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Generation of Primordial Magnetic Fields from QED and Higgs-like Domain Walls in Einstein–Cartan Gravity

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Abstract: Spacetime torsion is known to be highly suppressed at the end of inflation, which is called preheating. This result was recently shown in (EPJ C (2022)) in the frame of Einstein–Cartan–Brans–Dicke inflation. In this paper, it is shown that a torsionful magnetogenesis in QED effective Lagrangean drives a torsion damping in order to be subsequently amplified by the dynamo effect after the generation of these magnetic fields seeds. This damping on amplification would depend upon the so-called torsion chirality. Here, a cosmic factor gkK is present where K is the contortion vector and k is the wave vector which is connected to the inverse of magnetic coherence length. In a second example, we find Higgs inlationary fields in Einstein–Cartan gravity thick domain walls (DWs). Recently, a modified Einstein–Cartan gravity was given by Shaposhnikov et al. [PRL (2020)] to obtain Higgs-like inflatons as a portal to dark energy. In the case of thick DW, we assume that there is a torsion squared influence, since we are in the early universe where torsion is not so weak as in the late universe as shown by Paul and SenGupta [EPJ C (2019)] in a 5D brane-world. A static DW solution is obtained when the inflationary potential vanishes and Higgs potential is a helical function. Recently, in the absence of inflation, domain wall dynamos were obtained in Einstein–Cartan gravity (EC) where the spins of the nucleons were orthogonal to the wall.

Keywords: modified gravity theories; magnetogenesis; QED and preheating; domain walls; Higgs inflation

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1. Introduction

Recently, Shaposhnikov et al. [1] have investigated a metric-affine theory, alternative to general relativity, which is a generalization of the Einstein–Cartan theory of gravity (EC) [2]. This EC extension contains other invariants, besides the Ricci scalar in Riemann–Cartan spacetime, called Nieh–Yan and Host topological invariants. This is due to the fact that in EC gravity, the Riemann–Cartan curvature tensor is totally asymmetric and violates parity. Earlier, the author has showed [3] how to obtain magnetic seed fields from the electromagnetic gravity field Lagrangean, where the term $\epsilon^{ijkl}R_{ijkl}$ is added to the action. In the Shaposhnikov et al. Einstein–Cartan gravity, this term is also added to the action and called a Holst term. Their action, which is more complete than usual EC, gives rise to Higgs inflation. The simple presence of a scalar gravity sector is not enough to guarantee that the scalar field is a Higgs field [4]. Therefore, here, we shall investigate the non-minimal coupling of EC gravity to QED and scalar gravity sectors, via non-minimal coupling, and investigate static domain walls (DWs) in the Riemann-flat curvature plus torsionful spacetime. There, the QED gravity sector is also obtained from non-minimal coupling with torsion. Previously, Bassett et al. [5] investigated the variation of the magnetic energy density compared to electromagnetic radiation in a photon fluid. This has been done in a GR magnetogenesis frame. They found a cosmic dynamo as a metric perturbation at preheating ending of inflation. Since torsion is highly suppressed at that time epoch of the universe, one is led to think that torsion might not contribute to the magnetogenesis process [6]. Nevertheless, in this paper, we show that in EC [7] magnetogenesis, the torsion-damping effect affects the self-induction equation with non-adiabatic helical magnetic fields. This is made by decreasing the magnetic energy density to values compatible to astronomical observations [8]. Since EC gravity and GR are equivalent in the absence of matter sector [9], we must use in this QED the fermionic matter sector or the scalar gravity as a source of torsion. In the GR case, a similar result with helical fields has been established by Schober et al. [10]. Present results are accomplished by considering a semi-minimal coupling quantum electrodynamics (QED) on a torsionful inflationary case. We assume that torsion does not couple to the electromagnetic field minimally, in the QED case, and no massive photon to break gauge invariance is produced [11]. Recently, Khotari et al. [12] considered modifications of the minimal coupling of electromagnetism with torsion [12] with spin-one fields of non-Abelian nature, driven by torsionful magnetogenesis. This paper contains the derivation of a torsionful cosmic self-induced equation, which was obtained by variation of the torsionful QED effective Lagrangean. The value of torsion in terms of a cosmic time dependent factor is substituted into this cosmic self-induced equation and solved. Its solution is shown to depend upon a parameter β written in terms of torsion coupling, electric resistivity and the wave vector k. This results in a magnetic field damping, which is driven by torsion coupling of the magnetic field energy density. Results agree with the observations of modern astronomy. Several types of dynamos have been presented in the literature [13,14]. For example, chiral dynamos instabilities [15] are endowed with torsion. To our knowledge, this is the first time a cosmic preheting dynamo in a torsionful helical magnetogenesis is found to be compatible with observations with the aid of torsionful QED. Non-adiabatic and superadiabatic magnetic fields can be found from the present solutions. In addition to primordial magnetic field seeds in the QED sector, we also find seed fields from the presence of magnetic fields external to a static DW with torsion. Torsion at the early universe, where DWs are present, is of the order of 1 MeV [16], and this reasonably big order of magnitude allows us to show that torsion depends upon the DW equations from Euler–Lagrange equations. The paper is organized as follows: In Section 2, we present the effective QED Lagrangean with semi-minimal coupling and found a differential equation for the magnetic vector potential. In Section 3, we obtain galactic dynamo seeds from magnetic self-induction equation. In Section 4, we present the derivation of the magnetic wave equation in the background of torsionful QED cosmology. Solutions are found which depend upon torsion electric resistivity coupling. In Section 5, we investigate the role of a torsion on a static DW with scalar inflation in EC gravity without Nieh–Yan inflation or Holst invariant terms in the action, as considered by Karananas et al. [4]. In Section 6, we derive the self-induction equations in comoving coordinates of a DW static metric. Section 7 is left to discussions and conclusions. It is important to note that to consider dynamos in the early universe, we would need turbuence in plasmas, and since we do not address this theme here, we only consider dynamos to be used to amplify pre-existing magnetic fields generated by QED and other particle physics methods. Very recently S Capozziello et al. [17] has obtained a comparison between the gravitational contraction and dynamo effects as competing effects to amplify the magnetic fields.

