Growth of linear perturbations in a universe with superfluid dark matter

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The Lambda-Cold Dark Matter (ΛCDM) model agrees with most of the cosmological observations, but has some hindrances from observed data at smaller scales such as galaxies. Recently, Berezhiani and Khoury proposed a new theory involving interacting superfluid dark matter with three model parameters in,[1] which explains galactic dynamics with great accuracy. In the present work, we study the cosmological behaviour of this model in the linear regime of cosmological perturbations. In particular, we compute both analytically and numerically the matter linear growth factor and obtain new bounds for the model parameters which are significantly stronger than previously found. These new constraints come from the fact that structures within the superfluid dark matter framework grow quicker than in ΛCDM, and quite rapidly when the DM-baryon interactions are strong.

Keywords: Dark matter, cosmological perturbations

1. Introduction

With the advent of precision cosmology and satellites like Planck and WMAP, we have gained new insights about the evolution of the universe. Till date, Lambda-Cold Dark Matter (ΛCDM) provides the best fit to these available data and has been widely accepted as the standard model of cosmology.[2] The hypothesis of CDM, which are assumed to be collisionless non-relativistic particles, along with baryonic matter explains the CMB temperature anisotropy, matter power spectra, large scale galaxy distributions and lensing data remarkably well. In fact, the abundance of galaxy clusters and observed large scale structure formation history strongly supports the collisionless CDM scenario as opposed to any alternative theories to ΛCDM.[3–5] However, at smaller scales, CDM faces a number of challenges that need to be addressed.[6] For example, the Baryonic Tully-Fisher relation and the

corresponding tight correlation between the mass and dispersion velocity at the high-mass end can not satisfactorily be explained by CDM halo which predicts a larger scatter due to feedback processes in the galaxy.[7] Apart from this, there is another issue with the standard CDM picture in the galactic scale, known as the *cusp-core problem*.[8] The simulations of galactic halos with CDM produce a kink (cusp) at the center of the galaxy, whereas observations of various galactic density profile suggest a flat core. With improved observations of the faint dwarf galaxies and substructures within the galaxies like Milky Way and Andromeda, new set of discrepancies arise. While the missing satellite problem in dwarf galaxies[9] has been addressed to some extent, the *Too Big To Fail Problem*, arising from the prediction of satellites that are too massive and too dense by ΛCDM, compared to those observed, still remains unresolved.[10,11]

Due to the above unresolved issues, scientists have looked into other alternative explanations through modifications of General Relativity (GR). Several models have been proposed so far with the aim to explain existing data to the same degree of accuracy as ΛCDM as well as overcome its drawbacks. Many of them have already been ruled out or are highly constrained by the ongoing observations of gravitational waves, but some theories like $f(R)$, $f(T)$, $f(\mathcal{G})$, Scalar-tensor-vector theories of gravity etc. are still consistent with the data, and new observations are required to falsify these theories.[12–18] These theories are relativistic corrections of GR which modify the dynamics of spacetime through the modified field equations. The theory of Modified Newtonian Dynamics (MOND), on the other hand, is a modification to the Newtonian force law that changes the dynamics of interaction between two massive bodies in the non-relativistic limit.[19,20] MOND was first proposed in 1983 by Milgrom to account for the flattened galaxy rotation curves near the edge of the spiral galaxies like Milky Way. There is a universal acceleration scale a_0 in MOND, whose value is obtained as $10^{-8} cm/s^2$. For accelerations much lower than this scale, the Newtonian law is modified, and this explains the flat galaxy rotation curve data for a large number of galaxies.[21] Interestingly, the Baryonic Tully-Fisher relation in galaxies can exactly be derived from MOND where $M \propto v_c^4$. MOND can also explain several other galactic observations like the planar structure of galaxies, low merger rate etc.[22] Thus, we see that MOND, with just one free parameter, is a very well-behaved theory at the galactic scale. However, despite these successes, MOND faces several challenges in extragalactic and cosmological scales. Proper relativistic extension of MOND is not available.[23] Hence it cannot be applied at cosmological scales.

The effectiveness of MOND at small scales and success of ΛCDM at cosmological scales are the main motivations for scientists to look for models which are CDM-MOND hybrids i.e., theories that include usual cold dark matter as collisionless particles at cosmological scales, but give rise to a MOND-like modified force law at galactic scales such that they satisfy both sets of observations. This class of models take into consideration the interacting dark matter-baryon picture where

a MOND-like force is mediated through this new interaction term. Based on this idea, many models have been proposed which can reproduce both CDM features as well as MOND in their respective regime of validity.[1,24–27]

In this paper we shall focus on one such model proposed recently by Berezhiani and Khoury,[1] where CDM can form condensates at galactic scales depending upon the surrounding temperature and can behave as superfluid. It has already been shown by the authors that such model can explain a number of galactic scale observations due to their MONDian behaviour, which normal CDM fails to explain.[1,28–30] Although there are recent studies which suggest that the superfluid DM model is disfavoured compared to a spherically symmetric CDM halo at galactic scales as the superfluid DM overestimates the vertical acceleration which has to be counterbalanced to match the observations.[31,32] There are two free parameters in the theory which are assumed to be temperature dependent. It has been argued that at cosmological scales, the theory behaves as usual CDM and thus the background evolution and other cosmic histories remain unchanged as compared to ΛCDM. Here, we study the cosmological evolution of the background as well as the matter perturbations. We check whether the present model remains well-behaved at cosmological scales as has been claimed by the authors and compare our results with ΛCDM.

