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PAPER

# Relativistic quantum dynamics of scalar particles in the rainbow formalism of gravity

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## Abstract

In the present article, we investigate the Klein–Gordon equation (KGE) in a topologically trivial Gödel-type space-time in the context of rainbow gravity (RG). Exact solutions and energy spectrum of scalar particles are obtained for the considered model. Also, the same systems are studied with the existence of the Klein–Gordon oscillator (KGO) potential. Results are evaluated by considering two different rainbow functions and they are analyzed graphically. We observe that the energy spectrum of scalar particles is modified by rainbow functions compared to the solutions obtained via the ordinary general relativity (GR) theory.

## 1. Introduction

Einstein's GR theory is the ultimate 'classical' theory introduced to describe the nature of space-time and has many well-tested results [1–4]. According to the GR, the existence of matter and energy warps the geometry of space-time and alterations in the geometry influence the dynamics of the matter and fields. The curvature of geometry is equivalent to a gravitational potential and affects the motion of particles through the space-time. GR presents the most compatible results with the experimental data obtained for the novel effects of gravity such as gravitational waves, gravitational lensing and gravitational time dilation. Therewithal, GR has become a crucial tool for modern cosmology and astrophysics in the research of black holes. However, in addition to these achievements, GR alone is inadequate to examine the early universe and there are modified GR theories [5–11] in the literature in order to overcome difficulties such as inability to renormalize the quantum field theory form of gravitation and models predicting the slowdown of the Universe. These modified theories should be consistent with successful predictions of GR and the observational data [12–14] that reveal the acceleration of universe.

Modified theories aiming to investigate the quantum effects of gravity, in particular that should be active in the early stages of the Universe, have become a significant area of the modern physics. These proposed theories should be utilized to avoid the initial singularity through a potential barrier. Although there isn't a completely self-consistent theory for QG, semi-classical approaches have gained a lot of attention in recent years [15]. For this purpose, an approach presented by Amelino-Camelia known as 'Doubly Special Relativity' (DSR) addresses the minimum accessible regions in which the QG effect are dominant [16]. Consequently, it has been demonstrated in some observations [17] that at such ultra-high energy scales, the dynamics of particles are affected just as in curved space-time and depending on that the relativistic dispersion relation has to be modified accordingly [17, 18]. Afterwards, Magueijo and Smolin generalized DSR to 'Doubly General Relativity' (DGR) by considering gravity [19]. DGR implies that at high energy regimes geometry of classical space-time affected by probing particles with various energies and becomes a 'running geometry', namely the standard metric is deformed.

The RG [20, 21] has an increasing interest in recent years and offers results compatible with the predictions of the GR in understanding the quantum effects of gravity. The RG possesses the invariance of both the speed of light and Planck's energy [22, 23]. In the context of RG, plenty of studies have been performed regarding to cosmology and astrophysics. Among many of them, the thermodynamics of modified black holes are studied in the context of RG and temperature of black holes probed by particles is found to be energy-dependent for some specific rainbow metrics [24–27]. Also, RG is combined with $f(R)$ theory [28, 29] to study charged black holes. By using the RG formalism, Ali *et al* have shown that the time taken by both the in-going observer to cross the black hole horizon and the asymptotic observer will be finite when the measurements were performed to the Planck's scale. They showed that RG removes information paradox of black holes [30]. In another study, Ali has studied thermodynamical features of the rainbow black hole and found that the RG can cause a new mass-temperature relation and prevents black holes for entire evaporation [31]. Awad *et al* [32] and Hendi *et al* [33] have studied the general conditions of removing the big bang singularities via FRW cosmology in the context of RG and Gauss-Bonnet gravity. The effects of rainbow functions on the features of neutron stars such as the variations of their maximum mass have been studied by considering a modified spherical symmetric metric [34, 35]. Additionally, in [36] it has been examined to establish a connection between Hŏrava-Lifshitz gravity and RG.