2. QED Effective Lagrangean and Semi-Minimal Torsion–Photon Coupling RF^2

Though the torsion effects are highly suppressed, in comparison with curvature ones in the Einstein gravity sector, we do not consider Minkowski space here. This is due to the fact that as can be easily shown in this paper, from field equations, that torsion vanishes in Minkowski space. From Maziteli and Spedallieri [18], the QED effective Lagrangean is

$$S = \frac{1}{m^2}\int d^4x(-g)^{\frac{1}{2}}(-\frac{1}{4}F^2 + (m^2 + \epsilon R)\phi\bar{\phi} - D_j\phi D^j\bar{\phi}) \tag{1}$$

Operator, $D_i = \partial_i - ieA_i$ is the covariant derivative for the scalar fields. Maziteli and Spedalieri [18] have computed an effective Lagrangean for the e.m field by integrating the quantum scalar field. Via dimensional regularisation, they obtained the effective Lagrangean

$$\mathcal{L}_{eff} = -\frac{1}{4}F^2 + \frac{1}{2}\frac{1}{4\pi^{\frac{d}{2}}}(\frac{m}{\mu})^{d-4}\sum a_j(x)m^{4-2j}\Gamma(j-\frac{d}{2}) \tag{2}$$

The first Schwinger–De Witt (SDW) coefficients from Spedelieri et al. work are

$$a_0 = 1 \tag{3}$$

$$a_1 = -(\epsilon - \frac{1}{6})R \tag{4}$$

$$a_2 = \frac{1}{180}(R_{ijkl}R^{ijkl} - R_{ij}R^{ij}) + \frac{1}{2}(\epsilon - \frac{1}{6})^2R^2 + \frac{1}{6}(\epsilon - \frac{1}{5})R - \frac{e^2}{12}F^2 \tag{5}$$

$$a_3 = ... + \frac{e^2}{60}R_{ijkl}F^{ij}F^{kl} - \frac{e^2}{90}R_{ij}F^{ik}F^{kl} + (\frac{1}{6} - \epsilon)RF^2 + ... \tag{6}$$

