ROLE OF INELASTIC SURFACE EXCITATIONS AND THE ENERGY-DEPENDENT WOODS–SAXON POTENTIAL IN SUB-BARRIER FUSION OF $^{32,36}_{16}$S+$^{90}_{40}$Zr REACTIONS

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The static nucleus–nucleus potential and the energy-dependent nucleus–nucleus potential are used to address the sub-barrier fusion reactions. The static nucleus–nucleus potential systematically fails to recover the experimental data of $^{32,36}_{16}$S+$^{90}_{40}$Zr systems. However, the energy-dependent Woods–Saxon potential model (EDWSP model) in conjunction with the one-dimen-sional Wong formula accurately addresses the sub-barrier fusion enhancement of these systems. The role of the inelastic surface excitations of collision partners in the fusion dynamics is entertained within the context of coupled channel calculations performed by using coupled channel code CCFULL. It is worth noting here that the energy dependence in nucleus–nucleus potential simulates the effects of inelastic surface excitations of colliding nuclei in the sub-barrier fusion enhancement of $^{32,36}_{16}$S+$^{90}_{40}$Zr systems.

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1. Introduction

Heavy ion fusion reactions represent a spectroscopic tool to explore the role of nuclear structure and nuclear interaction between participating nuclei. These reactions are an intermediate step for the production of nuclei away from the valley of stability and superheavy elements. The one of the interesting aspects of the fusion reactions is the occurrence of substantially large fusion enhancement at sub-barrier energies over the expectations of one-dimensional barrier penetration model. This fusion enhancement can be correlated with the coupling of relative motion of reactants to inelastic surface excitations of projectile (target) or permanent deformations or nucleon (multi-nucleon) transfer channels. Indeed, the coupling between the elastic channel and intrinsic degrees of freedom leads to anomalously large sub-barrier fusion enhancement [1–6]. The impacts of static deformation and inelastic surface vibrations in the fusion process are adequately addressed by various theoretical approaches [1–8].

The nucleus–nucleus potential plays a central role in the exploration of the reaction mechanism and the complete knowledge of nucleus–nucleus potential is extremely desirable for good understanding of fusion dynamics. The success of any theoretical approach critically depends upon the choice of the optimum form of nucleus–nucleus potential. The presence of large ambiguities in this potential limits the basic understanding of the nuclear interactions. In this regard, different parameterizations of nuclear potential were used to explain the different nuclear phenomena in connection with heavy ion reactions. The various theoretical models employed the standard Woods–Saxon potential for description of the dynamics of sub-barrier fusion reactions [1–9]. The diffuseness parameter of static Woods–Saxon potential is related to the slope of nuclear potential in the tail region of Coulomb barrier. For heavy ion fusion reactions, a wide range of diffuseness parameter ranging from $a = 0.75$ fm to $a = 1.5$ fm was used to reproduce the sub-barrier fusion data. Surprisingly, such values are much larger than a value $a = 0.65$ fm extracted from the elastic scattering data [10–12]. This diffuseness anomaly, which might be an artifact of various static and dynamical physical effects, reflects the systematic failure of static Woods–Saxon potential for simultaneous exploration of the elastic scattering data and the fusion data [13–17]. The different kind of the channel coupling effects, which are occurring in the surface region of nuclear potential or in the tail region of Coulomb barrier is, in turn, responsible for modification in the value of the parameters of nuclear potential and hence the clarifications of these facts require more extensive investigations in the theoretical as well as experimental fronts.