2. Dark Matter Superfluid-Overview

The central idea of this model is that CDM is made up of particles which undergo phase transition below a particular critical temperature and becomes a superfluid. This requires that the particle CDM needs to be strongly interacting below a particular temperature. The superfluid behaviour depends on the strength of interaction and the mass of the particle. It has been shown in[1] that in order to form a Bose-Einstein Condensate (BEC) the following condition must be satisfied

$$m \lesssim \left(\frac{\rho}{v^3}\right)^{1/4} \tag{1}$$

where m and v corresponds to the mass and velocity of the particle respectively and ρ is the density of the condensate. Assuming virialization of dark matter halo at galactic scales, this gives an upper bound on the mass of the particle forming the halo

$$m \lesssim 2.3(1+z_{vir})^{3/8}\left(\frac{M}{10^{12}h^{-1}M_{\odot}}\right)^{-1/4}\text{eV} \tag{2}$$

Further assuming thermalization of CDM particles, one obtains the bound on interaction cross section as

$$\frac{\sigma}{m} \gtrsim 52(1+z_{vir})^{-7/2}\left(\frac{m}{eV}\right)^{4}\left(\frac{M}{10^{12}h^{-1}M_{\odot}}\right)^{2/3}\text{cm}^2\text{g}^{-1} \tag{3}$$

Using equipartition law, the critical temperature T_c of the CDM condensate can be obtained as

$$T_c = 6.5 \left(\frac{\text{eV}}{\text{m}}\right)^{5/3} (1 + z_{vir})^2 \text{mK} \tag{4}$$

It has been argued in[1] that the temperature of CDM at cosmological scales is much below the critical temperature ($\mathcal{O}(10^{-28})$ for $m \sim$ eV) which implies that the condensate behaves as a $T \approx 0$ superfluid at cosmological scales.

The description of superfluid dark matter is given in terms of a low energy effective theory with the Lagrangian of the form:

$$\mathcal{L} = \frac{2\Lambda(2m)^{3/2}}{3} \left(\dot{\theta} - m\Phi - \frac{(\nabla\theta)^2}{2m}\right)^{3/2} \tag{5}$$

Let us now understand the motivation of choosing such a Lagrangian. Here, θ is the phase of the wavefunction describing the superfluid phonon modes and Φ is the gravitational potential in which the DM particle sits and is given by the standard Newtonian potential in the usual non-relativistic case. This Lagrangian has a free parameter Λ which defines the strength of the superfluid (i.e. defined by the number of particles in the condensate state). The power of the Lagrangian is defined by the choice of the equation of state (EoS), and a fractional power of $5/2$ is indeed obtained in superfluids formed by ultra cold atoms. In the case of CDM superfluid, the choice of the power $3/2$ in the Lagrangian is somewhat arbitrary, but motivated by the fact that the superfluid DM should give rise to MOND-like dynamics at galactic scales when baryons are also included. This also corresponds to an equation of state $P \sim \rho^3$ which is suggestive of a dominant three-body interaction process. What kind of particles can lead to such a superfluid with this particular EoS and the physics of its formation has not been discussed earlier and is beyond the scope of this paper. For our purpose, we shall assume the form of this Lagrangian to study the characteristic features of the resultant superfluid DM model.

In the effective field theory formalism, the superfluid is described in terms of interacting phonon modes. The phonon modes can be described by the scalar field θ, which, at a constant chemical potential μ, can be expanded as,

$$\theta = \mu t + \phi$$

where ϕ denotes the excitation of the phonon modes.

The DM superfluid couples to the baryons through the phonon modes via an interaction given by the Lagrangian:

$$\mathcal{L}_{int} = -\alpha \frac{\Lambda}{M_{Pl}} \theta \rho_b \tag{6}$$

This kind of interaction ensures a MOND force. Here α is a dimensionless free parameter, which sets the interaction strength of the interaction, and ρ_b is the baryonic mass density.

Thus, the complete Lagrangian for an interacting superfluid DM is given by,

$$\mathcal{L} = \frac{2\Lambda(2m)^{3/2}}{3}\left(\dot{\theta} - m\Phi - \frac{(\nabla\theta)^2}{2m}\right)^{3/2} - \alpha\frac{\Lambda}{M_{Pl}}\theta\rho_b \tag{7}$$

It has been shown in[1] that the MONDian acceleration arises as a special case of the dynamics of the above Lagrangian. The validity of this model in solar system and Bullet cluster has also been discussed there.

In the cosmological context, although the authors in[1] discuss some general points regarding the background behaviour and the equation of state of this new superfluid dark matter, they do not shed much light on other important points such as growth of perturbations and structure formation. In the next sections, we solely focus on the cosmological aspects of this new theory.

3. Cosmological Solutions

In this section, we will study this theory in cosmological context. This is of particular interest since the theory also needs to be consistent with the present cosmological data.

