The tensor-kinetic field in nuclear collisions

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Abstract. The role of the tensor terms in the Skyrme interaction is studied for their effect in dynamic calculations where non-zero contributions to the mean-field may arise, even when the starting nucleus, or nuclei are even-even and have no active time-odd potentials in the ground state. We study collisions in the test-bed ^{16}O-^{16}O system, and give a qualitative analysis of the behaviour of the time-odd tensor-kinetic density, which only appears in the mean field Hamiltonian in the presence of the tensor force. We find an axial excitation of this density is induced by a collision.

1. Introduction

In its original presentation [1], the Skyrme interaction was motivated as a low-momentum expansion of a nuclear contact interaction, suitable for use as an effective nuclear potential. This original version of the potential, as posited by Skyrme, featured terms up to second order in the momentum, though he did write down higher order terms and consider their importance. Crucially, he included a simple three-body force (and explored a four-body force) which gave it enough degrees of freedom to do reproduce a wide range of data following Vautherin and Brink's first Hartree-Fock calculations with the Skyrme interaction [2]. The Skyrme force is now ubiquitous in nuclear structure [3], as well as being widely used in nuclear reaction calculations using Time-Dependent Hartree-Fock theory [4, 5, 6, 7].

The original version of the Skyrme force included a tensor term. Its initial exploration within the Hartree-Fock framework concentrated on spherical doubly-magic nuclei [8], in which the effect of the tensor force is limited to varying the spin-orbit splitting. The extra degree of freedom afforded by the tensor force was not sufficiently compelling to mandate its use for calculations of the time, and it was largely, though not completely, ignored in subsequent years.

The effective tensor force has undergone a revival in nuclear structure, particularly since the understanding of its role in changing magic numbers in light nuclei [9]. The Skyrme version has now been widely explored for its effect on the structure of nuclei across the periodic table [10, 11, 12].

In this paper, we explore the effect of the Skyrme tensor force in dynamic processes (namely heavy-ion reactions), in which parts of the mean field are brought into play that are not relevant for ground state properties of even-even nuclei. In the next section we summarise the relevant expression of the tensor force, following which a description and discussion of our calculations is given, followed by a short conclusion.

2. Tensor force and time-dependent Hartree-Fock

The tensor force as introduced by Skyrme, but written in contemporary notation ([12], to where we refer the reader unfamiliar with the symbols) reads

$$
\begin{aligned}
V_t(\boldsymbol{r}_1,\boldsymbol{r}_2) \quad & \frac{t_e}{2}\Big(\left[3(\hat{\boldsymbol{\sigma}}_1\cdot\boldsymbol{k}')(\hat{\boldsymbol{\sigma}}_2\cdot\boldsymbol{k}') - (\hat{\boldsymbol{\sigma}}_1\cdot\hat{\boldsymbol{\sigma}}_2)\boldsymbol{k}'^2\right]\delta(\boldsymbol{r}_1-\boldsymbol{r}_2) \\
& +\delta(\boldsymbol{r}_1-\boldsymbol{r}_2)\left[3(\hat{\boldsymbol{\sigma}}_1\cdot\boldsymbol{k})(\hat{\boldsymbol{\sigma}}_2\cdot\boldsymbol{k}) - (\hat{\boldsymbol{\sigma}}_1\cdot\hat{\boldsymbol{\sigma}}_2)\boldsymbol{k}^2\right]\Big) \\
& +t_o\left[3(\hat{\boldsymbol{\sigma}}_1\cdot\boldsymbol{k}')\delta(\boldsymbol{r}_1-\boldsymbol{r}_2)(\hat{\boldsymbol{\sigma}}_2\cdot\boldsymbol{k}) - (\hat{\boldsymbol{\sigma}}_1\cdot\hat{\boldsymbol{\sigma}}_2)\boldsymbol{k}'\delta(\boldsymbol{r}_1-\boldsymbol{r}_2)\boldsymbol{k}\right], \qquad (1)
\end{aligned}
$$

where t_e and t_o are parameters to be fitted to data, labelling the terms with odd and even powers, respectively, of the momentum operators $\boldsymbol{k}$ and $\boldsymbol{k}'$. For the calculations presented in this work, we use the SLy5 [13] parameter set for the basic Skyrme parameters, as augmented by tensor terms fitted to single particle spacings [10].

