Quantifying uncertainties in the high energy neutrino cross-section

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DOI: 10.7529/ICRC2011/V04/0890

Abstract: We compare predictions for high energy neutrino and anti-neutrino deep inelastic scattering cross-sections within the conventional DGLAP formalism of next-to-leading order QCD, using the latest parton distribution functions such as CT10, HERAPDF1.5 and MSTW08 and taking account of PDF uncertainties. From this we derive a benchmark cross-section and uncertainty which is consistent with the results obtained earlier using the ZEUS-S PDFs. We advocate the use of this for analysing data from neutrino telescopes, in order to facilitate comparison between their results.

Keywords: Deep Inelastic Scattering, Neutrino Physics, High Energy Cosmic Rays.

1 Introduction

Searches for high-energy cosmic neutrinos rely on predictions for the neutrino cross-section at high energies. These have however sizeable uncertainties deriving from the uncertainties on the parton distribution functions (PDFs) of the nucleon. Conventional PDF fits use the next-to-leading-order (NLO) DGLAP formalism [1, 2, 3, 4] of QCD to make predictions for DIS cross-sections of leptons on hadrons. At low x it may be necessary to go beyond the DGLAP formalism in order to sum $\ln(1/x)$ diagrams, as in the BFKL formalism [5, 6, 7] (for recent work see Refs. [8, 9]), or to even consider non-linear terms as in the colour glass condensate model [10, 11]. While the exact theoretical framework at low x is still contested it has been suggested [13, 12, 14, 15, 16] that observations of ultra high-energy neutrinos could itself be used to measure the cross-section thereby constraining the models. It is therefore important to not only consider the prediction for the cross-section but also to estimate their uncertainties in the conventional NLO DGLAP formalism.

In the framework of the quark-parton model, high energy neutrino deep inelastic scattering (DIS) accesses large values of Q^2, the invariant mass of the exchanged vector boson, and small values of Bjorken x, the fraction of the momentum of the incoming nucleon taken by the struck quark. Thus in evaluating uncertainties on high energy neutrino DIS cross-sections it is important to use the most up-to-date information from the experiments at HERA, which have accessed the lowest x and highest Q^2 scales to date. H1 and ZEUS have now combined the data collected in the years 1994–2000 to give very accurate inclusive cross-sections in the range $6 \times 10^{-7} < x < 0.65$ and $0.045 < Q^2 < 30000$ GeV^2 [17]. These data have not been available (or not been used) in previous predictions [18, 19, 20]. It is the purpose of the present paper to re-evaluate the high energy cross-sections using the most up-to-date PDF sets, with particular emphasis on those which do use these precise, combined HERA data. The calculation is made using PDFs which were evaluated in NLO DGLAP fits, and our calculation of the neutrino structure functions and cross-sections is also made consistently at NLO. For further details, we refer the interested reader to Ref. [21].

2 Formalism

The kinematics of lepton hadron scattering is described in terms of the variables Q^2, Bjorken x, and y which measures the energy transfer between the lepton and hadron systems. The double differential charged current (CC) cross-section for neutrino and anti-neutrino production on isoscalar nucleon targets is given by [22]

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}x\,\mathrm{d}Q^2} = \frac{G_\mathrm{F}^2 M_W^4}{4\pi(Q^2+M_W^2)^2 x}\sigma_\mathrm{r},$$

where the reduced cross-section $\sigma_\mathrm{r}(\nu(\bar{\nu})N)$ is

$$\sigma_\mathrm{r} = \left[Y_+ F_2^\nu(x,Q^2) - y^2 F_\mathrm{L}^\nu(x,Q^2) + Y_- x F_3^\nu(x,Q^2)\right],$$

and F_2, xF_3 and F_L are related directly to quark momentum distributions, with $Y_\pm = 1 \pm (1-y)^2$.

The QCD predictions for these structure functions are obtained by solving the DGLAP evolution equations at NLO in the $\overline{\mathrm{MS}}$ scheme with the renormalisation and factorization scales both chosen to be Q^2. These equations yield the PDFs at all values of Q^2 provided these distributions have been input as functions of x at some input scale Q_0^2.