Keeping this idea, the recent work undertook several efforts by analyzing the large set of the experimental data within the framework of energy-dependent Woods–Saxon potential model (EDWSP model) [18–29]. The closely similar physical effects that arise due to the internal structure of colliding pairs can be induced by entertaining the energy dependence in real part of nucleus–nucleus potential in such a way that it becomes more attractive at sub-barrier energies. This energy-dependent nucleus–nucleus potential will effectively decrease the interaction barrier between fusing nuclei and hence predicts substantially larger sub-barrier fusion cross sections with reference to the energy-independent one-dimensional barrier penetration model as evident from the earlier work (EDWSP model). The introduction of the energy dependence in real part of nucleus–nucleus potential is also evident from the nucleon–nucleon interactions as well as from the non-local quantum effects [18–29]. The fusion dynamics of ${}^{32,36}_{16}\mathrm{S}+{}^{90}_{40}\mathrm{Zr}$ systems which are well studied in literature has been analyzed in the present work [30, 31]. In the fusion of these systems, the effects of multi-phonon vibrational state are dominant and coupling to such channels produce large fusion enhancement at sub-barrier energies as compared with expectations of the one-dimensional barrier penetration model. Theoretical calculations are performed by using the energy-independent Woods–Saxon potential as well as energy-dependent Woods–Saxon potential model in conjunction with the one-dimensional Wong formula [32]. The role of inelastic surface vibrations of colliding pairs is addressed within the coupled channel code CCFULL [33] wherein the static Woods–Saxon potential is used. The static Woods–Saxon potential in conjunction with one-dimensional Wong formula fails to explain the complete description of fusion enhancement of ${}^{32,36}_{16}\mathrm{S}+{}^{90}_{40}\mathrm{Zr}$ systems which reflects the inconsistency of static Woods–Saxon potential for explanation of fusion dynamics of various projectile–target combinations. However, the EDWSP model adequately explains the fusion enhancement of ${}^{32,36}_{16}\mathrm{S}+{}^{90}_{40}\mathrm{Zr}$ systems wherein the energy dependence in the Woods–Saxon potential simulates various channels coupling effects that arise due to internal structure of colliding nuclei.

2. Theoretical formalism

2.1. One-dimensional Wong formula

The fusion cross section within partial wave analysis is given by the following expression

$$\sigma_{\mathrm{F}} = \frac{\pi}{k^2}\sum_{\ell=0}^{\infty}(2\ell+1)T_\ell^{\mathrm{F}} . \tag{1}$$

Hill and Wheeler proposed an expression for tunneling probability (T_ℓ^{F}) which is based upon the parabolic approximation wherein the effective interaction between the collision partners has been replaced by an inverted parabola [34]

$$T_\ell^{\mathrm{HW}} = \frac{1}{1+\exp\left[\frac{2\pi}{\hbar\omega_\ell}\left(V_\ell - E\right)\right]} . \tag{2}$$

This approximation was further simplified by Wong using the following assumptions for the barrier position, barrier curvature and barrier height [32]

$$\begin{aligned} R_\ell &= R_{\ell=0} = R_B ,\\ \omega_\ell &= \omega_{\ell=0} = \omega ,\\ V_\ell &= V_B + \frac{\hbar^2}{2\mu R_B^2}\left[\ell+\frac{1}{2}\right]^2 . \end{aligned} \tag{3}$$

By applying these assumptions and Eq. (2) into Eq. (1), the fusion cross section can be written as

$$\sigma_{\rm F} = \frac{\pi}{k^2}\sum_{\ell=0}^{\infty}(2\ell+1)\frac{1}{\left[1+\exp\frac{2\pi}{\hbar\omega}(V_\ell - E)\right]}\,. \tag{4}$$

Wong assumes that infinite number of partial waves contribute to the fusion process, so changing the summation over ℓ into integral with respect to ℓ in Eq. (4) and by solving the integral, one can obtain the following expression of the Wong formula [32]

$$\sigma_F = \frac{\hbar\omega R_B^2}{2E}\ln\left[1+\exp\left(\frac{2\pi}{\hbar\omega}(E-V_B)\right)\right]\,. \tag{5}$$

2.2. Energy-dependent Woods–Saxon potential model (EDWSP model)

The nucleus–nucleus potential is the fundamental characteristic of heavy ion fusion reactions. The present work uses the static Woods–Saxon potential and energy-dependent Woods–Saxon potential in conjunction with the one-dimensional Wong formula for theoretical calculations of fusion dynamics of ${}^{32,36}_{16}\mathrm{S}+{}^{90}_{40}\mathrm{Zr}$ systems [30, 31]. The form of static Woods–Saxon potential is defined as

$$V_N(r) = \frac{-V_0}{\left[1+\exp\left(\frac{r-R_0}{a}\right)\right]} \tag{6}$$

