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Neutron stars in modified theories of gravity with scalar field

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Abstract. Neutron stars offer the opportunity to probe both the properties of matter and the nature of gravity in the strong-field regime. However, uncertainties in the equation of state and in possible modifications to general relativity often lead to qualitatively and quantitatively similar observational signatures, making them difficult to distinguish. In this talk, we will focus on scalar-tensor extensions of gravity and highlight some effects that cannot be mimicked by uncertainties in the equation of state.

1 Introduction

Neutron stars (NS) provide us with the opportunity to test both the strong gravity regime and the properties of matter at extreme densities. Unfortunately, a degeneracy between modified gravity effects and effects from the nuclear matter equation of state uncertainty exists. This degeneracy is both on a qualitative and quantitative level and it covers a variety of NS observables related to the NS structure and dynamics. Thus, it is often very difficult to test generalized theories of gravity and high-density matter behavior independently.

One way to overcome this problem is to find qualitatively new features and effects in modified theories of gravity which cannot be easily mimicked by changing the NS matter EoS. It is important also to understand up to what extend the degeneracy exist. A very interesting effect observed for some theories of gravity are the first-order-like gravitational phase transitions [1, 2, 3], and some of the effects reported in [4]. Note that up to some extend, such phase

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transitions could be partially mimicked by matter phase transition in the star. In the present talk we will extend the study carried in [4] and proceed with the exploration of the solutions space in extended scalar-tensor theories (ESTT) of gravity. More precisely, we will focus on a combination between pure scalar-Gauss-Bonnet (sGB) gravity [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23] and classical scalar-tensor theories (STT) [24]. For the motivation behind this combination we refer the reader to the introduction section in [4]. In the particular study we will concentrate on Gauss-Bonnet coupling which allows for spontaneous scalarization [17, 18, 19, 20, 21] (for a review on scalarization see [25]) in pure sGB theory and include in addition an Einstein frame matter coupling $A(\varphi)$ which contains both linear and quadratic terms in the scalar field. Note that this matter coupling is actually the conformal factor relating the Einstrain frame and the physical Jordan frame spacetime metric.

2 Theory framework

In the present talk we consider extended scalar-tensor theory with the following general form of the Einstein frame action

$$S = \frac{1}{16\pi G_*}\int d^4x\sqrt{-g}\Big[R - 2\nabla_\mu\varphi\nabla^\mu\varphi - V(\varphi) + \lambda^2 f(\varphi)\mathcal{R}^2_{GB}\Big] + S_{\text{matter}}(A^2(\varphi)g_{\mu\nu}, \chi), \quad (1)$$

where R denotes the Ricci scalar with respect to the Einstein frame metric $g_{\mu\nu}$, ∇_μ is the covariant derivative with respect to $g_{\mu\nu}$ and $f(\varphi)$ is the GB coupling function for the scalar field φ. $V(\varphi)$ is the potential of the scalar field and in the present talk we adopted the case $V(\varphi) = 0$. The GB coupling constant λ has dimension of *length* and $\mathcal{R}^2_{GB}$ denotes the Gauss-Bonnet invariant defined by $\mathcal{R}^2_{GB} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}$ where $R_{\mu\nu}$ and $R_{\mu\nu\alpha\beta}$ are the Ricci tensor and the Riemann tensor respectively with respect to the Einstein frame metric $g_{\mu\nu}$. S_{matter} is the Einstein frame matter action where the matter fields are collectively denoted by χ and $A(\varphi)$ is the Einstein frame conformal coupling function between the scalar field and the matter. This general formulation of the theory has already been examined in [4] and in the present talk we extend that study. The theory has the classical scalar-tensor theory as a particular case when $f(\varphi) = const$ and the pure scalar-Gauss-Bonnet gravity for $A(\varphi) = 1$.

In the present paper, we are interested in static and spherically symmetric neutron star solutions in asymptotically flat spacetime. Hence we consider general static and spherically symmetric ansatz for the Einstein frame metric, namely:

$$ds^2 = -e^{2\Phi(r)}dt^2 + e^{2\Lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2). \quad (2)$$

The explicit form of the field equations for the above ansatz for the metric and the transformations of all quantities between the mathematically more convenient Einstein frame and the physical Jordan frame one can find in [4].

If all quantities appearing in the field equations are expanded in power series around the center of the star, the requirement for regularity of the equations at the center not only determines the initial conditions but also yields an additional relation among the expansion coefficients and the parameters of the theory. This relation serves as an existence condition. Its derivation and general form for the theory defined by the action (1) can be found in [4]. Usually, the existence condition is violated for large neutron star masses and strong scalar fields

leading to premature termination of the branches of solutions at some central energy density which depends on the coupling parameters of the theory.