3.1. *Background Solutions*

For the background cosmology, we have $\theta = \theta(t)$. In the FLRW background with a scale factor a, the equation of motion for θ can be derived from the action as,

$$\frac{d}{dt}\left[(2m)^{3/2}a^3\dot{\theta}^{1/2}\right] = -\frac{\alpha}{M_{Pl}}a^3\rho_b \tag{8}$$

Assuming the evolution of baryons i.e. $\rho_b \propto 1/a^3$ as in standard ΛCDM, we get,

$$\rho_m = -\frac{\alpha\Lambda}{M_{Pl}}m\rho_b t + \frac{m\Lambda C}{a^3} \tag{9}$$

Here C is an integration constant which has to be determined from the present DM density. The second term (ρ_{dust}) corresponds to the dust like evolution. The form of the density is similar to those obtained in dynamical space-time theories as discussed in.[33–35] For the second term to dominate (such that ρ_m behaves as dust), it can be shown that one needs to satisfy the following constraint:

$$\frac{\alpha\Lambda\rho_b}{M_{Pl}\rho_{dust}}mt_0 \leq 1 \tag{10}$$

where t_0 is the present age of the universe.

Bounds on the model parameters:

- **From the EoS**- Equation of state for the DM superfluid (assuming negligible interaction) is given by,

$$w = \frac{\rho_{dust}^2}{12\Lambda^2 m^6} \tag{11}$$

For DM to behave as dust at the background level, Λ should be bounded from below,

$$\Lambda \gg 0.1 \left(\frac{m}{\text{eV}}\right)^{-3} \text{eV} \tag{12}$$

- **From coupling to baryons**- From (10) and (12), and assuming a constant baryon-to-DM ratio ($\rho_{dust}/\rho_b = 6$), we get,

$$\alpha \ll 2.4 \times 10^{-4} \left(\frac{m}{\text{eV}}\right)^2 \tag{13}$$

These bounds are different from the bounds obtained for galaxies, as discussed in.[1,28]

3.2. *Perturbations*

Study of linear perturbation theory in the context of ΛCDM has been an important step towards understanding the evolution of the universe. CMB spectra carries information about the inhomogeneities present in the early universe. Hence, any cosmological model needs to satisfy the CMB data to a high degree of accuracy. This requires analysing the matter power spectrum resulting from the initial density perturbations. In this section, we examine the growth of cosmological perturbations in DM superfluid model at linear order.

The Lagrangian of the theory in an FLRW matter dominated universe is given as,

$$\mathcal{L} = c_1 \left(\dot{\theta} - \frac{(\nabla\theta)^2}{2m} - m\Phi\right)^{3/2} - c_2\rho_b\theta \tag{14}$$

where c_1, c_2 are constants expressed as,

$$\begin{aligned} c_1 &= \frac{2\Lambda(2m)^{3/2}}{3} \\ c_2 &= \alpha\frac{\Lambda}{M_{Pl}} \end{aligned} \tag{15}$$

Here, $\theta(x,t) = \bar{\theta}(t) + \delta\theta(x,t)$ is a scalar field which is a fucntion of both space and time.

We can find the Euler-Lagrange equation from the above equation as:

$$\frac{d}{dt}\left(\frac{\partial\mathcal{L}}{\partial\dot{\theta}}\right) - \frac{\partial\mathcal{L}}{\partial\theta} = 0 \tag{16}$$

This gives us the background equation of motion as shown in the previous subsection as well as the first order perturbation equation of the dark matter density. But this single governing equation is inadequate to obtain the complete numerical solution which requires a complete set of differential equations.

In order to get the full set of perturbation equations, we start with the fluid equations that govern the dynamics of the dark matter superfluid. The fluid equations, namely the continuity equation and the Navier-Stokes equation can be derived using the Hamiltonian formalism, as described in.[36] In,[36] the authors work out the fluid equations for an interacting two-component BEC dark matter. Here in this work, we follow the same prescription for a superfluid dark matter which interacts with the baryonic matter. The corresponding Lagrangian is given by (14).

From the Lagrangian, we get the conjugate momentum as,

$$\begin{aligned}\Pi_\theta &= \frac{\partial \mathcal{L}}{\partial \dot{\theta}} \\ &= \Lambda (2m)^{3/2} \left[\dot{\theta} - m\Phi - \frac{(\nabla\theta)^2}{2m}\right]^{1/2} \end{aligned} \tag{17}$$

The Hamiltonian H describing the superfluid can be obtained as,

$$H = \Pi_\theta \dot{\theta} - \mathcal{L} \tag{18}$$

Since, $\dot{\theta} = m\Phi + \frac{(\nabla\theta)^2}{2m} + \frac{\Pi_\theta^2}{\Lambda^2(2m)^3}$ from (17), we get the Hamiltonian H as follows,

$$H = \frac{\Pi_\theta^3}{3\Lambda^2(2m)^3} + \left(m\Phi + \frac{(\nabla\theta)^2}{2m}\right)\Pi_\theta + \frac{\alpha\Lambda}{M_{pl}}\rho_b\theta \tag{19}$$

3.2.1. *Hamilton's equation of motion*

The Hamilton's equations of motion are :

$$\dot{\theta} = \frac{\partial H}{\partial \Pi_\theta} \tag{20}$$

and

$$\dot{\Pi}_\theta = -\frac{\partial H}{\partial \theta} \tag{21}$$

For this model, the two equations become, respectively,

$$\dot{\theta} = \frac{\Pi_\theta^2}{\Lambda^2(2m)^3} + m\Phi + \frac{(\nabla\theta)^2}{2m} \tag{22}$$

and

$$\dot{\Pi}_\theta = \frac{1}{m}\nabla\cdot(\Pi_\theta\nabla\theta) - \frac{\alpha\Lambda}{M_{pl}}\rho_b \tag{23}$$