with $R_0 = r_0(A_{\rm P}^{1/3} + A_{\rm P}^{1/3})$. The quantity V_0 is depth and a is the diffuseness parameter of nuclear potential. In EDWSP model, the depth of real part of the Woods–Saxon potential is defined as [18–29]

$$\begin{aligned} V_0 &= \left[A_{\rm P}^{2/3} + A_{\rm T}^{2/3} - (A_{\rm P}+A_{\rm T})^{2/3}\right] \\ &\quad\times\left[2.38 + 6.8(1+I_{\rm P}+I_{\rm T})\frac{A_{\rm P}^{1/3}A_{\rm T}^{1/3}}{\left(A_{\rm P}^{1/3}+A_{\rm T}^{1/3}\right)}\right]\ \mathrm{MeV}\,, \end{aligned} \tag{7}$$

where

$$I_{\rm P} = \left(\frac{N_{\rm P}-Z_{\rm P}}{A_{\rm P}}\right) \quad \text{and} \quad I_{\rm T} = \left(\frac{N_{\rm T}-Z_{\rm T}}{A_{\rm T}}\right)$$

are the isospin asymmetry of collision partners. The present parameterization of depth is based upon the reproduction of fusion excitation function data of wide range of projectile–target combinations ranging from $Z_{\rm P}Z_{\rm T} = 84$ to $Z_{\rm P}Z_{\rm T} = 1640$ [18–29]. The first term in the square bracket of Eq. (7)

is directly proportional to the surface energy of nucleus and hence strongly depends on the collective motion of all the nucleons inside the nucleus. The various channel coupling effects, which are responsible for fusion enhancement at sub-barrier energies, are the surfacial effects which, in turn, modify the surface diffuseness as well as the surface energy of collision partners. For instance, the colliding nuclei overlap in the neck region wherein the densities of collision partners get fluctuated. These kinds of fluctuation in densities are dynamical physical effects which are responsible for modification of diffuseness parameter and hence bring the necessity of larger value of diffuseness parameter ranging from $a = 0.75$ fm to $a = 1.5$ fm for reproduction of fusion excitation function data [13–17]. The first term inside the square bracket of Eq. (7) accommodates all such physical effects. The second term inside the square bracket of Eq. (7) is directly related to the isospin asymmetry effects of colliding nuclei. The isospin asymmetry is different for different isotopes of a particular element and hence isotopic effects are also included in the nucleus–nucleus potential via this term. In literature, an abnormally large value of diffuseness parameter ranging from $a = 0.75$ fm to $a = 1.5$ fm has been used to account the fusion dynamics of wide range of projectile–target combinations. This abnormally large diffuseness might be an artifact of various kinds of static and dynamical physical effects such as fluctuation of densities and surface energy of colliding pairs. In this regard, the energy dependence in the Woods–Saxon potential is introduced via its diffuseness parameter and hence given by the following expression [18–29]

$$a(E) = 0.85\left[1 + \frac{r_0}{13.75\left(A_{\mathrm{P}}^{-1/3} + A_{\mathrm{T}}^{-1/3}\right)\left(1 + \exp\left(\frac{\frac{E}{V_{B0}} - 0.96}{0.03}\right)\right)}\right]\ \mathrm{fm}\,. \quad (8)$$

In the above expression, the range parameter (r_0) is treated as free parameter and varied to reproduce the fusion data. The value of range parameter strongly depends upon the nature of fusing systems under consideration. In EDWSP model calculations, the above expression provides a wide range of diffuseness depending upon the value of r_0 and the bombarding energy of collision partners. The coupled channel calculations using the static Woods–Saxon potential with large diffuseness parameter ranging from $a = 0.75$ fm to $a = 1.5$ fm has an effect that is closely similar to that of shallow M3Y+repulsion potential in low energy region [35–41]. Ghodsi *et al.* [41] have showed that the M3Y+repulsion and static Woods–Saxon potential with large diffuseness parameter adequately reproduce the fusion dynamics of various heavy ion fusion reactions and M3Y+repulsion can be accurately reproduced in sub-barrier energy region by the static Woods–Saxon potential with large diffuseness parameter. In the present work, it was found that the predictions of energy-dependent Woods–Saxon potential have the close resemblance with that of static Woods–Saxon potential with large diffuseness. Furthermore, one can easily noticed that the theoretical calculations based on static Woods–Saxon potential (CCFULL calculations) must include the couplings to inelastic surface excitations of colliding nuclei or other static and dynamical effects to explain the fusion data. However, the energy dependence in the nucleus–nucleus potential introduces similar kind of channel coupling effects that arise due to intrinsic degrees of freedom of colliding nuclei and hence, reasonably reproduces the experimental data of ${}^{32,36}_{16}\mathrm{S}+{}^{90}_{40}\mathrm{Zr}$ systems.