In recent decades, the dynamics of scalar (spin-0 mesons via the KGE), fermionic (spin-1/2 electrons via the Dirac equation) and vector particles (spin-1 such as $W^{\pm}$, $Z^0$ bosons and photons via the Duffin-Kemmer-Peatiau equation) have been extensively studied in curved space-time models [37–45]. Also, relativistic form of the harmonic oscillator [46] has been included in these equations as a linear interaction [47–52].

Recently, a great attention has been given to the solutions of Klein–Gordon and Dirac equations in the context of RG. In a study presented in [53], Santos *et al* obtained the energy spectrum of scalar particles in the cosmic string space-time for the KGO and a vector potential of the Coulomb-type for two different choices of rainbow functions. Bakke and Mota [54] studied the energy spectrum of fermionic particles in the same space-time via RG formalism described by two different rainbow functions. Also, Bezerra *et al* [55] investigated the energy spectrum of relativistic and non-relativistic scalar particles and analyzed the Landau levels in the cosmic string space-time in the framework of RG. In these studies, they showed that modified theory, the RG, leads to alterations in the usual energy levels obtained in considered space-time.

In the present study, our goal is also to investigate the energy spectrum of KGO in a Gödel-type metric in the context of RG described by two rainbow functions. This problem has previously been studied in [56] based on the GR formalism. We solve exactly the KGE for the KGO in a topologically trivial Gödel-type space-time deformed by the rainbow functions and obtain the energy levels. We compare the results obtained with those obtained in the absence of rainbow functions and analyze the energy levels graphically.

The structure of the paper will be as follows: In subsequent section, we summarize fundamentals of the RG framework and present some preliminary calculations. In section 3, we find exact solutions of the KGE and obtain a relation for the quantized energy levels of the scalar particles. Then, in section 4, we analyze quantum dynamics of the KGO in the RG formalism. Finally, we discuss the obtained results in section 5. Throughout the study, we use natural units $G = \hbar = c = 1$.

## 2. Preliminaries

The RG is a semi-classical approach to search for a quantum gravity theory at ultra-high energy regime. In the context of the theory, Lorentz symmetry breakdown occurs at this energy scale by introducing the rainbow functions in the energy-momentum dispersion relation [19, 21]. Accordingly, the metric is also modified as to be energy-dependent. In the modified metric, QG corrections are also represented by the rainbow functions. Therefore, definition of these rainbow functions has very important theoretical and phenomenological outcomes, such as removing the initial singularity in the cosmic period with rescaling the general form of the FRW metric in accordance with RG [57]. As the energy of probing particle moving in the space-time approaches to the Planck scale, the energy-dependence of the metric becomes stronger. The modified dispersion relation is given in the following form,

$$f^2(\chi)E^2 - g^2(\chi)p^2 = m^2, \tag{1}$$

where, $m$ is mass of particle, $\chi = \frac{E}{E_{Pl}}$ is the ratio of energy of the probing particle to the Planck's energy $E_{Pl}$, $f(\chi)$ and $g(\chi)$ are called as 'rainbow functions'. For the infrared energy regimes, the rainbow functions obey the below expression,

$$\lim_{\chi\to 0} f(\chi) = \lim_{\chi\to 0} g(\chi) = 1, \tag{2}$$

and usual dispersion relation and ordinary GR are recovered.

According to perspective of the RG, the modified equivalence principle suggests that metrics are given in terms of energy-dependent tetrads [58]

$$g_{\mu\nu}(E) = e^i_\mu(E) \otimes e^j_\nu(E)\eta_{ij}, \tag{3}$$

with

$$e^0_0(E) = \frac{1}{f(\chi)}\tilde{e}^0_0, \quad e^i_i(E) = \frac{1}{g(\chi)}\tilde{e}^i_i, \tag{4}$$

where the tilde refers to the energy-independent tetrads.