The tensor terms give rise to a contribution to the Skyrme energy density functional (EDF) of the form [12, 15, 16]:

$$
\begin{aligned}
E_{sk}^{tensor} = & \int \Bigg\{ \frac{3}{16}\left[(3t_e - t_o)\,(\boldsymbol{\nabla}\cdot\boldsymbol{S}(\boldsymbol{r}))^2 - (3t_e+t_o)\sum_q(\boldsymbol{\nabla}\cdot\boldsymbol{S}_q(\boldsymbol{r}))^2\right] \\
& -\frac{1}{4}\Bigg[(t_e+t_o)\left(\boldsymbol{S}(\boldsymbol{r})\cdot\boldsymbol{T}(\boldsymbol{r}) - \sum_{\mu\nu}J_{\mu\nu}(\boldsymbol{r})^2\right) \\
& \quad -(t_e-t_o)\sum_q\left(\boldsymbol{S}_q(\boldsymbol{r})\cdot\boldsymbol{T}_q(\boldsymbol{r}) - \sum_{\mu\nu}J_{q,\mu\nu}(\boldsymbol{r})^2\right)\Bigg] \\
& +\frac{3}{4}\Bigg[(t_e+t_o)\left(\boldsymbol{S}(\boldsymbol{r})\cdot\boldsymbol{F}(\boldsymbol{r}) - \frac{1}{2}\sum_{\mu\nu}J_{\mu\nu}(\boldsymbol{r})J_{\nu\mu}(\boldsymbol{r})\right) \\
& \quad -(t_e-t_o)\sum_q\left(\boldsymbol{S}_q(\boldsymbol{r})\cdot\boldsymbol{F}_q(\boldsymbol{r}) - \sum_{\mu\nu}J_{q,\mu\nu}(\boldsymbol{r})J_{q,\nu\mu}(\boldsymbol{r})\right)\Bigg] \\
& +\frac{1}{16}\Bigg[(3t_e-t_o)\,\boldsymbol{S}(\boldsymbol{r})\cdot\boldsymbol{\nabla}^2\boldsymbol{S}(\boldsymbol{r}) \\
& \quad -(3t_e+t_o)\sum_q\boldsymbol{S}_q(\boldsymbol{r})\cdot\boldsymbol{\nabla}^2\boldsymbol{S}_q(\boldsymbol{r})\Bigg]\Bigg\}\,d\boldsymbol{r}. \qquad (2)
\end{aligned}
$$

Here, aside from t_o and t_e, the quantities are densities and currents, derived from the scalar and vector densities (see [12] for full details). We mention in particular the term including $\boldsymbol{S}\cdot\boldsymbol{F}$. It, alone of all terms in (2) does not feature in the density functional due to other parts of the Skyrme force, and in particular the so-called *tensor-kinetic* field,

$$F_\mu(\boldsymbol{r}) = \frac{1}{2}\sum_{\nu=x}^{z}\left(\nabla_\mu\nabla'_\nu + \nabla'_\mu\nabla_\nu\right)S_\nu(\boldsymbol{r},\boldsymbol{r}')|_{\boldsymbol{r}'=\boldsymbol{r}}, \qquad (3)$$

does not couple to the nucleus within the Skyrme EDF framework without active tensor parameters. We therefore study the appearance of this field in the present work.

To perform our calculations, we use time-dependent Hartree-Fock (TDHF), as originally conceived by Dirac [14], and now realised in a symmetry-unrestricted three-dimensional cartesian grid code, with the full version of the Skyrme force, including tensor terms.