3 Setting the theory and numerical results

The present talk extends on the results form [4] and explores the solutions space of theory which is a nonlinear combination between scalar-Gauss-Bonnet gravity and classical scalar-tensor theory.

In order to obtain the equilibrium spherically symmetric solutions, we need to impose the following natural boundary conditions, namely regularity at the center of the star:

$$\Lambda|_{r\to 0} \to 0, \quad \left.\frac{d\Phi}{dr}\right|_{r\to 0} \to 0, \quad \left.\frac{d\varphi}{dr}\right|_{r\to 0} \to 0 \tag{3}$$

and asymptotic flatness at infinity:

$$\Lambda|_{r\to\infty} \to 0, \quad \Phi|_{r\to\infty} \to 0, \quad \varphi|_{r\to\infty} \to 0. \tag{4}$$

The central value for the scalar field, $\varphi(0)$, and the metric function $\Phi(0)$, are calculated as a result of a shooting procedure.

An important characteristic of the neutron star solutions is the scalar charge D which is defined through the leading order expansion of the scalar field at $r \to \infty$

$$\varphi \simeq \frac{D}{r} + O(1/r^2). \tag{5}$$

Regarding the specific form of the theory, it is fixed by the particular choice of the coupling functions. For the GB coupling we chose a coupling function which obeys the conditions for spontaneous scalarization $\frac{df}{d\varphi}(0) = 0$ and $\frac{d^2f}{d\varphi^2}(0) \neq 0$ in the following form:

$$f(\varphi) = -\frac{1}{2\beta}\left[1 - e^{-\beta\varphi^2}\right]. \tag{6}$$

For the Einstein frame matter coupling we use what is effectively a combination between Damour-Esposito-Farese (DEF) [26, 27] and Brans-Dicke (BD) [28] coupling in the form:

$$A(\varphi) = e^{\frac{1}{2}\gamma\varphi^2+\alpha\varphi}, \;\; \kappa(\varphi) = \frac{d\ln A(\varphi)}{d\varphi} = \gamma\varphi + \alpha. \tag{7}$$

Due to the linear term in the Einstein frame matter coupling, the trivial case $\varphi = 0$, hence general relativity (GR), is not a solution of the theory, therefore, only scalarized solutions are possible.

Both the pure sGB theory with the coupling (6) and pure classical STT with the coupling (7) having $\alpha = 0$ (namely, the DEF theory) are symmetric with respect to the sign of the scalar field. In those cases solutions with both positive and negative scalar field always exist for the same set of parameters. Those solutions have the same magnitude of the scalar field, and except for the opposite signs of the field, they are identical. The linear term, however, breaks this symmetry. It is possible that solutions only with positive or only with negative scalar field exist, and even if both types exist for a given set of parameters they are in general different. One

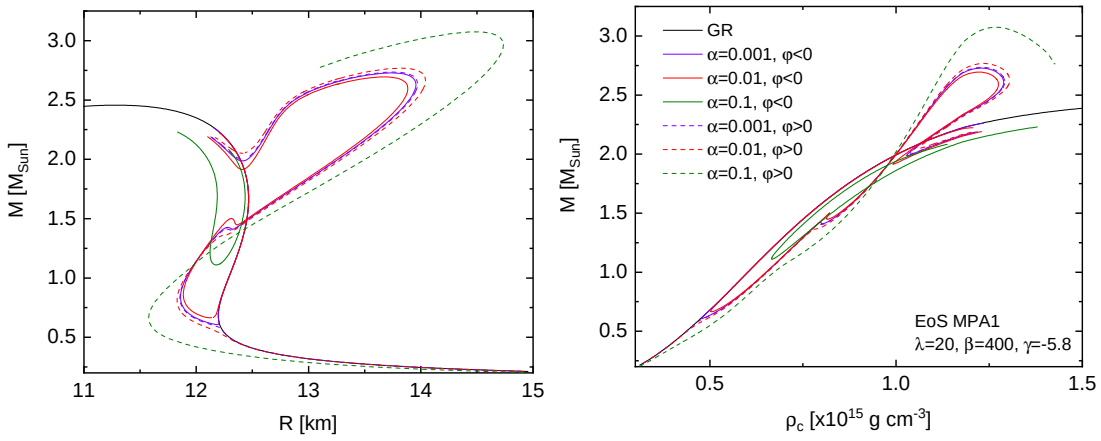

Figure 1: *(left)* Mass as a function of the radius of the star, *(right)* mass as a function of the central energy density. The positive scalar field branches are marked with continuous lines and the negative one with dashed lines. Different values of α are in different colors. The GR branch is displayed only as a reference since it is not a solution of the field equations for $\alpha \neq 0$.

more important consequence of the linear term is the existence of additional scalar radiation polarization, the so-called breathing modes [29].