In GR, exact solutions of the field equations are crucial to our comprehension of real physical universe. However, Einstein's field equations also allow the existence of non-physical solutions, and the behavior of these solutions violates very important fundamental physical requirements such as the causality. Even so, these anomalous solutions provide a philosophical insight into the properties of general relativity [59]. The most famous of these types of solutions is Gödel's metric [60]. In that study, Gödel investigates a non-expanding but rotating solution of the Einstein field equations. The Gödel's metric contains closed time-like and closed null curves, so the metric is acausal [61]. It allows for timelike curves but does not include timelike geodesics. The so-called Gödel universe has no singularity or horizon [62, 63]. Later, Gödel also investigated the features of expanding and rotating spatially homogeneous solutions of field equations [64]. In the GR, attempts have been made on the generalization of Gödel's metric to eliminate closed timelike and closed null curves [65–67].

In a study performed by Ahmed [68], a topologically trivial Gödel-type space-time is introduced as

$$ds^2 = -dt^2 + dx^2 + (1 - \alpha^2x^2)dy^2 - 2\alpha x dt dy + dz^2, \tag{5}$$

where $\alpha > 0$ and $-\infty < t, x, y, z < \infty$ . As we mentioned in the previous section, energy spectrum of the scalar particles for this background is obtained via the ordinary GR formalism in [56]. In the RG gravity, with the help of relations (3) and (4), the line-element describing Gödel-type space-time should be modified as follows

$$ds^2 = -\frac{dt^2}{f^2(\chi)} + \frac{1}{g^2(\chi)}[dx^2 + (1 - \alpha^2x^2)dy^2 + dz^2] - \frac{2\alpha x}{f(\chi)g(\chi)}dtdy. \tag{6}$$

So, the covariant and contravariant forms of the metric tensor are written as

$$g_{\mu\nu} = \begin{pmatrix} -\frac{1}{f^2(\chi)} & 0 & -\frac{\alpha x}{f(\chi)g(\chi)} & 0 \\ 0 & \frac{1}{g^2(\chi)} & 0 & 0 \\ -\frac{\alpha x}{f(\chi)g(\chi)} & 0 & \frac{(1-\alpha^2x^2)}{g^2(\chi)} & 0 \\ 0 & 0 & 0 & \frac{1}{g^2(\chi)} \end{pmatrix}, \tag{7}$$

$$g^{\mu\nu} = \begin{pmatrix} -f^2(\chi)(1-\alpha^2x^2) & 0 & -f(\chi)g(\chi)\alpha x & 0 \\ 0 & g^2(\chi) & 0 & 0 \\ -f(\chi)g(\chi)\alpha x & 0 & g^2(\chi) & 0 \\ 0 & 0 & 0 & g^2(\chi) \end{pmatrix}. \tag{8}$$

## 3. Quantized energy for scalar particles

The covariant form of the equation of relativistic scalar particles in the presence of external vector fields in a curved space-time is written as [56]

$$\left[-\frac{1}{\sqrt{-g}}\nabla^{(+)}_\mu g^{\mu\nu}\sqrt{-g}\nabla^{(-)}_\nu + m^2\right]\psi = 0, \tag{9}$$

where, $m$ is mass of particle and

$$\sqrt{-g} = \sqrt{-detg_{\mu\nu}}, \tag{10}$$

$$\nabla^{(\pm)}_\mu = \partial_\mu \pm \Gamma_\mu + iA_\mu. \tag{11}$$

Also, $\Gamma_\mu$ and $A_\mu$ denote vector potentials coupled non-minimally and minimally to the KGE, respectively. For the considered metric, we have

$$\sqrt{-g} = \frac{1}{f(\chi)g^3(\chi)}. \tag{12}$$

Now, we are in a position to discuss exact solutions of the KGE and energy levels of scalar particles in a Gödel-type via the RG framework.