In QCD at leading order, the structure function F_L is identically zero, and the structure functions F_2 and xF_3 for charged current neutrino interactions on isoscalar targets can be identified with quark distributions. At NLO these expressions must be convoluted with appropriate coefficient functions in order to obtain the structure functions (and F_L is no longer zero) but these expressions still give us a good idea of the dominant contributions. Cross-sections for neutral current (NC) and anti-neutrino interactions are calculated in a similar way.

3 Parton Density Functions

Uncertainties on PDFs derive from two sources: experimental errors and parametrisation uncertainties. To allow for the estimation of the error induced in the predicted observable, i.e. cross-sections in the present case, modern PDF sets provide not only the best-fit PDF but also variants that reflect these different uncertainties. For experimental errors a set of variant PDFs, so-called eigenvectors, is obtained after diagonalisation of the error matrix. The eigenvectors are linearly independent such that the individual experimental errors can be added in quadrature. The variants for the parametrisation uncertainties are obtained from fits by varying certain parameter values (e.g. the starting scale Q_0^2 for evolution and the value of $\alpha_\mathrm{s}(M_Z)$) or the parametrisation for the input PDF parametrisation at Q_0^2.

The PDF4LHC group has recently benchmarked modern parton density functions [23]. Since our concern is with high energy neutrino cross-sections, rather than with LHC physics, we focus on PDF sets which make use of the newly combined accurate HERA data [17]. Of all the PDFs considered by the PDF4LHC only HERAPDF1.0 [17]) and NNPDF2.0 [24] used these data. However there has been a subsequent update of the CTEQ6.6 [25] PDFs to CT10 [26] which does use these data, while HERAPDF1.0 has recently updated to HERAPDF1.5 [27] using an preliminary combination of HERA data from 2003–2007 as well as the published combined data. We will utilise the CT10 and HERAPDF1.5 PDFs for the present study; we also consider the MSTW2008 PDFs in order to compare with other recent calculations of high energy neutrino cross-sections [20], although we caution that these have *not* included the most accurate HERA low x data relevant to the present study.

4 Results

The calculation of the CC and NC cross-sections in NLO has been performed using `DISPred` [28]. The PDFs have been implemented through the LHAPDF interface [29]. Particular care has been exercised to perform a self-consistent calculation. For example the PDFs from LHAPDF are mostly defined for a limited range in Q^2 and x and "freeze" beyond this range, which would result in underestimation of the cross-section at high energies; therefore we have used other implementations [30, 31]. Naturally, the cross sections have been calculated at a consistent order with respect to the PDFs.

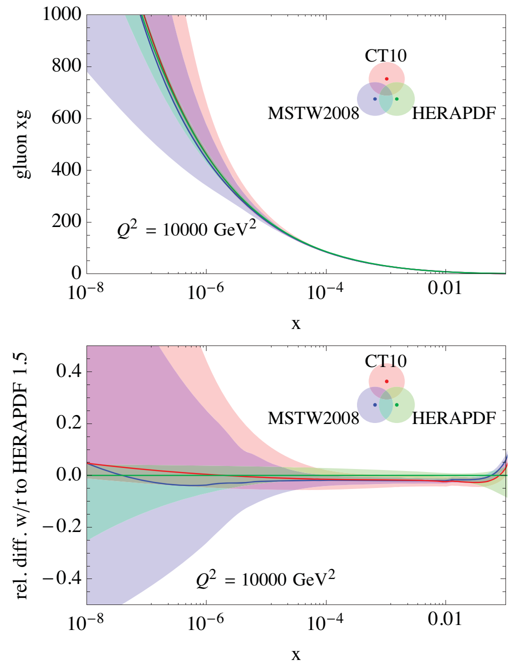

Figure 1: **Top panel:** Gluon structure function at $Q^2 = 10^4\,\mathrm{GeV}^2$ for the three PDF sets used. **Bottom panel:** The relative deviations and uncertainties (at 68% c.l.) with respect to the central value of HERAPDF1.5. The uncertainty bands are shown *with* member 9 for HERAPDF1.5 and member 52 for CT10.

