OVERVIEW OF MUON COOLING*

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Abstract

Muon cooling techniques are surveyed, along with a concise overview of relevant recent R&D.

INTRODUCTION

Muon cooling enables muon colliders and neutrino factories, and enhances low-energy muon experiments. At high energies, use of muons rather than electrons substantially suppresses radiative processes ($\propto m_{\rm lepton}^{-4}$), allowing acceleration and collision in rings — greatly reducing lepton-collider footprint and cost — as well as more-monochromatic collisions and feasibility at much higher energies (10 TeV or more) [1]. While muon decay (mean lifetime = 2.2 μs) complicates beam handling, it enables stored-muon-beam neutrino factories, the most capable technique for precision measurements of neutrino oscillation [2].

Figure 1 schematically compares these two types of high-energy muon facility, for both of which the performance and cost depend on how well a muon beam can be cooled. They are seen to have much in common:

- In both designs, a high-intensity (MW-scale), medium-energy "proton driver" illuminates a high-power capable target in a heavily shielded enclosure, copiously producing charged pions, which decay into intense broad-band muon beams.

- Bunching and "phase rotation" (reducing the energy spread by accelerating slower muon bunches and decelerating faster ones) prepare the muon beams to be cooled. The "initial cooling" stage completes the facility "front end" [3], which is similar if not identical in the two cases.

- After cooling, the beams are accelerated to the desired energy and injected into storage rings, where they circulate for $O(10^3)$ turns.

Rubbia has emphasized the importance of muon colliders for Higgs-boson studies [4]. To test for physics beyond the standard model (SM) requires sub-percent measurements of Higgs branching ratios as well as a precision scan of the resonance lineshape, possible only with s-channel production at a 125 GeV muon collider. Studies of the Higgs self-coupling are also needed, requiring a $\gtrsim$ TeV muon collider.[1] Hints are emerging for possible new physics in the $\gtrsim$ 2 TeV region [5]. Above $\approx$ 1–2 TeV, the muon collider is arguably the most capable and cost-effective lepton collider [6].

A natural muon collider staging plan thus emerges [6, 7]:

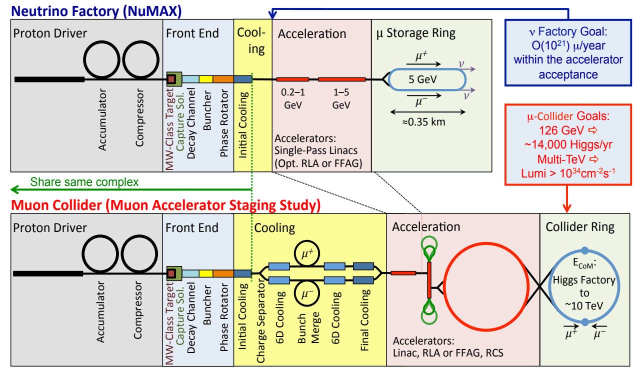

Figure 1: Neutrino factory (top) / muon collider (bottom) comparison. The "front end" (muon production, collection, bunching, phase rotation, and initial cooling) can be very similar for both. It is followed in a neutrino factory by acceleration of the muons to multi-GeV energy and injection into a storage ring, with long straight sections in which muon decay forms intense neutrino beams aimed at near and far detectors. For a muon collider, it is followed by 6-dimensional cooling, bunch coalescence, acceleration (e.g., to 62.5 GeV for a precision "Higgs Factory"), and injection into a collider ring, where μ^+ and μ^- bunches circulate for $\sim 10^3$ turns.

1. Start by building a neutrino factory,[2] which can do competitive physics with no cooling, and ultimately requires only "initial" cooling by a factor of ~ 10 in six-dimensional (6D) emittance.

2. Then upgrade the facility to a 125 GeV "Higgs Factory" muon collider, requiring $O(10^6)$ or more emittance reduction.

3. Then upgrade to a $\gtrsim$ TeV collider.

At low energies, cooling can give smaller and more intense stopping-muon beams [8, 9]. A subject of ongoing R&D at the Paul Scherrer Institute, it may enable enhanced studies of muonium spectroscopy, searches for muon–electron conversion and muonium–antimuonium oscillations, and a test of antimuon gravity [10], among other measurements [11].

BRIEF HISTORY

Muon colliders have been discussed since the 1960s [12, 13]. The key idea enabling high luminosity — ionization cooling — came later [14, 15], and its theory was not fully understood until the 1990s [16].

In the mid-1990s the ("grass roots") Muon Collider Collaboration formed, producing a report on muon colliders

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1 As time passes and nothing below 1 TeV besides the Higgs is seen at LHC, comparable measurements with electrons seem increasingly unlikely.

2 The nuSTORM short-baseline muon storage-ring facility, aimed at precision cross-section measurements and sterile-neutrino searches, requires no cooling and no new technology and has been proposed as an even earlier step [18].

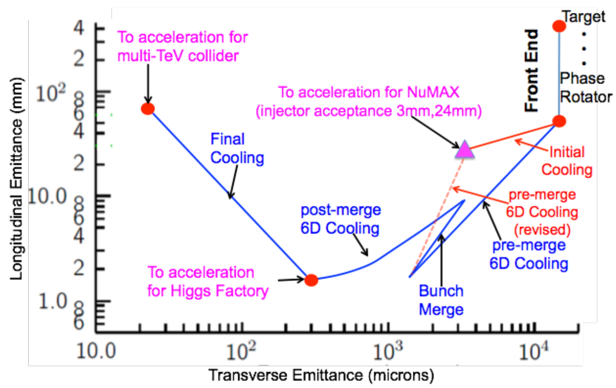

Figure 2: Cooling "trajectory" in longitudinal and transverse emittance, with red points showing MAP emittance goals.

for the 1996 Snowmass meeting [19]. The following year the neutrino factory concept was born [20], leading to a collaboration expansion and change of name (to the Neutrino Factory and Muon Collider Collaboration, NFMCC) [21], and stimulating a series of neutrino factory feasibility studies [22–25], workshops [26], and the development of the Muon Ionization Cooling Experiment [27], among others [28, 29]. In 2006 a directed effort to develop a site-specific muon collider proposal, the Muon Collider Task Force (MCTF) [30], was initiated at Fermilab. This led the DOE to request a concerted effort, the Muon Accelerator Program (MAP) [31], which began in 2011. Sadly, MAP is now a casualty of the P5 process [32] and is in the midst of a funding rampdown.

EMITTANCE GOALS

It is useful to enumerate briefly the emittance targets that have been identified for various physics goals. These are best understood in terms of "cooling trajectories" on the longitudinal vs. transverse normalized-emittance plane (Fig. 2) [33].