In the absence of vector potentials, equation (9) takes the following form in view of the metric (6)

$$\left[\frac{d^2}{dx^2} - (Ax^2 + Bx + C)\right]\phi(x) = 0, \tag{13}$$

where

$$A = E^2\alpha^2\frac{f^2(\chi)}{g^2(\chi)}, \tag{14}$$

$$B = 2EP_y\alpha\frac{f(\chi)}{g(\chi)}, \tag{15}$$

$$C = -E^2\frac{f^2(\chi)}{g^2(\chi)} + (P_y^2 + P_z^2) + \frac{m^2}{g^2(\chi)}. \tag{16}$$

Thus, we can write

$$\psi = e^{i(zP_z + yP_y - Et)}\phi(x), \tag{17}$$

since the Hamiltonian of the system is independent of $y$, $z$ and $t$ coordinates. Introducing a new variable $\xi = x + \frac{B}{2A}$ reduces equation (13) to the form of Weber equation [69]:

$$\frac{d^2\phi(\xi)}{d\xi^2} - \left[A\xi^2 - \frac{B^2}{4A} + C\right]\phi(\xi) = 0, \tag{18}$$

which is providing the conditions

$$A = k^2 > 0, \tag{19}$$

$$-\frac{B^2}{4A} + C = -(2n + 1)k. \tag{20}$$

As a result, the corresponding solutions are obtained as

$$\phi(\xi) = e^{-\frac{1}{2}k\xi^2}H_n(\sqrt{k}\,\xi), \tag{21}$$

where $k = \frac{E\alpha f(\chi)}{g(\chi)}$ and $H_n$ represent Hermite polynomials with $n = (1, 2, 3, \ldots)$. Making use of the conditions given in equations (19) and (20) one can obtain the energy spectrum of the probing particle as

$$E_{\pm} = \frac{g(\chi)}{f(\chi)}\left\{N \pm \sqrt{N^2 + \left[P_z^2 + \frac{m^2}{g^2(\chi)}\right]}\right\}, \tag{22}$$

where

$$N = \left(n + \frac{1}{2}\right)\alpha. \tag{23}$$

Under the infrared energy limit defined by equation (2), this energy expression reduces to the exact form obtained in [56]. Hence, energy levels can be evaluated exactly by focusing on two rainbow functions. On this purpose, we can consider two different scenarios for the rainbow functions:

$$1^{\text{st}}\ \text{Scenario:}\quad f(\chi) = g(\chi) = \frac{1}{1 - \lambda\chi}, \tag{24}$$

$$2^{\text{nd}}\ \text{Scenario:}\quad f(\chi) = 1,\quad g(\chi) = \sqrt{1 - \lambda\chi^2}. \tag{25}$$

where $\lambda$ is a dimensionless parameter. These forms of the rainbow functions have been taken into account previously in [15, 16, 18] and [70, 71] in studying of black hole physics, black hole thermodynamics, initial singularity problem and in solving the Dirac equation. The quantum effects are inserted in the rainbow functions and these functions represent the deformation of the early spacetime geometry as a result of the motion of

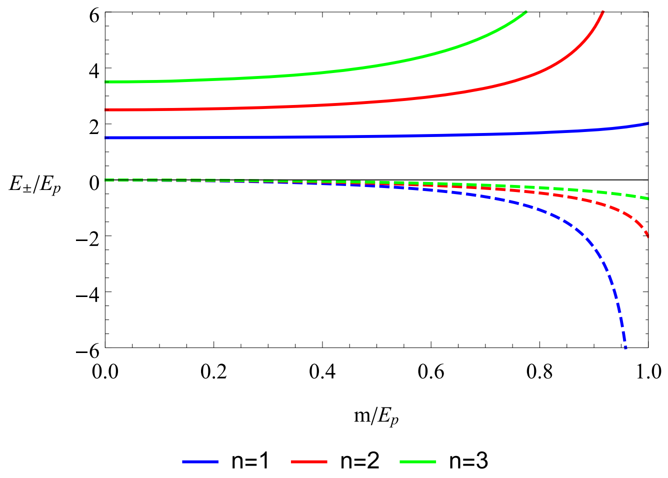


**Figure 1.** Evolution of $\frac{E_+}{E_{Pl}}$ (positive energy, upper side) and $\frac{E_-}{E_{Pl}}$ (negative energy, lower side) for $n = 1$ (blue), $n = 2$ (red) and $n = 3$ (green) energy levels with $E_{Pl} = 1$ (∼1.2∗$10^{17}$ GeV), $P_z = 0.1$ (in units of the Planck's energy), $\lambda = 1$ and $\alpha = 0.5$ according to the first scenario of the rainbow functions.