3.2.2. *Fluid equations*

In order to get the fluid equations from the above Hamilton's equations of motion, we identify the terms as corresponding hydrodynamical variables. We define the mass density term (as the co-efficient of Φ in the Hamiltonian) and the four-velocity of the fluid, $\vec{u}$ as

$$\rho_m = m\Pi_\theta, \quad \vec{u} = -\frac{\nabla\theta}{m}. \tag{24}$$

Using the above definitions, we get the fluid equations from equation (23) and (22) as follow,

$$\dot{\rho}_m + \nabla\cdot(\rho_m\vec{u}) = -\frac{\alpha\Lambda m}{M_{pl}}\rho_b \tag{25}$$

$$\dot{\vec{u}} + (\vec{u}\cdot\nabla)\vec{u} = -\frac{\rho_m\nabla\rho_m}{4\Lambda^2 m^6} - \nabla\Phi \tag{26}$$

These are the two fluid equations: Continuity equation and Navier-Stokes equation.

Now, the Poisson's equation can be written as

$$\nabla^2\Phi = 4\pi G(\bar{\rho} + \delta\rho) \tag{27}$$

Integrating twice and substituting the background density using Friedmann equations, we get the potential as:

$$\Phi = -\frac{1}{2}(\dot{H} + H^2)l^2 + \phi \tag{28}$$

where l is the proper distance defined as $\vec{l} = a(t)\vec{x}$ and ϕ is the potential due to inhomogeneities.

Similarly, the four-velocity $\vec{u}$ can be split into two parts, Hubble flow and a peculiar velocity $\vec{v}$ as follows:

$$\vec{u} = H\vec{l} + \vec{v} \tag{29}$$

Expressing everything in comoving co-ordinates $\vec{x}$ and using $\nabla_l = \frac{1}{a(t)}\nabla_x$, we get,

$$\dot{\rho}_m + 3H\rho_m + \frac{1}{a}\nabla\cdot(\rho_m\vec{v}) = -\frac{\alpha\Lambda m}{M_{pl}}\rho_b \tag{30}$$

and

$$\dot{\vec{v}} + H\vec{v} + \frac{1}{a}(\vec{v}\cdot\nabla)\vec{v} = -\frac{\rho_m\nabla\rho_m}{4a\Lambda^2 m^6} - \frac{\nabla\phi}{a} \tag{31}$$

These are the two fluid equations of motion that we shall use for the rest of our calculations.

3.2.3. *Evolution of perturbations*

The total DM density ρ_m and the baryonic density ρ_b can be split into two parts: background and perturbation:

$$\rho_m = \bar{\rho}_m + \delta\rho_m, \quad \rho_b = \bar{\rho}_b + \delta\rho_b$$

respectively.

We define, the relative density perturbations for these two components as,

$$\delta_m = \frac{\delta\rho_m}{\bar{\rho}_m + \bar{\rho}_b} \quad \text{and} \quad \delta_b = \frac{\delta\rho_b}{\bar{\rho}_m + \bar{\rho}_b}.$$

In the linear perturbation regime, we treat $\delta\rho_m$, $\delta\rho_b$ and $\vec{v}$ to be small, and hence, neglect the higher orders of these terms. Perturbing the two fluid equations in the linear regime gives:

$$\dot{\delta}_m + \frac{\bar{\rho}_m}{a\bar{\rho}}\nabla\cdot\vec{v} = -\frac{\alpha\Lambda m}{M_{pl}}\delta_b \tag{32}$$

and

$$\dot{\vec{v}} + H\vec{v} = -\frac{\bar{\rho}_m\nabla\delta\rho_m}{4a\Lambda^2 m^6} - \frac{1}{a}\nabla\phi \tag{33}$$

By using the above equations along with the Poisson's equation and assuming $\bar{\rho}_m \sim \bar{\rho}$, we get the evolution equation for δ_m as follows:

$$\ddot{\delta}_m + 2H\dot{\delta}_m - \frac{\bar{\rho}_m\delta_m}{2M_{pl}^2} - \frac{\bar{\rho}_m^2\nabla^2\delta_m}{4a^2\Lambda^2 m^6} = -\frac{\alpha\Lambda m\dot{\delta}_b}{M_{pl}} - 2H\frac{\alpha\Lambda m\delta_b}{M_{pl}} + \frac{\bar{\rho}_b\delta_b}{2M_{pl}^2} \tag{34}$$

This is a second order differential equation. The coefficient of the spatial derivative ∇^2 gives the square of the sound speed c_s. Thus, we get,

$$c_s^2 = \frac{\bar{\rho}_m^2}{4\Lambda^2 m^6} \tag{35}$$

Below in Fig. 1, we show the plot for c_s^2 vs the redshift z for $m = 1$ eV and $\Lambda = 500$ eV. We take the time evolution of the background density $\bar{\rho}_m$ as

$$\bar{\rho}_m = \frac{0.4(1+z)^3}{(1+1000)^3} \tag{36}$$

where the value of $\bar{\rho}_m$ at equality ($z = 1000$) is set as 0.4 eV^4 ([36]). As evident from the plot, the sound speed is very small (compared to the speed of light $c = 1$).

3.3. *Analytical Solution*

We now have the perturbation equations (32) and (33) supplemented by the Poisson equation. In the absence of baryons, we recover the usual evolution of CDM as a non-relativistic fluid. This can be seen as follows.