1. As mentioned, an initial neutrino factory configuration without cooling, producing $O(10^{20})$ neutrinos/yr, can be cost-effectively upgraded to $O(10^{21})$ neutrinos/yr (in the so-called NuMAX configuration) by adding an order of magnitude of 6D cooling [6]. This works together with a dual-use linac that accelerates protons from 3.0 to 6.75 GeV as part of the proton driver and then accelerates cooled muons from 1.25 to 5 GeV, requiring muon input transverse and longitudinal emittances of $\approx 3\pi$ mm·rad for full acceptance.

2. A Higgs Factory muon collider requires exquisite energy spread to support a precision Higgs-lineshape energy scan ($\Gamma^{SM}_{\rm Higgs} = 4$ MeV). The MAP goal is transverse/longitudinal emittances of $\approx 0.3/1.5\pi$ mm·rad, achieved in a series of "6D" cooling channels, enabling 5×10^{31} cm^{-2} s^{-1} luminosity and 5 MeV energy spread.

3. Above 1 TeV collision energy the MAP goal is transverse/longitudinal emittances of $\approx 0.025/70\pi$ mm·rad, enabling $\gtrsim 10^{34}$ cm^{-2} s^{-1} luminosity. Following the 6D cooling channels, these parameters are achieved by means of "final cooling," incorporating significant transverse→longitudinal emittance exchange.

PRINCIPLES OF MUON COOLING

The short lifetime of the muon vitiates all beam-cooling methods currently in use.[3] However, a method almost uniquely applicable to the muon — ionization cooling [15] — seems equal to the challenge. In this, muons are made to pass through an energy absorber of low atomic number (Z) in a suitable magnetic focusing field; the normalized transverse emittance $\epsilon_{\perp,n}$ then obeys [16]

$$\frac{d\epsilon_{\perp,n}}{ds} \approx -\frac{1}{\beta^2}\left\langle\frac{dE_\mu}{ds}\right\rangle\frac{\epsilon_{\perp,n}}{E_\mu} + \frac{1}{\beta^3}\frac{\beta_\perp(0.014\,\mathrm{GeV})^2}{2\,E_\mu m_\mu X_0}\,, \quad (1)$$

where βc, $\langle dE_\mu/ds\rangle$, $\beta_\perp$, m_μ, and X_0 are the muon velocity, average energy loss per unit length, betatron function at the absorber, muon mass, and absorber material radiation length. (This is the expression appropriate to the cylindrically symmetric case of solenoidal focusing, for which $\beta_x = \beta_y \equiv \beta_\perp$ and cooling occurs equally in the x-x' and y-y' phase planes.) The first term in Eq. 1 is the cooling term, and the second describes heating due to multiple scattering.[4] The heating term is minimized via small $\beta_\perp$ and large X_0 (low-Z absorber material). For a given cooling-channel design, an equilibrium emittance is reached when the heating and cooling terms balance, after which a revised design with lower $\beta_\perp$ is required if cooling is to continue.

Somewhat counterintuitively, the optimal momentum for cooling is found to be ≈ 200 MeV/c [16], near the *minimum* of the ionization energy-loss ("dE/dx") curve in matter [17]. This is a compromise between the heating effects of the "straggling tail" at higher momentum and the negative slope of the dE/dx curve below the ionization minimum (creating problematic, *positive* feedback for energy-loss fluctuations).

The physics of Eq. 1 is well established, yet engineering details — or poorly modeled tails of distributions — could profoundly affect ionization cooling-channel cost and performance. This motivates an effort to build and test a realistic section of cooling channel: the international Muon Ionization Cooling Experiment (MICE) [34], discussed in detail elsewhere [35].

STAGES OF MUON COOLING

Bunching and Phase Rotation

Before the muon beam is cooled one wants to reduce its ~100% initial energy spread. First an energy–time correlation is developed within an RF-free drift region, then the beam is bunched, then the faster bunches decelerated and

[3] I.e., electron and stochastic, laser cooling being in any case inapplicable to an object without internal degrees of freedom, and synchrotron radiation being negligible by virtue of the muon's large mass.

[4] There is a direct analogy to synchrotron-radiation cooling, in which energy loss likewise provides cooling, while the heating is caused by quantum fluctuations.

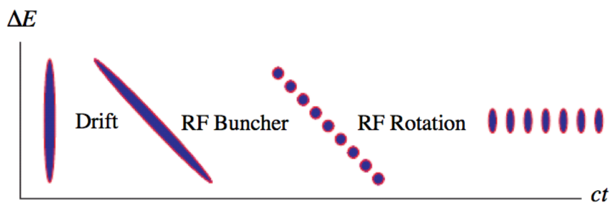

Figure 3: Cartoon of bunching and phase-rotation process.

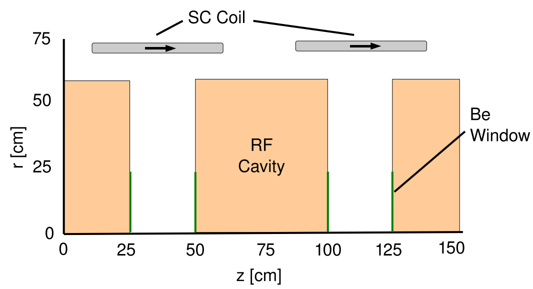

Figure 4: Lattice cell used for bunching and phase rotation (accelerating gradient doubled via thin Be cavity windows).

the slower ones accelerated ("phase-rotation") (Fig. 3) [3]. The lattice of Fig. 4 is used, with a series of RF cavities of decreasing frequency ranging from ≈ 500 down to 325 MHz.

Initial Cooling

Successful, purely transverse ionization-cooling lattices were developed by the year 2000 [36], when Neutrino Factory Feasibility Study II (FS2) was carried out [24]. The FS2 design employed two magnetic-field harmonics, allowing small $\beta_\perp$ to be achieved by working between the resulting "π" and "$\pi/2$" resonances. A simplified, more cost-effective design (Fig. 5) was adopted by the International Design Study for the Neutrino Factory (IDS-NF) [37]. In contrast to the bunching and phase rotation lattice of Fig. 4, in these cooling lattices, alternating solenoid-field directions prevent the buildup of a net canonical angular momentum due to energy loss and re-acceleration within a solenoidal field. (Since solenoids focus in both transverse directions, these lattices are generically referred to as "FOFO," in contrast to FODO alternating-gradient quadrupole lattices.)