Here, we note that due to the use of semi-minimal coupling, where torsion is also our gravitational field, it appears only in a_2 as a first term. Since in the semi-minimal coupling torsion does not appear, in the covariant derivative, it consequently does not appear in the electromagnetic field. Accordingly, torsion appears only in the curvature for the first time in a_2. Moreover, from semi-minimal coupling, I shall consider the following effective Lagrangean in Riemann–Cartan spacetime

$$\mathcal{L}_{eff} = -\frac{1}{4}F^2(1 + \frac{b}{m^2}R) \tag{7}$$

where we have taken n = 1 such as in Widrow and Turner [19]. From this effective Lagrangean, we obtain the field equations for the Friedmann spatially flat metric

$$ds^2 = a^2(-d\eta^2 + dx^2) \tag{8}$$

as

$$\partial^i(F_{ij}(1 + \frac{bR}{m^2})) = 0 \tag{9}$$

Expanding this last equation, one obtains

$$\ddot{A}_k - \nabla^2 A_k + \frac{\dot{R}}{R}\dot{A}_k = 0 \tag{10}$$

where to obtain this equation, we use the gauges $A_0 = 0$ and $divA = 0$. Moreover, we assume here that in the Riemannian case inflationary epoch $R >>> m^2$, so this would reduce the last equation to

$$[\ddot{A}_k + \frac{\dot{R}}{R}\dot{A}_k] = 0 \tag{11}$$

where R is the Ricci scalar. This shows that although there is no inflation here, we consider that torsion has a similar behavior, so actually $\dot{K} >>> m^2$.

3. Galactic Dynamo Seeds in RF^2 Semi-Minimal Coupling

In this section, we shall solve Equation (11) in the case of curved spacetime with torsion, and performing the semi-minimal coupling where the Ricci scalar is approximated taken as $2\dot{K}$, where K is the time component K^0, which to simplify matters is the only homogeneous component of contortion, an algebraic combination of torsion. Here, we assume linearization of the Ricci–Cartan scalar

$$R = g_{ij}R^{ij} = R^* + 2\nabla_i K^i - K^2 \tag{12}$$

where $K^j = K^{rj}{}_r$ represents the trace of torsion tensor, R^* is the Riemannian Ricci scalar that shall be taken as a constant as in de Sitter or Einstein space. Let us now perform the variation of the Lagrangean density $\sqrt{g}\mathcal{L}$ with respect to the scale cosmological factor a and contortion K. This would complete the Einstein–Cartan–Maxwell equations systems with propagating torsion. This can be completed easily by computing the Euler–Lagrange equations

$$\frac{d}{dt}\frac{\partial\sqrt{g}\mathcal{L}}{\partial\dot{a}} - \frac{\partial\sqrt{g}\mathcal{L}}{\partial a} = 0 \tag{13}$$

$$\frac{d}{dt}\frac{\partial\sqrt{g}\mathcal{L}}{\partial\dot{K}} - \frac{\partial\sqrt{g}\mathcal{L}}{\partial K} = 0 \tag{14}$$

The last equation determines contortion K in terms of the scale factor a. This yields

$$K = -\frac{3\dot{a}}{a} \tag{15}$$

Before applying this result to the expression for the Ricci–Cartan scalar, let us express this scalar in terms of the scalar a and torsion K. Then, we are left with the following expression

$$R = g_{ij}R^{ij} = R^* + 2\dot{K} - K^2 + \partial_t ln\sqrt{g}K \tag{16}$$

or

$$R = g_{ij}R^{ij} = -6[\frac{\ddot{a}}{a} + (\frac{\dot{a}}{a})^2] + 2\dot{K} - K^2 + (\partial_t lna^3)K \tag{17}$$

which yields

$$\dot{R} = \dot{R^*} + \ddot{K} + (\frac{\dot{a}}{a} + 2K)\dot{K} + 3[\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2}]K \tag{18}$$

The expression for $\ddot{K}$ is

$$\ddot{K} = -3[\frac{\dddot{a}}{a} - 3\frac{\ddot{a}\dot{a}}{a^2} + (\frac{\dot{a}}{a})^3] \tag{19}$$