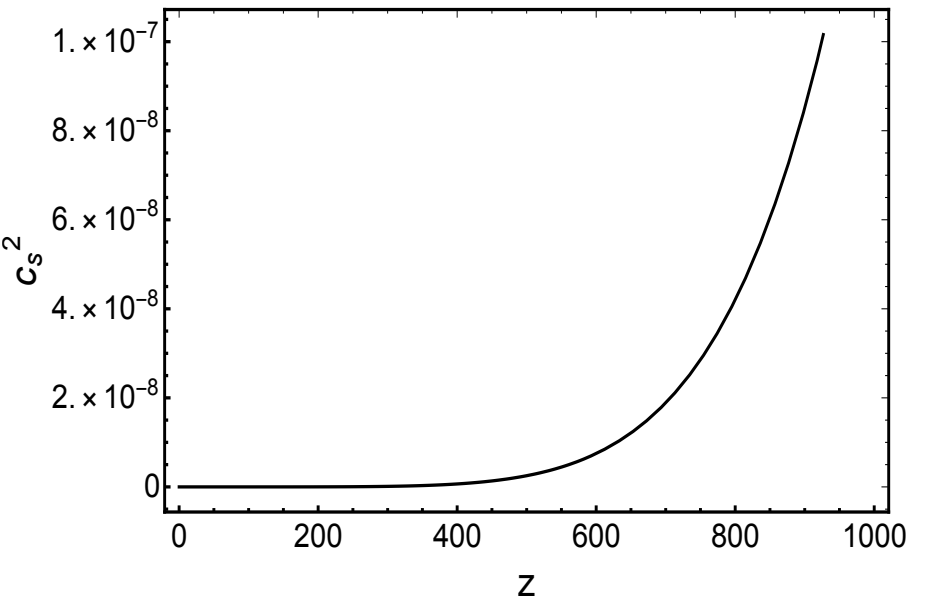

Fig. 1. Plot for c_s^2 vs z. As can be seen, the sound speed is very small compared to the speed of light ($c = 1$ in this case) at all times.

Without baryons:
In the absence of baryons, i.e. by setting $\bar{\rho}_b = 0$ and $\delta\rho_b = 0$ in equations (32) and (33), we obtain a set of equations as follows:

$$\dot{\delta}_m + \frac{1}{a}\nabla \cdot \vec{v} = 0 \tag{37}$$

and

$$\dot{\vec{v}} + H\vec{v} = -\frac{\bar{\rho}_m \nabla \delta\rho_m}{4a\Lambda^2 m^6} - \frac{1}{a}\nabla\phi \tag{38}$$

For $\Lambda \gg 1$, i.e, when the sound speed c_s is taken to be very small, the above set of equations reduce to the usual perturbation evolution equations in a ΛCDM model in the matter-dominated regime.[37] Thus, in the absence of baryons, usual CDM like evolution is recovered at the background level as well as for the perturbations. This can also be seen from the second order differential equation governing the evolution. From equation (34), in the absence of baryons, we obtain,

$$\ddot{\delta}_m + 2H\dot{\delta}_m - \frac{\bar{\rho}_m \delta_m}{2M_{pl}^2} - \frac{\bar{\rho}_m^2 \nabla^2 \delta_m}{4a^2\Lambda^2 m^6} = 0 \tag{39}$$

As expected, the above equation is the usual evolution equation for CDM in a flat ΛCDM universe, with a small sound speed given by (35). Since the sound speed

is very small, for all practical purposes, this term can be neglected and we end up with the following second order differential equation:

$$\ddot{\delta}_m + 2H\dot{\delta}_m - \frac{\bar{\rho}_m \delta_m}{2M_{pl}^2} = 0 \tag{40}$$

This is the evolution equation of non-relativistic CDM in the ΛCDM model.[38] The growing solution of this equation is the usual CDM like evolution, which is $\delta \propto a$ for a matter dominated universe.[37,38] In the absence of baryons, the superfluid dark matter thus behaves exactly like CDM and the growth of perturbations follow the ΛCDM-like evolution.

With baryons:
We now investigate how the perturbations evolve when both baryons and baryonic interactions with superfluid dark matter is present. A complete solution is to be found numerically for different values of the model parameters. This will be pursued in the next section. For now, we try to find the generic nature of the DM perturbations growth with some basic assumptions regarding the baryonic density evolution and the interaction strength between baryons and the superfluid. We assume that the baryon perturbation δ_b follows the same rate of growth as in ΛCDM i.e. at late times $\delta_b \propto a$ as obtained from the observed power spectrum. We now consider equation (34) which is the single second order evolution equation. We are only interested in the temporal behaviour of δ_m. With the assumption that $c_s^2 \ll 1$, equation (34) can be analytically solved for two limiting cases. For the first case when the interaction strength is negligible, one can set $\alpha = 0$. With this condition, the only remaining term in the RHS of (34) is the last term. The resulting equation has a power law solution of the form $\delta \propto t^{2/3} \propto a$ in the matter dominated era. This is the usual CDM-like behaviour as expected when the interaction strength is negligible. The other limiting case solution can be obtained by setting a very strong interaction strength i.e. by taking a large enough α such that the first two terms in the RHS of (34) become dominant. In this case, the solution for the matter dominated universe comes out to be of the form

$$\delta_m \propto t^{5/3} \propto a^{5/2} \tag{41}$$

The time evolution of The DM perturbation growth in the two cases is thus captured by the following form:

$$\delta_m \propto a \qquad \text{for no interaction} \tag{42}$$

$$\delta_m \propto a^{5/2} \qquad \text{for strong interaction} \tag{43}$$

Important distinct features arise when we look at the time evolution of δ_m for each mode. During the matter dominated era, δ_m grows as a in ΛCDM whereas in this model, it grows as $a^{5/2}$ i.e. at a much faster rate compared to ΛCDM.

