based on the $SU(2)_L \times U(1)_Y$ electro-weak group) must obviously be somehow extended in order to properly address some experimental challenges such as the dark matter puzzle, the neutrino oscillation phenomenon (implying tiny, but massive, neutrinos), the particular electric charge quantization observed in nature, the number of precisely three fermion generations, the recently reported muon $g - 2$ discrepancies, etc. The present letter simply aims at obtaining the neutral charges, computed as the couplings of the fermion fields to specific vector gauge bosons that mediate the weak interactions in the abovementioned $SU(5)_L \times U(1)_Y$ model. Consequently, we argue that the outcome is viable from a phenomenological standpoint. Our approach relies on the method for treating generalized $SU(n)_L \times U(1)_Y$ gauge models with spontaneous symmetry breaking (SSB) conceived some years ago by Cotăescu [5] and recently developed further by the author [6].

This letter is organized into five sections, each dealing with some particular aspects of the extended SM to the $SU(3)_c \times SU(5)_L \times U(1)_Y$ gauge group (in short, the 3-5-1 model). In Sect. 2 we briefly review the particle content of the model under consideration here (lepton and quark irreducible representations), the gauge fields (with a focus on the interactions mediated by the four neutral vector bosons Z, Z', Z'', Z'''), and the scalar sector (responsible, via a specific Higgs mechanism, for the SSB). Section 3 deals properly with the mass matrix of the abovementioned four neutral vector bosons and its diagonalization, which finally supplies, by employing the appropriate $\omega \in SO(4)$ matrix, the corresponding neutral charge operators. In Sect. 4 these operators are computed in detail for all fermion representations, and hence their precise quantization is obtained. Section 5 is reserved for some concluding remarks.

2. $SU(5)_L \times U(1)_Y$ gauge model

The particle content of the 3-5-1 model at hand is displayed below. There are three left-handed generations of leptons and quarks (see Ref. [1]), occurring in the following distinct left-handed quintuplets:

$$L_{iL} = \begin{pmatrix} N''_i \\ N'_i \\ N_i \\ \nu_i \\ e_i \end{pmatrix}_L, \quad Q_{1L} = \begin{pmatrix} U'' \\ U' \\ U \\ u \\ d \end{pmatrix}_L, \quad Q_{2L,3L} = \begin{pmatrix} D''_{2,3} \\ D'_{2,3} \\ D_{2,3} \\ d_{2,3} \\ u_{2,3} \end{pmatrix}_L, \quad (1)$$

with $i = 1, 2, 3$, and the corresponding right-handed singlet partners.

Their irreducible representations with respect to the model's whole gauge group $SU(3)_c \times SU(5)_L \times U(1)_Y$ are summarized below:

$$L_{iL} \sim \left(\mathbf{1}, \mathbf{5}, -\frac{1}{5} \right), \quad e_{iR} \sim (\mathbf{1}, \mathbf{1}, -1), \quad \nu_{iR}, N_{iR}, N'_{iR}, N''_{iR} \sim (\mathbf{1}, \mathbf{1}, 0), \quad (2)$$

$$Q_{1L} \sim \left(\mathbf{3}, \mathbf{5}, \frac{7}{15} \right), \quad Q_{kL} \sim \left(\mathbf{3}, \mathbf{5}^*, -\frac{2}{15} \right), \quad (3)$$

$$u_{kR}, u_R, U_R, U'_R, U''_R \sim \left(\mathbf{3}, \mathbf{1}, \frac{2}{3} \right), \quad d_R, d_{kR}, D_{kR}, D'_{kR}, D''_{kR} \sim \left(\mathbf{3}, \mathbf{1}, -\frac{1}{3} \right), \quad (4)$$

with $k = 2, 3$.

These assignments are not arbitrary at all, but inferred [1,6] by imposing the renormalization criteria that require all the axial anomalies to be canceled. The general method [5] was proved to predict, when this strict requirement is fulfilled, (i) precisely three fermion generations [6] (if the number of colors in the $SU(3)_c$ of the quantum chromodynamics is kept, as usual, to three), and (ii) the electric charge quantization [6] observed in nature. This seems to be a common result of many sorts of SM extensions emerging at not very high scales above the TeV threshold. For example, $SU(3)_L \times U(1)_Y$ gauge models with [7–12] or without [13–26] exotic electric charges, or (more recently) those based on the $SU(4)_L \times U(1)_Y$ gauge group [27–51] have been considered as plausible scenarios for particle physics. The richer phenomenology all these models exhibit and can predict has been extensively discussed in the literature. Yet, to the best of our knowledge, there has still been no attempt to properly address the larger possible [6] SM extension, namely the one based on the electro-weak $SU(5)_L \times U(1)_Y$ gauge group, except for a short letter [52] considering only a very particular such model involving exotic electric charges. As we have already stated, our approach is different and considers only the case with no exotic electric charges, in realistic connection to the experimental observations to date.