The expression for Ricci–Cartan scalar $\sqrt{g}R$ is

$$a^3R = -3[3\ddot{a}a^2 + 7\dot{a}^2a] \tag{20}$$

Substitution of this expression into the Euler–Lagrange equation above leads to

$$\ddot{a}a - 4\ddot{a}\dot{a} = 0 \tag{21}$$

By making use of the ansatz $a \sim t^n$, where n is a real number, one obtains the following algebraic equation

$$n(n-2) - 4n^2 = 0 \tag{22}$$

which yields immediatly $n = -\frac{2}{3}$, and $a \sim t^{-\frac{2}{3}}$, which represents a contracting phase of the cosmological model with torsion. Therefore, from the above expression for K, one obtains $K \sim a^{\frac{3}{2}}$. In terms of cosmic time, the contortion and cosmic factor is $K \sim -\frac{2}{3}t^{-1}$ and $a \sim t^{-\frac{2}{3}}$. This shows that in the contracting phase of the cosmological bouncing model, the contraction of the universe goes faster than torsion, whereas in the expansion inflationary factor, the torsion decays faster than cosmic expansion, showing that torsion is really highly suppressed by inflation. Now, to investigate how the magnetic field can be highly compressed in the contracting phase giving rise to a kind of dynamo action, one simply compute the ratio $\frac{\dot{R}}{R}$ as

$$\frac{\dot{R}}{R} = \frac{[3\ddot{K} - \frac{2}{3}(K^2)^{\cdot}]}{[3\dot{K} - \frac{2}{3}K^2]} \tag{23}$$

Being a very weak field at the preheating phase, we may approximate torsion in order that

$$\frac{\dot{R}}{R} \approx \frac{\ddot{K}}{\dot{K}} \tag{24}$$

This expression then yields

$$\frac{\dot{R}}{R} \approx 2t^{-1} \tag{25}$$

Therefore, substitution of this value into the Fourier transformed equation for A_k above one obtains

$$\ddot{A}_k + 2t^{-1}\dot{A}_k = 0 \tag{26}$$

solution of this differential equation is easy if we assume the ansatz $A_k = A^0{}_k t^n$ where n is an integer and 0-index denotes an initial value. The substitution of this ansatz in the expression (26) yields the algebraic characteristic equation

$$n(n-1) + 2n = 0 \tag{27}$$

which trivially yields $n = -1$; then, going back to the solution ansatz, one obtains

$$A_k \sim A_{k(0)} t^{-1} \tag{28}$$

The magnetic field $B_{seed} = ikA$ and $B_{seed} \sim B_G t^{-1}$ and $B_G = 10^{-6} G$, this implies that $B_{seed} \sim 10^{-24} Gauss$, which is strong enough to be able to seed the galactic dynamo [20].

4. Torsion Chirality QED Magnetogenesis

In this section, the derivation of a dynamo equation from the electromagnetic equations is given where torsion is introduced by the coupling of the new torsional covariant derivative: $\nabla_i = \partial_i + igK_i$ where K^i is the contortion vector assumed in previous sections. Here, g is the torsion coupling. One notices that the partial differentiation substitutes the usual Riemann covariant operator, since we have assumed here that torsion coupling is not present in principle in the definition of electromagnetic field 2-tensor F_{ij}. Hence, the Maxwell equations in a Friedmann torsionful universe are given by

$$\nabla \times (a^2 E) = \partial_t (a^2 B) \tag{29}$$

which is the modified Faraday equation in curved Riemann Friedmann spacetime, whereas the Ampere modified equation is

$$\nabla \times (a^2 B) = a^2 J \tag{30}$$

whereas the absence of a monopole equation is given by

$$\nabla . (a^2 B) = 0 \tag{31}$$

Since a cosmic factor depends only upon cosmic time t, this expression is equivalent to the regular Maxwell one $divB = 0$ given in Riemann-flat spacetime without torsion. By taking the coupling derivative above, we obtain

$$[\nabla + igK] \times (a^2 B) = a^2 J \tag{32}$$

and

$$\nabla \times [v \times (a^2 B)] + ig[K \times B] = \sigma[-\partial_t (a^2 B) + a^2 \nabla \times (v \times B] \tag{33}$$

where we have used in this computation the Ohm s law

$$J = \sigma E + v \times B \tag{34}$$

By taking the hypothesis of amagnetic helical dynamos where

$$\nabla \times B = \lambda B \tag{35}$$

and that the term which will appear in the expansion $\nabla.v = \alpha$ which would be connected with the mean field dynamo term or electromotive term αB and the Hubble constant $H = \frac{\dot{a}}{a}$ yields the equation of the cosmic dynamo in QED torsionful magnetogenesis as