The neutrino factory and muon collider designs can be better unified by employing a six-dimensional (rather than

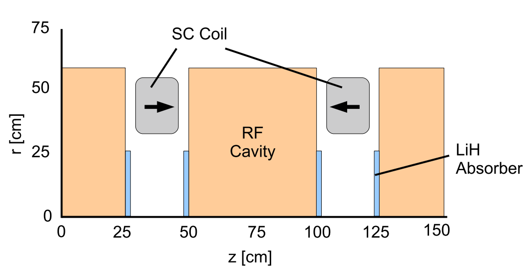

Figure 5: IDS-NF transverse cooling lattice cell, with alternating solenoids to create low-β regions between RF cavities and thin, Be-coated LiH absorbers as cavity windows.

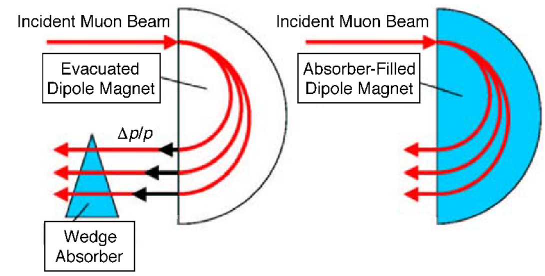

Figure 6: Two emittance exchange approaches: small beam with nonzero momentum spread is converted into more monoenergetic beam with transverse position spread. The reverse occurs in "Final Cooling." (Figure: Muons, Inc.)

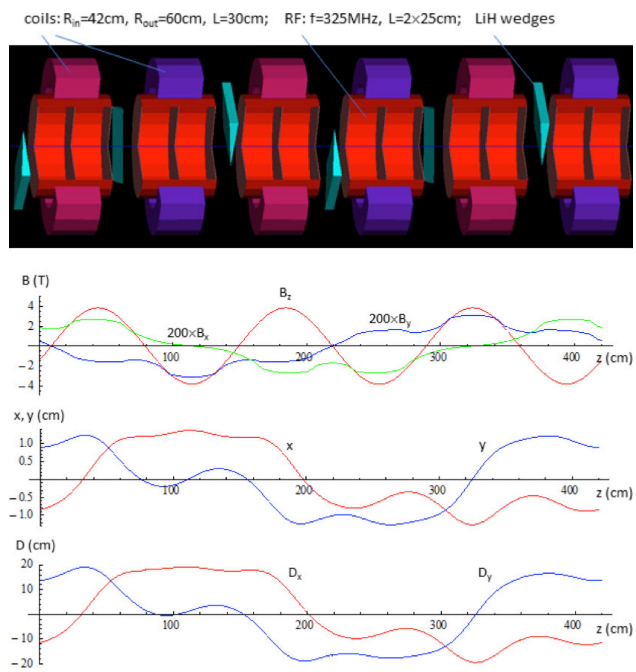

Figure 7: (top) HFOFO Snake lattice scheme, combining tilted solenoids with LiH-wedge and gaseous-hydrogen absorbers and RF cavities; (bottom) B_z, beam positions, and dispersion vs. distance along beam axis [38].

transverse) initial cooling lattice, which (as mentioned) also permits cost savings by allowing a dual-use (proton–muon) linac. The purely transverse ionization-cooling effect can be shared among the transverse and longitudinal phase planes in a lattice in which dispersion causes momentum-dependent path-length through an absorbing medium ("emittance exchange"), e.g., as depicted in Fig. 6. Since, at the large transverse emittance of the initial beam, charge separation would be challenging, a 6D cooling lattice that works simultaneously for both muon charge signs is desirable. This design challenge is met by the "HFOFO Snake" (Fig. 7) [3, 38], in which small tilts of the solenoids relative to the beam axis, in orientations that rotate about the beam axis by 120° per step, create a small, rotating-dipole field component. This

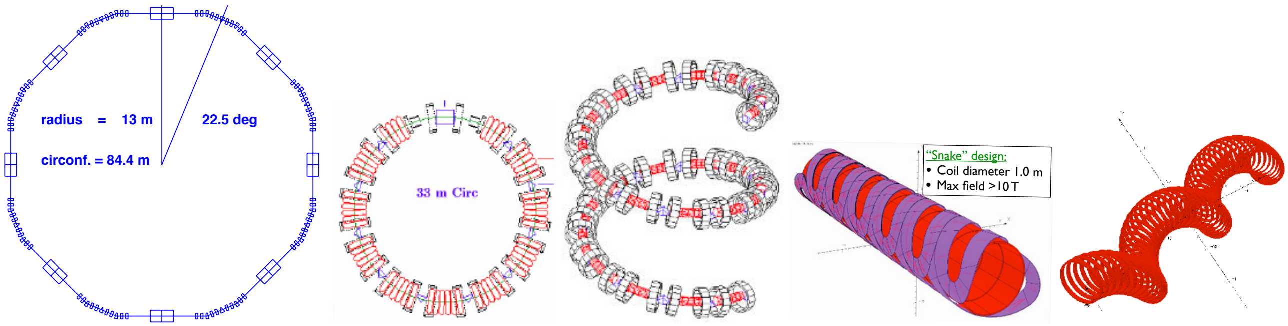

Figure 8: Examples of 6D cooling apparatus that have been shown to work in simulation: (left to right) quadrupole–dipole ring, "RFOFO" solenoid-focused ring, "RFOFO Guggenheim" helix, helical cooling channel (two versions).

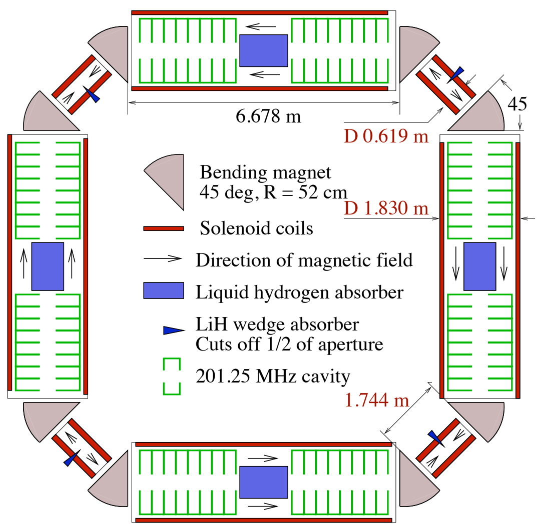

Figure 9: First successful 6D cooling design: Balbekov's "tetra" ring cooler [43].

creates periodic orbits and dispersion that are isomorphic, with a half-period offset, for the two charges.