2.3. Coupled channel model

Theoretically, the coupled channel calculations can be used to address the effects of coupling between relative motion and nuclear structure degrees of freedom. In this section, a brief review of the coupled channel model is presented [33, 42]. Therefore, the set of coupled channel equation can be written as

$$\left[\frac{-\hbar^2}{2\mu}\frac{d^2}{dr^2} + \frac{J(J+1)\hbar^2}{2\mu r^2} + V_N(r) + \frac{Z_\mathrm{P} Z_\mathrm{T} e^2}{r} + \varepsilon_n - E_\mathrm{cm}\right]\Psi_n(r) + \sum_m V_{nm}(r)\Psi_m(r) = 0\,. \tag{9}$$

Here, $\vec{r}$ is the radial coordinate for the relative motion between fusing nuclei, μ is defined as the reduced mass of the projectile and target system. The quantities E_cm and ε_n represent the bombarding energy in the centre-of-mass frame and the excitation energy of the n^th channel respectively. The V_{nm} is the matrix elements of the coupling Hamiltonian, which in the collective model consists of Coulomb and nuclear components. For the coupled channel calculations, the code CCFULL [33], wherein the coupled channel equations are solved numerically, has been used. The set of coupled channel equations is solved by using two basic approximations. The first approximation is no Coriolis or rotating frame approximation that has been used for reducing the number of the coupled channel equations [33, 42]. In the condition of no transfer of the angular momentum from relative motion between reactants to their intrinsic motion, the total orbital angular momentum quantum number L can be replaced by the total angular momentum quantum number J. The ingoing wave boundary conditions (IWBC), which are well applicable for heavy ion reactions, are another approximations used to solve the coupled channel equations. According to IWBC, there are only incoming waves at $r = r_\mathrm{min}$, which is taken as the minimum position of the Coulomb pocket inside the barrier and there are only outgoing waves at infinity for all channels except the entrance channel ($n = 0$) [33, 42]. The code CCFULL [33] makes the use of static Woods–Saxon potential for addressing the role of internal structure degrees of freedom of colliding pairs such as inelastic surface vibrations, rotational states and multi-nucleon transfer channels. By including all the relevant channels, the fusion cross section can be written as

$$\sigma_{\mathrm{F}}(E) = \sum_J \sigma_J(E) = \frac{\pi}{k_0^2} \sum_J (2J+1) P_J(E)\,, \tag{10}$$

where $P_J(E)$ is the total transmission coefficient corresponding to the angular momentum J.

3. Results and discussion

Recently, the EDWSP model was successfully used to describe the dynamics of heavy ion fusion reactions. The present work is motivated to address the relative importance of the static Woods–Saxon potential and energy-dependent Woods–Saxon potential by analyzing the fusion dynamics of ${}^{32,36}_{16}\mathrm{S}+{}^{90}_{40}\mathrm{Zr}$ systems. The colliding nuclei are spherical in shape and possessing the low-lying surface vibrational states only. The values of deformation parameter and the corresponding excitation energy of low-lying 2^+ and 3^- vibrational states of all nuclei are listed in Table I. In Table II, the values of potential parameters as required in the EDWSP model calculations for ${}^{32,36}_{16}\mathrm{S}+{}^{90}_{40}\mathrm{Zr}$ systems are listed.

TABLE I

The deformation parameter (β_λ) and the corresponding excitation energy (E_λ) of the low lying quadrupole and octupole vibrational states of colliding nuclei.