The electro-weak interactions in the model are mediated by the vector bosons supplied by the adjoint representation of the semi-simple gauge group employed above, namely

$$A_\mu = \begin{pmatrix} D_\mu^1 & Y_\mu^{''0} & Y_\mu^{''0} & Y_\mu^0 & Y_\mu^{''+} \\ Y_\mu^{''0*} & D_\mu^2 & X_\mu^{''0} & X_\mu^0 & Y_\mu^{'+} \\ Y_\mu^{''0*} & X_\mu^{''0*} & D_\mu^3 & X_\mu^0 & Y_\mu^+ \\ Y_\mu^{0*} & X_\mu^{''0*} & X_\mu^{0*} & D_\mu^4 & W_\mu^+ \\ Y_\mu^{''-} & Y_\mu^{'-} & Y_\mu^- & W_\mu^- & D_\mu^5 \end{pmatrix}, \quad (5)$$

with the diagonal entries (corresponding to the Cartan subalgebra of the $su(5)_L \times u(1)_Y$ algebra) considered in order as:

$$\begin{aligned} D_\mu^1 &= \frac{1}{2}A_\mu^3 + \frac{1}{2\sqrt{3}}A_\mu^8 + \frac{1}{2\sqrt{6}}A_\mu^{15} + \frac{1}{2\sqrt{10}}A_\mu^{24} + YB_\mu^0, \\ D_\mu^2 &= -\frac{1}{2}A_\mu^3 + \frac{1}{2\sqrt{3}}A_\mu^8 + \frac{1}{2\sqrt{6}}A_\mu^{15} + \frac{1}{2\sqrt{10}}A_\mu^{24} + YB_\mu^0, \\ D_\mu^3 &= -\frac{1}{\sqrt{3}}A_\mu^8 + \frac{1}{2\sqrt{6}}A_\mu^{15} + \frac{1}{2\sqrt{10}}A_\mu^{24} + YB_\mu^0, \\ D_\mu^4 &= -\frac{3}{2\sqrt{6}}A_\mu^{15} + \frac{1}{2\sqrt{10}}A_\mu^{24} + YB_\mu^0, \\ D_\mu^5 &= -\frac{2}{\sqrt{10}}A_\mu^{24} + YB_\mu^0. \end{aligned} \quad (6)$$

The off-diagonal entries can be put as $B_\mu^{\alpha\beta} = \frac{1}{\sqrt{2}}(A_\mu^\alpha \pm iA_\mu^\beta)$ with $\alpha, \beta = 1, 2, 3, 4, 5, \alpha \neq \beta$. Obviously, the off-diagonal entries correspond either to charged bosons (if $\alpha = 5$ or $\beta = 5$), or to neutral bosons (if simultaneously $\alpha \neq 5$ and $\beta \neq 5$). That means there are no exotic electric charges allowed by this model, since all the gauge bosons exhibit only 0, $\pm e$ charges.

SSB is achieved by means of an appropriate scalar sector consisting of the five scalar quintuplets

$$\phi^{(k)} = \begin{pmatrix} \phi_1^{(k)} \\ \phi_2^{(k)} \\ \phi_3^{(k)} \\ \phi_4^{(k)} \\ \phi_5^{(k)} \end{pmatrix} \sim \left(\mathbf{1}, \mathbf{5}, -\frac{1}{5} \right), \quad k = 1, \dots, 4, \quad \phi^{(5)} = \begin{pmatrix} \phi_1^{(5)} \\ \phi_2^{(5)} \\ \phi_3^{(5)} \\ \phi_4^{(5)} \\ \phi_5^{(5)} \end{pmatrix} \sim \left(\mathbf{1}, \mathbf{5}, \frac{4}{5} \right), \quad (7)$$

developing for each of them its own vacuum expectation value (VEV), in the manner $\langle \phi^{(i)} \rangle = \eta_i V$, due to a set of real parameters ($\eta_i \in (0, 1)$) once a unique overall scale V in the model is assumed. The parameters can be grouped into a 5×5 diagonal matrix (η) whose entries obey (according to the general method [5]) a restrictive trace condition $\text{Tr}(\eta^2) = 1$, i.e. $\eta_1^2 + \eta_2^2 + \eta_3^2 + \eta_4^2 + \eta_5^2 = 1$, so that the relation among all five VEVs $\langle \phi^{(1)} \rangle^2 + \langle \phi^{(2)} \rangle^2 + \langle \phi^{(3)} \rangle^2 + \langle \phi^{(4)} \rangle^2 + \langle \phi^{(5)} \rangle^2 = V^2$ holds. For our purpose we employ [1] the parameter matrix