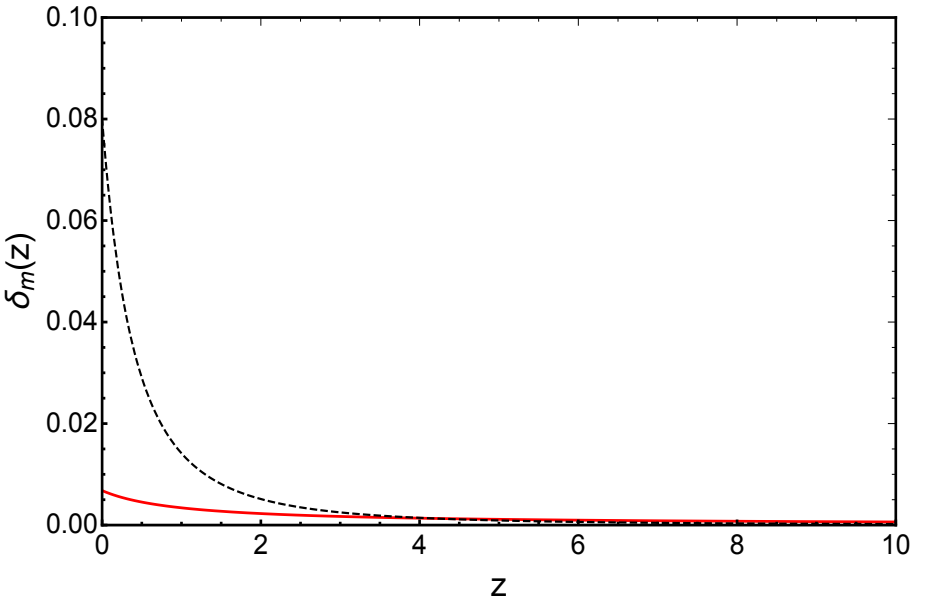

Fig. 2. Growth of δ_m with respect to z. The red solid line represents the growth for ΛCDM and the black dashed line corresponds to the growth for superfluid for $\alpha = 10^{-6}$. The growth in case of superfluid DM is dominated by the term proportional to $a^{5/2}$ and is higher compared to ΛCDM. The higher the value of α, i.e. the stronger the superfluid DM-baryonic interaction, the steeper is the growth rate (as will be discussed in the next section).

For convenience, we write the evolution of δ_m in terms of the redshift:

$$\delta_m \propto \frac{1}{(1+z)^{5/2}} \tag{44}$$

Fig. 2 shows the nature of growth in both the models (red solid curve representing ΛCDM, black dashed curve representing superfluid DM).

A proper way to find the full solution for the perturbed quantities is to solve coupled differential equations using a numerical approach. In the next section we solve the perturbation equations numerically in the linear regime and look for any possible deviations from ΛCDM.

4. Numerical Solution

In order to obtain the solutions for δ_m, we rewrite equations (32) and (33) in the Fourier domain in physical co-ordinate as,

$$\dot{\delta}_m + \frac{\bar{\rho}_m}{\bar{\rho}}(ikv) = -\frac{\alpha \Lambda m}{M_{pl}}\delta_b \tag{45}$$

and

$$ik\dot{v} + ikHv = \frac{k^2\bar{\rho}_m\bar{\rho}\delta_m}{4\Lambda^2 m^6} + \frac{a^2}{2M_{pl}^2}\left(\delta\rho_m + \delta\rho_b + \frac{3iaH\bar{\rho}v}{k}\right) \tag{46}$$

To solve the above equations, we write them in terms of redshift,

$$-H(1+z)\frac{d\delta_m}{dz} + \frac{\bar{\rho}_m}{\bar{\rho}}(ikv) = -\frac{\alpha\Lambda m}{M_{pl}}\delta_b \tag{47}$$

and

$$-ikH(1+z)\frac{dv}{dz} + ikHv = \frac{k^2\bar{\rho}_m\bar{\rho}\delta_m}{4\Lambda^2 m^6} + \frac{\bar{\rho}}{2M_{pl}^2(1+z)^2}\left(\delta_m + \delta_b + \frac{3iHv}{k(1+z)}\right) \tag{48}$$

Parameters and initial conditions:

The model parameters involved are m, Λ and α. We take $m = 1$ eV and $\Lambda = 500$ eV while keeping the parameter α as free parameter which is varied to check where the model deviates from flat ΛCDM.

We integrate the perturbation equations using the following initial conditions at the epoch of equality $z = 1000$: We set $\delta_b(z = 1000) = \delta_m(z = 1000) = 10^{-5}$ and $H(z = 1000) = m = 1$ eV.

Since $\bar{\rho_m} \gg \bar{\rho}_b$, we assume $\bar{\rho} = \bar{\rho}_m + \bar{\rho}_b \approx \bar{\rho}_m$ as given in (36).

The initial value of v at $z = 1000$ is chosen to be around 1. For the time evolution of the background density and Hubble parameter, we take the usual ΛCDM evolution of these quantities in matter-dominated era, i.e. $\bar{\rho}_m \propto 1/a^3$ and $H \propto 1/a^{3/2}$. Furthermore, we take $\delta_b \propto a$. We keep the wavenumber k fixed at 0.0001 eV, although the nature remains same for larger values of k.