$$[\partial_t B + 2\frac{\dot{a}}{a}B] = -[\eta\lambda^2 - g[a^2 K \times B] = J \tag{36}$$

where we have used in the divergence of velocity the term is $\nabla \rightarrow ik$. The special term $gk.K$ is the coupling of torsion with the chiral torsion helicity given by the scalar dot of the wave vector k and the contortion vector K. A similar situation has been also addressed by Karananas and Tsagas [21,22]. To further simplify this equation, one takes the ansatz for the magnetic field as $B = B_{seed} exp(\gamma t)$, where γ if positive is the amplification factor of the magnetic field, one obtains

$$\gamma = [g(k.K) - (2H + \lambda^2\eta + \alpha] \tag{37}$$

where $H = H_0$ in de Sitter expansion to simplify matters. Then, if $\gamma = g(k.K) - (2H + \lambda^2\eta + \alpha)$, this means that for the greater sign, we obtain the dynamo action, whereas the equal sign represents the saturation of the dynamo where the magnetic field remains the same without growing in time. Associated with a plus or minus sign, respectively, is either spin or torsion vector flips, and these are parallel or antiparallel. Here, the electric resistivity is $\eta = \sigma^{-1}$, where σ is the electric conductivity parameter. However, Equation (37) and the magnetic field ansatz yields

$$B \sim B_{seed} e^{[gkK - (2H_0 + \eta\lambda^2 + \alpha)]t} \tag{38}$$

where g is the torsion coupling constant. Therefore, whether the magnetic field is enhanced or damping by chiral torsional effects depends only upon torsion chirality. Now, with this solution at hand, we may perform the astrophysical analysis of this solution. We notice that if the chirality kK term is negative, therefore, we might have a decaying of the magnetic field on a strong suppression as inflationary ends. However, on the contrary, if it is positive or right-handed, the torsion chirality induces a dynamo action. Now, let us drop the hypothesis of the Hubble constant de Sitter cosmology and write H in terms of the $a(t)$ cosmic scale factor. Then, the magnetic field above takes a more interesting solution where the adiabatic magnetic field is now modified by an exponential term which is fundamental. The general B field is

$$B \sim B_{seed} e^{[gkK - (\eta\lambda^2 + \alpha)]t} a^{-2} \tag{39}$$

So, even if the adiabaticity expression tends to damp the magnetic field with cosmic expansion, the mere existence of a positive or right-handed torsion chirality makes the dynamo action possible. Furthermore, one could even notice that in the absence of torsion, the magnetic field decays always here even in the presence of a convective term. If the electromagnetic mean field force α is negative and dominates the magnetic helicity term times the electric resistivity, which is possible since the early universe is positive, the convection term does contribute to the dynamo onset. Deu to torsion, magnetic energy density is $\rho_B \sim B^2 \sim a^{-2} exp[gkK - (\eta\lambda^2 + \alpha)]a^{-2}$ which shows that torsion may really damp the magnetic field via chiral torsion factor, unless it is positive.

5. Scalar Gravity–Torsion Sector in Static DW

In this section, we assume there is a distinct source for torsion, namely a scalar field which is given by a DW with torsion. In the example of a fermionic sector, we have already obtained [23] a DW in EC gravity, where the spin of nucleons are fermionic sources of torsion and are polarized along orthogonal directions to the planar DW. In this case, we assume that the present domain wall is static and a solution is naturally obtained from the Lagrangean. In general, EC gravity is sourced by the fermionic sector, and its energy momentum tensor gives rise naturally to torsion. The scalar torsionless sector DW Lagrangean is given by

$$S = \int d^4x(-g)^{\frac{1}{2}}[g^{ij}\partial_i\phi\partial_j\phi - \frac{\lambda}{2}(\phi^2 - \eta^2)^2] \tag{40}$$