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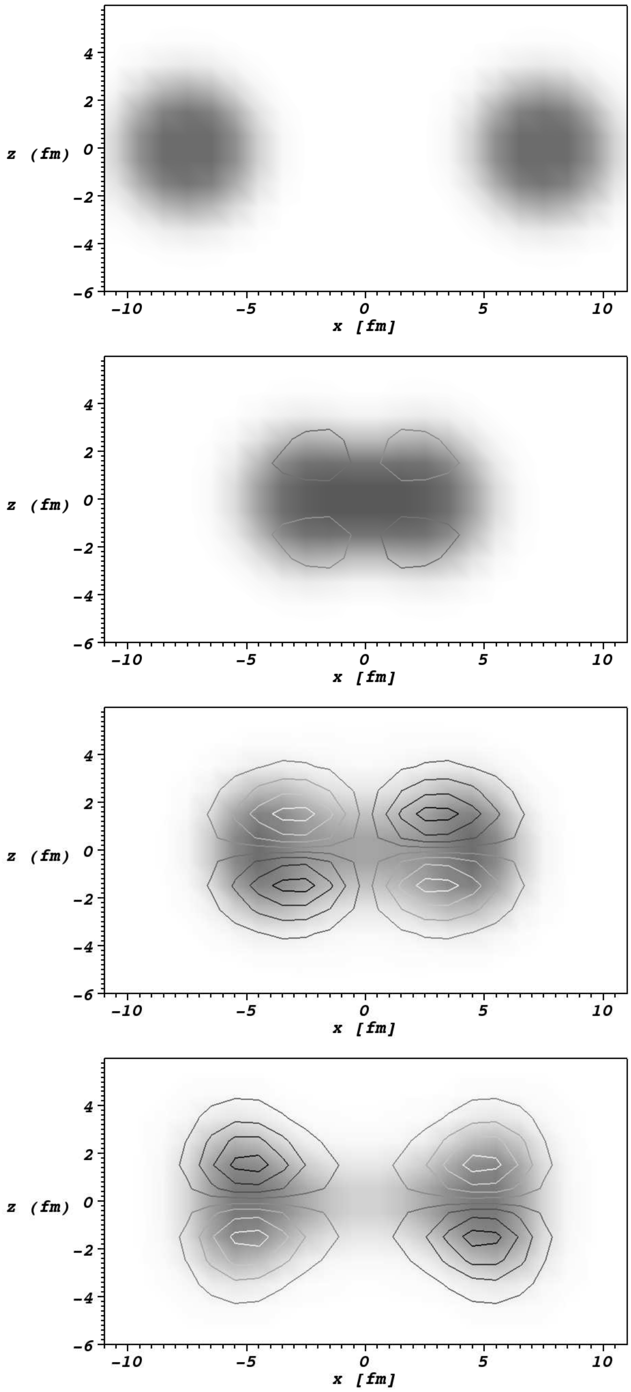

Figure 1. The densities (shading) and component of the tensor-kinetic density out of the x-z reaction plane at 4 instants in the central collision of ^{16}O + ^{16}O at 100 MeV. The contours representing the tensor-kinetic density run from -0.006 fm^{-1} (darkest contour) to +0.006 fm^{-9} (lightest contour). The times are, from top to bottom 0 fm/c, 68 fm/c, 120 fm/c and 160 fm/c. A fuller description of the dynamics is given in the text.

3. Results

As an example, we choose the collision of two ^{16}O nuclei at a centre of mass energy of 100 MeV. This has become a kind of standard reference point for TDHF calculations, following historical precedent [17].

Figure 1 shows snapshots of the time-evolution of the system set up as described. The shaded density gives an indication of the position of the nuclei. The calculation was carried out in a $(24\,\mathrm{fm})^3$ box, and we zoom-in and slice across the x-z reaction plane.

Narrating from top to bottom as time increases, we see the initial nuclei, separated at $t = 0$ with no active F_y density, until the nuclei collide. At 68 fm/c the first excitation of the tensor-kinetic density is seen. As the nuclei pass through each other (above about 65 MeV centre of mass energy for this Skyrme force, the nuclei have too much energy to fuse) a stronger excitation is seen. This consists of each fragment nucleus having an axial excitation of the $\boldsymbol{F}$ density, in opposite directions (dark contours going in to the paper / screen, light contours coming out).

This kind of excitation was seen in the spin density $\boldsymbol{S}$ in previous calculations using the SkM* force [18], and seems to be a general phenomenon. Here we are observing a derivative of the spin density, but since it is oscillatory, so too is its derivative. The oscillatory nature of the excitation can be seen in the last frame of Figure 1, since between 120 and 160 fm/c the excitation reverses direction. Effectively the spin-polarisation of the nuclear matter is excited into ring-like structures, which are then in a Lenz's law-like way opposed, resulting in a reversal of direction, which continues until the nucleus de-excites.

What is new in the present calculation is that the $\boldsymbol{F}$ density couples to the mean field and the density functional and affects the overall dynamics. The effect is small, as might be expected by the dominance of the non-tensor terms in the bulk nuclear properties. A guide to its size is given in Figure 2, in which the contribution to the total energy from the $\boldsymbol{S} \cdot \boldsymbol{F}$ term is shown as a function of time for the collision shown in Figure 1.