Figure 1 compares the gluon PDF and its uncertainty at $Q^2 = 10000\,\mathrm{GeV}^2$ for the three PDFs which we consider. This value of Q^2 is in the middle of the range which contributes significantly to the neutrino cross-sections. We see that the central values of the gluon PDFs are all very similar, whereas the uncertainty estimates differ. The CT10 and HERAPDF1.5 uncertainties are actually very similar if we leave out member 52 from the CT10 error set. This error set was introduced into the CT10 analysis to allow for a larger uncertainty at low x [32]. Previous CTEQ analyses such as CTEQ6.6 [25] do not have such an extreme error set. The problem with such an *ad hoc* introduction of a steeply increasing gluon PDF is that at low x it leads to a very strong rise of the unphysical.

The larger error band of MSTW2008 is partly due to the fact that it does not include the most up to date HERA data, which have significantly reduced errors at low x. However the more striking difference between MSTW2008 and both

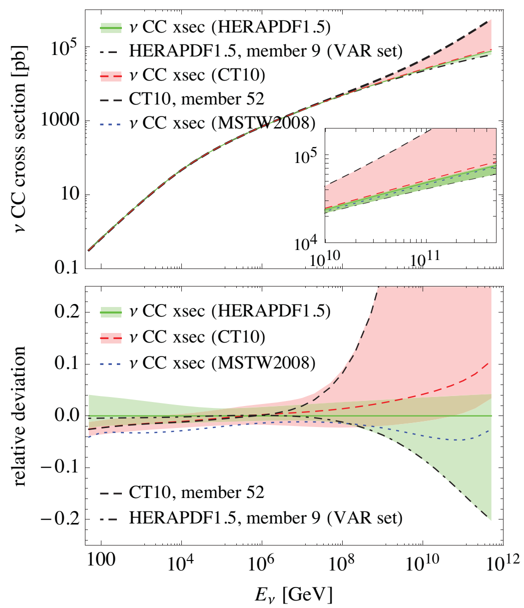

Figure 2: Comparison of the total cross-section (top panels) and uncertainties (bottom panels) for CC scattering as predicted by the HERAPDF1.5, CT10 and MSTW2008 (central member only) PDF sets. The cross-sections and deviations for member 9 of HERAPDF1.5 and member 52 of CT10 are indicated by the dashed and dot-dashed lines, respectively.

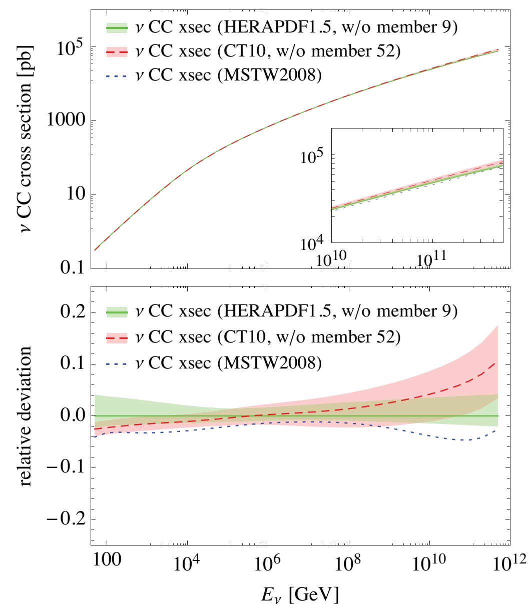

Figure 3: Same as Fig. 2, but excluding member 9 of the HERAPDF1.5 set and member 52 of the CT10 set.

HERAPDF1.5 and CT10 is the downward divergence of its error band which is due to the gluon becoming negative at low x, Q^2. At NLO the gluon PDF does not have to be positive, although one might consider that it going negative signals a breakdown of the DGLAP formalism. However measurable quantities such as the longitudinal structure function F_L, which is closely related to the gluon at small x, *must* be positive. The CT(EQ) analyses do not allow such negative gluon variants. We have checked for HERAPDF1.5 that the (moderately) negative gluon does not lead to negative F_L. The MSTW2008 set however includes member PDF sets with negative gluons that do lead to negative F_L and are thus unphysical.