In what follows, the star parameters will be presented in the physical Jordan frame. For the Gauss-Bonnet constant we use the following dimensionless normalization:

$$\lambda \to \frac{\lambda}{R_0} \tag{8}$$

where $R_0 = 1.47664$ km is a half of the gravitational radius of the Sun.

In order for us to solve the field equations and obtain the neutron star solutions, we need to supply the EoS for the neutron star matter. In this talk we adopt a realistic one, namely EoS MPA1 [30] via its piecewise polytropic approximation [31]. The theory we examine has four free parameters. The approach we chose is to fix the parameters associated with the sGB part and the parameter in the quadratic term of the Einstein frame matter coupling and concentrate on the effect of the linear term by varying the corresponding constant.

The numerical solutions for the theory discussed above with $\lambda = 20, \beta = 400, \gamma = -5.8$ (chosen in correlation with [4]) and different values for α ($\alpha > 0$) are presented in Fig. 1. In the left panel, we plot the mass of the neutron star as a function of its radius and in the right panel, the mass as a function of the central energy-density. On both panel the GR branch is shown only as a reference, since it is not a solution of the field equations. We were able to find branches of solutions with both positive and negative scalar field, which, since the theory is not symmetric with respect to the sign of the scalar field, are qualitatively different and we will discus them both.

The behavior of the solutions branches observed in the mass of radius panel depends on the magnitude of α. When α is small, a process resembling spontaneous scalarization is observed. Let us describe it more thoroughly: for small masses the solutions are weakly scalarized and the solution branch visually overlaps with the GR one. At a given mass this branch sharply

transitions into its strongly scalarized part. This process looks identical to the spontaneous scalarization observed in the DEF and in the pure sGB theories. In those cases the scalarized branch emanates form the trivial one, $\varphi = 0$, at a given mass (bifurcation point). In our case we will call the point at which the scalarized branch makes a sharp transition between its weakly and strongly scalarized parts bifurcation-like point and the corresponding process – secondary scalarization. Below the bifurcation-like point only the branch with negative scalar field exist and it overlaps with the GR one. Parts of both the $\varphi < 0$ and $\varphi > 0$ branches almost overlap above the bifurcation-like point. The positives scalar field branch, though, is characterized by a minimal mass which is slightly above the bifurcation-like point, and below it no solutions exist. From that minimal mass point two positive scalar field subbranches emanate. One with weak scalar field which overlaps with the GR branch and a second one with stronger scalar field which closely follows the negative scalar field branch in its secondary scalarized part. The first positive scalar field subbranch is terminated before it reaches a turning point and the strongly scalarized one is terminated at the same point as the negative scalar field one.

When α increases the branches corresponding to the two signs of the scalar field deviate from each other. For the branches with negative scalar field, the bifurcation like point disappears and the transition between the different parts of the brach get smoother. For big enough α the scalarized branch is smooth. The low mass stars have smaller radius and the high mass ones have larger radius compared to the same mass GR solutions. The maximal mass is higher compared to the GR one as well. Those branches we will call BD-like. The branches with positive scalar field get smoother and shorter with the increase of α. For big enough α both the weakly and the strongly scalarized solutions always have smaller radius compared to the same mass GR solutions. In addition the strongly scalarized solutions are always with smaller radius compared to the weakly scalarized ones. The minimal mass increases significantly with the increase of α as well. The described behavior is observed in the mass of central energy density panel as well.

Both types of branches demonstrate very interesting and nontrivial behavior for small and intermediate α. In both cases we observe what in [4] is described as GB-like behavior at low masses which gets overpowered by DEF-like behavior at some point along the branch. The effect of α on this transition is very interesting. For very small α, the two branches more or less overlap and demonstrate the behavior described in [4]. It is as follow: the GB like branch reaches a local maximal mass, where an intermediate branch appears along which the mass decrease until it reaches a local minima. This minima is followed by another turning point at which the DEF-like behavior stars dominating and the mass increases again up to the global maximum. It is interesting that with the increase of α this transition gets smoother and the intermediate branch disappears.

Past the turning point, associated with the global maximal mass, the mass of the solutions starts decreasing down to a new local minima, followed by another turning point past which the mass starts increasing again. This final section is terminated before reaching new turning point. Based on the common understanding for the stability of neutron star solutions, we expect that all parts of the branches for which the mass of the star deceases, to be potentially unstable. Similarly, in the positive scalar field case, the weakly scalarized branches are more likely unstable as well.