Six-Dimensional Cooling

A high-luminosity Higgs Factory ($\mathcal{L} \gtrsim 10^{32}$ cm^{-2} s^{-1}) or TeV muon collider ($\mathcal{L} \gtrsim 10^{34}$ cm^{-2} s^{-1}) requires a cooling scheme that reduces both transverse and longitudinal emittances by an overall factor of 10^6 or more in 6D emittance. Various approaches to this goal [1, 39, 40] were developed by the NFMCC, MCTF, and two small R&D firms with SBIR/STTR funding: Muons, Inc. [41] and Particle-Beam Lasers (PBL) [42]. Under the MAP program [31], this work has been continued by many of the same people. Three approaches were shown to work in simulation (Fig. 8): rings, helices, and snakes (a fourth, the "Rectilinear FOFO," is discussed below). Like transverse-cooling lattices, most 6D-cooler designs employ superconducting-solenoid focusing and benefit from the ability of such solenoids to accommodate a large beam, generate low β, and focus simultaneously in both x and y, enabling compact lattices that minimize muon decay in flight.

The earliest successful example of a 6D cooling channel was the 4-sided solenoid-focused ring of Balbekov [36, 43], but it was so tightly packed as to lack space for beam injection and extraction (Fig. 9). This first "in principle" success led to the development of the approaches depicted in Fig. 8: rings with space allocated for the above functions [44, 45], and helices [46, 47], which can embody the symmetries of rings, but are open at the ends for beam ingress and egress.

Helical channels, through which each bunch passes only once, reduce beam loading on absorbers and RF cavities. They can also provide faster cooling via "tapering": increasing the focusing strength along the channel, thereby decreasing the equilibrium emittance as the beam is cooled. The Helical Cooling Channel (HCC), based on a Hamiltonian theory [46], uses a combination of "Siberian snake"-like helical dipole and solenoid fields; it also employs a continuous, high-pressure, gaseous-hydrogen absorber so as to minimize both the deleterious effects of windows and (via pressurized RF cavities, discussed below) the length of the channel. Following the HCC's invention, its (required) solenoid, helical dipole, and (desired, for increased acceptance) helical quadrupole field components were shown to arise naturally from a simple sequence of offset current rings (Fig. 8, far right) [48], which follow the winding path of the beam envelope around the helix [49]. Simulations of a series of HCCs have demonstrated cooling to 0.6/0.9 mm·rad emittances, close to the MAP Higgs Factory goal [49]. The helical geometry requires two sets of 6D cooling channels for a muon collider, one for each muon charge sign. The "HFOFO Snake" channel (Fig. 7) [38], the "least circular" of these approaches, as mentioned can simultaneously cool muons of both signs and would thus be followed by a charge separator in order to feed the 6D channels.

While helical designs have been shown to perform well, their engineering could be challenging. For example, "Guggenheim" channels could fill a large volume (Fig. 10) and require magnetic shielding between turns. This led to a search for alternatives. Surprisingly, a rectilinear geometry promising the same cooling performance was found [50], and its performance borne out by detailed simulation studies [51]. Figure 11 shows the geometry and a representative performance plot [52]; a 10^5 6D cooling factor is achieved,

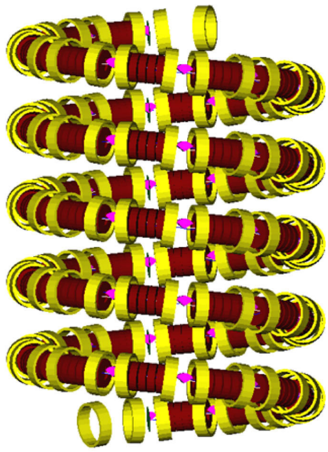

Figure 10: To-scale rendering of five periods of Guggenheim 6D cooler; vertical extent is about 20 m.

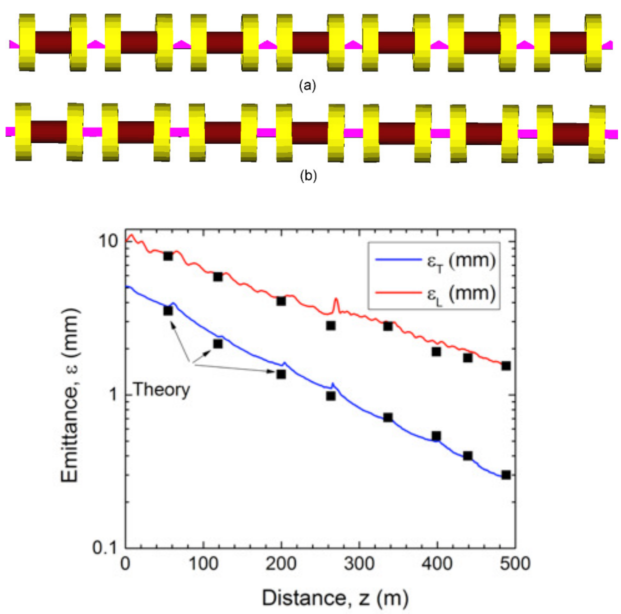

Figure 11: "Rectilinear FOFO" lattice scheme and example of its cooling performance [52].

with final emittances 0.28/1.5 mm·rad, exceeding the MAP Higgs Factory goal.

Final Cooling

After 6D cooling, at the lowest point of the Fig. 2 curve (labeled "To acceleration for Higgs Factory"), the transverse emittance is about an order of magnitude too large, and the muon bunches shorter than necessary, for a high-luminosity TeV collider; the 6D emittance is an order of magnitude larger than desired. In principle this gap can be closed with "final cooling" (Fig. 12) in extremely high-field (30–40 T), narrow-bore solenoids enclosing LH_2 absorbers, in which transverse cooling can be carried out as the muon momentum is allowed to fall towards the Bragg peak of the dE/dx curve [17], while the longitudinal emittance grows due to

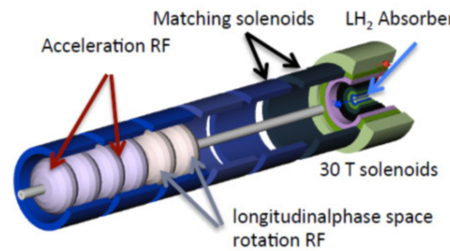

Figure 12: High-field-solenoid "final cooling" lattice cell.

dE/dx positive feedback. Such solenoids should be feasible using high-temperature superconductor (e.g., Bi-2223 tape) operated at LHe temperature. Despite promising initial results obtained [53] by a PBL-BNL magnet R&D effort, given magnetic force and quench issues, further R&D will be required in order to realize a complete magnet system. (Very high-field dipoles are also desired in muon collider rings in order to increase the average luminosity via smaller ring circumference, giving more collisions per muon lifetime.)

Such a channel has been shown in simulation to approach the MAP final-emittance goals, falling short by a factor ≈ 2 in transverse emittance if 30 T solenoids are used [54]. This emittance gap might be closed with the use of higher field. Alternative final-cooling ideas are also under study [55, 56]. These include reverse emittance exchange in wedge absorbers and transverse ionization cooling in quadrupole-focused channels [56], which can achieve $\beta^* < 1$ cm.