$$\eta^2 = \text{Diag} \left(\frac{1-a}{3}, \frac{1-a}{3}, \frac{1-a}{3}, \frac{a-b}{2}, \frac{a+b}{2} \right), \quad (8)$$

which fulfills the trace requirement. It also splits the VEVs, as one can tune $a, b \rightarrow 0$ (very small) so that $\langle \phi^{(1)} \rangle, \langle \phi^{(2)} \rangle, \langle \phi^{(3)} \rangle \sim V$ and $\langle \phi^{(4)} \rangle, \langle \phi^{(5)} \rangle \sim v_{\text{SM}}$.

The detailed procedure and the resulting Higgs spectrum are presented in Ref. [1, Appendix B].

3. Boson mass spectrum

According to the general method [5], with the above parameter choice one gets (once the SSB is achieved) the following mass matrix for the Hermitian bosons that mediate the weak interactions:

$$M^2 = \begin{pmatrix} M^2(Z''') & 0 & 0 \\ 0 & M^2(Z'') & 0 \\ 0 & 0 & M_{2 \times 2}^2(Z, Z') \end{pmatrix}. \quad (9)$$

As expected, two of the heavier bosons are completely decoupled; only one of them (Z') mixes with the SM neutral Z boson. So, only the 2×2 mass matrix

$$M_{2 \times 2}^2(Z, Z') = \frac{m^2}{2} \begin{pmatrix} \frac{1}{3} \left(1 + \frac{7}{2}a - \frac{9}{2}b\right) & \frac{1}{\sqrt{15}\cos\theta} \left(1 - \frac{5}{2}a + \frac{3}{2}b\right) \\ \frac{1}{\sqrt{15}\cos\theta} \left(1 - \frac{5}{2}a + \frac{3}{2}b\right) & \frac{1}{5\cos^2\theta} \left(1 + \frac{15}{2}a + \frac{15}{2}b\right) \end{pmatrix} \quad (10)$$

actually goes through the diagonalization procedure. Here, θ stands for the rotation angle of a generalized Weinberg transformation (see Ref. [5, Sect. 5]) that separates the massless electromagnetic direction in the parameter space. It is connected to the SM Weinberg angle (θ_W) in our particular 3-5-1 model [1] in the manner $\sin\theta = 2\sqrt{\frac{2}{5}}\sin\theta_W$.

Now, one has to precisely enforce the mass $m\sqrt{a}/\cos\theta_W$ of the SM neutral boson Z ($\simeq 91.2$ GeV [53]) as an eigenvalue of the matrix in Eq. (10). In our parametrization, $M(W^\pm) = m\sqrt{a}$ ($\simeq 80.4$ GeV [53]), with the notation $m^2 = \frac{1}{4}g^2V^2$ used throughout the proceedings.

Hence, one gets a restriction [1] on the two free parameters a and b , namely

$$(a \tan^2\theta_W + b)^2 = 0. \quad (11)$$

Under these circumstances the boson mass matrix $M_{2 \times 2}^2(Z, Z')$ to be diagonalized becomes the one-parameter matrix

$$\frac{m^2}{2} \begin{pmatrix} \frac{1}{3} \left[1 + a \frac{(7+2\sin^2\theta_W)}{2(1-\sin^2\theta_W)}\right] & \frac{1}{\sqrt{3(5-8\sin^2\theta_W)}} \left[1 - a \frac{(5-2\sin^2\theta_W)}{2(1-\sin^2\theta_W)}\right] \\ \frac{1}{\sqrt{3(5-8\sin^2\theta_W)}} \left[1 - a \frac{(5-2\sin^2\theta_W)}{2(1-\sin^2\theta_W)}\right] & \frac{1}{5-8\sin^2\theta_W} \left[1 + a \frac{(15-30\sin^2\theta_W)}{2(1-\sin^2\theta_W)}\right] \end{pmatrix}. \quad (12)$$

The diagonalization of the matrix in Eq. (9) is performed simply by employing the $SO(4)$ matrix

$$\omega = \begin{bmatrix} 1_{2 \times 2} & 0 \\ 0 & \frac{1}{2\sqrt{2}\cos\theta_W} \begin{pmatrix} -\sqrt{3} & \sqrt{5-8\sin^2\theta_W} \\ -\sqrt{5-8\sin^2\theta_W} & -\sqrt{3} \end{pmatrix} \end{bmatrix} \quad (13)$$

in the manner

$$\omega M^2 \omega^T = \text{Diag}[M^2(Z'''), M^2(Z''), M^2(Z), M^2(Z')]. \quad (14)$$