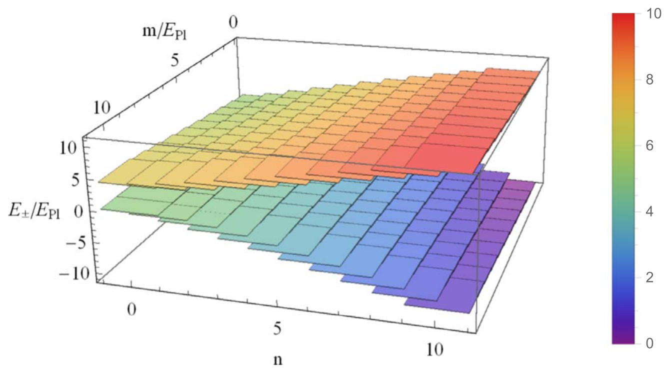


**Figure 2.** Behavior of $\frac{E_+}{E_{Pl}}$ (positive energy, upper side) and $\frac{E_-}{E_{Pl}}$ (negative energy, lower side) for the first scenario of the rainbow functions with assumptions $E_{Pl} = 80$ (∼9.8∗$10^{18}$ GeV), $P_z = 0.4$ (in units of the Planck's energy),$\lambda = 1$ and $\alpha = 0.2$.

probing particles. Therefore, defining suitable rainbow functions is significant in view of both theoretical and phenomenological studies.

Considering the first scenario of the rainbow functions leads to the following energy spectrum for the scalar particles

$$E_\pm = \frac{\widetilde{N} \pm \sqrt{\widetilde{N}^2 + (P_z^2 + m^2)\left(1 - \frac{\lambda^2 m^2}{E_{Pl}^2}\right)}}{\left(1 - \frac{\lambda^2 m^2}{E_{Pl}^2}\right)}, \tag{26}$$

where

$$\widetilde{N} = N - \frac{\lambda m^2}{E_{Pl}}. \tag{27}$$

We see that the energy spectrum depends on the rotation parameter $\alpha$ defined in the Gödel-type spacetime and Planck's energy. In figure 1, we depict the energy spectrum for the first three levels depending on the particle mass. One can see that there exist a symmetry breaking in the energy levels due to the rainbow functions. The spectrum is also illustrated in figure 2 as a function of $m$ and $n$, in which also the symmetry breaking is seen clearly.

For the second scenario of the rainbow functions presented in equation (25), energy levels are calculated as follows

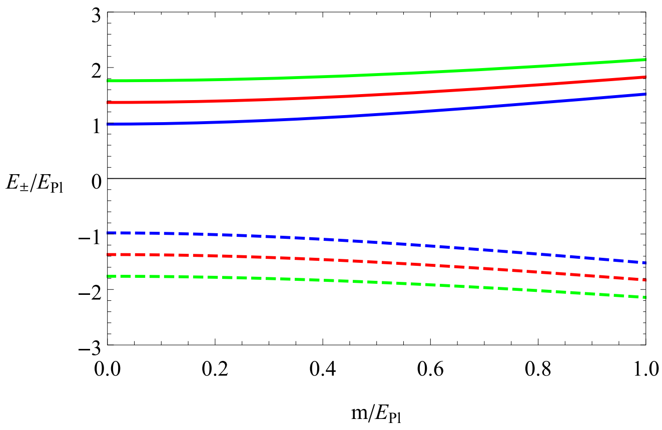


**Figure 3.** Evolution of $\frac{E_+}{E_{Pl}}$ (positive energy, upper side) and $\frac{E_-}{E_{Pl}}$ (negative energy, lower side) for $n = 1$ (blue), $n = 2$ (red) and $n = 3$ (green) energy levels with $E_{Pl} = 5$ ($\sim 6*10^{17}$ GeV), $P_z = 0.5$ (in units of the Planck's energy), $\lambda = 1$ and $\alpha = 0.25$ according to the second scenario of the rainbow functions.

