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Aspects of Quantum Gravity in de Sitter Space

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Dissertation presented in partial fulfillment of the requirements for the degree of Doctor of Science (PhD): Physics

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Preface

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Several chapters of this thesis are essentially reprints of existing papers [1–6], with minor modifications for the cohesion of the text. As copyright is retained

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Abstract

Our universe is expanding. Even more dramatically, the rate of expansion is increasing. The simplest example of a universe with an accelerating expansion is a de Sitter spacetime. Our universe seems to be asymptotically evolving to de Sitter space. This process is associated with the largest cosmological scales. The reader may associate quantum mechanics with small scales and be surprised to hear that quantum effects play an important role in this cosmological evolution. In part I, we will begin by arguing for the importance of quantum gravity in a cosmological context. We will then overview some properties of de Sitter space and some of the quantum gravity that will play a role in our work.

The title of this thesis is *Aspects of Quantum Gravity in de Sitter Space*. We will look at three such aspects.

In part II we will use holography. This is a technique to study quantum gravity by using quantum field theory in one less spacetime dimension. We will holographically construct a theory with a supersymmetric de Sitter vacuum and compute its no-boundary wavefunction. The no-boundary wavefunction is a proposal for the ground state of a theory of quantum gravity. By interpreting this wavefunction, one can gauge some properties of the quantum gravity theory such as the stability of the would-be ground state and expectation values of quantum fields.

In part III, we will study string compactifications. Superstring theory is the leading candidate for a complete theory of quantum gravity. However, superstring theory is most naturally formulated in ten spacetime dimensions. We will investigate how to construct vacua where the extra dimensions are sufficiently small such that a large observer (for instance a human) will only experience four large spacetime dimensions. We then further look at the possibility of making these four large dimensions a de Sitter space. As we will see, this is no easy task and no consensus has yet been reached on whether it is possible to realize a four-dimension de Sitter vacuum in superstring theory.

Part IV is concerned with the swampland program. This program is concerned with constraining low energy effective field theories by using quantum gravity. After an overview of the swampland programme, we will study the decay of charged black holes in de Sitter space and from the consistency of this decay argue for a bound on the spectrum of allowed charged particles in de Sitter space. After this we take a different approach and look at an alternative cosmological model known as quintessence, where the cosmological evolution is driven by a rolling scalar field. We argue that if for some reason de Sitter space is fundamentally inconsistent in a theory of quantum gravity, this bounds the allowed couplings in quintessence models. In particular, it bounds the allowed time-variation of the fine-structure constant.

We do not want to leave the reader with the impression that this thesis consists of three disparate pieces. In part V, we will conclude. We will argue that these three aspects are part of a whole and each aspect might be informative of the others. We will speculate on possible conclusions to draw from these aspects and give an outlook on possible future work.

Beknopte samenvatting

Ons universum breidt zich uit. Het tempo waaraan ons universum zich uitbreidt versnelt steeds. Het eenvoudigste voorbeeld van een ruimtetijd die zich aan een versnellend tempo uitbreid is de Sitter-ruimtetijd. Ons universum lijkt asymptotisch naar een de Sitter-ruimtetijd toe te evolueren. Dit proces is geassocieerd met de grootste kosmologische afstanden in ons universum. De lezer associeert kwantummechanica misschien met dingen op kleine afstanden en is misschien verbaasd dat kwantumeffecten een belangrijke rol spelen in deze kosmologische evolutie. We zullen in deel I het belang van kwantumgravitationele effecten in een kosmologische context beargumenteren. Daarna zullen we een overzicht geven van enkele eigenschappen van de Sitter-ruimtetijd en kwantumgravitatie die een rol zullen spelen in ons werk.

De titel van dit thesis is *Kwantumgravitationele aspecten van de Sitter ruimte*. We zullen drie zulke aspecten bestuderen.

In deel II zullen we holografie gebruiken. Dit is een techniek om kwantumgravitatie te bestuderen gebruikmakende van kwantumveldentheorie met een ruimtetijddimensie minder. We zullen holografisch een supersymmetrische theorie in de Sitter-ruimtetijd construeren en de Hartle-Hawking golffunctie van deze theorie berekenen. De Hartle-Hawking golffunctie is een voorstel voor de grondtoestand in een theorie van kwantumgravitatie. Door deze golffunctie te interpreteren kan men enkele eigenschappen van de kwantumgravitatietheorie inschatten zoals de stabiliteit van de voorgestelde grondtoestand een de verwachtingswaarden van velden in de theorie.

In deel III zullen we compactificaties van snaartheorie bestuderen. Supersnaartheorie is de voornaamste kandidaat voor een complete theorie van kwantumgravitatie. Supersnaartheorie is echter het meest natuurlijk te formuleren in tien ruimtetijdsdimensies. Wij zullen onderzoeken hoe men vacua kan construeren waarbij de extra dimensies voldoende klein zijn zodat deze op voor de mens relevante schalen niet te zien zijn. We bekijken in hoe

verre het mogelijk is om een de Sitter-ruimtetijd te verwezenlijken in de vier macroscopische ruimtetijdsdimensies. We zullen zien dat dit geen eenvoudig taak is en dat er nog geen consensus bestaat over of het mogelijk is een de Sitter-ruimtetijd te verwezenlijken in supersnaartheorie.

Deel IV gaat over het *swampland program*. Het doel van dit programma is gebruik maken van kwantumgravitatie om de eigenschappen van effectieve theorieen op lage energie te bepalen. Na een overzicht te geven van het *swampland program* zullen we de verdamping van geladen zwarte gaten in een de Sitter-ruimtetijd bestuderen. Wij zullen beargumenteren dat eisen dat deze verdamping op een consistente manier gebeurt het toegelaten spectrum van geladen deeltjes in de theorie beperkt. Hierna zullen we een andere aanpak volgen en een alternatieve theorie van kosmologie, kwintessens, bekijken. Hier wordt de kosmologische evolutie gedreven door een rollend scalair veld. We zullen beargumenteren dat als de Sitter-ruimtetijd om een bepaalde reden inconsistent zou zijn in een theorie van kwantumgravitatie, dit de toegelaten koppelingen in kwintessens-modellen beperkt. In het bijzonder zullen we zien dat dit de toegelaten tijdsvariatie van de fijnstructuurconstante beperkt.

We willen de lezer niet achterlaten met de indruk dat deze drie aspecten van kwantumgravitatie in de Sitter-ruimtetijd totaal gedissocieerd van elkaar zijn. In deel V geven we ons besluit en zullen we de drie aspecten aaneenrijgen. We geven ook een zicht op mogelijk toekomstig werk dat het werk in dit thesis kan uitbreiden.

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Part I

Prolegomena

Chapter 1

Introduction

At the largest cosmological scales, our universe in its current stage of evolution approximately resembles a spacetime known as de Sitter space. As time goes on, the universe is expected to resemble de Sitter space more and more. The goal of this thesis is to understand some aspects of quantum gravity in de Sitter space. The universe is very large and on average very cold. One expects quantum corrections to gravity to become relevant at high energy and short length scales. Why then should we care about quantum gravity in de Sitter space? Let us start by giving some motivation.

1.1 Our universe

Let us start by overviewing some cosmology, see for instance [7] for a more detailed review. We will take for granted that the reader has a general understanding of quantum field theory and general relativity and set set $\hbar = c = 1$.

If one assumes that for the purposes of dynamics at large enough scales the universe is spatially isotropic and homogeneous, its metric is the Friedmann–Lemaître–Robertson–Walker (FLRW) metric

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -dT^2 + g_{ij}dx^i dx^j = -dT^2 + a(T)^2 d\Sigma_3^2\,, \tag{1.1.1}$$

with $d\Sigma_3^2$ the metric on the spatial slices

$$d\Sigma_3^2 = \frac{1}{1-kr^2}dr^2 + r^2 d\Omega_2^2\,, \tag{1.1.2}$$

with $d\Omega_2^2$ the metric on a unit size two-sphere. Here, $k = 0$ for flat spatial slices, $k = 1$ for hyperspherical spatial slices, and $k = -1$ for hyperbolic spatial slices.

The rate of expansion (or contraction) per $a(T)$ volume is then given by the hubble parameter

$$H(T) \equiv \frac{\dot{a}(T)}{a(T)} \,. \tag{1.1.3}$$

In general relativity, the dynamics of spacetime is determined by the energy-momentum $T_{\mu\nu}$

$$\mathcal{R}_{\mu\nu} - g_{\mu\nu}\mathcal{R} = 8\pi G_N T_{\mu\nu} \,, \tag{1.1.4}$$

with G_N Newton's constant, $\mathcal{R}_{\mu\nu}$ and $\mathcal{R}$ the Ricci tensor and scalar. Applying this to the FLRW metric, the dynamics of the spacetime are then determined by

$$H^2 + \frac{k^2}{a^2} = 8\pi G_N \frac{\rho}{3} \,, \tag{1.1.5}$$

and

$$\dot{\rho} = -3H(p + \rho) \,, \tag{1.1.6}$$

with $\rho(T) = T_{00}$ the energy density and $p(T) : T_{ij} = g_{ij}p$ the pressure of the contents of the spacetime. Equation 1.1.6 follows from energy conservation and tell us how the matter in the spacetime dilutes as space expands. For dust $p = 0$ and for radiation $p = \rho/3$. Equation 1.1.5 then tells us how the spacetime expands in response to the matter present.

