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Softening of gravitational effect on noncommutative spacetime

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Abstract. A noncommutative gravitational theory constructed by applying Moyal deformation quantization and the Seiberg–Witten map to teleparallel gravity shows that gravity acts repulsively in an extreme region where its quantum effects become prominent. The problems involved with the order of products and the relationship between the metric and the Moyal product can be solved by assigning their roles, such that the former is responsible for the rule of the inner product and the latter is in charge of tensor and field noncommutativity. As a result, it is found that the cosmic evolution of the very early stage of the universe was modified as if the noncommutative effect works similarly to a cosmological constant. Furthermore it was clarified that the degree of gravitational redshift of a spherically symmetric space with central mass becomes smaller, which means the attractive force is effectively weakened. Such consequences indicate that in the noncommutative spacetime, the gravitational interaction is softened compared to the classical gravity.

1. Introduction

General relativity is the most successful gravitational theory so far and it has been confirmed by various experiments and observations such as the detection of gravitational wave [1] and the imaging of a black hole shadow [2]. The development of detailed understanding of cosmic evolution or event horizon could enable us to find a deviation from classical general relativity, that is, a quantum gravitational effect. A reliable quantum gravitational theory is a long-standing task to be established in modern physics. It will unify four fundamental interactions including gravity and is believed to become the ultimate physical theory.

As an attempt to construct a quantum gravity, a Moyal-deformation quantization has been applied to teleparallel gravity on the noncommutative spacetime [3] in this study. The noncommutative spacetime characterized by a nontrivial commutation relation [4] naturally appears from string theory [5] and the noncommutative geometry would be the basis of quantum theory [6, 7]. The Moyal product [8] is a general method of the deformation quantization and is conventionally used to reproduce general relativity on the noncommutative spacetime. Teleparallel gravity is a gauge theory on local translational symmetry [9–12] and is essentially equivalent to the general relativity. The method employed in [3] has an advantage that the tetrad and the metric are provided naturally by introducing gauge fields. In order to avoid the difficulty with the product order, we have determined that the metric and the tetrad are responsible for the inner product concerning the sum over the subscripts, while only the Moyal product expresses the noncommutativity of spacetime. Consequently, our formulation can be applied to cosmological and astrophysical phenomena in regemes where the quantum nature of spacetime should not be neglected, such as in the early universe and around black holes.

2. Formulation

Using geometrical units with $c = G = 1$, the Lagrangian in the teleparallel gravity is described as

$$\mathcal{L}_{TG} = \frac{h}{2\kappa}\left(\frac{1}{4}\eta_{ab}g^{\mu\rho}g^{\nu\sigma} + \frac{1}{2}h_a{}^{\sigma}h_b{}^{\nu}g^{\mu\rho} - h_a{}^{\nu}h_b{}^{\sigma}g^{\mu\rho}\right)F^a{}_{\mu\nu}F^b{}_{\rho\sigma} \ , \tag{1}$$

where $\kappa = 8\pi$ is the coupling constant, the Greek alphabet denotes indices related to spacetime and the Latin alphabet corresponds to tangent space. Accordingly, x^μ is the spacetime coordinate and x^a is the tangent-space coordinate. Then $g_{\mu\nu}$ is the usual spacetime metric tensor and $\eta_{ab} = \mathrm{diag}(-1, +1, +1, +1)$ is the Minkowski metric of the tangent space. The tetrad, $h^a{}_\mu$, satisfies formulae as

$$g_{\mu\nu} = \eta_{ab}h^a{}_\mu h^b{}_\nu \ , \qquad \eta_{ab} = g_{\mu\nu}h_a{}^\mu h_b{}^\nu \ , \tag{2}$$

and $h = \det\,(h^a{}_\mu)$ is a volume element. Finally the field strength is written as

$$F^a{}_{\mu\nu} = \partial_\mu B^a{}_\nu - \partial_\nu B^a{}_\mu = \partial_\mu h^a{}_\nu - \partial_\nu h^a{}_\mu \ , \tag{3}$$

where $B^a{}_\mu$ is a gauge field of local translational transformation. Note that the Lagrangian (1) is identical to that in the general relativity except for surface terms [9].

In the deformation quantization by the Moyal product method, the product of arbitrary functions $f(x), g(x)$ must obey

$$f \star g = \exp\left[\frac{i}{2}\theta^{\mu\nu}\frac{\partial}{\partial x^\mu}\frac{\partial}{\partial y^\nu}\right]f(x)\ g(y)\bigg|_{x=y} = fg + \frac{i}{2}\theta^{\mu\nu}\partial_\mu f\ \partial_\nu g + \cdots \ , \tag{4}$$

whereby the spacetime coordinate satisfies the nontrivial commutation relation as

$$[x^\mu, x^\nu]_\star = x^\mu \star x^\nu - x^\nu \star x^\mu = i\theta^{\mu\nu} \ , \tag{5}$$

where $\theta^{\mu\nu}$ is the noncommutative parameter and is estimated to be the order of the square of Planck length, l_{pl}, if the spacetime noncommutativity comes from the quantum effect.