The three new eigenvalues, specific to our 3-5-1 model, are now in order:

$$M^2(Z') = m^2 \left[\frac{4}{3} \left(\frac{1 - \sin^2\theta_W}{5 - 8\sin^2\theta_W} \right) + \frac{a(5 - 10\sin^2\theta_W - 4\sin^4\theta_W)}{3(5 - 8\sin^2\theta_W)(1 - \sin^2\theta_W)} \right], \quad (15)$$

$$M^2(Z'') = M^2(Z''') = m^2 \left(\frac{2}{3} \right) (1 - a), \quad (16)$$

all of them much heavier than Z , as shown in Ref. [1] if one considers the parameter a very small, say with order of magnitude $\underline{O}(10^{-3})$ or smaller. Such a tuning ensures an overall scale V of around 10 TeV or higher, according to $v_{\text{SM}} = \sqrt{a}V$.

4. Weak charges

Once we have identified the ω matrix, all the weak charge operators $Q^\rho(Z_i)$ can be computed, according to the prescriptions of the general method in Ref. [5], as:

$$Q^\rho(Z_i) = g \left[D_{\hat{k}}^\rho - v_{\hat{k}} (D^e v) (1 - \cos\theta) - v_{\hat{k}} \frac{g'}{g} Y^\rho \sin\theta \right] \omega_{\hat{i}}^{\hat{k}}, \quad (17)$$

where the versors $v_{\hat{k}}$ are associated to the Hermitian diagonal generators of the gauge group. As proved in Ref. [1], in order to avoid exotic electric charges one must select for the model at

hand $v_{n^2-1} = v_{24} = 1$ and simultaneously impose the vanishing of the other three versors $v_3 = v_8 = v_{15} = 0$. In a way, one can say that these versors properly discriminate among the various models based on the same gauge group.

Thus, in our particular 3-5-1 model, the neutral charge operators become

$$Q^\rho(Z) = \frac{e}{s_W c_W} \left[-\sqrt{\frac{3}{8}} T_{15}^\rho + \sqrt{\frac{5-8s_W^2}{8}} \left(T_{24}^\rho \sqrt{\frac{5-8s_W^2}{5}} - \frac{2\sqrt{2}s_W^2}{\sqrt{5-8s_W^2}} Y^\rho \right) \right], \quad (18)$$

$$Q^\rho(Z') = \frac{e}{s_W c_W} \left[-\sqrt{\frac{5-8s_W^2}{8}} T_{15}^\rho - \sqrt{\frac{3}{8}} \left(T_{24}^\rho \sqrt{\frac{5-8s_W^2}{5}} - \frac{2\sqrt{2}s_W^2}{\sqrt{5-8s_W^2}} Y^\rho \right) \right], \quad (19)$$

$$Q^\rho(Z'') = g T_8^\rho = \frac{e}{2\sqrt{3}s_W} \text{Diag}(1, 1, -2, 0, 0), \quad (20)$$

$$Q^\rho(Z''') = g T_3^\rho = \frac{e}{2s_W} \text{Diag}(1, -1, 0, 0, 0), \quad (21)$$

where we made use of the notations $s_W = \sin \theta_W$ and $c_W = \cos \theta_W$, along with the identification $e = g \sin \theta_W$ (once we established that the coupling g of $SU(5)_L$ is identical to the g of $SU(2)_L$ in the SM).

Now, with a little algebra, the neutral charges are inferred straightforwardly for all the irreducible representations in our model. We opt to express these charges, as usual, in $e/2s_W c_W$ units in order to easily compare them to the well-known SM-predicted values.

The resulting couplings with the SM neutral vector boson Z are

$$Q^{(5, -\frac{1}{3})}(Z) = \begin{pmatrix} 0 & & & & \\ & 0 & & & \\ & & 0 & & \\ & & & 1 & \\ & & & & -1 + 2s_W^2 \end{pmatrix} \quad (22)$$

for the lepton sector, and

$$Q^{(5, \frac{7}{15})}(Z) = \begin{pmatrix} -\frac{4s_W^2}{3} & & & & \\ & -\frac{4s_W^2}{3} & & & \\ & & -\frac{4s_W^2}{3} & & \\ & & & 1 - \frac{4s_W^2}{3} & \\ & & & & -1 + \frac{2s_W^2}{3} \end{pmatrix}, \quad (23)$$

$$Q^{(5^*, -\frac{2}{15})}(Z) = \begin{pmatrix} \frac{2s_W^2}{3} & & & & \\ & \frac{2s_W^2}{3} & & & \\ & & \frac{2s_W^2}{3} & & \\ & & & -1 + \frac{2s_W^2}{3} & \\ & & & & 1 - \frac{4s_W^2}{3} \end{pmatrix} \quad (24)$$

for the quark sector.