Figure 3 shows the evolution of the DM density perturbation δ_m with respect to the redshift z for different values of $\alpha = 10^{-8},\ 10^{-7},\ 10^{-6},\ 10^{-4}$ and also for ΛCDM corresponding to $\alpha = 0, \Lambda \to \infty$. As expected, the smaller the value of α, the closer the resemblance with ΛCDM-like evolution. As we see in Fig. 3, the plot for $\alpha = 10^{-8}$ coincides with ΛCDM. When α is large enough, the growth is very steep. This is because a large enough α implies large interaction strength between the superfluid phonons and baryons, ensuring that structure formation takes place at an earlier epoch as compared to ΛCDM.

In Figure 4, we plot the relative differences between the perturbation growth in ΛCDM model and superfluid DM model for different values of α in terms of $\delta_{\text{superfluid}}/\delta_{\Lambda\text{CDM}}$. As expected, the ratio is very high at a lower redshift. As we go to higher redshifts, the ratio tends to 1 i.e., they eventually agree with ΛCDM at very high redshifts and matches exactly at $z = 1000$ where we set our initial conditions. The ΛCDM model corresponds to $\alpha = 0$. For $\alpha = 10^{-8}$, the deviation from ΛCDM at low redshift goes up to 0.13% at $z = 0.01$. The larger the value of α, the higher is the ratio, implying a stronger deviation from ΛCDM at low enough redshifts. As α is increased to 10^{-7}, the deviation from ΛCDM becomes much larger ($\sim 62\%$).

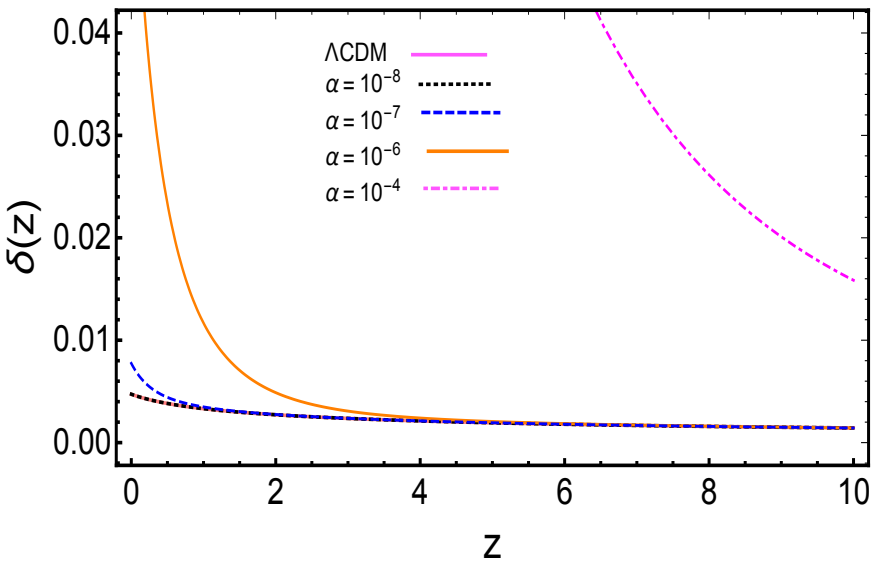

Fig. 3. Plot for δ_m vs z for different values of α as obtained numerically. The different curves correspond to different α as shown in the figure. The curve corresponding to $\alpha = 10^{-8}$ coincides with ΛCDM. For higher values of α, the deviation from ΛCDM increases gradually. The growth rate at a given redshift is maximum for $\alpha = 10^{-4}$ for the cases considered here.

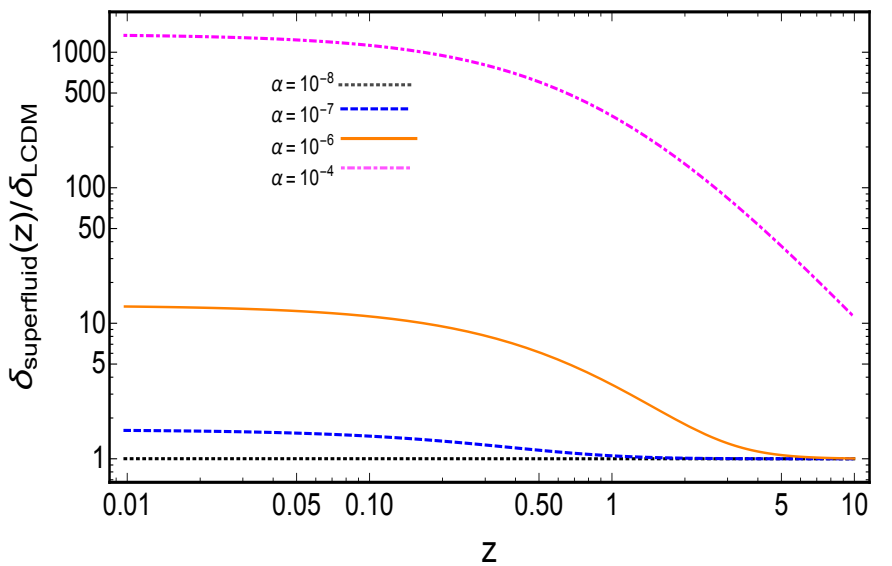

Fig. 4. Plot for $\delta_{\text{superfluid}}/\delta_{\Lambda\text{CDM}}$ vs z for different values of α. For an exact coincidence with ΛCDM, this ratio should be 1. Values > 1 signify larger deviation from ΛCDM. For $\alpha = 10^{-8}$, the ratio is almost nearly 1 showing a deviation only up to 0.13% at $z = 0$. For $\alpha = 10^{-7}$, the deviation increases up to 62% at the same redshift.