The universe presently appears to be very flat [8]. Either $k = 0$ or k/a^2 is very small and we will neglect it for now. Radiation then evolves as $\rho_R \propto a^{-4}$ and dust as $\rho_D \propto a^{-3}$. The universe had an early radiation dominated era. As the universe expanded and ρ_R diluted the expansion became dominated by dust. But ρ_D too dilutes as the universe expands. It seems the current expansion of the universe is driven by a very different contribution, dark energy, which obeys $\rho_{DE} = -p_{DE}$ [9, 10]. This contribution does not dilute as the universe expands $\dot{\rho}_{DE} = 0$, it provides a constant energy per unit volume and its observed value[1] is $\rho_\Lambda \approx 10^{-47}\text{GeV}^4 > 0$. From eq. (1.1.5) one sees that $H > 0$ is constant when $\rho_\Lambda > 0$ dominates. The spacetime with only $\rho_\Lambda > 0$ contributing to energy density and constant H is known as de Sitter space[2].

What is the source of this dark energy? We know that in quantum mechanics or quantum field theory (QFT) the ground state can have a zero-point energy

[1] Recent observation see e.g. [11–14] have cast some doubt on this, and are suggestive of $p_{DE} < -\rho_{DE}$. We will not get involved in this controversy as it is a matter for observational astronomers to resolve. It would anyway be good to understand quantum gravity in pure de Sitter space first as the $p_{DE} < -\rho_{DE}$ is even more confusing.

[2] Named after Willem de Sitter who constructed this spacetime in [15] and tried to fit ρ_{DE} with the expansion of distant galactic objects.

associated to it. This provides a constant energy density as desired. In QFT this statement is unphysical as only differences in energy are measurable. But gravity couples directly to the energy through eq. (1.1.4), making the energy itself physically meaningful. The QFT zero point energy one obtains from the standard model is many orders of magnitude too large for ρ_{DE}, at best 10^{-6}GeV^4 if one takes the QCD scale as the QFT cut-off scale and even 10^{71}GeV^4 if one takes the Planck scale M_p as cut-off [16]. As a QFT calculation this zero point energy is perhaps rather meaningless as we can simply add a constant term to the standard model Lagrangian to shift the energy as desired. After coupling to gravity, the action is then

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{16\pi G_N}\left(\mathcal{R} - 2\Lambda\right) + \mathcal{L}_{\text{matter}}\right], \tag{1.1.7}$$

with $\mathcal{L}_{\text{matter}}$ the matter content of the theory. One then sets Λ, the cosmological constant term[3], such that part of it cancels the QFT vacuum energy and the remained provides the observed dark energy. From a pure low energy effective field theory (EFT) perspective this is just one more tunable parameter in the theory that we set to the observed value. But is this explanation really satisfactory?

We cannot strictly claim that this low energy EFT purist picture is wrong, but it requires more magic than appears at first sight. The reader may have thought it odd that the QFT vacuum energy changes so drastically depending on whether one takes the cut-off scale the QCD scale or the Planck scale. In fact the vacuum energy, like the Higgs mass, is a relevant operator and heavily dependent on the UV details of the theory. For a theory with UV cut-off Λ_{UV} with a single free scalar of mass $m_{IR} \ll \Lambda_{UV}$ one can show [16]

$$\langle\rho_{DE}\rangle = \int_0^{\Lambda_{UV}} \frac{4\pi k^2 dk}{(2\pi)^3}\frac{\sqrt{k^2+m_{IR}^2}}{2} \approx \frac{\Lambda_{UV}^4}{16\pi^2}\,. \tag{1.1.8}$$

This leads to a rather disconcerting situation. Imagine on has a theory with two free[4] scalars of mass m_{IR}, m_{UV}, with $m_{IR} \ll m_{UV}$, coupled only to gravity. There is a low-energy EFT with just the IR scalar and this predicts the scalar contributes

$$\langle\rho_{DE}\rangle \sim m_{UV}^4\,, \tag{1.1.9}$$

to the dark energy. Suppose one adds an intermediate free scalar with mass m_{yellow} such that $m_{IR} \ll m_{yellow} \ll m_{UV}$. Now the low-energy EFT should predict

$$\langle\rho_{DE}\rangle \sim m_{yellow}^4\,, \tag{1.1.10}$$

[3] In a dynamical theory of gravity this is no longer a constant term in the action because of the $\sqrt{-g}$ prefactor.

[4] Everything we say here only becomes much worse for interacting fields.

but of course the low-energy theory shouldn't know that a new scalar has been introduces and that the high-energy cut-off needs to be changed. The low-energy EFT seems useless for predicting $\langle \rho_{DE} \rangle$.

If we have a QFT that we trust up to the Planck scale we would get

$$\langle \rho_{DE} \rangle \sim M_p \left(c_0 + \sum_{i=0}^{\infty} c_i \right) , \tag{1.1.11}$$

with c_0 such that one gets the $\langle \rho_{DE} \rangle$ predicted at lowest order in perturbation theory and c_i higher corrections. For a low-energy approximation to be valid, one needs $\sum_{i=1}^{\infty} c_i \ll c_0$. We will come back to this point (much) later in chapter 13.

So far, we have been working in a semiclassical picture where we mix classical general relativity and QFT for the standard model fields. Ideally, we would want to quantize gravity. General relativity is famously nonrenormalizable and we would need to develop a quantum theory of gravity. This is a noble goal, but does it matter for understanding ρ_{DE} and de Sitter space? It is the late universe where ρ_{DE} is dominant and quantum corrections hardly seem relevant for describing late universe cosmology, which happens at low energies.

There is commonly believed to be an era of quasi-de Sitter evolution preceding the radiation-dominated era. This is the inflationary period and it has been proposed as the mechanism which drove the universe to be very homogeneous, isotropic, and spatially flat. The dilution of matter in the inflationary period has also been used as an explanation for why magnetic monopoles which appear in extensions of the standard model have not been observed. But while ρ_{DE} during inflation is believed to be much larger than the current value of the dark energy, it is still nowhere near large enough for quantum gravity to become relevant.

In fact, this picture of higher order quantum corrections becoming relevant at certain energy scales can be deceptive in quantum gravity. Famously, the quantum behavior of black hole event horizons has been puzzling people for decades despite the fact that for large enough black holes the curvature near the event horizon becomes negligible. In quantum gravity, de Sitter space will present its own set of puzzles and we will encounter some throughout the thesis. One is that de Sitter space possesses cosmic horizons akin to black hole event horizons. Another is that when one evolves time forwards the contents of de Sitter space are diluted due to the expansion. Evolving time backwards things are then focused together and the typical energy scales increases. In a pure de Sitter space a low energy state at some point in time is then deeply sensitive to the high UV physics of the theory in the far past. One cannot cleanly separate

energy scales and deal with a low energy EFT once long enough time scales are involved.

More generally than just de Sitter space, quantum gravity seems to have a certain unreasonable effectiveness in constraining effective field theories. It might seem that any effective field theory that is well-behaved below its UV cut-off scale should be able to couple to quantum gravity. However, it has been argued that coupling EFTs to quantum gravity can lead to paradoxes and inconsistencies. Avoiding these inconsistencies leads to much stronger constraints on the EFTs than one may expect and it leads on constraints far below the energy scale where one expects quantum gravity to become relevant. The program of constraining EFTs in this manner is known as the swampland program and we will have much more to say on it in part IV.

1.2 Quantum gravity

There are two parts to describing a physical system, dynamics and the boundary conditions.

The most successful proposal for the dynamics of a UV-complete theory of quantum gravity is string theory, which we will describe in chapter 3. The dynamics of string theory at low energy are captured by Einstein gravity, which makes it very appealing as a theory of quantum gravity for our world. But as we will see, making contact with the real word is not so easy. It is not straightforward to get four spacetime dimensions in string theory and obtaining a positive cosmological constant is an especially difficult, parhaps impossible, task.