Table 1. Couplings of the SM fermions.

Couplings $\left(\times \frac{e}{2s_W c_W}\right)$	Z	Z'	Z''	Z'''
e_L, μ_L, τ_L	$-1 + 2s_W^2$	$\frac{(1-2s_W^2)\sqrt{3}}{\sqrt{5-8s_W^2}}$	0	0
$\nu_{eL}, \nu_{\mu L}, \nu_{\tau L}$	1	$\frac{(1-2s_W^2)\sqrt{3}}{\sqrt{5-8s_W^2}}$	0	0
e_R, μ_R, τ_R	$2s_W^2$	$\frac{-2\sqrt{3}s_W^2}{\sqrt{5-8s_W^2}}$	0	0
$\nu_{eR}, \nu_{\mu R}, \nu_{\tau R}$	0	0	0	0
u_L, c_L	$1 - \frac{4}{3}s_W^2$	$\frac{(-1+\frac{4}{3}s_W^2)\sqrt{3}}{\sqrt{5-8s_W^2}}$	0	0
t_L	$1 - \frac{4}{3}s_W^2$	$\frac{(1-\frac{2}{3}s_W^2)\sqrt{3}}{\sqrt{5-8s_W^2}}$	0	0
d_L, s_L	$-1 + \frac{2}{3}s_W^2$	$\frac{(-1+\frac{4}{3}s_W^2)\sqrt{3}}{\sqrt{5-8s_W^2}}$	0	0
b_L	$-1 + \frac{2}{3}s_W^2$	$\frac{(1-\frac{2}{3}s_W^2)\sqrt{3}}{\sqrt{5-8s_W^2}}$	0	0
u_R, c_R, t_R	$-\frac{4}{3}s_W^2$	$\frac{4s_W^2}{\sqrt{3}\sqrt{5-8s_W^2}}$	0	0
d_R, s_R, b_R	$\frac{2}{3}s_W^2$	$-\frac{2s_W^2}{\sqrt{3}\sqrt{5-8s_W^2}}$	0	0

For the Z' neutral vector boson, the couplings are, up to a factor $\frac{\sqrt{3}}{\sqrt{5-8s_W^2}}$,

$$Q^{(5, -\frac{1}{5})}(Z') = \begin{pmatrix} -\frac{2c_W^2}{3} & & & & \\ & -\frac{2c_W^2}{3} & & & \\ & & -\frac{2c_W^2}{3} & & \\ & & & 1 - 2s_W^2 & \\ & & & & 1 - 2s_W^2 \end{pmatrix} \quad (25)$$

for the lepton sector, and

$$Q^{(5, \frac{7}{15})}(Z') = \begin{pmatrix} -\frac{2(1-3s_W^2)}{3} & & & & \\ & -\frac{2(1-3s_W^2)}{3} & & & \\ & & -\frac{2(1-3s_W^2)}{3} & & \\ & & & 1 - \frac{2s_W^2}{3} & \\ & & & & 1 - \frac{2s_W^2}{3} \end{pmatrix}, \quad (26)$$

$$Q^{(5^*, -\frac{2}{15})}(Z') = \begin{pmatrix} \frac{2(1-2s_W^2)}{3} & & & & \\ & \frac{2(1-2s_W^2)}{3} & & & \\ & & \frac{2(1-2s_W^2)}{3} & & \\ & & & -1 + \frac{4s_W^2}{3} & \\ & & & & -1 + \frac{4s_W^2}{3} \end{pmatrix} \quad (27)$$

for the quark sector.

The results for the neutral charges of the SM fermions are summarized in Table 1. It is now evident that the couplings connecting any SM fermion to the neutral SM vector boson (Z) are utterly recovered, meaning that the SM is not altered at all at tree level. At the same time, the heavier Z'' and Z''' , being completely decoupled, exhibit no interactions with the SM fermions.

Table 2. Couplings of the non-SM fermions.