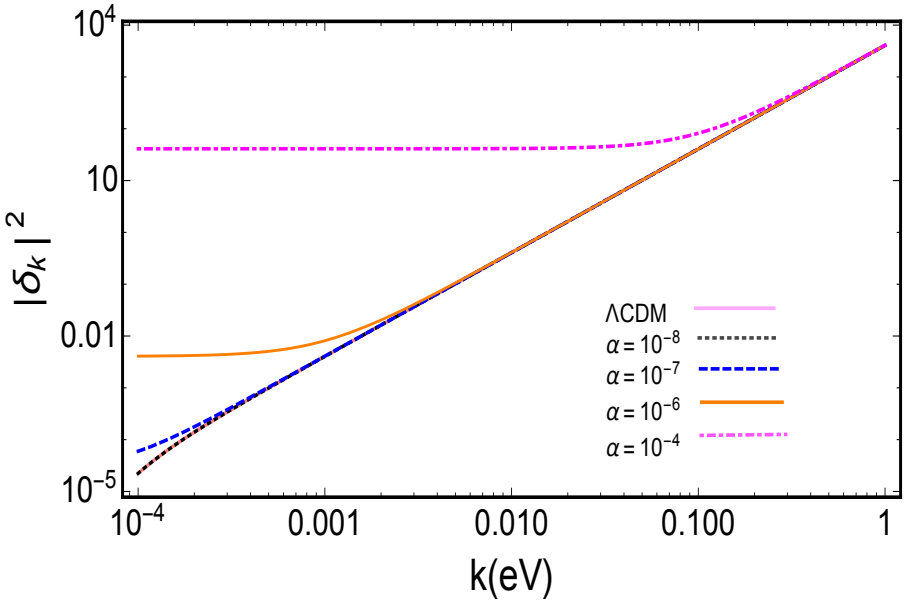

Fig. 5. Plot for $|\delta_k|^2$ vs k at $z = 0$. This is the matter power spectrum up to a constant factor. The plot reaffirms our previous results. In general the power increase approximately linearly with k. However, for large enough α, it becomes relatively flatter.

We can also plot the matter power spectrum $P(k)$ as a function of k at $z = 0$. The matter power spectrum $P(k) \propto |\delta_m(k)|^2$. In Fig. 5, we plot $|\delta_m(k)|^2$ vs. k which shows how the power varies for different values of α. As shown in the figure, the power spectrum for $\alpha = 10^{-8}$ matches with the ΛCDM prediction. As can be seen, the power increases for larger values of α at a given value of k. This is because the perturbation growth is stronger for large α as discussed earlier.

5. Results and Discussions

The superfluid dark matter model is a very promising and newly emerging model of cosmology combining together the rich physics of condensed matter, particle physics and cosmology. In view of its success in explaining a number of observations within the galaxies where ΛCDM fails to provide a satisfactory explanation, this model can be said to offer a greater understanding of the universe. In their earlier works, Khoury and his collaborators have investigated the implications of this model at galactic scales. However, a complete study of cosmological implications have not been performed earlier. In this paper, we have tried to investigate, both analytically and numerically, whether the predicted cosmology of the model tallies well with the observations and how different the predictions are from that of ΛCDM. In the realm of non-relativistic low energy effective theory of superfluid, the background cosmology agrees with the predictions of ΛCDM, and this gives a constraint on the

two model parameters α and Λ which turn out to be different than their galactic scale constraints. This result has also been discussed in.[1] At the level of first order perturbation, we find that the above constraints lead to a cosmology which differ significantly from ΛCDM. In particular, our analytical results suggests that the growth of density perturbations of dark matter superfluid roughly goes as $a^{5/2}$, which is much higher compared to the ΛCDM picture ($\delta_m \propto a$). This might be due to the strong interaction between superfluid phonons and baryonic matter. This behaviour has also been verified from the numerical solutions. For the numerical calculations, in particular, we have kept two of the model parameters m and Λ fixed at 1 eV and 500 eV respectively. This gives an upper bound on the third parameter: $\alpha \leq 10^{-8}$ corresponding to just 0.13% deviation from ΛCDM. This is different from the value quoted in.[1] The bound obtained in,[1] for $m = 1$ eV, is $\alpha \leq 10^{-4}$, which, even though predicts the correct background evolution, strongly deviates from ΛCDM in the context of perturbation growth in the present epoch. This can be seen in Figs. 3, 4 and 5. In our analysis, we have assumed the baryonic component to follow standard dust evolution ($\propto \frac{1}{a^3}$). In the absence of baryons, however, this model successfully reproduces the usual non-relativistic CDM evolution as obtained in case of a flat matter dominated ΛCDM universe, both at the background as well as first order perturbation level.

A more complete analysis of the perturbation growth should rely on the proper relativistic extension of the theory, which has not been attempted in this paper. Some relativistic models have been discussed in the original paper,[1] however a rigorous analysis is still lacking. We hope to address the same in a future work. Our work looks into the solution in the linear regime where perturbations are taken to be small. In future, we plan to extend our analysis to the non-linear regime and study the structure formation through spherical collapse. It would also be interesting to see how well this model predicts the CMB or the halo mass function.

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