Clearly, the contributions from the $\boldsymbol{S} \cdot \boldsymbol{F}$ term are small in this case, given that the total

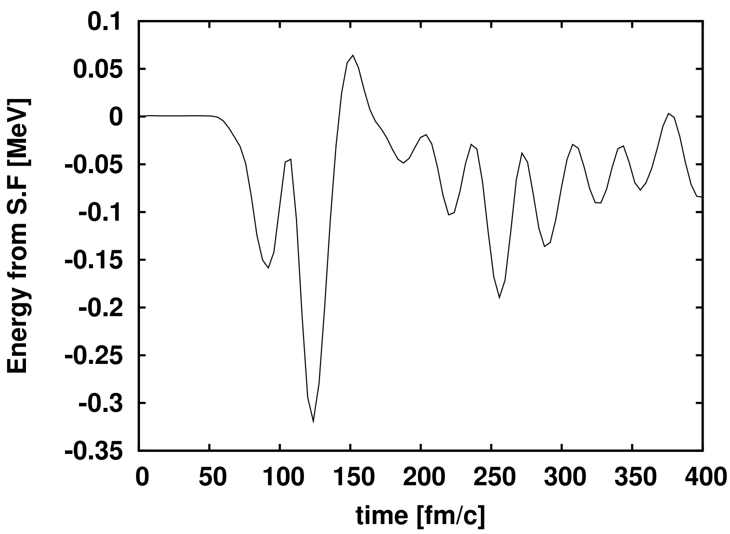

Figure 2. The contribution to the total energy from the $\boldsymbol{S} \cdot \boldsymbol{F}$ terms in the energy density functional.

energy of the system is around 400 MeV, but the inclusion of the tensor terms overall will have a combined effect from changing the ground state structure, which may be evident when moving away from doubly-magic nuclei, as well as in effects that depend on small changes to the overall energy, and in detailed structure effects. In particular, the location of the fusion window is sensitive to the addition of the tensor force by several MeV [16].

4. Conclusions

The full tensor force has been added to a full time-dependent Hartree-Fock model, and used to study heavy-ion collisions above the fusion window. The newly-active terms in the mean-field contribute during the collision thanks to the dynamic excitation of a non-zero spin polarization within the fragments.

The contribution of the new terms to the overall energy is small, and future work should evaluate the role of the tensor term in different dynamic observables. We are preparing follow-up work on the effect of the tensor parameterisations of the Skyrme force on the location of the fusion window and cross-sections. This complements other work on the effect of the tensor terms on giant resonances [19].

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References

[1] Skyrme T H R 1959 *Nucl. Phys.* **9** 615
[2] Vautherin D and Brink D M 1972 *Phys. Rev.* C **5** 626-647
[3] Bender M, Heenen P-H and Reinhard P-G 2003 *Rev. Mod. Phys.* **75** 121-180
[4] Golabek C and Simenel C 2009 *Phys. Rev. Lett.* **103** 042701
[5] Nakatsukasa T and Yabana K 2005 *Phys. Rev.* C **71** 024301
[6] Umar A S and Oberacker V E 2006 *Phys. Rev.* C **74** 024606
[7] Maruhn J A, Reinhard P-G, Stevenson P D, Rikovska Stone J and Strayer M R 2005 *Phys. Rev.* **71** 064328
[8] Stancu F, Brink D M and Flocard H 1977 *Phys. Lett.* B **68** 108
[9] Otsuka T, Suzuki T, Fujimoto R, Grawe H and Akaishi Y 2005 *Phys. Rev. Lett.* **95** 232502
[10] Golò G, Sagawa H, Fracasso S and Bortignon P F 2007 *Phys. Lett.* B **646** 227-231
[11] Suckling E B and Stevenson P D 2010 *EPL* **90** 12001
[12] Lesinski T, Bender M, Bennaceur K, Duguet T and Meyer J 2007 *Phys. Rev.* C **76** 014312
[13] Chabanat E, Bonche P, Haensel P, Meyer J and Schaeffer R 1998 *Nucl. Phys.* A **635** 231-256
[14] Dirac P A M 1930 *Proc. Camb. Philos. Soc.* **26** 376
[15] Perlińska E, Rohoziński S, Dobaczewski J and Nazarewicz W 2004 *Phys. Rev.* C **C 69** 014316
[16] Suckling E B 2011 *PhD thesis University of Surrey*
[17] Umar A S and Oberacker V E 2006 *Phys. Rev.* C **73** 054607
[18] Maruhn J A, Reinhard P-G, Stevenson P D and Strayer M R *Phys. Rev.* C **74** 027601
[19] Fracasso S, Stevenson P D and Suckling E B *in preparation.*