Most days, we do not violently explode in rays of high-energy physics. It is then reasonable to assume that the universe nowadays either is in a local equilibrium ground state or at least not too drastically dissimilar from such a state. That is, any nonequilibrium fluctuations are on energy scales well below the quantum gravity scale. As part of the boundary conditions for our theory of quantum gravity, we need to set a state at one spatial slice of the theory, from which to evolve. String theory does not provide us with a unique vacuum. We will see in part III that it is possible to construct many metastable vacua for string theory. One proposal to select a quantum gravity ground state in a cosmological context is the Hartle-Hawking state, which we will discuss in section 2.3 .

1.3 Overview

In the remainder of part I we will give a general introduction to some concepts that will be relevant later. In chapter 2 we discuss some properties of de Sitter space and quantum cosmology. We then outline the basic principles of string theory and holography in chapter 3.

The next three parts make up the bulk of the thesis. Each of these make up one aspect, one way of studying quantum gravity in de Sitter space and cosmological spacetimes generally. Each part starts with an introductory chapter containing background material, followed by the chapters containing original research.

In part II, we use holographic techniques to study quantum gravity in de Sitter space. The focus here is on a theory known as higher-spin gravity. While this is a different theory from Einstein gravity, we will argue that the lessons learned here can to an extend be applied to Einstein gravity.

Then, in part III, we study string compactifications. String theory appears most naturally in more than four dimensions. If one wishes to work towards the real world, one must then compactify these extra dimensions to get a theory where only four dimensions are large enough that a human-sized observer can experience them. We will not get so far as to be able to actually construct compactifications where one has a positive cosmological constant in four dimensions, but we will have some things to say about attempting to construct four-dimensional de Sitter vacua.

In part IV, we study consistency conditions for low energy effective field theories (EFTs) coming from quantum gravity in de Sitter space and cosmological spacetimes generally. In chapter 11, we study the decay of charged black holes in de Sitter space and argue that there are bounds on the spectrum of charged particles allowed in the theory for this decay to happen consistently. In chapter 12, we look at quintessence models, where one does not have a cosmological constant. Instead, the cosmological parameter slowly changes over time.

These three parts provide three different roads towards studying quantum gravity. Since these are three ways of studying the same things, these parts are not entirely disconnected and interconnections will show up throughout. In the conclusion, part V, we will draw these parts together more explicitly into a coherent whole.

Chapter 2

de Sitter space

This chapter overviews some properties of de Sitter space.Some valuable general discussion of this topic can be found in [17–19]

2.1 Classical

In this section we will describe some basic properties of de Sitter space as a classical solution to Einstein gravity. A de Sitter space is the maximally symmetric solution to Einstein gravity with a positive cosmological constant Λ eq. (1.1.7) .

A natural geometric picture of $d-$dimensional de Sitter space is as an hyperbola of characteristic length l embedded in $d+1-$dimensional Minkowski space given by

$$-X_0^2 + \sum_{i=1}^{d} X_i^2 = l^2 \,, \tag{2.1.1}$$

with X the cartesian Minkowski coordinates. Because of this, the isometries of de Sitter space are $SO(d,1)$. Here, l is the de Sitter radius, with

$$\Lambda = \frac{(d-2)(d-1)}{2l^2} \,. \tag{2.1.2}$$

de Sitter space has a constant positive internal curvature. Its Ricci scalar is given by

$$\mathcal{R} = \frac{d(d-1)}{l^2} \,. \tag{2.1.3}$$

A manifold can be parametrized with many different coordinate systems. We will go over some of the most instructive ones for de Sitter space.

Global coordinates are given by

$$ds^2 = -d\tau^2 + l^2 \cosh^2(\tau/l) d\Omega_{d-1}^2 \,. \tag{2.1.4}$$

In these coordinates, the spatial slices are $S^{d-1}-$spheres. They have minimal radius l at $\tau = 0$ and grow unbounded towards the past and future from $\tau = 0$. One can show that there are conformal boundaries in the asymptotic past I^- and future I^+. From this, the conformal diagram of de Sitter space is given by section 2.1 .

Global coordinates inform us of the global geometry of de Sitter space. The coordinates most physically relevant to an observer are the static patch coordinates. These cover the region of de Sitter space that a single geodesic observer can see. Static patch coordinates are given by

$$ds^2 = -\left(1 - \frac{r^2}{l^2}\right) dt^2 + \left(1 - \frac{r^2}{l^2}\right)^{-1} dr^2 + r^2 d\Omega_{d-2}^2 \,. \tag{2.1.5}$$

The observer to which these static coordinates "belong" sits at $r = 0$. There is a coordinate singularity at $r = l$. Indeed, the structure of this metric seems very similar to the Schwarzschild metric for a black hole. In fact, $r = l$ is a null hypersurface that surrounds the observer. Objects that pass beyond $r = l$ can never return to the observer. There is a cosmological horizon, analogous to a black hole horizon, at $r = l$. This can also be seen from the Penrose diagram section 2.1 . Finally, there are coordinates that describe only a part of de Sitter space known as the planar patch. These are given by

$$ds^2 = \frac{l^2}{\eta^2} \left(-d\eta^2 + \sum_{i=1}^{d-1} dx^i dx^i \right) \,, \tag{2.1.6}$$

with $\eta = -e^{-T/l}$ the conformal time coordinate. This is equivalently given by

$$ds^2 = -dT^2 + e^{2T/l} \left(\sum_{i=1}^{d-1} dx^i dx^i \right) \,. \tag{2.1.7}$$

Here, the spatial slices are infinite flat $d-1$ dimensional planes. One recognizes this as the FLRW metric with flat spatial slicings and $a = e^{T/l}$ from section 1.1 . This parametrization is thus of great interest in cosmological and inflationary contexts.

Yet different parametrizations of de Sitter space exist, see for instance, [19].

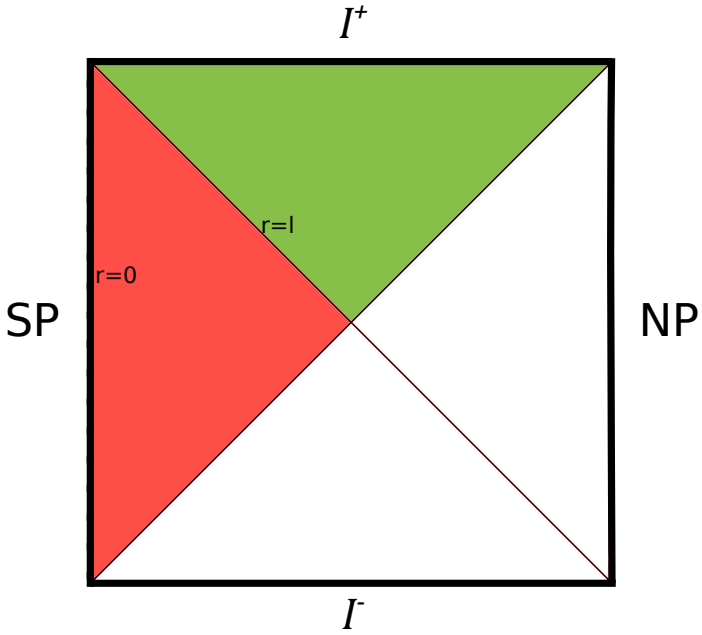


Figure 2.1: The conformal diagram of de Sitter space. Time is the vertical direction, 45 degree diagonals are lightlike. Horizontal slices are S^1 circles with the south pole (SP) on the left and the north pole (NP) on the right side. There is an S^{d-2}-sphere at each point of the diagram. The static patch for an observer standing still at the south pole is shown in red. The combination of the red and green areas makes up a planer patch.

The metric of a neutral, nonrotating black hole in de Sitter space is given by

$$ds^2 = -\left(1 - \frac{r^2}{l^2} - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{r^2}{l^2} - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\Omega_{d-2}^2 \,. \quad (2.1.8)$$

There is now both a cosmological horizon and a black hole horizon. As the mass M of the black hole increases, not only does the size of the black hole horizon increase, but the cosmological horizon also shrinks. Since the black hole must fit inside the cosmic horizon, there is a largest possible black hole in de Sitter space, the Nariai black hole. We will have a great deal more to say on black holes and de Sitter space in chapter 11.

2.2 Quantum

We will now consider de Sitter space on a quantum level, without access to a full quantum gravity, but from the perspective of QFT around a de Sitter background. For the rest of the chapter, we focus on the four-dimensional case.

As we saw in section 2.1, each observer experience a cosmic horizon beyond which the universe is inaccessible. This is a situation familiar from black hole physics. In the case of a flat space black hole, one associates to this black hole an entropy

$$S_{BH} = \frac{A}{4G_N}\,, \tag{2.2.1}$$

with A the black hole area. There is a temperature associated to the black hole, for a Schwarzschild black hole

$$T_{BH} = \frac{1}{8\pi GM} \tag{2.2.2}$$

This temperature appears as the blackbody spectrum of the Hawking radiation of particles emitted by the black hole when one does quantum field theory (QFT) in a classical black hole background.