Variation of the action above yields the following wave equation for the scalar field

$$[\partial^2{}_t - \nabla^2]\phi + 2\lambda(\phi^2 - \eta^2)\phi = 0 \tag{41}$$

In the static case, DW this wave equation reduces to

$$\frac{d^2\phi}{dz^2} - 2\lambda(\phi^2 - \eta^2)\phi = 0 \tag{42}$$

which yields the following classical solution for the Minkowski DW as

$$\phi(z) = \eta tanh(\frac{z}{\delta_0}) \tag{43}$$

where $\delta_0 = \frac{1}{\eta\sqrt{\lambda}}$ is the thickness of the wall in flat spacetime. Since the denominator is essentially the mass of the Higgs-like boson, as observed by Dolgov et al. [24], this δ_0 is microscopically small; otherwise, it is cosmologically large. Therefore, this boson would have a tiny mass, and it would generate long-range forces which would contradict LHC experiments. The torsion mass would reach 600 GeV for example [25]. Now, let us use a non-minimal coupling between torsion and the Higgs-like field to consider the action

$$S = \int d^4x(-\frac{\lambda}{2}(\phi^2 - \eta^2)^2 + D_j\phi D^j\phi) \tag{44}$$

Here, $D_i = \partial_i - iS_i$ is the covariant derivative for the scalar fields containing the minimal coupling, this time between torsion axial vector of components S_i and the Higgs-like field. Let us express the last action explicitly in terms of the torsion by minimal coupling. This yields

$$\mathcal{L}_{S^2\phi} = -\frac{1}{2}g^{ij}[\partial_i\phi\partial_j\phi + 2iS_i\phi\partial_j\phi - S^2\phi^2 - \frac{\lambda}{2}(\phi^2 - \eta^2)^2] \tag{45}$$

By applying the Euler–Lagrange equation to this Lagrangean with respect to the Higgs-like scalar field and axial torsion pseudo-vector, respectively, yields

$$\partial_k[\frac{\partial\mathcal{L}}{\partial\partial_k\phi}] - \frac{\partial\mathcal{L}}{\partial\phi} = 0 \tag{46}$$

and

$$\partial_k[\frac{\partial\mathcal{L}}{\partial\partial_k S_i}] - \frac{\partial\mathcal{L}}{\partial S_i} = 0 \tag{47}$$

From the Lagrangean, for the interaction between torsion and Higgs-like fields in DW, one obtains the following field equations: First, the one which gives the torsion axial pseudo-vector sourced by Higgs-like field as

$$S_k = i\partial_k ln\phi \tag{48}$$

This shows that the axial torsion pseudo-vector sourced by a Higgs-like field is a complex torsion. This kind of affine connection appears in loop quantum gravity. Let us now solve the remaining Equation (46). This yields the following equation

$$\partial_k \partial^k \phi - [\frac{\lambda}{2}(\phi^2 - \eta^2)^2 + S^2]\phi = 0 \tag{49}$$

where $S^2 = S_k S^k$, therefore, by plugging Equation (47) into this last wave equation for Higgs-like field in DWs, we obtain

$$\partial_k \partial^k \phi - [\frac{\lambda}{2}(\phi^2 - \eta^2)^2 + \partial_k ln\phi \partial^k ln\phi]\phi = 0 \tag{50}$$

which can approximated by

$$\partial_k \partial^k \phi - [\frac{\lambda}{2}(\eta^2)^2 - \frac{\partial_k \phi \partial^k \phi}{\phi^2}]\phi = 0 \tag{51}$$

Recently, [15] we showed that since, by the end of inflation, torsion is highly suppressed in scalar-torsion gravity sector, one may approximate this equation, in order to obtain the same solution as before for a flat DW, which shows from expression (48) that the z-component of torsion is given by

$$S_z = iS_0 sech^2(\frac{z}{\delta_0}) \tag{52}$$

Now, let us examine this solution for torsion more closely from the physical point of view. Since hyperbolic secant has an accute Gaussian format, it is squared secant, and so is the torsion vector component in the direction of the z-coordinate orthogonal to DW. This seems to indicate that both torsions sourced either by the fermionic sector or scalar Higgs-like sector have a similar behavior orthogonally to the flat torsional DW. If δ_0 is small microscopically but finite, then when we approach the wall as $z \to \infty$, the Gaussian function tends to be a delta Dirac distribution. This could for example represent a string crossing a DW. Primordial magnetic DW with Debye screening has been recently investigated which gives also motivation to obtain the dynamo equation in comoving coordinates in the next section as we did for the QED sector dynamo.