Couplings ($\times \frac{e}{2s_W c_W}$)	Z	Z'	Z''	Z'''
$N_{eL}, N_{\mu L}, N_{\tau L}$	0	$\frac{2c_W^2}{\sqrt{3}\sqrt{5-8s_W^2}}$	$-\frac{2c_W}{\sqrt{3}}$	0
$N'_{eL}, N'_{\mu L}, N'_{\tau L}$	0	$\frac{2c_W^2}{\sqrt{3}\sqrt{5-8s_W^2}}$	$\frac{c_W}{\sqrt{3}}$	$-c_W$
$N''_{eL}, N''_{\mu L}, N''_{\tau L}$	0	$\frac{2c_W^2}{\sqrt{3}\sqrt{5-8s_W^2}}$	$\frac{c_W}{\sqrt{3}}$	c_W
$N_{eR}, N_{\mu R}, N_{\tau R}$	0	0	0	0
$N'_{eR}, N'_{\mu R}, N'_{\tau R}$	0	0	0	0
$N''_{eR}, N''_{\mu R}, N''_{\tau R}$	0	0	0	0
U_{1L}	$-\frac{4}{3}s_W^2$	$-\frac{2(1-3s_W^2)}{\sqrt{3}\sqrt{5-8s_W^2}}$	$-\frac{2c_W}{\sqrt{3}}$	0
D_{2L}, D_{3L}	$\frac{2}{3}s_W^2$	$\frac{2(1-2s_W^2)}{\sqrt{3}\sqrt{5-8s_W^2}}$	$\frac{2c_W}{\sqrt{3}}$	0
U'_{1L}	$-\frac{4}{3}s_W^2$	$-\frac{2(1-3s_W^2)}{\sqrt{3}\sqrt{5-8s_W^2}}$	$\frac{c_W}{\sqrt{3}}$	$-c_W$
D'_{2L}, D'_{3L}	$\frac{2}{3}s_W^2$	$\frac{2(1-2s_W^2)}{\sqrt{3}\sqrt{5-8s_W^2}}$	$-\frac{c_W}{\sqrt{3}}$	c_W
U''_{1L}	$-\frac{4}{3}s_W^2$	$-\frac{2(1-3s_W^2)}{\sqrt{3}\sqrt{5-8s_W^2}}$	$\frac{c_W}{\sqrt{3}}$	c_W
D''_{2L}, D''_{3L}	$\frac{2}{3}s_W^2$	$\frac{2(1-2s_W^2)}{\sqrt{3}\sqrt{5-8s_W^2}}$	$-\frac{c_W}{\sqrt{3}}$	$-c_W$
$U_{1R}, U'_{1R}, U''_{1R}$	$-\frac{4}{3}s_W^2$	$\frac{4s_W^2}{\sqrt{3}\sqrt{5-8s_W^2}}$	0	0
$D_{2R}, D_{3R}, D'_{2R}, D'_{3R}, D''_{2R}, D''_{3R}$	$\frac{2}{3}s_W^2$	$-\frac{2s_W^2}{\sqrt{3}\sqrt{5-8s_W^2}}$	0	0

This means that the two bosons do not interfere with the established SM phenomenology. Only Z' could eventually somehow influence the SM phenomenology. Therefore, it deserves distinct consideration in which the necessary corrections are properly performed in order to provide us with some restrictions regarding the parameters of this model.

For the the heavier Z'' and Z''' neutral vector bosons the computation is much simpler, since their couplings are connected only to their associated diagonal generators. That is, only T_3 accounts for Z'' couplings and T_8 accounts for Z''' couplings, with no admixture at all. The resulting values are summarized in Table 2, once the explicit expressions are computed in the following.

In the case of the Z'' , the couplings are inferred as

$$Q^{(\mathbf{5}, -\frac{1}{5})}(Z'') = \frac{c_W}{\sqrt{3}} \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & -2 & \\ & & & 0 \\ & & & & 0 \end{pmatrix} \quad (28)$$

for the lepton sector, and

$$Q^{(\mathbf{5}, \frac{7}{15})}(Z'') = \frac{c_W}{\sqrt{3}} \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & -2 & \\ & & & 0 \\ & & & & 0 \end{pmatrix}, \quad (29)$$

$$Q^{(\mathbf{5}^*, -\frac{2}{15})}(Z'') = \frac{c_W}{\sqrt{3}} \begin{pmatrix} -1 & & & \\ & -1 & & \\ & & 2 & \\ & & & 0 \\ & & & & 0 \end{pmatrix} \quad (30)$$

for the quark sector.