It is then tempting to suggest that a similar thermodynamic interpretation can be associated to a de Sitter horizon. This was proposed in [20], where it was argued that when one does QFT in a de Sitter background, a particle detector observes isotropic thermal radiation of particles at a temperature

$$T_{dS} = \frac{1}{2\pi l}\,. \tag{2.2.3}$$

To the de Sitter horizon, [20] associate an entropy

$$S_{dS} = \frac{A}{4G_N} = \frac{\pi l^2}{G_N}\,. \tag{2.2.4}$$

Since the entropy inside a static patch is finite, physics inside the static patch should be described by a finite-dimensional Hilbert space. This should come as a shock from a pure QFT perspective. One can expand a field in the QFT in modes and there are an infinite number of such modes, even in a compact geometry. However, this story radically changes in a theory with dynamical gravity. In this case, too large an energy in one place will produce a black hole, providing a cut-off for the QFT description [21]. There is then a contribution to the number of states coming from the IR modes of QFT fields and coming from black holes. Since there is a maximum black hole size, the total number of modes is bounded.

There are some notable differences between a black hole horizon and a cosmological horizon.

A black hole horizon has objective meaning in the sense that each observer outside the black hole agrees on its location. Meanwhile, each observer in de Sitter space has their own subjective horizon. Since each observer is in the center of their horizon, it is impossible to go near the horizon and probe it.

While the size of a black hole increases as mass is added to it, from eq. (2.1.8) and below, a cosmological horizon shrinks as mass is added inside it.

Most striking perhaps is the difference in how the produced radiation backreacts. For black holes, it is generally agreed that emitting Hawking radiation causes the black hole to evaporate. For de Sitter space, the situation is less clear, as the produced radiation remains inside the de Sitter space. There have been for instance claims that the cosmic horizon recedes and one evolves towards flat space as a result of this evaporation, e.g. [22–24]; claims that de Sitter space is stable aginst the backreaction of this particle productions, e.g. [25–28]; and claims the the cosmological horizon shrinks and one evolves towards a singular geometry where the semiclassical description breaks down, e.g. [29]. One of the major issues in this discussion is that in QFT on a curved background there is no clear unique choice for how to define the vacuum state. The most common vacuum choice is known as the Bunch-Davies vacuum, which was recently argued to be stable in [30]. This is clearly an important point, but we will not say more about it directly here.

Much of what we discussed in this section will be crucial for part IV.

2.3 Quantum cosmology

In the previous section, we 'went quantum'. But we restricted ourselves to discussing QFT on a classical gravitational background and approximately discussing how this background would classically backreact to QFT effects. From a field theory perspective, gravity is just another field in the Lagrangian and one expects that it should be quantized.

In quantum mechanics or QFT, the dynamics can be described by the Schrödinger equation

$$H|\Psi\rangle = i\frac{\partial}{\partial t}|\Psi\rangle\,, \tag{2.3.1}$$

with $|\Psi\rangle$ any state in the theory and H the canonically quantized Hamiltonian.

As discussed in chapter 1, there are issues with quantizing Einstein gravity. Still, one can formally go through the procedure of canonically quantizing Einstein gravity with Hamiltonian $\mathcal{H}$. When one does this for spacetimes whose spatial slices are closed and without boundary, one finds the Wheeler-deWitt equation [31]

$$\mathcal{H}|\Psi\rangle = 0\,, \tag{2.3.2}$$

instead of the Schrödinger equation. The difference is that the right hand side is zero as a result of the time parametrization invariance of general relativity. This equation is a constraint on the states in the quantum theory.

Now one could attempt to solve Wheeler-deWitt equation directly. This is clearly a difficult task, but one also expects this equation to generally have many solutions. Clearly, it would be desirable to have some mechanism to select some preferred state. A state of great interest is the Hartle-Hawking state [32, 33]. This is an attempt to define a ground state in quantum gravity despite notions of energy being ambiguous there.

To start, let us consider quantum mechanics for a single particle with position $x(t)$, with dynamics described by an action $S[x(t)]$. Starting in an initial state $|x', t'\rangle$ at time t', the wavefunction Ψ of this particle at a time t can be computed in the path integral formalism as

$$\Psi(x,t) = \langle x,t|x',t'\rangle = N\int_{\mathcal{C}}[\mathcal{D}x]e^{iS[x(t)]}\,, \tag{2.3.3}$$

Where N is a normalizing factor and one integrates over paths with the right initial and final conditions. Now consider $t = 0$, $x' = 0$ and insert a complete set of energy eigenstates E_n with wavefunction $\phi_n(x)$,

$$\langle x,0|x',t'\rangle = \sum_n \langle x,0|E_n\rangle\langle E_n|0,t'\rangle = \sum_n \varphi_n(x)\overline{\varphi_n(0)}e^{iE_nt'}\,. \tag{2.3.4}$$

One then wick rotates into euclidean time, $t' \to -i\tau'$. Taking the limit $\tau' \to -\infty$, the state of lowest energy dominates over all others, which disappear. One then prepares the ground state at time zero in this way. Translating this Wick rotation back in path integral language eq. (2.3.3), one prepares the ground state $\psi_0(x)$ as

$$\Psi_0 = N\int_{\mathcal{C}}[\mathcal{D}x]e^{-I[x(\tau)]}\,, \tag{2.3.5}$$

where I is the euclidean action, obtained from S by sending $t \to -i\tau$ and taking out an overall factor of i. One integrates over paths which have a vanishing action in the far past.

This same procedure works to define the ground state for QFT in flat space[1].

[1] When one follows this procedure for QFT on a fixed de Sitter background, one obtains the Bunch-Davies vacuum mentioned in section 2.2.

It is then reasonable to hope that, by analogy, it also defines the ground state in quantum gravity. This defines the Hartle-Hawking state Ψ_{HH}. For Einstein gravity with a cosmological constant Λ, and optionally also matter fields, which we generically denote as ϕ, one then has

$$\Psi_{HH}(h_{ij}, \phi_0) = \int_{\mathcal{C}} [\mathcal{D}g][\mathcal{D}\phi] e^{-I[g,\phi]} \,, \tag{2.3.6}$$

where h_{ij} is the three-metric and ϕ_0 the field configuration on the final spatial surface where Ψ_{HH} is defined. The euclidean action on a manifold $\mathcal{M}$ is given by

$$\begin{aligned} I[g,\phi] = &- \int_{\mathcal{M}} d^4x\sqrt{g} \left[\frac{1}{16\pi G_N} (\mathcal{R} - 2\Lambda) + \mathcal{L}_{\text{matter}}(\phi) \right] \\ &+ \int_{\partial\mathcal{M}} d^3x \sqrt{g_b} \frac{1}{8\pi G_N} K \,. \end{aligned} \tag{2.3.7}$$

Here, $\mathcal{L}_{\text{matter}}(\phi)$ is the matter field lagrangian. The final term in this action is the Gibbons-Hawking-York (GHY) boundary term, with g_b the metric and K the extrinsic curvature on the boundary of the manifold $\partial\mathcal{M}$. We should include such a term for each boundary to ensure the Einstein equations of motion are reproduced correctly from varying this action. In the future, we may ofter suppress the GHY term.

One can show that this wavefunction indeed satisfies the Wheeler-deWitt equation, $\mathcal{H}\Psi_{HH} = 0$.

We have not yet specified over what geometries, and with what boundary conditions, we should integrate eq. (2.3.6). It was argued in [32, 33] that one should integrate over all four-geometries that (i) are compact and (ii) have no boundary, except for the surface on which Ψ_{HH} is defined. Since there are no boundaries, there is also no need to make a choice of boundary conditions for g and ϕ. Because of this, Ψ_{HH} is also called the no-boundary wavefunction.

The space of all possible configurations of h_{ij} and ϕ_0 is known as superspace[2]. In practice, it is not possible to compute eq. (2.3.6) for all field configurations. Indeed, if we knew how to do this consistently, Einstein gravity + matter would provide a consistent theory of quantum gravity. Instead, one restricts to computing Ψ_{HH} for a subset of particularly symmetric field configurations where computations become more manageable. One refers to this subset as a minisuperspace. Further, one usually performs a saddle-point approximation of eq. (2.3.6) to compute Ψ_{HH}. In this thesis, we will not be directly computing

[2]In a cruel twist of fate, the term superspace is also used for an unrelated concept related to supersymmetry and we will have need of superspace in both meaning in part II.

Ψ_{HH} through eq. (2.3.6). Instead, we will use holography, a procedure we outline in chapter 4.