Hereafter $\hat{}$ means the quantity defined in the noncommutative spacetime. The most important assumption in this study is that the Moyal product should be applied only to the inner product of vectors $\hat{u}$ or $\hat{v}$ in the noncommutative spacetime and a vector $\hat{s}$ in the tangent space as

$$g_{\mu\nu}u^\mu v^\nu \longrightarrow \hat{g}_{\mu\nu}(\hat{u}^\mu \star \hat{v}^\nu) \ , \qquad h^a{}_\mu u^\mu s_a \longrightarrow \hat{h}^a{}_\mu(\hat{u}^\mu \star \hat{s}_a) \ , \tag{6}$$

and the relations between the metric and the tetrad is unchanged as

$$\hat{g}_{\mu\nu} = \eta_{ab}\hat{h}^a{}_\mu \hat{h}^b{}_\nu \ , \qquad \eta_{ab} = \hat{g}_{\mu\nu}\hat{h}_a{}^\mu \hat{h}_b{}^\nu \ . \tag{7}$$

The resulting Lagrangian of noncommutative gravitational theory is calculated as

$$\mathcal{L}_{NCG} = \frac{\hat{h}}{2\kappa}\left(\frac{1}{4}\eta_{ab}\hat{g}^{\mu\rho}\hat{g}^{\nu\sigma} + \frac{1}{2}\hat{h}_a{}^{\sigma}\hat{h}_b{}^{\nu}\hat{g}^{\mu\rho} - \hat{h}_a{}^{\nu}\hat{h}_b{}^{\sigma}\hat{g}^{\mu\rho}\right)(\hat{F}^a{}_{\mu\nu} \star \hat{F}^b{}_{\rho\sigma}) \ . \tag{8}$$

Here the noncommutative field strength is derived as

$$\hat{F}^a{}_{\mu\nu} = \partial_\mu \hat{B}^a{}_\nu - \partial_\nu \hat{B}^a{}_\mu - \frac{i}{2}[\hat{B}^b{}_\mu, \hat{B}^c{}_\nu]_\star \gamma^a{}_{bc} \ , \tag{9}$$

where $\gamma^a{}_{bc}$ is the structure constant of the anti-commutator for the local translation generator.

3. Result and Discussion

Now we define time-space and space-space noncommutativity as

$$[t, x^j]_\star = i\Sigma \ , \qquad [x^i, x^j]_\star = i\Theta \ , \tag{10}$$

where $x^0 = t$ is the time component of the coordinate. Then the scale factor in the noncommutative version of the radiation-dominant homogeneous and isotropic spacetime can be shown as

$$\frac{a(t)}{a(t_{eq})} = \tau^{1/2} - \frac{1}{96}\sigma(165\tau^{1/2} + 23\tau^{-1/2} + 10\tau^{-1/2}\ln\tau - 188) \ , \tag{11}$$

where t_{eq} is the epoch of matter-radiation equality, $\tau \equiv t/t_{eq}$ and $\sigma \equiv \Sigma/(l_{pl}t_{eq})$. It is evident that the size of the universe diverges at $\tau \to 0$, which suggests the existence of the bounce process to evade the initial singularity of the universe as if there were a negative pressure energy like a cosmological constant. In addition, the noncommutative gravitational redshift parameter, z, in the spherically symmetric spacetime with central mass, M, can be expressed as

$$z = \frac{1 + M/2r}{1 - M/2r}\left\{1 + \frac{\Sigma}{l_{pl}}\frac{16}{r}\left(\frac{M}{2r}\right)^2 \frac{1 + M/2r}{1 - M/2r}\left(1 + \frac{M}{2r}\right)^{-3}\right\}^{-1/2} - 1 \ , \tag{12}$$

where r is the distance from the central mass in a usual Schwarzschild metric. Since the noncommutative terms in (12) makes z smaller, it seems that the noncommutativity works repulsively against the classical gravitational force.

Both applications imply that the quantum-gravitational effect based on a noncommutative spacetime would soften the attractive interaction. Developing experiments and observations will reveal unknown nature of the beginning of the universe and/or the vicinity of the event horizon so that they could demonstrate that our claims are reasonable. Further detailed analysis of the noncommutative spacetime would contribute to the progress of quantum gravity and it must be useful for our understanding of unified theories and cosmic origin. As a comment, we would like to note that consistent noncommutativity can be obtained independent of the coordinate system in the spacetime defined by the generalized Moyal product [13]. Then, for example, although the expression (12) is derived based on the Cartesian coordinate, it can be justified without any coordinate ambiguity.

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