"Advanced" Cooling Ideas

While the scheme of Fig. 12, and others that have been studied within MAP, can move from the "Higgs Factory" emittance point towards larger longitudinal and smaller transverse emittances, they have not been shown to provide the higher luminosity along with small energy spread at the Higgs called for by Rubbia [4]. One scheme that could potentially satisfy Rubbia's requirements is Derbenev's "Phase-resonant Ionization Cooling" (PIC), which has been shown to work in principle but still requires a detailed aberration correction scheme to be worked out [57]. PIC is based on the "inverse" of slow extraction: a resonance is used to drive the beam towards small displacement and large angle, with ionization-cooling absorbers providing beam angular stabilization (Fig. 13). Other schemes for reaching smaller emittances have also been proposed [58–60]. Rubbia's goal provides an excellent challenge for next-generation studies.

FRICTIONAL COOLING

The schemes described above all work at momenta well above the dE/dx Bragg peak, and most operate at $\gamma\beta \approx 2$, near the ionization *minimum*. An entirely different approach seeks to exploit the much higher ionization energy-loss rate near the Bragg peak ($\gamma\beta \approx 0.01$) [17] but must cope with significant challenges (e.g., sufficiently rapid acceleration, making windows thin enough to overcome multiple scattering, and avoiding high-voltage breakdown in gas, or elec-

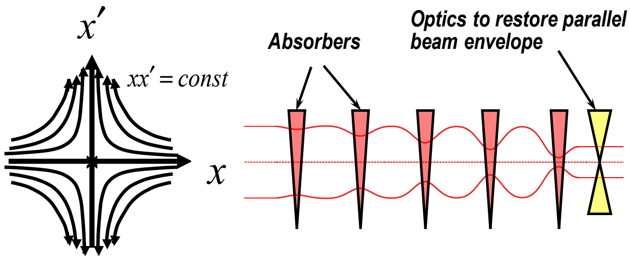

Figure 13: (Left) PIC-induced hyperbolic beam motion in horizontal plane; (right) PIC lattice concept.

tron multiplication in thin foils). This "frictional cooling" regime has been studied experimentally [61] and R&D continues [11,62]. A potential conceptual advance, the "particle refrigerator" [63], seeks to increase frictional cooling channel energy acceptance by two to three orders of magnitude, could lead to very compact high-flux muon sources, and may also be applicable to decelerating and cooling other particle species besides muons [64]. In contrast to the schemes discussed previously, by taking advantage of the *positive* slope of the dE/dx curve just below the Bragg peak, frictional cooling can cool directly in 6D, without emittance exchange.

R&D ISSUES

High-Field Solenoids

Since muon ionization cooling depends on strongly focusing muons as they traverse material, high-field magnets are a requirement for low emittance to be achieved. The 6D cooling channels studied by MAP achieve their goals using solenoids wound with NbTi or Nb_3Sn conductor. To go beyond those goals will require high-temperature (HTS) magnets (as already discussed in the "final cooling" context). The PBL-BNL progress on such magnets [53] (achieving 15 T — a world record for an all-HTS magnet) is thus quite encouraging and bodes well both for final cooling and for extension of existing 6D cooling-lattice designs to yet lower emittances. The continued development of such magnets is anticipated for other purposes [65] and will be closely watched by muon-cooling proponents.

RF Technology

A "cost driver" for muon accelerator facilities is RF acceleration. Most ionization cooling channel designs require operation of RF cavities in multi-tesla fields, precluding the use of superconducting cavities. To accommodate the large initial beam sizes, the lowest cavity frequencies are typically in the ballpark of 200–325 MHz; however, much of the R&D is done on "1/4-scale" (805 MHz) prototypes. (These are not only easier to fabricate, test, and modify, but are also similar to those that would be used in later stages of the cooling system, where the beam is smaller.) Cavity electrical efficiency is maximized by "pillbox" geometry, with apertures closed by thin conductive windows (Fig. 14) — a technique suitable

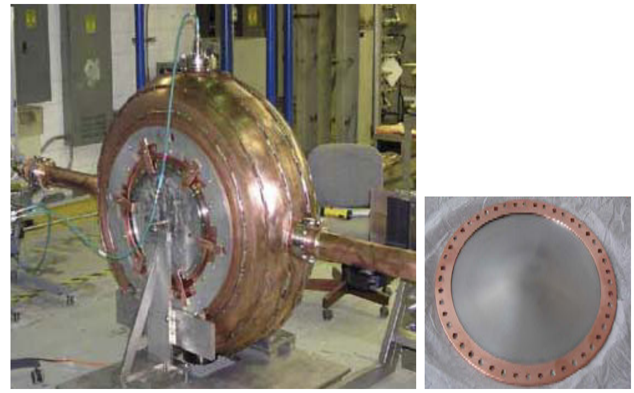

Figure 14: (left) MICE prototype 201 MHz cavity; (right) photo of curved beryllium window for 805 MHz cavity.

only for muons. For a given input power or maximum surface electric field, closed-aperture cavities have twice the on-axis accelerating gradient of standard, open-cell cavities. They incur the possible penalty of focused surface-emitted electrons from one window being accelerated across the gap and damaging the window opposite.

While the maximum magnetic field on the RF-cavity windows in the MICE cooling lattice is about 2 T, in later cooling stages, where lower equilibrium emittance requires stronger focusing, the field is many times stronger. Early data obtained by the NFMCC on an 805 MHz copper cavity operated in a solenoidal magnetic field [66] indicated that beyond a limiting accelerating gradient, damaging sparks occurred, degrading the cavity conditioning. The observed loss in accelerating gradient ranged up to a factor ≈ 3 at 4 T. However, more recent cavities display a far less egregious behavior [67], and the early results now appear to have been related to coupler arrangement and other design details that have since been improved. Thus cavity operation in multi-tesla magnetic field seems no longer a potential showstopper [68]. This is one of the major pieces of recent progress in muon cooling R&D.

Cavities pressurized with hydrogen gas were initially proposed as a means of raising operating gradients via the Paschen effect [69]. They were subsequently found to mitigate magnetic-field-induced gradient degradation as well (Fig. 15) [70]. As mentioned, used aggressively, they enable the continuous, "combined-function," HCC cooling channels in which the ionization energy loss and re-acceleration take place simultaneously throughout the length of the channel [46, 69]. A less ambitious application has also been suggested: using them in a "conventional" ionization-cooling channel (e.g., those of Figs. 7 and 11) with just enough hydrogen pressure to overcome any magnetic-field-induced degradation [71]. In pressurized cavities a potential pitfall is cavity loading due to acceleration of ionization electrons [72]; theory and experimental studies show that this can be overcome via a small (0.01%) admixture of electronegative gas [67, 72, 73].