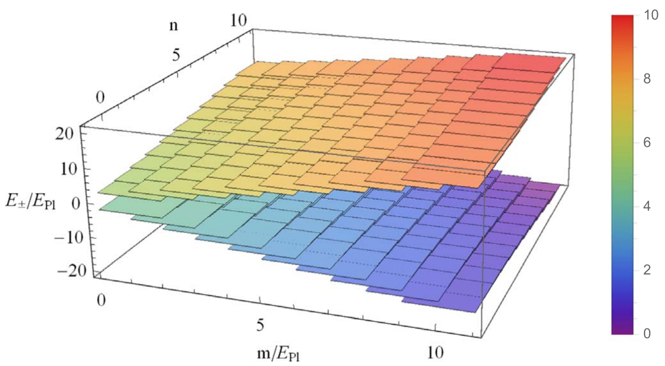


**Figure 4.** Behavior of $\frac{E_+}{E_{Pl}}$ (positive energy, upper side) and $\frac{E_-}{E_{Pl}}$ (negative energy, lower side) for the second scenario of the rainbow functions with assumptions $E_{Pl} = 90$ ($\sim 1.1*10^{19}$ GeV), $P_z = 0.4$ (in units of the Planck's energy), $\lambda = 1$ and $\alpha = 0.8$.

$$E_\pm = \pm\sqrt{\frac{\beta + \sqrt{\beta^2 - 4\gamma\tau}}{2\gamma}} \tag{28}$$

where

$$\gamma = 1 + \frac{P_z^2\lambda\left(2 + \frac{\lambda P_z^2}{E_{Pl}^2}\right) + 4\lambda N^2}{E_{Pl}^2} \tag{29}$$

$$\beta = 2(m^2 + P_z^2 + 2N^2) + \frac{2\lambda P_z^2(m^2 + P_z^2)}{E_{Pl}^2} \tag{30}$$

and

$$\tau = (m^4 + P_z^4 + 2m^2P_z^2) \tag{31}$$

We immediately realize that energy spectrum for the second scenario also contains the terms related to the geometry of the space-time and Planck's energy. Figure 3 shows the dependence of the energy on the mass of the particle for first three levels. Subsequently, in figure 4, the energy spectrum can be seen as a function of $m$ and $n$.

## 4. Energy relation for the KGO

The KGO is a linear interaction defined for spin-0 particles and resembles to the Dirac oscillator for spin-1/2 particles. It is a relativistic type of the harmonic oscillator and has been studied widely in literature. Some examples containing the KGO can be sorted such as its investigation in cosmic strings space-times for existance of a uniform magnetic field [47], the KGO coupled to the curved background within the Kaluza-Klein theory [72] and in a Gödel-type space-time [56]. Also, recently, it has been studied in the cosmic string space-time via the RG formalism [53].

In the present case, we aim to study exact solutions of KGE from another point of view and to obtain the energy spectrum for KGO in a Gödel-type RG formalism defined by the line-element (6). For this purpose, we take $A_\mu = 0$ and assume

$$\Gamma_\mu = (0,\ m\omega_0 x,\ 0,\ 0). \tag{32}$$

Substituting these forms of vector potentials into the KGE (9) yields the subsequent differential equation:

$$\left[\frac{d^2}{dx^2} - (\widetilde{A}x^2 + \widetilde{B}x + \widetilde{C})\right]\widetilde{\phi}(x) = 0, \tag{33}$$

where

$$\psi = e^{i(zP_z + yP_y - Et)}\widetilde{\phi}(x), \tag{34}$$

and below definitions are made

$$\widetilde{A} = E^2\alpha^2\frac{f^2(\chi)}{g^2(\chi)} + m^2\omega_0^2, \tag{35}$$

$$\widetilde{B} = 2EP_y\alpha\frac{f(\chi)}{g(\chi)}, \tag{36}$$

$$\widetilde{C} = -E^2\frac{f^2(\chi)}{g^2(\chi)} + P_y^2 + P_z^2 + m\omega_0 + \frac{m^2}{g^2(\chi)}. \tag{37}$$