6. Decay of Chiral Magnetic Field Seeds in Scalar Higgs-like Sector Einstein–Cartan Static DWs

In this section, we shall obtain the magnetic dynamo equation in the background of a planar thick static wall in comoving coordinates, which are given by the metric line element

$$ds^2 = e^{cz}(dt^2 - dz^2) - e^{-cz}(dx^2 + dy^2) \tag{53}$$

This metric by assuming just one non-trivial component of the magnetic field orthogonal to the wall as B_z, then, we have from the chiral dynamo [26] equation without the convection term the following equation

$$\partial_t B^z - \eta \partial^2 B^z = \mu_5 B^z \tag{54}$$

Note that by considering the metric (53) and the comoving coordinates for the contravariant component of the magnetic field as B^z transforms as $e^{-az} B_z$, where c is an integration constant, one obtains the chiral dynamo equation, even in the absence of convective terms in the explicit form as

$$\partial_t (e^{-2cz} B_z) - \eta \partial^2{}_z (e^{-2cz} B_z) = \mu_5 (e^{-2cz} B_z) \tag{55}$$

Here, μ_5 is the chiral chemical potential, which is fundamental for a chiral magnetic effect. By the way, this example here is a new example of the torsional CME (TCME) investigated by Imaki and Qiu [27]. Furthermore, we note that we are not considering the convective term here that depends on the velocity of the chiral plasma for example, but yet we shall obtain c. As usual, the z-component of the magnetic field depends upon the z and t-cosmic coordinates. Then, the solution of this equation is given by

$$B_z = B_{(0)} e^{-2c(2c\eta+\mu_5)t} \tag{56}$$

which decays because we do not have the convective dynamo term. To complete the solution, one simply multiply this magnetic field component by the comoving component to obtain

$$B^z = B_{(0)} e^{-2c(2c\eta+\mu_5)t+z} \tag{57}$$

Therefore, one notices that the domain wall sourced by the Higgs-like field is compatible with the one we have obtained previously in the fermionic sector where the nucleons spins were polarized along the orthogonal direction to the DW.

7. Conclusions and Outlook

In this paper, we showed that torsional Ohmic effects also affect the amplification of magnetic fields during preheating. However, despite the weakness of the fields we obtained here of the order of $B_{torsion} = 10^{-24}G$, this estimate is still able to seed galactic dynamo. This result, which is our main goal in the paper, has been obtained by finding a solution of QED equations with torsion semi-minimally coupled with an electromagnetic field. This does not introduce a massive photon as in some previous investigations such as in Proca magnetogenesis recently investigated by Garcia de Andrade [6] and Adelberger et al. [28]. Effective Lagrangean can be used to determine the torsion which can be used to seed galactic dynamos. From a cosmic self-induced, torsionful QED background equation, we show that torsion contributes the magnetic field as a damping coupling with Ohmic resistivity to decrease the values of the magnetic densities in several situations of astrophysically interest and to provide results compatible with astronomical observations. The motivation from this study came from some work by Campanelli et al. [29], where an investigation of similar issues was undertaken in general relativistic backgrounds of Riemannian geometry, and by the work of Salim et al. [30,31] on the amplification of the magnetic field in bouncing cosmological models. Another aspect of magnetogenesis can be found in Pandey [32]. We also showed that the torsion decouples from the expansion of a magnetic field. Recently, Banerjee [33] has informed us that values of $B - field$ as low as $10^{-32} Gauss$ may also seed galactic dynamos. Probably, one of the most interesting features of introducing torsion here is that since dark energy is repulsive gravity [2] and dark matter is attractive, the slow down in the magnetic field growing discussed above may be due to torsion contributions to dark energy. A more detailed investigation on that matter must appear in a forthcoming paper. Previously, Gasperini [34] showed that it is possible to regularize the curvature singularity in a radiation dominated universe by the repulsive effect of spin–spin interaction in Einstein–Cartan gravity. Other types of dynamos in topological defects such as domain walls can be found recently in the literature [7]. Magnetogenesis is such an important subject that appeared very recently on a scientific magazine for popular science [25]. Following the evolution of a double thick domain wall, one containing matter and the other containing antimatter, given by Dolgov et al. [24] in the context of an inflationary universe, one may further generalize the ideas discussed here in Section 5. This is work under progress.

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