The couplings for Z'' yield

$$Q^{(\mathbf{5}, -\frac{1}{5})}(Z''') = c_W \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & 0 & \\ & & & 0 \\ & & & & 0 \end{pmatrix} \quad (31)$$

for the lepton sector, and

$$Q^{(\mathbf{5}, \frac{7}{15})}(Z''') = c_W \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & 0 & \\ & & & 0 \\ & & & & 0 \end{pmatrix}, \quad (32)$$

$$Q^{(\mathbf{5}^*, -\frac{2}{15})}(Z''') = c_W \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 0 & \\ & & & 0 \\ & & & & 0 \end{pmatrix} \quad (33)$$

for the quark sector.

Let's turn now to the fermion singlets. Their weak charges are computed in a simpler way, since one takes into account only Y and none of the T generators. Hence, there are no interactions at all for the right-handed singlets with the heavier Z'' and Z''' . At the same time, as expected, all the neutral right-handed fermions have no weak interactions regardless of the SM or non-SM bosons.

For the sake of completeness we display below the weak interactions of the right-handed representations explicitly. In the case of charged leptons we get the weak couplings

$$Q^{(\mathbf{1}, -1)}(Z) = 2s_W^2, \quad Q^{(\mathbf{1}, -1)}(Z') = -\frac{2\sqrt{3}s_W^2}{\sqrt{5-8s_W^2}}, \quad (34)$$

while all kinds of right-handed neutrinos are sterile,

$$Q^{(\mathbf{1}, 0)}(Z) = 0, \quad Q^{(\mathbf{1}, 0)}(Z') = 0, \quad (35)$$

as expected.

In the quark sector, the right-handed up-type quark couplings yield

$$Q^{(\mathbf{1}, 2/3)}(Z) = -\frac{4s_W^2}{3}, \quad Q^{(\mathbf{1}, 2/3)}(Z') = \frac{4s_W^2}{\sqrt{3(5-8s_W^2)}}, \quad (36)$$

while the right-handed down-type quarks interact weakly in the manner

$$Q^{(1,-1/3)}(Z) = \frac{2s_W^2}{3}, \quad Q^{(1,-1/3)}(Z') = -\frac{2s_W^2}{\sqrt{3(5-8s_W^2)}}. \quad (37)$$

5. Concluding remarks

As one can easily observe from the results derived and presented above, the SM fermions preserve their couplings predicted by the SM. Moreover, they have no couplings at all with the new bosons Z'' and Z''' , but only with Z' (≥ 5.1 TeV [50]), whose influence (due to the mixing with Z in the diagonalization procedure) can (in a future work) be estimated. If worked out properly such corrections could enforce restrictions on the scale of the model and other parameters as well (for instance, the appropriate Yukawa couplings in the neutrino or quark sectors). At the same time, a particular feature of our 3-5-1 model is that the SM Z boson has vector interactions with all heavier fermions (other than SM fermions), while Z' makes no distinction on the electric charge basis when it comes to the left-handed fermions in the same doublet from the SM. At the same time, this kind of model supplies a plethora of suitable candidates for cold dark matter, since a lot of neutral fermions, neutral scalars, and even neutral vector bosons are not interacting with ordinary matter particles, as the new scale V is, properly speaking, completely decoupled from the SM's scale. Thus, such elusive particles could naturally have escaped any observation until now, and their relic density can in principle be estimated.

Having said all this, we consider that the particular 3-5-1 electro-weak unification with no exotic charges is a viable SM extension candidate, so that it truly deserves attention and further study for deepening its phenomenological investigation.

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References

- [1] A. Palcu, Prog. Theor. Exp. Phys. **2021**, 103B02 (2021).
- [2] S. Weinberg, Phys. Rev. Lett. **19**, 1264 (1967).
- [3] S. Glashow, Nucl. Phys. **20**, 579 (1961).
- [4] A. Salam, in Elementary Particle Theory: Relativistic Groups and Analyticity (Nobel Symposium No. 8), ed. N. Svartholm Svartholm (Almqvist and Wiksell, Stockholm, 1968).
- [5] I. I. Cotăescu, Int. J. Mod. Phys. A **12**, 1483 (1997).
- [6] A. Palcu, J. Phys. G: Nucl. Part. Phys. **48**, 055003 (2021).
- [7] F. Pisano and V. Pleitez, Phys. Rev. D **46**, 410 (1992).
- [8] P. H. Frampton, Phys. Rev. Lett. **69**, 2889 (1992).
- [9] I. I. Cotăescu and A. Palcu, Mod. Phys. Lett. A **23**, 1011 (2008).
- [10] G. de Conto and V. Pleitez, Phys. Rev. D **91**, 015006 (2015).
- [11] G. de Conto and V. Pleitez, J. High Energy Phys. **1705**, 104 (2017).
- [12] A. C. B. Machado, J. Montano, and V. Pleitez, J. Phys. G: Nucl. Part. Phys. **46**, 115005 (2019).
- [13] H. N. Long, Phys. Rev. D **53**, 437 (1996).
- [14] H. N. Long, Phys. Rev. D **54**, 4691 (1996).
- [15] A. Palcu, Mod. Phys. Lett. A **23**, 387 (2008).
- [16] A. Palcu, Prog. Theor. Exp. Phys. **2013**, 0903B03 (2013).
- [17] A. J. Buras, F. de Fazio, and J. Girrbach-Noe, J. High Energy Phys. **1408**, 039 (2014).
- [18] F. S. Queiroz, C. Siqueira, and J. W. F. Valle, Phys. Lett. B **763**, 269 (2016).
- [19] A. J. Buras and F. de Fazio, J. High Energy Phys. **1603**, 010 (2016).
- [20] D. T. Huong and P. V. Dong, Eur. Phys. J. C **77**, 204 (2017).