Chapter 3

String theory

In this chapter, we will give a brief introduction to string theory. The goal is to provide a top-down view of the general properties of string theory, not to immediately make contact with the world we inhabit. Once we have these general properties in hand, we will spend the next three parts, the bulk of the thesis, applying these properties and working towards saying something about our world. It is not realistic to give a complete picture of string theory in the space available here. We will only give a quick overview of what will be needed for later, mainly in words and with equations that are meant to be suggestive rather than part of any kind of derivation. We also dare not give a complete bibliography here of what is a synthesis of many decades of research. Many excellent texts on string theory are available, amongst them [34–38], to which we refer the interested reader for further information and a more complete bibliography. Further, [39, 40] were very useful in writing this chapter.

3.1 A theory of strings

3.1.1 The basics

In nonrelativistic quantum mechanics with a fixed number of particles, one usually takes the (generalized) positions and momenta of these particles as the variables to be quantized. However, when one quantizes a relativistic theory where particles can be created or destroyed, the standard approach is to introduce fields that fill space-time. One then quantizes these fields and their conjugate momenta. It is not strictly necessary to do this. One can continue to

work with a theory of particles. To start, the worldline action of a single free particle of mass m without spin degrees of freedom in D-dimensional space is given by

$$S = -m \int d\tau \sqrt{-G_{MN} \frac{\partial X^M}{\partial \tau} \frac{\partial X^N}{\partial \tau}} \,, \tag{3.1.1}$$

with G_{MN} the metric in D-dimensional space and τ the proper time of the particle. The equations of motion from this action then produce the geodesic motion of a free particle. Very coarsely speaking, one can now quantize by defining a partition function by path-integrating this action. Actions can be introduced into the theory by choosing to integrate the action not just over line segments, but over graphs. The types of graphs one includes in the path-integral now determine the type of theory one obtains. For instance, if one only allows graphs whose nodes have four edges, one would end up with a ϕ^4-type theory in QFT language. We have given only the roughest sketch of this procedure here. The interested reader can find more details in e.g. [41].

For particles, QFT is generally the preferred procedure when quantizing. However, when we applying the procedure outlined above to a theory of strings, one discovers some very nice things. The worldsheet action for a Bosonic string is

$$S = -\frac{T_S}{2} \int d\tau d\sigma \sqrt{-\gamma} \gamma_{ab} G_{MN} \partial^a X^M \partial^b X^M \tag{3.1.2}$$

Here, γ_{ab} is the worldsheet metric, the metric on the two-dimensional surface that a string sweeps out in spacetime. The string tension T_S is the higher-dimensional analogon of the mass m in the particle action. The Regge slope α' is defined by

$$T_S = \frac{1}{2\pi\alpha'} \,. \tag{3.1.3}$$

The string length l_s is the typical length scale set by these scales[1], $l_s \equiv \sqrt{\alpha'}$. One can vary this action to find that its equations of motion lead to the geodesic motion of a free string.

When path-integral quantizing this action, there is much less freedom than in the particle case. There, we could choose different classes of graphs. Here, we can only choose different classes of smooth two-dimensional surfaces. We can distinguish surfaces without a boundary, giving closed strings[2], and surfaces with a boundary, giving an open string[3] with the end-points of the string corresponding to the boundary. Further, we can also distinguish between orientable and non-orientable surfaces. We will not go deeply into this here,

[1] Different definitions of the string length are in use, varying by an $\mathcal{O}(1)$ factor. Luckily T_S and α' tend to be used consistently throughout the literature.

[2] Picture a rubber band.

[3] Picture a piece of twine.

except to say that one can have a theory with only closed strings, but a theory with open strings must necessarily also include closed strings.

When quantizing the string, one finds states of different spin in the theory. In the critical spacetime dimension[4], $D = 26$, closed strings have massless states of spin two in their spectrum. This means that by quantizing the closed string, we automatically obtain a quantum theory of gravity as part of the string theory! Promisingly, in the classical and point-particle limit, this theory of gravity is effectively Einstein gravity.

The full massless spectrum of the closed Bosonic string consists of the graviton G_{MN}, a totally antisymmetric tensor field B_{MN} and a scalar field, the dilaton Φ. The field B_{MN} is the potential for a gauge field with three-form field strength $H_3 = dB_2$. In addition, there is an infinite tower of massive fields with their mass scale set by $\sqrt{T_S}$. That is, there are states of mass $\sqrt{T_S}$, of mass $\sqrt{2T_S}$, et cetera. More disconcertingly, there is a tachyonic scalar with $m^2 \propto -T_S$, pointing to an instability of the vacuum around which one usually quantizes Bosonic string theory.

The open string has a gauge field in the massless sector, a tower of massless fields, and again a tachyon.

So far, string theory has some disconcerting tachyonic instabilities and it lacks fermions in its spectrum. Both these problems are resolved by going to the superstring.

3.1.2 Superstring theory

Supersymmetry is an enhancement of the Poincaré symmetry group. This enhancement turns the Poincaré group into a supergroup. That is, it adds transformations which mix fermionic and bosonic degrees of freedom, whose generators have anticommutation rather than commutation relations. One can can have different amounts of supersymmetry, different extensions of the Poincaré group. The amount of supersymmetry is indicated by $\mathcal{N}$. We will not give further details here.

There are five main superstring theories that can be constructed. There are three $\mathcal{N} = 1$ superstring theories: type I, heterotic $SO(32)$, and heterotic $E_8 \times E_8$. There are two $\mathcal{N} = 2$ superstring theories, type IIA and type IIB. These are all free of tachyons. Their critical dimension is $D = 10$.

[4]String theory is most straightforwardly treated in this critical dimension. Different dimensions are an interesting subject, but more subtle. Throughout this thesis, we work in critical-dimension string theory.

The two superstring theories that will mainly be used in this thesis are the IIA and IIB theories. We will use these in their low-energy supergravity limit. That is, we restrict to the fields in the massless sector and assume the physical processes we will look at are at energies below $\sqrt{T_S}$ and the geometries we look at have typical length scales larger than l_s. In this case, we can write down an effective field theory action for the lightest sector. We will describe the Bosonic part of this action.

The IIA and IIB actions have a common sector called the NS-NS sector, with action

$$S_{NS} = \frac{1}{2\kappa_0^2} \int d^{10}x \sqrt{-G} e^{-2\Phi} \left(\mathcal{R} + 4\partial_\mu \Phi \partial^\mu \Phi - \frac{1}{2} |H_3|^2 \right) \tag{3.1.4}$$

Where $2\kappa_0^2 = (2\pi)^7 \alpha'^4$. In addition, there is the R-R sector, which differs for the IIA and IIB theory. This consists of p-form gauge fields F_p with potentials C_{p-1}. For IIA there are gauge fields for all even p, for IIB for all odd p. Finally, there is a topological Chern-Simons term formed from a combination of the NS-NS and R-R gauge fields.

Note that apart from T_S, there are no tunable couplings in the theory. Instead, the background values of the fields themselves set the values of the couplings. One calls $g_s = e^\Phi$ the string coupling.

Looking around, we notice that there are four spacetime dimensions rather than the ten predicted by semiclassical superstring theory. One could respond to this in several ways. One is to relegate string theory to the dustbin of history. This is not the approach we will take here. Alternatively, one can try to somehow get four large spacetime dimensions in string theory. One could look at noncritical string theory in four dimensions or look for critical string theory solutions with four large spacetime dimensions and six very small highly nonclassical dimensions. Such approach certainly are interesting, but the approach we will take here is to start from the semiclassical low-energy ten-dimensional actions and look for solutions with four large dimensions and the remaining dimensions small enough that a macroscopic observer cannot see them, but still in a regime where there is as much semiclassical control as one can hope to get. One then also hopes that one can find a four-dimensional solution with a positive cosmological constant. We will turn to this matter in part III.

3.2 A theory of strings?

We have so far seen string theory as a few different theories describing dynamical strings. There is much more than this, however.

Firstly, there are dualities. These relate different theories in different regimes. One such duality is T-duality. As an example of T-duality, one can make one of the spacetime directions periodic, a circle with radius R, by imposing $X \sim X + 2\pi R$. T-duality then is the statement that IIA string theory with circle radius R is equivalent to IIB string theory with radius α'/R. There are several such dualities, relating the different superstring theories.

Second, string theory permits additional objects. These are generalized membranes called p-branes. We will see what types of branes can arise and how they arise in the following paragraphs. Here, p stands for the number of extended spatial dimensions of the object. In addition to spatial extension, the objects are also extended in time. A particle is a 0-brane and a string is a 1-brane. In fact, string theory does not merely permit these objects, they are necessarily part of the theory and can be spontaneously produced for instance through dualities. A p-brane can be electrically charged under a gauge field with a $p+1$-form potential and magnetically charged under a field with a $d-p-3$-form potetnial, with d the spacetime dimensions.