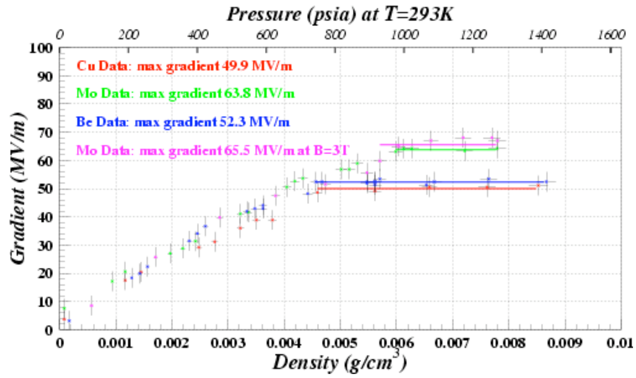

Figure 15: Observed dependence of maximum safe surface electric field in GH_2-pressurized 805 MHz cavity vs. hydrogen density and pressure for various electrode materials. Molybdenum electrodes were tested without (green points) and with (magenta) a 3 T axial field, with no observed degradation in maximum safe electric field [70].

System Tests

As mentioned, the MICE experiment is assembling a cooling cell and testing it in a muon beam, which will go a long way towards establishing the feasibility of the ionization cooling technique. Beyond MICE, it will be desirable, or even essential, to demonstrate the performance of the chosen 6D cooling lattice, as well as PIC or whichever advanced technique is chosen to go beyond the MAP emittance specs.

CONCLUSIONS

Muon cooling looks feasible, both for neutrino factories and muon colliders. Promising designs for these facilities have been conceived. The neutrino factory has been shown to be the best future facility for the precision study of neutrino oscillation and the search for non-standard neutrino physics. Muon colliders remain compelling and have been proposed by Rubbia as especially well suited for the precision study of the Higgs, provided the luminosity can be increased over that of the MAP Higgs Factory design. The latter would thus benefit from new cooling ideas that go beyond "conventional" ionization cooling; appealing solutions have been proposed and are the subject of ongoing work. A premature end of this U.S. R&D program has however been dictated by the P5 committee. In light of interest in Europe [74], it is hoped that muon-cooling research will nevertheless continue to be pursued.

ACKNOWLEDGMENTS

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REFERENCES

[1] R. Palmer, "Muon Colliders," Rev. Accel. Sci. Tech. **7** (2015) 137.

[2] S. Geer, "Neutrino beams from muon storage rings: Characteristics and physics potential," Phys. Rev. D **57**, 6989 (1998); *ibid.* **59**, 039903E (1999);
C. T. Rogers *et al.*, "Muon front end for the neutrino factory," Phys. Rev. ST Accel. Beams **16**, 040104 (2013);
S. Choubey *et al.* (IDS-NF Collaboration), "International Design Study for the Neutrino Factory, Interim Design Report," arXiv:1112.2853 [hep-ex] (Mar. 2011);
A. Bandyopadhyay *et al.*, "Physics at a future Neutrino Factory and super-beam facility," Rep. Prog. Phys. **72**, 106201 (2009), arXiv:0802.4023 [physics.acc-ph];
M. Lindner, "The physics potential of future long baseline neutrino oscillation experiments," in *Neutrino Mass*, ed. G. Altarelli, K. Winter, Springer Tracts in Modern Physics **190**, 209 (2003);
M. Apollonio *et al.*, "Oscillation Physics with a Neutrino Factory," CERN-TH-2002-208 (Oct. 2002);
C. Albright *et al.*, "Physics at a Neutrino Factory," FERMILAB-FN-692 (May 2000), hep-ex/0008064.

[3] D. Neuffer, "Progress of Front End and HFOFO Snake," this workshop.

[4] C. Rubbia, "A complete demonstrator of a muon cooled Higgs factory," arXiv:1308.6612 (2013), and presentation to Muon Accelerator Program Spring 2015 Meeting, Fermilab, May 18–23, 2015, `https://indico.fnal.gov/conferenceDisplay.py?confId=9752` and `http://tinyurl.com/oe9yesf` (2015).

[5] K. Lane, "Is the Standard Model about to crater?," CERN Courier, Nov., 2015, p. 21, `http://cerncourier.com/cws/article/cern/62874`.

[6] J.-P. Delahaye *et al.*, "A Staged Muon Accelerator Facility for Neutrino and Collider Physics," Proc. IPAC2014 Conference, Dresden, Germany, paper WEZA02 (2014).

[7] "Enabling Intensity and Energy Frontier Science with a Muon Accelerator Facility in the U.S.: A White Paper Submitted to the 2013 U.S. Community Summer Study of the Division of Particles and Fields of the American Physical Society," ed. J.-P. Delahaye *et al.*, arXiv:1308.0494 (2013).

[8] See e.g. D. Taqqu, "Ultraslow Muonium for a Muon beam of ultra high quality," Phys. Procedia **17** (2011) 216.

[9] Mu2e home page: `http://mu2e.fnal.gov/`;
C. M. Ankenbrandt *et al.*, "Intense Stopping Muon Beams," presented at PAC09 (*op. cit.*), paper MO6RFP080;
K. Yonehara *et al.*, "MANX, A 6-D Muon Beam Cooling Experiment for RAL," *ibid.*, paper WE6PFP090;
S. A. Kahn *et al.*, "Integrating the MANX 6-D Muon Cooling Experiment into the MICE Spectrometers," *ibid.*, paper WE6PFP095.

[10] D. M. Kaplan *et al.*, "Measuring Antimatter Gravity with Muonium," Proc. New Frontiers in Physics 2014, EPJ Web Conf. **95** (2015) 05008.

[11] A. Knecht, this workshop.

[12] F. F. Tikhonin, "On the Effects with Muon Colliding Beams," JINR Report P2-4120 (Dubna, 1968);
G. I. Budker, "Accelerators and Colliding Beams," in Proc.

7th Int. Conf. (Yerevan, 1969); extract available in AIP Conf. Proc. **352**, 4 (1996).

[13] D. V. Neuffer, R. B. Palmer, "A high-energy high-luminosity μ^+-μ^- collider," Proc. 1994 Eur. Particle Accelerator Conf. (EPAC94), p. 52;
R. Palmer *et al.*, Nucl. Phys. Proc. Suppl. **51A** (1996) 61;
C. M. Ankenbrandt *et al.*, "Status of Muon Collider Research and Development and Future Plans," Phys. Rev. ST Accel. Beams **2**, 081001 (1999);
D. Ayres *et al.*, "Expression of Interest in R&D towards a Neutrino Factory Based on a Storage Ring and a Muon Collider," arXiv:physics/9911009.