In Fig 2 (top panel) we compare the CC cross-sections, along with their total uncertainties (including that coming from the variation of $\alpha_s(M_Z)$), as predicted by HERAPDF1.5 and CT10. The MSTW2008 central prediction is also included for comparison. In Fig 2 (bottom panel) we emphasize the small differences in the central values of the PDFs and their relative uncertainties. In order to highlight the effect of the extreme members of HERAPDF1.5 and CT10 in Figs 3, we show these plots without member 9 of the HERAPDF15 variations (which allows for the gluon to become negative at low x and Q^2) and without member 52 for CT10 (the cross-section for which rises $\propto E_\nu^{0.7}$ whereas for the central member it rises $\propto E_\nu^{0.3}$). However *any* power-law rise in the cross-section will eventually violate the Froissart bound, which requires the rise to be no faster than $\log^2 s$ [33]. This should result in a reduction of the cross-section at high energies, by a factor of ~ 2 at $E_\nu = 10^{12}$ GeV [34] and perhaps even more [35].

5 Conclusions

We find that the predictions of high energy neutrino DIS cross-sections from the central values of HERAPDF1.5, CT10 and MSTW2008 PDFs are very similar. However the predictions for the uncertainties (deriving from the uncertainties on the input PDFs) differ quite strongly. If we exclude error sets which either lead to too steep a rise in the cross-section, or allow the low x gluon to be negative at low Q^2, then we find that the uncertainty estimates of HERAPDF1.5 and CT10 — both of which use the most up-to-date, accurate HERA data — are remarkably consistent. In particular, we find the uncertainties to be much smaller than claimed recently [20].

Our results for the high energy neutrino and anti-neutrino CC and NC DIS cross-sections and their uncertainties using HERAPDF1.5 at NLO are shown in Fig. 4. The general trend of the uncertainties can be understood by noting that as one moves to higher neutrino energy one also moves to lower x where the PDF uncertainties are increasing. The PDF uncertainties are smallest at $10^{-2} \lesssim x \lesssim 10^{-1}$, corresponding to $s \sim 10^5$ GeV^2. When the high x region becomes important the neutrino and anti-neutrino cross-sections are different because the valence contribution to xF_3 is now significant. This is seen in Fig. 4, as is the onset

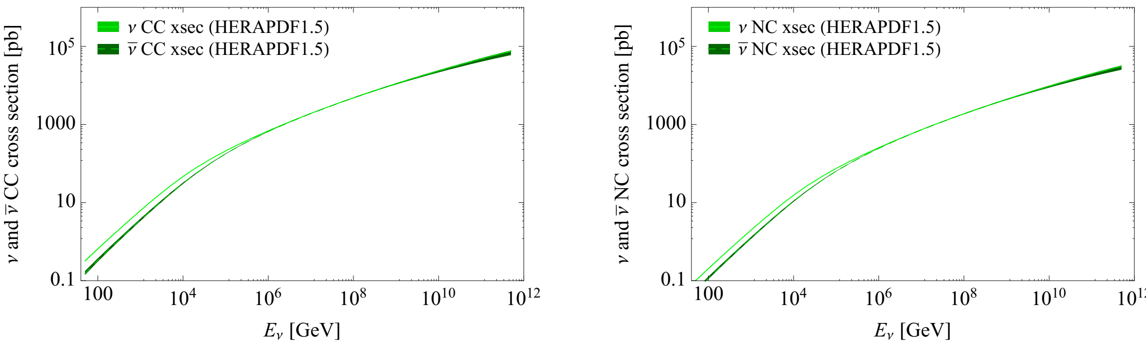

Figure 4: Neutrino and anti-neutrino cross-sections on isoscalar targets for CC and NC scattering for HERAPDF1.5.

of the linear dependence of the cross-sections for $s < M_W^2$. Note that our predictions are made for $Q^2 > 1$ GeV^2 since perturbative QCD cannot sensibly be used at lower values. For higher energies, we intend to upgrade ANIS [36] to use the HERAPDF1.5 (differential) cross-sections. Meanwhile, the tabulated cross sections for protons, neutrons and isoscalar targets are available from a webpage [37]; differential cross sections are available upon request. Any measured deviation from these values would signal the need for new physics beyond the DGLAP formalism.

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