The type II string theories hypothetically without branes would consist only of closed strings. However, there can be strings with their end points stuck to the surface of branes known as D-branes[5], which can be charged under the RR gauge fields. These open string degrees of freedom then provide the dynamics of the D-brane. D-branes can be stacked on top of each other. A stack of N D-branes will result in the open string degreees of freedom including a rank N gauge group, $U(N)$ for the type II theories. A Dp-brane has unit charge under under the RR-gauge field with $p+1$ form potential. Negatively charged D-branes are called anti D-branes.

Let us give a few more specifics. Consider a stack of N Dp-branes[6] in type II string theory. This can be described in two ways at low energies.

On the one hand, it can be described from a string dynamics perspective. There is the dynamics of the open strings on the branes, giving a $p+1$-dimensional gauge theory coupled to fermions and scalars, S_{brane}. There are the closed strings away from the branes captured by a ten-dimensional gravitational theory,

[5] The D stands for Dirichlet, referring to the change in boundary conditions they impose on the strings.

[6] We take $p < 7$ in the following to ensure field strengths generated by the brane fall off rapidly away from the brane.

S_{closed}. Finally, the endpoints of an open string on the branes can come together to form a closed string, and analogously closed string can land on the branes to become open strings. This leads to an interaction term $S_{\text{interaction}}$ between the open and closed string sectors. In total, one then has an effective description given by

$$S = S_{\text{brane}} + S_{\text{interaction}} + S_{\text{closed}} \tag{3.2.1}$$

Branes have a tension, capturing how heavy they are, analogously to strings. For the branes in this stack, one can show that it is given by

$$T_D = \frac{1}{(2\pi\alpha')^2 g_{\text{gauge}}^2}\,, \tag{3.2.2}$$

with the coupling strength of gauge theory on the brane given by

$$g_{\text{gauge}}^2 = (2\pi)^{p-2} g_s {\alpha'}^{\frac{p-3}{2}}\,. \tag{3.2.3}$$

Note that in the regime of small string coupling g_s, where string perturbration theory is reliable, D-branes become very heavy.

On the other hand, one can start in the supergravity description of type II string theory and construct a solution with the right RR charge, tension, and symmetries to match a Dp-brane stack. Such a solution exists and it has metric

$$ds^2 = f(r)^{-1/2}(-dt^2 + dx_p^2) + f(r)^{1/2} dx_{9-p}^2\,, \tag{3.2.4}$$

with dx_p the p spatital directions along the brane, dx_{9-p} the spatial direction transverse to the brane to the brane, r the radial direction perpendicular to the brane and

$$f(r) = 1 + \left(\frac{R}{r}\right)^{7-p}\,, \tag{3.2.5}$$

with

$$R^{7-p} = 2^{5-p}\pi^{\frac{5-p}{2}}\Gamma\left(\frac{7-p}{2}\right) g_s N {\alpha'}^{\frac{7-p}{2}}\,. \tag{3.2.6}$$

This brane is charged under the RR fields with $p+2$-form field strength, producing the field profile

$$F_{p+2} = d\left(f^{-1}(r) \star_{p+1} 1\right)\,. \tag{3.2.7}$$

Finally, there is a profile for the dilaton[7],

$$e^{\phi} = f(r)^{\frac{3-p}{4}}\,. \tag{3.2.8}$$

[7] One defines $\phi \equiv \Phi - \Phi_0$, the fluctuation of the dilaton.

One can compute the RR charge by integrating R_{p+2} over a hypersphere around the brane and the tension or mass by computing the ADM mass asymptotically and show that these match those computed from string perturbation theory.

A different type of brane is the NS5 or F5 brane. The strings that make up the dynamical degrees of freedom of string theory are called F1 strings, F for fundamental. The F1 strings are electrically charged under the H_3 field. There is then a solotonic object magnetically charged under the H_3 field, a higher-dimensional analogue of how Maxwell theory in four spacetime dimensions has pointlike magnetic monopole solitons. This object is the NS5 brane, called so because it is charged under the gauge field in the NS sector of string theory.

Further, there are corners of the theory where there is no description in terms of strings at all. For instance, there is a duality known as S-duality which exchanges strong and weak couplings. When S-duality is applied to IIA string theory, one discovers M-theory, an eleven-dimensional theory whose dynamical objects are 2-branes and 5-branes called M-branes[8]. Semiclassically, this theory is described by the unique eleven dimensional supergravity, with bosonic action

$$S = \frac{1}{2\kappa_{11}^2}\int d^{11}x\sqrt{-G}\left(\mathcal{R} - \frac{1}{48}|F_4|^2\right) - \frac{1}{12\kappa_{11}^2}\int A_3 \wedge F_4 \wedge F_4\,. \qquad (3.2.9)$$

Note that dimensionally reducing this action produces the IIA action. A fuller view of this web of dualities can be seen in fig. 3.1 . Not pictured is F-theory, which can to an extend be though of as a twelve-dimensional interpretation of how S-duality acts on IIB string theory.

In conclusion, if one thing is clear about string theory, it's that it's not (just) a theory of strings.

3.3 Holography

Holography is the statement that a theory of quantum gravity can be understood in terms of a QFT in a different number of dimensions. This general statement was very concretely understood in string theory in terms of the AdS/CFT correspondence. This relates quantum gravity in an anti-de Sitter background to a conformal field theory in one dimension less[9].

[8] The unification of all these theories through dualities as whole is sometimes also referred to as M-theory. When we speak of M-theory, we will be thinking of the eleven-dimensional limit specifically.

[9] This statement is strictly a lie. The quantum gravity is often really on $AdS_D \times X$, with X a manifold such that the total spacetime dimension is that of either critical superstring theory or M-theory.

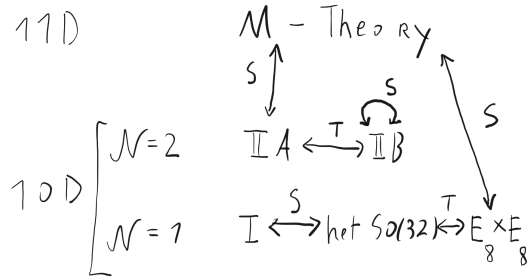


Figure 3.1: The web of string dualities. We only show sufficient dualities to demonstrate that all superstring theories are connected by duality. The $E_8 \times E_8$ heterotic string can be obtained by carefully compactifying M-theory on a line segment.

To see this more concretely realized, let us return to the stack of branes and focus more concretely on a stack of N D3-branes.

Let us think again about this theory first from the string perspective. The whole theory is captured by eq. (3.2.1). The gauge theory living on the brane is in this case $\mathcal{N} = 4$ Super Yang-Mills (SYM) in four spacetime dimensions. Let us take the limit $\alpha' \to 0$ while keeping g_{gauge} fixed[10]. The strength of the interaction term in the action is set by[11] $g_s \alpha'^2$ and so vanishes in this limit. The gravitational theory of closed strings away from the brane becomes free in this limit.

We are then left with two theories, decoupled from each other, in this limit. There is $\mathcal{N} = 4$ SYM in four dimensions on the brane and a free ten-dimensional gravitational theory.

Now let us look at this same limit from the supergravity perspective. Looking at eq. (3.2.4), we can on the one hand look at the $r \to \infty$ limit and get supergravity in ten dimensional flat space. On the other hand, we can look at the near-brane

[10]Note that in the D3 brane case, this also implies g_s fixed via eq. (3.2.3).

[11]As a heuristic argument for why this is the interaction strength: branes are higher-dimensional analogues of black holes, certainly when viewed as supergravity solutions. Open strings being emitted from the brane as closed strings can then be thought of as Hawking radiation and its interaction strength is set by the gravitational coupling which can be seen in eq. (3.1.4).

limit $r \to 0$. In this limit, the metric becomes

$$ds^2 = \frac{r^2}{R^2}\left(-dt^2 + dx_3^2\right) + \frac{R^2}{r^2}dr^2 + +R^2 d\Omega_5^2 \tag{3.3.1}$$

which is[12] $AdS_5 \times S^5$. For the supergravity approximation to be valid, the AdS radius must be much larger than the string length, $R \gg l_s$. This implies $g_s N \gg 1$. This describes an infinitely long, smooth, throat as one approaches the stack of D3 branes. Carefully taking the $\alpha' \to 0$ limit, the low energy modes inside the throat decouple from the low-energy modes in the flat geometry away from the brane. We then again have two decoupled systems. Clearly one would then like to identify these two systems with the two decoupled systems in the stringy perspective. The supergravity in $AdS_5 \times S^5$ is then identified with the $\mathcal{N} = 4$ SYM theory on the brane. The procedure we have outlined descriptively was first performed in depth in [42].