Nucleus	β_2	E_2 [MeV]	β_3	E_3 [MeV]	Reference
${}^{32}_{16}\mathrm{S}$	0.32	2.230	0.40	5.006	[30]
${}^{36}_{16}\mathrm{S}$	0.16	3.291	0.38	4.192	[31]
${}^{90}_{40}\mathrm{Zr}$	0.09	2.186	0.22	2.748	[30, 31]

TABLE II

Range, depth and diffuseness of Woods–Saxon potential used in the EDWSP model calculations for various systems [18–29].

System	r_0 [fm]	V_0 [MeV]	$\frac{a^{\mathrm{Present}}}{\mathrm{Energy\ range}}\left[\frac{\mathrm{fm}}{\mathrm{MeV}}\right]$
${}^{32}_{16}\mathrm{S}+{}^{90}_{40}\mathrm{Zr}$	1.120	91.36	$\frac{0.97\ \mathrm{to}\ 0.85}{65\ \mathrm{to}\ 100}$
${}^{36}_{16}\mathrm{S}+{}^{90}_{40}\mathrm{Zr}$	1.105	106.40	$\frac{0.97\ \mathrm{to}\ 0.85}{65\ \mathrm{to}\ 100}$

Since the collision partners are spherical nuclei and involves a common doubly magic target, the fusion mechanism is expected to be very simple. These nuclei have well known vibrational spectra and inelastic surface vibrations of colliding pairs are dominating in the enhancement of sub-barrier fusion cross sections over the expectations of one-dimensional barrier penetration model. Furthermore, these fusing systems are well studied so they are chosen for the present analysis. Since the target is common to both projectiles, the distinguishable features of sub-barrier fusion cross section data of ${}^{32,36}_{16}\mathrm{S}+{}^{90}_{40}\mathrm{Zr}$ reactions arise because of different nuclear structure and different collective properties of projectiles. The lighter projectile (${}^{32}_{16}\mathrm{S}$) is expected to possess a strong quadrupole vibration and therefore coupling of 2^+ vibrational state displays more pronounced effects in the sub-barrier fusion enhancement of ${}^{32}_{16}\mathrm{S}+{}^{90}_{40}\mathrm{Zr}$ reaction with respect to ${}^{36}_{16}\mathrm{S}+{}^{90}_{40}\mathrm{Zr}$ reaction [30, 31].

Before discussing the details of coupled channel calculations, the fusion dynamics of ${}^{32,36}_{16}\mathrm{S}+{}^{90}_{40}\mathrm{Zr}$ reactions is discussed within the context of the energy-independent and energy-dependent Woods–Saxon potential model in conjunction with the one-dimensional Wong formula. The experimental data of ${}^{32,36}_{16}\mathrm{S}+{}^{90}_{40}\mathrm{Zr}$ reactions are substantially larger than the calculations based upon static Woods–Saxon potential in the Wong formalism. The failure of static Woods–Saxon potential to give the adequate explanation of fusion data suggests the modifications in the values of potential parameters and hence mirror the significance of introducing the energy dependence in Woods–Saxon potential. In EDWSP model, the energy-dependent diffuseness parameter produces a spectrum of barrier of varying heights. The barriers whose heights are lower than that of uncoupled classical barrier are responsible for the maximum flux lost from elastic channel to fusion channel. This ultimately brings the larger sub-barrier fusion cross section data over the predictions of energy-independent one-dimensional Wong formula as evident from Fig. 1. In EDWSP model, the fluctuation of diffuseness parameter is effectively equivalent to the increase of capture radii of colliding nuclei which, in turn, suggests that the fusion process starts at much larger internuclear separation between the collision partners [20]. The similar kinds of static and dynamical physical effects are evident from the coupled channel analysis of these systems which will be discussed in Fig. 2. In the coupled channel calculations, the fusion data are substantially larger than no coupling calculations based on the static Woods–Saxon potential obtained by using the coupled channel code CCFULL (see Fig. 2). This unambiguously reflects the inconsistency of static Woods–Saxon potential for description of the fusion dynamics of ${}^{32,36}_{16}\mathrm{S}+{}^{90}_{40}\mathrm{Zr}$ systems.