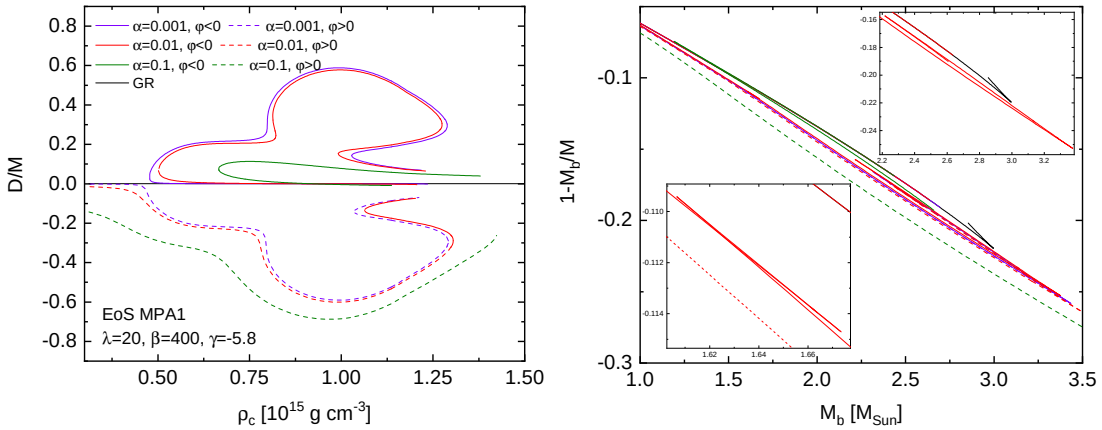

Figure 2: *(left)* The scalar charge of the star, normalized to its mass, as a function of the central-density, *(right)* the binding energy, as a function of the baryon mass of the star. The positive scalar field branches are marked with continuous lines and the negative one with dashed lines. Different values of α are in different colors. The GR branch is displayed only as a reference since it is not a solution of the field equations for $\alpha \neq 0$.

In Fig. 2 we proceed with two important characteristics of the solutions. In the left panel we plot the normalized scalar charge of the star as a function of the central energy density and in the right panel, the binding energy as a function of the baryon mass of the star. The scalar charge follows the behavior of the branches described above. One should point out that its magnitude increases with α for the solutions with negative scalar field but it decreases for the solutions with positive scalar field. The binding energy is a good indication for the stability of the solutions. In the right panel one can see the typical cusps associated with the change of stability at all local and global maxima and minima observed in the mass of radius and mass of central energy density figures, hence all subbranches for which the mass decreases are potentially unstable. The fact that the stable solution branches are discontinuous can have interesting astrophysical implications. Regarding the branches with positive scalar field, the weakly scalarized subbranches have lower, in absolute value, binding energy compared to the strongly scalarized ones. Hence, the latter ones are potentially stable and the former ones, potentially unstable.

4 Conclusions

In the present talk we studied some nontrivial features of neutron stars in a particular class of extended scalar-tensor theory. The specific theory we examine represents a nonlinear combination between a pure scalar-Gauss-Bonnet gravity and classical scalar-tensor theory. For the sGB sector we use a theory which allows for spontaneous scalarization and we supplement it with an Einstein frame matter coupling (conformal factor) which contains both linear and quadratic terms in the scalar field. The theory with that particular choice for the conformal factor, in the absence of sGB term, reduces to a combination between DEF and BD scalar-tensor theories. The presence of a linear BD type term is important because it allows the emission of additional polarization of scalar radiation, the so-called breathing modes.

The theory under consideration has four parameters. We chose to fix the two associated with the sGB part and the constant in the quadratic term in the Einstein frame matter coupling and study the change in the solutions when the BD constant in the linear term is varied. Since the theory is not symmetric with respect to the sign of the scalar field, the positive and negative scalar field solutions differ. However, the intermediate and high central energy density (intermediate and high mass) part of these two branches visually overlap for small BD constant, but they significantly deviate when its value is increased. Based on the binding energy, the stable branches are identified and some interesting features can be observed. The branches with positive scalar field have minimal mass below which no solutions exist, and its value increases with the increase of the BD constant. For both positive and negative scalar field branches, for small and intermediate values for the linear term constant, there are local minima and maxima in the mass of radius diagrams which are connected with intermediate potentially unstable branches. Hence, in some regions the branches will be discontinuous and transitioning along them will be possible only with a jump, which could have an observational footprint.

Features like the existence of minimal allowed mass and discontinued branches of solutions which allow for first-order-like gravitational phase transitions cannot be accounted for by the uncertainty in the pure nuclear matter equations of state inside the neutron star. We should note, though, that there are similarities with the matter phase transitions from nuclear to deconfined quark matter. It is already clear that such effects are internal characteristics of many modified theory of gravity [1, 2, 3, 4, 32, 33] and not only a result of some finetuning of parameters. Therefore, the astrophysical signatures of such modified theories deserves further study.

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