- [21] M. Reig, J. W. F. Valle, and C. A. Vaquera-Araujo, J. High Energy Phys. **1705**, 100 (2017).
- [22] K. Huitu and N. Koivunen, Phys. Rev. D **98**, 011701R (2018).
- [23] K. Huitu and N. Koivunen, J. High Energy Phys. **1910**, 065 (2019).
- [24] C. A. de Sousa Pires et al., Phys. Lett. B **797**, 134827 (2019).
- [25] H. N. Long et al., Nucl. Phys. B **943**, 114629 (2019).
- [26] A. Palcu, Mod. Phys. Lett. A **35**, 2050083 (2020).
- [27] R. Foot, H. N. Long, and T. A. Tran, Phys. Rev. D **50**, R34 (1994).
- [28] L. A. Sanchez, F. A. Perez, and W. A. Ponce, Eur. Phys. J. C **35**, 259 (2004).
- [29] Fayyazuddin and Riazuddin, J. High Energy Phys. **0412**, 013 (2004).
- [30] W. A. Ponce, D. A. Gutierrez, and L. A. Sanchez, Phys. Rev. D **69**, 055007 (2004).
- [31] W. A. Ponce and L. A. Sanchez, Mod. Phys. Lett. A **22**, 435 (2007).
- [32] L. A. Sanchez, L. A. Wills-Toro, and J. I. Zuluaga, Phys. Rev. D **77**, 035008 (2008).
- [33] Riazuddin and Fayyazuddin, Eur. Phys. J. C **56**, 389 (2008).
- [34] A. Palcu, Mod. Phys. Lett. A **24**, 1247 (2009).
- [35] A. Palcu, Mod. Phys. Lett. A **24**, 1731 (2009).
- [36] J. L. Nisperuza and L. A. Sanchez, Phys. Rev. D **80**, 035003 (2009).
- [37] A. Palcu, Mod. Phys. Lett. A **24**, 2175 (2009).
- [38] A. Palcu, Mod. Phys. Lett. A **24**, 2589 (2009).
- [39] S. Villada and L. A. Sanchez, J. Phys. G: Nucl. Part. Phys. **36**, 115002 (2009).
- [40] A. Palcu, Int. J. Mod. Phys. A **24**, 4923 (2009).
- [41] A. Jaramillo and L. A. Sanchez, Phys. Rev. D **84**, 115001 (2011).
- [42] A. Palcu, Phys. Rev. D **85**, 113010 (2012).
- [43] J. M. Cabarcas and J.-Alexis Rodriguez, Mod. Phys. Lett. A **29**, 1450032 (2014).
- [44] A. G. Dias et al., Ann. Phys. **349**, 232 (2014).
- [45] D. Cogollo, Int. J. Mod. Phys. A **30**, 1550038 (2015).
- [46] D. Cogollo, Int. J. Mod. Phys. A **30**, 1550187 (2015).
- [47] H. N. Long, L. T. Hue, and D. V. Loi, Phys. Rev. D **94**, 015007 (2016).
- [48] G. Pallacio, Int. J. Mod. Phys. A **31**, 1650142 (2016).
- [49] A. Palcu, Int. J. Theor. Phys. **56**, 403 (2017).
- [50] D. T. Binh, V. V. On, and H. N. Long, Int. J. Mod. Phys. A **34**, 1950062 (2019).
- [51] D. Cogollo et al., Int. J. Mod. Phys. A **35**, 2050126 (2020).
- [52] P. H. Frampton et al., Phys. Lett. B **715**, 275 (2012).
- [53] P. A. Zyla et al. [Particle Data Group], Prog. Theor. Exp. Phys. **2020**, 083C01 (2020).