In the fusion dynamics of ${}^{32,36}_{16}\mathrm{S}+{}^{90}_{40}\mathrm{Zr}$ systems, no coupling calculations wherein both colliding systems are taken as inert are significantly smaller than those of experimental data. In the fusion of ${}^{32}_{16}\mathrm{S}+{}^{90}_{40}\mathrm{Zr}$ system, the inclu-

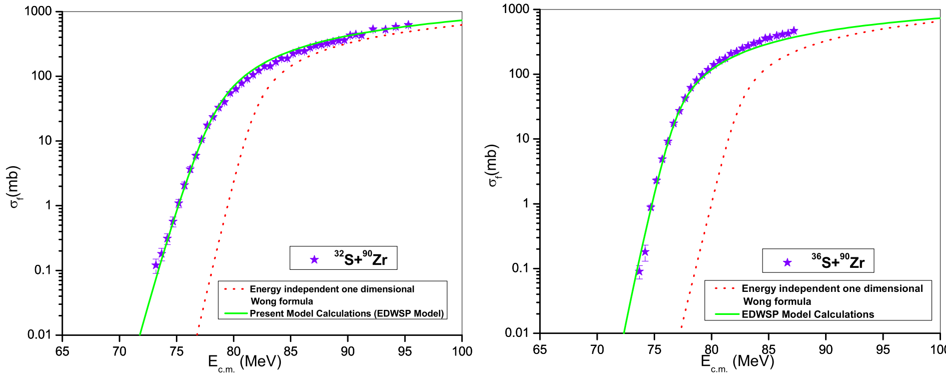

Fig. 1. The fusion excitation functions of $^{32,36}_{16}$S+$^{90}_{40}$Zr systems obtained by using the static Woods–Saxon potential model and energy-dependent Woods–Saxon potential model (EDWSP model) [18–29]. The results are compared with available experimental data (*) taken from Ref. [30, 31].

sion of single-phonon or two-phonon or three-phonon 3^- vibrational states alone in target is unable to reproduce the sub-barrier fusion data. This suggested that more intrinsic degrees of freedom play a significant role in the fusion dynamics of this system. The lighter projectile exhibits strong quadrupole vibrations and its effects are expected to be more pronounced. The coupling to one-phonon 2^+ and 3^- vibrational states of target along with their mutual couplings in projectile and two-phonon 3^- vibrational states in target adequately account the sub-barrier fusion enhancement of $^{32}_{16}$S+$^{90}_{40}$Zr system (Fig. 2 (left)). For $^{36}_{16}$S+$^{90}_{40}$Zr system (Fig. 2 (right)), if colliding nuclei are considered as inert, the experimental data are substantially larger than the theoretical predictions. The coupling to one-phonon 2^+ vibrational state in projectile as well as one-phonon 2^+ and 3^- vibrational states of target along with their mutual couplings strongly enhance the fusion cross section as compared to no coupling calculations but fail to account the experimental data in whole range of energy. The addition of higher phonon states of target like two-phonon 3^- vibrational states improves the coupled channel predictions but still there remain large discrepancies between theoretical calculations and experimental data. The inclusion of one-phonon 2^+ vibrational state in projectile, one-phonon 2^+ vibrational state and three-phonon 3^- vibrational states of target along with the mutual excitations such as $(3^-)^3, (2^+ \otimes (3^-)^2)$ states produce the close agreement between coupled channel calculations and fusion data (Fig. 2 (right)). For $^{32,36}_{16}$S+$^{90}_{40}$Zr systems, the ground state Q-values are negative for all neutron transfer channels which suggest that the effects of neutron transfer channel seem to be undesirable. Therefore, the relative fusion enhancement of sub-barrier fusion cross section data with respect to one-dimensional barrier penetration model can be attributed to the presence of multi-phonon vibrational states as evident from Fig. 2.