Now, changing the variable by assuming $\widetilde{\xi} = x + \frac{\widetilde{B}}{2\widetilde{A}}$ transforms equation (32) again to the form of Weber equation [69]:

$$\frac{d^2\phi(\widetilde{\xi})}{d\widetilde{\xi}^2} - \left[\widetilde{A}\widetilde{\xi}^2 - \frac{\widetilde{B}^2}{4\widetilde{A}} + \widetilde{C}\right]\phi(\widetilde{\xi}) = 0, \tag{38}$$

Considering the conditions

$$\widetilde{A} = \widetilde{k}^2 > 0 \tag{39}$$

and

$$-\frac{\widetilde{B}^2}{4\widetilde{A}} + \widetilde{C} = -(2n+1)\widetilde{k}, \tag{40}$$

the solutions are obtained in terms of Hermite polynomials as

$$\widetilde{\phi}(\widetilde{\xi}) = e^{-\frac{1}{2}\widetilde{k}\widetilde{\xi}^2}H_n(\sqrt{\widetilde{k}}\,\widetilde{\xi}), \tag{41}$$

where

$$\widetilde{k} = \sqrt{\frac{E^2\alpha^2 f^2(\chi)}{g^2(\chi)} + m^2\omega_0^2}, \tag{42}$$

with $n = (1, 2, 3, \ldots)$. By using the condition given in equation (39), we achieve the following equation for the energy levels

$$\Omega^2\left(1 + \frac{P_y^2\alpha^2}{\Omega^2\alpha^2 + m^2\omega_0^2}\right) - \theta^2 = (2n+1)\sqrt{\Omega^2\alpha^2 + m^2\omega_0^2}, \tag{43}$$

where

$$\Omega = E\frac{f(\chi)}{g(\chi)}, \tag{44}$$

$$\theta = \sqrt{P_y^2 + P_z^2 + m\omega_0 + \frac{m^2}{g^2(\chi)}}. \tag{45}$$

One can easily conclude that the energy levels depend on the oscillator frequency, rotation parameter $\alpha$ and Planck's energy. For the infrared limit, it can be seen that the energy spectrum reduces to the result obtained via the GR theory in [56].

## 5. Conclusion

In this study, exact solutions of the KGE are obtained in the context of RG for a Gödel-type space-time. Passage from the ordinary GR to RG via redefinition of the metric provides to understand the dynamics of the particle on an ultra-energetic scale. In the redefinition of the metric, the rainbow functions are involved in the metric. The mathematical form of the obtained analytical solutions are same for GR and RG formalism and they both Hermite polynomials with different arguments. In the limit cases of rainbow functions, the solutions obtained in RG are reduced to results obtained in ordinary GR.

Energy levels of the probing particle are investigated for the absence of a vector potential and for the presence of the KGO by considering two rainbow functions. In both cases, it is seen that the results obtained here can be reduced to ones obtained in the GR under the limit $E_{Pl} \to 0$ [56].

In the infrared energy limit and for the case $P_z = m = 0$, equation (22) is reduced to the energy spectrum of the harmonic oscillator, whose frequency is $4\omega$ in $(d-1)$ dimension [56]. Also, energy spectra of both KGE and KGO are discrete and obtained depending on the vorticity parameter that is related to the angular velocity of rotation of space-time as $\omega = \frac{\alpha}{2}$, where the rotation vector is $\chi_\mu = -\frac{\alpha}{2}\delta_3^\mu$ and vorticity scalar is $\omega = (\chi_\mu \chi^\mu)^{\frac{1}{2}}$. As a result of vorticity of the Gödel-type universe, these spectra show that the angular velocity coupling takes place in $P_z$ direction.

In order to construct a relation between energy and mass of the Klein–Gordon particle, we take certain values for some parameters contained in the spectra such as $E_{Pl}$ and $\alpha$. As a result, symmetry breaking occurs in the usual KGE energy spectra due to rainbow functions. In the case of KGO, since the energy spectrum is obtained in a more complex form, a graphical analysis can not be performed easily due to the non-diagonal metric structure and the evolution of energy spectrum should be evaluated numerically.

## Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

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