It seems that both of these theories describe the same physical system, they are dual to each other. We have only argued this in the $\alpha' \to 0$, $N \to \infty$ limit where gravity is weakly coupled. However, the the hope and belief is that this duality between the two theories is true generally. In this case, quantum gravity in anti-de Sitter space[13], in this specific theory can, be understood purely in terms of $\mathcal{N} = 4$ SYM, a conformal quantum field theory. As further evidence that these two theories describe the same physics, they posses the same symmetries. For instance, the isometries of AdS_5 are $SO(3, 2)$, which is the same group as the four-dimensional conformal group.

The concept of AdS/CFT has been extended to many different cases beyond this initial example. This framework has undergone many tests and seems very robust. In general, a supersymmetric gravity theory in AdS_D is believed to be dual to a CFT in $D - 1$ dimensions with the same symmetries. Note that $SO(D - 2, 2)$ is both the $D - 1$ dimensional conformal group and the isometry group of AdS_D. The general statement that these theories are dual is that their partition functions are the same,

$$Z_{CFT} = Z_{Gravity}\,. \tag{3.3.2}$$

One often works in the limit where the gravity theory is semiclassical and its partition function can be approximated by a saddle-point approximation.

AdS_D has a $D - 1$ dimensional conformal boundary at spatial infinity. The CFT_{D-1} is then often thought of as if it lives on this boundary. The gravity theory is then called the bulk theory. Fields in the bulk are dual to operators

[12] To see this, change coordinates as $r/R \equiv R/z$ to get AdS_5 in the Poincaré patch.

[13] One often unspokenly restricts to studying only the part of the theory that comprises the zero modes on the S^5.

in the CFT with the same representations under the symmetries of the theory. Fields ϕ in the gravity theory have an asymptotic behavior as one approaches the conformal boundary, $\phi \sim x_0^{\Delta}\phi_0$. Such a field is then dual on the CFT side to an operator $\mathcal{O}_\Delta$ with conformal scaling dimension Δ. Giving a value to ϕ_0 in the gravity theory corresponds to sourcing $\mathcal{O}_\Delta$ in the CFT. If a Lagrangian desciption of the CFT exists, this means adding a term $+\phi_0\mathcal{O}_\Delta$ to the Lagrangian. The AdS/CFT dictionary is then.

$$Z_{CFT}[\phi_0] = Z_{Gravity}[\phi_0]\,. \tag{3.3.3}$$

One can vary such expressions to obtain correlation functions in the CFT $< \mathcal{O}_1(x_1)..\mathcal{O}_n(x_n) >$ and correlation functions of bulk fields on the boundary surface . The AdS/CFT dictionary can be tested by computing such correlation functions both directly in the CFT and through the AdS/CFT dictionary and seeing whether they agree.

As a concrete example, for free scalar fields in AdS_D of mass m, there are two different asymptotic fall-offs, $\Delta_\pm$ with value

$$\Delta_\pm = \frac{D-1}{2} \pm \sqrt{\frac{(D-1)^2}{4} + m^2R^2}\,. \tag{3.3.4}$$

One can then source two different operators in the CFT, $\mathcal{O}_{\Delta_+}$ and $\mathcal{O}_{\Delta_-}$, corresponding to two different boundary asymptotics for the bulk scalar.

Clearly, it would be very useful to have a formalism analogous to AdS/CFT for quantum gravity in de Sitter space. In this case, there is no direct argument for holography from string theory completely understood. We will turn to this matter in part II.

Th example of AdS/CFT we have discussed has $\mathcal{N} = 4$ supersymmetry. Because of supersymmetry, precise QFT computations are possible. We will see in part III that supersymmetry is also of great value for finding vacua in gravitational theories. But supersymmetry buys us even more: stability. In fact, [43] has claimed that any nonsupersymmetric AdS vacuum is unstable against brane nucleation. This instability is not necessarily a great concern for a local observer in AdS so long as the nucleation rate per unit volume is low. It is however catastrophic for AdS/CFT. As one goes towards the boundary, the volume of a radial slice of AdS space increases unboundedly. The vacuum-destroying brane nucleation then happens immediately with certainty near the boundary. The claim of [43] is an example of a swampland conjecture, of which we will see more of the swampland program in chapter 10. If this conjecture is true, AdS/CFT only makes sense in a supersymmetric context.

Part II

de Sitter Holography

Chapter 4

Intoduction to dS/CFT

In 3.3 we provided an introduction to holography in the context of string theory. The form of holography we discussed there was AdS/CFT. To understand quantum gravity both in our current cosmological era and in a possible inflationary period, it would be interesting to understand if such a form of holography exists for de Sitter space.

4.1 Holography and de Sitter space

Like AdS, de Sitter space has a conformal boundary. It seems natural to hope that some holographic structure analogous to AdS/CFT exists for de Sitter space [44]. In fact, this analogy can be made quite strong, as one can analytically continue from AdS to de Sitter. One can for instance starts from AdS_D in the Poincaré patch[1]

$$ds^2 = \frac{l_{AdS}^2}{z^2}\left(-dt^2 + dz^2 + \sum_{i=1}^{D-2} dx^i dx^i\right) . \tag{4.1.1}$$

One wick rotates the time direction $t = ix^{D-1}$ to obtain the metric

$$ds^2 = \frac{l_{AdS}^2}{z^2}\left(dz^2 + \sum_{i=1}^{D-1} dx^i dx^i\right) . \tag{4.1.2}$$

This is a metric on Euclidean AdS (EAdS) (the hyperbola).

[1] Which we encountered for AdS_5 in section 3.3.

One now sets $z = i\eta$ and $l_{AdS} = -il_{dS}$ to obtain the planar patch of de Sitter space eq. (2.1.6),

$$ds^2 = \frac{l_{dS}^2}{\eta^2}\left(-d\eta^2 + \sum_{i=1}^{D-1} dx^i dx^i\right) . \tag{4.1.3}$$

This same procedure of analytic continuation works globally.

The "naive" way to obtain a form of dS/CFT is then to start with the prescription for AdS/CFT described in eq. (3.3.3) and define dS/CFT through the analytic continuation of this. One then obtains $Z_{\text{CFT}}[h_{ij}, \phi^0] = Z_{\text{de Sitter}}[h_{ij}, \phi^0]$, with h_{ij} the metric and ϕ^0 the field values on this boundary surface. But what exactly are these analytic continuations of partition functions? One still has a path integral of some sort on each side of the equation. A de Sitter space has a conformal boundary, as we saw in section 2.1. Globally, this boundary is an S^{D-1}-sphere, on which one could define a partition function. In de Sitter, we are path-integrating our theory up to this final surface. We encountered precisely such a path-integral in section 2.3. It prepares our theory in a state, the Hartle-Hawking state. One then has [45, 46]

$$Z_{\text{CFT}}[h_{ij}, \phi^0] = \Psi_{HH}[h_{ij}, \phi^0] . \tag{4.1.4}$$

Possible issues with this procedure become apparent when one analytically continues some of the AdS/CFT dictionary entries into dS/CFT. Consider for instance massive scalar bulk fields, eq. (3.3.4). For dS/CFT, this would become

$$\Delta_\pm = \frac{D-1}{2} \pm \sqrt{\frac{(D-1)^2}{4} - m^2 R^2} . \tag{4.1.5}$$

This formula is worrying. For large conformal dimension operators in the CFT, the m^2 of the corresponding bulk field becomes negative and one gets tachyonic fields that one would expect to destabilize the bulk theory.

If the bulk theory has heavy fields with $m^2 > 0$, the conformal scaling dimension of the boundary operator becomes complex-valued, which is not permitted for a unitary CFT. Heavy bulk fields will generically be present in string theory, both from the tower of higher string excitations[2] and the tower of Kaluza-Klein modes[3].

But is this second point truly a concern? In AdS/CFT, unitary time evolution in the boundary theory is dual to unitary time evolution in the bulk theory and one needs a well-behaved CFT for a well-behaved gravity theory. However, in the de Sitter case, the boundary theory 'lives on' the future boundary and is

[2] See section 3.1.
[3] See section 7.1.

Euclidean. Time evolution in the bulk does not dualize to time evolution on the boundary.

Perhaps it is wrong to speak of a boundary CFT in this case, and one should instead speak merely of a conformal structure on the boundary. What one wants is functions of the boundary coordinates x, $h_{ij}(x)$ and $\phi_0(x)$. One then constructs a functional $Z[h_{ij}, \phi_0]$, which depends on this functions in such a way that all the same symmetries and representations as the bulk theory, in particular conformal symmetry on S^{D-1}, are respected. This Z does not need to make sense directly in terms of a dynamical system on S^{D-1}, it only needs to do the job for the holographic dictionary eq. (4.1.4). We will continue to refer to this boundary object as a CFT.