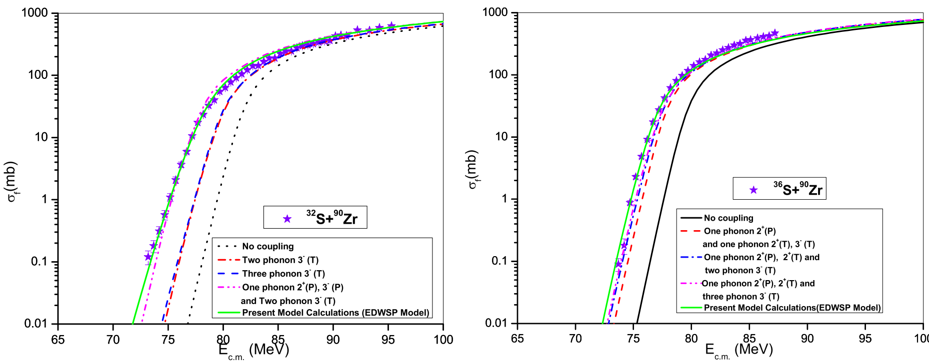

Fig. 2. The fusion excitation functions of ${}^{32,36}_{16}$S+${}^{90}_{40}$Zr systems obtained by using the EDWSP model and the coupled channel code CCFULL. The results are compared with available experimental data (*) taken from Ref. [30, 31].

It is well known fact that the couplings between the relative motion of colliding nuclei and the internal structure degrees of freedom of fusing nuclei result in a distribution of barriers of varying heights and the passage through the barriers whose heights are smaller than that of the uncoupled fusion barrier is more probable. This distribution of barriers is a direct manifestation of enhancement of sub-barrier fusion cross section data by several orders of magnitude over the expectations of the one-dimensional barrier penetration model. In the same analogy, the EDWSP model produces a spectrum of barriers of varying heights and reasonably addresses the fusion enhancement at sub-barrier energies as already discussed in Fig. 1. In EDWSP model calculations, $a = 0.97$ fm is the largest value of diffuseness parameter resulting in the lowest fusion barrier. This lowest fusion barrier can cause the maximum flux lost from the elastic channel to fusion channel. As the incident energy increases, the value of diffuseness parameter decreases resulting in an increase of the height of the corresponding fusion barrier. In the above barrier energy regions, wherein the fusion cross section is almost independent of different channel coupling effects (internal structure of colliding nuclei), the value of diffuseness parameter gets saturated to its minimum value ($a = 0.85$ fm). At this diffuseness parameter, the corresponding fusion barrier is highest. Furthermore, the energy dependence in Woods–Saxon potential simulates the effects of internal structure degrees of freedom and consequently raises number of questions on the validity of static Woods–Saxon potential for description of fusion process. Therefore, the various kind of channel coupling effects with regard to the sub-barrier fusion enhancement whether mirrors the true picture of fusion process or simply mocks up the inconsistency of static Woods–Saxon potential are still not clear and hence more intensive investigations are required on theoretical as well as experimental fronts.

4. Conclusions

The present work is motivated to track the limitations of static Woods–Saxon potential model and the applicability of the EDWSP model for complete description of fusion dynamics by considering the fusion of $^{32,36}_{16}\mathrm{S}+^{90}_{40}\mathrm{Zr}$ systems. The couplings to inelastic surface excitations of colliding nuclei are found to be dominating in the fusion enhancement of these systems and the coupled channel calculations including the effects of multi-phonon vibrational state of colliding pairs reasonably account the sub-barrier fusion data. The static Woods–Saxon potential in conjunction with one-dimensional Wong formula systematically fails to reproduce the sub-barrier fusion data of $^{32,36}_{16}\mathrm{S}+^{90}_{40}\mathrm{Zr}$ systems. However, the energy-dependent Woods–Saxon potential model (EDWSP model) in conjunction with the one-dimensional Wong formula accurately describes the fusion dynamics of these systems. The close resemblance of the predictions of EDWSP model and coupled channel model for the fusion of $^{32,36}_{16}\mathrm{S}+^{90}_{40}\mathrm{Zr}$ systems unambiguously suggested that energy dependence in the Woods–Saxon potential mimics the effects of dominant internal structure degrees of freedom of collision partners. In the EDWSP model calculations, a wide range of diffuseness parameter ranging from $a = 0.85$ fm to $a = 0.97$ fm is required for reproduction of the fusion enhancement of various heavy ion fusion reactions.

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