[14] Y. M. Ado, V. I. Balbekov, "Use of ionization friction in the storage of heavy particles," At. Energ. **31**(1) 40 (1971), English translation in Atomic Energy (Springer) **31**(1) 731, http://www.springerlink.com/content/v766810126338571/.

[15] G. I. Budker, A. N. Skrinsky, Sov. J. Part. Nucl. **21**, 277 (1978);
A. N. Skrinsky, V. V. Parkhomchuk, Sov. J. Part. Nucl. **12**, 223 (1981);
D. Neuffer, "Principles and Applications of Muon Cooling," Part. Accel. **14** (1983) 75;
D. Neuffer, AIP Conf. Proc. **156**, p. 201 (1987).

[16] D. Neuffer, "$\mu^+\mu^-$ Colliders," Yellow Report CERN-99-12 (1999);
R. C. Fernow, J. C. Gallardo, Phys. Rev. E **52**, 1039 (1995).

[17] See e.g. "Passage of Particles Through Matter," in K. A. Olive *et al.* [Particle Data Group], Chin. Phys. C **38**, 090001 (2014).

[18] P. Kyberd *et al.*, "nuSTORM: Neutrinos from STORed Muons," arXiv:1206.0294 (2012).

[19] J.C. Gallardo *et al.*, "Muon Muon Collider: Feasibility Study," prepared for. 1996 DPF/DPB Summer Study on New Directions in High-Energy Physics (Snowmass96), BNL-52503, available from http://www.cap.bnl.gov/mumu/pubs/snowmass96.html

[20] S. Geer, in **Workshop on Physics at the First Muon Collider and Front-End of a Muon Collider**, S. Geer, R. Raja, eds., AIP Conf. Proc. **435** (American Institute of Physics, New York, 1998), p. 384.

[21] Neutrino Factory and Muon Collider Collaboration home page: http://cap.bnl.gov/mumu.

[22] B. Autin, A. Blondel, J. Ellis, "Prospective study of muon storage rings at CERN," Tech. Rep. CERN 99-02, CERN, Geneva, Switzerland, April 1999.

[23] D. Finley, N. Holtkamp (eds.), "Feasibility Study on a Neutrino Source Based on a Muon Storage Ring," SLAC-REPRINT-2000-054, FERMILAB-PUB-00/108-E (2000).

[24] S. Ozaki, R. Palmer, M. Zisman, J. Gallardo (Eds.), "Feasibility Study-II of a Muon-Based Neutrino Source," BNL-52623 (2001), available from http://www.cap.bnl.gov/mumu/studyii/FS2-report.html.

[25] Y. Kuno et al., "A feasibility study of a neutrino factory in Japan," HTTP://WWW-PRISM.KEK.JP/NUFACTJ/, May 2001.

[26] See NuFact 2015 website, https://indico.fnal.gov/conferenceDisplay.py?ovw=True&confId=8903, and links therefrom.

[27] G. Gregoire *et al.*, "An International Muon Ionization Cooling Experiment (MICE)," Proposal to Rutherford Appleton Laboratory, http://mice.iit.edu/micenotes/public/pdf/MICE0021/MICE0021.pdf;

[28] MERIT home page: http://proj-hiptarget.web.cern.ch/proj-hiptarget/.

[29] S. Machida *et al.*, Nat. Phys., March 2012 **8** 3 243;
T. R. Edgecock *et al.*, "EMMA – the World's First Non-scaling FFAG," Proc. 11th Eur. Particle Accelerator Conf. (EPAC 08), Magazzini del Cotone, Genoa, Italy, 23–27 June 2008, p. 3380.

[30] Muon Collider Task Force (MCTF) home page: https://mctf.fnal.gov/.

[31] Muon Accelerator Program home page: http://map.fnal.gov.

[32] Particle Physics Project Prioritization Panel, "Building for Discovery: Strategic Plan for U.S. Particle Physics in the Global Context," http://science.energy.gov/~/media/hep/hepap/pdf/May%202014/FINAL_P5_Report_053014.pdf

[33] Such plots have been in use for years within the muon collider R&D program; see, e.g., P. Snopok, "Muon Cooling," ICFA Beam Dyn. Newslett. **65**, 45 (2014).

[34] MICE website: http://mice.iit.edu.

[35] C. Rogers, "Status, Recent Results and Prospect of the International Muon Ionization Cooling Experiment (MICE)," this workshop.

[36] M. M. Alsharo'a *et al.*, "Recent progress in neutrino factory and muon collider research within the Muon Collaboration," Phys. Rev. ST Accel. Beams **6**, 081001 (2003).

[37] The ISS Accelerator Working Group, "Accelerator design concept for future neutrino facilities," JINST **4**, P07001 (2009);
S. Choubey *et al.* (IDS-NF Collaboration), "International Design Study for the Neutrino Factory, Interim Design Report," arXiv:1112.2853 [hep-ex] (Mar. 2011).

[38] Y. Alexahin, "Helical FOFO Snake for Initial 6D Cooling of Muons," ICFA Beam Dyn. Newslett. **65**, 49 (2014).

[39] R. B. Palmer *et al.*, "A Complete Scheme of Ionization Cooling for a Muon Collider," Proc. 2007 Particle Accelerator Conf. (PAC07), p. 3193 (2007), arXiv:0711.4275 [physics.acc-ph].

[40] R. P. Johnson, Ya. Derbenev, "Low Emittance Muon Colliders," in Proc. PAC07 (*op. cit.*), p. 706.

[41] Muons, Inc. home page: http://www.muonsinc.com.

[42] A now-defunct SBIR-funded R&D company led by D. Cline.

[43] V. Balbekov *et al.*, "Muon Ring Cooler for the MUCOOL Experiment," Proc. 2001 Particle Accelerator Conf. (PAC01), p. 3867.

[44] R. B. Palmer *et al.*, "Ionization cooling ring for muons," Phys. Rev. ST Accel. Beams **8**, 061003 (2005).

[45] X. Ding *et al.*, "6D 'Garren' snake cooler and ring cooler for $\mu^\pm$ cooling of a muon collider," Nucl. Instrum. Meth. A **768** (2014) 9;
D. Cline *et al.*, "Comparison of 6D Ring Cooler Schemes and Dipole Cooler for $\mu^+\mu^-$ Collider Development," Proc. PAC07 (*op. cit.*), p. 3038;

H. Kirk *et al.*, "Muon Storage Rings for 6D Phase-Space Cooling," Proc. 2003 Particle Accelerator Conf. (PAC03), p. 2008.

[46] Ya. Derbenev, R. P. Johnson, "Six-dimensional muon beam cooling using a homogeneous absorber: Concepts, beam dynamics, cooling decrements, and equilibrium emittances in a helical dipole channel," Phys. Rev. ST Accel. Beams **8**, 041002 (2005).

[47] P. Snopok, G. Hanson, A. Klier, "Recent progress on the 6D cooling simulations in the Guggenheim channel," Int. J. Mod. Phys. A **24**, 987 (2009).

[48] S. A. Kahn *et al.*, "Magnet System for Helical Muon Cooling Channels," Proc. PAC07 (*op. cit.*), p. 443;
V. Kashikhin *et al.*, "Magnets for the Manx 6-D Muon Cooling Demonstration Experiment.," *ibid.*, p. 461.

[49] K. Yonehara, "Study of Helical Cooling Channel for Intense Muon Source," this workshop.

[50] V. Balbekov, "R_FOFO snake channel for 6D muon cooling," `http://map-docdb.fnal.gov/cgi-bin/ShowDocument?docid=4365`.

[51] D. Stratakis, R. Palmer, "Rectilinear six-dimensional ionization cooling channel for a muon collider: A theoretical and numerical study," Phys. Rev. ST Accel. Beams **18**, 031003 (2015).

[52] D. Stratakis, "Rectilinear Channel for Muon Cooling Towards Micron Scale Emittances," this workshop.

[53] R. Gupta *et al.*, "High Field HTS Solenoid for a Muon Collider–Demonstrations, Challenges, and Strategies," IEEE Trans. Appl. Supercond. **24** (2014) 4301705.

[54] H. K. Sayed, R. Palmer, D. Neuffer, "High Field - Low Energy Muon Ionization Cooling Channel," Phys. Rev. ST Accel. Beams **18**, 091001 (2015).

[55] D. Neuffer, "Final Cooling for a High Energy High Luminosity Collider," this workshop;
D. Neuffer *et al.*, "Final Cooling for a High Energy High-Luminosity High-Energy Lepton Collider," Proc. PAC2015, paper TUBD2.

[56] J. G. Acosta, "Final Muon Ionization Cooling Channel using Quadrupole Doublets for Strong Focusing," this workshop.

[57] V. Morozov, "Progress on Parametric-resonance Ionization Cooling," this workshop;
A. Afanasev *et al.*, "Epicyclic Helical Channels for Parametric Resonance Ionization Cooling," presented at PAC09 (*op. cit.*), paper FR5RFP012.

[58] Y. Fukui *et al.*, "Progress in Designing a Muon Cooling Ring with Lithium Lenses," Proc. 2003 Particle Accelerator Conf. (PAC03), p. 1787.

[59] A. Zholents, M. Zolotorev, W. Wan, "Optical stochastic cooling of muons," Phys. Rev. ST Accel. Beams **4**, 031001 (2001).

[60] See e.g. V. Litvinenko, "Recent Progress in the Coherent Electron Cooling Experiment," this workshop.

[61] M. Mühlbauer *et al.*, Hyperfine Interact. **119**, 305 (1999).

[62] D. Greenwald, Y. Bao, A. Caldwell, "Frictional Cooling Scheme for Use in a Muon Collider ," Proc. NuFact09, AIP Conf. Proc. **1222** (2010) 293;
D. J. Summers *et al.*, "6D muon ionization cooling with an inverse cyclotron," Proc. Int. Workshop on Beam Cooling and Related Topics (COOL05), AIP Conf. Proc. **821**, 432 (2006).

[63] T. J. Roberts, D. M. Kaplan, "Particle Refrigerator," presented PAC09 (*op. cit.*), paper WE6PFP096.

[64] For example, 40% survival of antiprotons with deceleration from several MeV down to 40 keV has been demonstrated in simulation studies [63].

[65] See e.g. H. W. Weijers *et al.*, "Tests of HTS insert coils above 30 T," Int. Symp. on Superconductivity, Tsukuba, Japan, 27–29 Oct. 2008, `http://www.magnet.fsu.edu/library/presentations/NHMFL_Presentation-4065.pdf`;
D. Hazelton, "Continued developments in high magnetic fields enabled by econd-generation high-temperature superconductors," Magnetics Conf,, Chicago, IL, 15–16 Apr. 2009, `http://www.superpower-inc.com/system/files/2009_0415_Magnetics_Conf_2G_Coils_DWH.pdf`;
D. C. Larbalestier *et al.*, Nat. Mat. **13** (2014) 375, `http://www.nature.com/nmat/journal/v13/n4/full/nmat3887.html`.

[66] D. Huang *et al.*, "RF Studies at Fermilab MuCool Test Area," presented at PAC09 (*op. cit.*), paper TU5PFP032;
D. Huang, "Vacuum RF Tests," 2009 NFMCC Collaboration Meeting, available from `http://www.cap.bnl.gov/mumu/conf/MC-090125/agenda.html`.

[67] B. Freemire, " Rf Technologies for Ionization Cooling Channels," this workshop.

[68] M. Palmer, "The Muon Accelerator Program: R&D Towards Future Neutrino Factory and Lepton Collider Capabilities," this workshop.

[69] R. P. Johnson *et al.*, "Gaseous Hydrogen and Muon Accelerators," AIP Conf. Proc. **671**, 328 (2003);
R. Johnson, D. M. Kaplan, "A Gaseous Energy Absorber for Ionization Cooling of Muon Beams, MuCool Note 195, `http://nfmcc-docdb.fnal.gov/cgi-bin/RetrieveFile?docid=195&version=1&filename=muc0195.pdf` (2001).

[70] P. M. Hanlet *et al.*, "Studies of RF Breakdown of Metals in Dense Gases," Proc. 2005 Part. Accel. Conf. (PAC 05), Knoxville, Tennessee, 16–20 May 2005, p. 3259.

[71] J. C. Gallardo, M. Zisman, "A Possible Hybrid Cooling Channel for a Neutrino Factory," Proc. IPAC'10, paper WEPE074;
J. C. Gallardo, M. Zisman, "Thoughts on Incorporating HPRF in a Linear Cooling Channel," Proc. NuFact09, AIP Conf. Proc. **1222** (2010) 308.

[72] A. Tollestrup, "Handbook for Gas Filled RF Cavity Aficionados," FERMILAB-TM-2430-APC, May 2009, `http://www-spires.fnal.gov/spires/find/hep?key=8276862`.

[73] M. Chung *et al.*, "Pressurized H2 RF Cavities in Ionizing Beams and Magnetic Fields," Phys. Rev. Lett. **111** (2013) 184802.

[74] "Discussion of the scientific potential of muon beams," CERN, 17 Nov. 2015, `https://indico.cern.ch/event/450863/timetable/#20151118`.