In de Sitter space, there are two conformal boundaries, one in the far past I^- and one in the far future I^+, see section 2.1 for reference. Do we not then expect two boundary theories? Throughout this thesis, we will avoid this issue, initially by focusing on the planar patch which only has access to one boundary, later by using eq. (4.1.4) which defines one state at one moment in time. To say just a few words now, let us note that there is "too much" de Sitter space. There is a great deal of de Sitter space which an observer cannot access. To the observer at the south pole, the right quadrant in section 2.1 is entirely inaccessibly, information can only be sent into the top quadrant, information can only be received from the bottom quadrant, and the left quadrant is fully interactive. If one only takes what the observer can use to to be physically relevant, only half of de Sitter space is relevant. Willem de Sitter was already aware of this point [15] and proposed quotienting de Sitter space. Unfortunately, his proposed quotient breaks the de Sitter isometries. More recently, a $dS_2/\mathbb{Z}_2$ quotient space which does preserve the de Sitter isometries have been considered [47–49]. This quotient identifies $X \sim -X$ in eq. (2.1.1). As a result, I^+ is identified with I^-, "mirrored" and there is only a single conformal boundary. There is no longer a global arrow of time in this spacetime. This is perhaps no great concern as there are no closed timelike curves of finite length.

4.2 Higher spin theory and holography

As we saw, analytically continuing from AdS/CFT to dS/CFT seems to produce some oddities for massive bulk fields. The procedure might go more smoothly in case the bulk theory only has light fields. An example of such a theory is higher spin theory, also known as Vasiliev theory [50]. We will focus on the case where the bulk is four-dimensional. We will not focus on these bulk higher spin theories, which require a deal of formal machinery to properly describe, see

e.g. [51,52] for reviews. We also gave a more detailed overview of the higher-spin theory relevant for this part in [53]. We give only the basic facts and focus on the dual CFT description.

First, we will consider the free field case in flat space. An Abelian spin-1 gauge field A^μ has equation of motion

$$\partial_\mu \partial^\mu A^\nu - \partial_\mu \partial^\nu A^\mu = 0\,. \tag{4.2.1}$$

This is invariant under a gauge transformation

$$\delta A_\mu = \partial_\mu \epsilon\,. \tag{4.2.2}$$

The equation of motion for a spin s- gauge field is the Fronsdal equation [54]. The spin-s field $\phi_{\mu_1..\mu_s}$ should be totally symmetric, doubly traceless

$$g^{\mu_1\mu_2} g^{\mu_3\mu_4} \phi_{\mu 1 \mu_2 \mu_3 \mu_4 .. \mu_s} = 0\,, \tag{4.2.3}$$

and should obey the equation of motion[4]

$$\partial_\rho \partial^\rho \phi_{(\mu_1..\mu_s)} - \partial_\rho \partial_{(\mu_1} \phi^\rho{}_{\mu_2..\mu_s)} + \partial_{(\mu_1} \partial_{\mu_2} \phi^\rho{}_{\mu_3..\mu_s)\,\rho} = 0\,. \tag{4.2.4}$$

This has a gauge symmetry

$$\delta \phi_{(\mu_1..\mu_s)} = \partial_{(\mu_1} \beta_{\mu_2..\mu_s)}\,, \tag{4.2.5}$$

with gauge parameter $\beta_{\mu_1..\mu_{s-1}}$ traceless

$$\beta^\rho{}_{\rho \mu_1..\mu_{s-3}} = 0\,. \tag{4.2.6}$$

How do we go from a free theory of a spin-s field with $s > 2$ to an interacting theory? There are a number of restrictions that need to be overcome, reviewed in [52]. For instance, the higher spin gauge symmetries would constitute an extension of the spacetime symmetries. The Coleman-Mandula theorem [55] guarantees that all extensions of the spacetime symmetries in flat space, other than supersymmetry, lead to a trivial scattering matrix. One can then only add free higher spin gauge fields in flat space.

Such no-go restrictions are evaded by Vasiliev theory. Vasiliev theory, the higher spin theory that we focus on, does not exist on a flat space background but on an (Anti) de Sitter background. We focus for now on the AdS case. There are a variety of different Vasiliev-type theories. In the simplest case, one has a

[4] The brackets denote symmetrization with unit weight, dividing each term by the number of permutations.

conformally massive scalar, $m^2 = -2/l_{AdS}^2$, and an interacting gauge field for every even spin. Since there is a dynamical spin-two field this is a gravitational theory, though very distinct from Einstein gravity. Even in this simplest case, there is a parameter space of different theories distinguished by their parity. Of note are type A Vasiliev theory theory, where the scalar is parity even, and type B Vasiliev theory, where the scalar is parity odd. Both theories have fields of every even spin. The gauge symmetires of all the fields, including the graviton, combine into one group. Its algebra is denoted $ho(3,2)$ in the AdS case and $ho(4,1)$ in the dS case. These contain the isometries of the spacetime, $so(3,2)$ for AdS and $so(4,1)$ for dS respectively, as a subalgebra.

Since AdS has a conformal spatial boundary, one still needs to set boundary conditions for the fields to fully determine the theory. Most boundary conditions will break the higher spin gauge symmetry. we focus on the scalar field ϕ. Recall from section 3.3 that in the Poincaré patch eq. (4.1.1), there are leading and subleading terms to the scalar field's behavior towards the conformal boundary $z \to 0$ namely $\phi \sim z^{\Delta}\phi_0$ with Δ given by eq. (3.3.4) [56]. In this case, $\Delta = 1$ or $\Delta = 2$. In the type A Vasiliev theory, turning a source ϕ_0 on for only the $\Delta = 1$ fall-off preserves the higher spin symmetry. In the type B Vasiliev theory, turning a source ϕ_0 on for only the $\Delta = 2$ fall-off preserves the higher spin symmetry. All other choices of boundary condition and all choices of parity other than type A or B break the higher spin symmetry.

We focus now on AdS/CFT for type A Vasiliev theory with $\Delta = 1$ boundary conditions. Higher spin AdS/CFT is reviewed in [57]. Due to the additional higher spin symmetries, one expects that the boundary theory should have the full $ho(3,2)$ higher spin symmetry in top of the $so(3,2)$ conformal symmetry. This is indeed the case. The CFT dual is the singlet sector of the free $O(N)$ model of N real scalar fields ϕ^i, $i = 1, .., N$, [58, 59]

$$S = \int d^3x \sqrt{h} \delta_{ij} \partial_\mu \phi^i \partial^\mu \phi^j \, . \tag{4.2.7}$$

Having $ho(3,2)$ symmetry in the lower-dimensional field theory is equivalent to being a free theory [60, 61]. There are conserved currents associated to the symmetries, schematically given by

$$J_{(s)} = J_{\mu_1..\mu_s} = \sum_{k=0}^{s} c_{(s)\,k} \delta_{ij} \partial_{\{\mu_1}..\partial_{\mu_k} \phi^i \partial_{\mu_{k+1}}..\partial_{\mu_s\}} \phi^j \, . \tag{4.2.8}$$

The curly brackets denote traceless symmetrization. The coefficients $c_{(s)k}$ are such that the currents are conserved, primary, and in an irreducible spin-s representation. A generating function is known that determines these coefficients for any spin [62, 63]. These currents are zero for all odd spins, in agreement with there only being even spin bulk fields.

Sourcing such a conserved current in the boundary theory corresponds to turning on the field of the corresponding spin in the bulk theory. In particular, the spin-zero current[5] is given by

$$J_{(0)} = \delta_{ij}\phi^i\phi^j \,. \tag{4.2.9}$$

Since ϕ has $\Delta = 1/2$ as it is a free scalar in three dimensions, this operator has $\Delta = 1$ as desired for the dual operator to the bulk scalar field with our boundary conditions.

The factor of N in the $O(N)$ model is related to the bulk Newton's constant and cosmological constant as

$$G_N = \frac{1}{N\Lambda} \tag{4.2.10}$$

Holding Newton's constant in the bulk fixed, Going from AdS to dS, $\Lambda \to -\Lambda$ then corresponds in the boundary theory to $N \to -N$. We thus get the $O(-N)$ model. What are we to make of this? This will be the subject of the next section.

We comment briefly on AdS/CFT for different Vasiliev theories. Type B Vailiev theory with $\Delta = 2$ boundary conditions has unbroken higher spin symmetry, so the CFT dual should be free. The CFT dual is the $O(N)$ model of N free Fermionic spinors. Type A with $\Delta = 2$ breaks higher spin symmetry. Its proposed CFT dual is the $O(N)$ model of N real critically coupled scalars with action

$$S = \int d^3x\sqrt{h}\left[\delta_{ij}\partial_\mu\phi^i\partial^\mu\phi^j + \frac{\lambda}{N}\left(\delta_{ij}\phi^i\phi^j\right)^2\right] . \tag{4.2.11}$$

The coupling λ is at the Wilson-Fisher CFT fixed point value. Since this theory is no longer free, it breaks higher spin symmetry as desired. Type B with $\Delta = 1$ has as proposed CFT dual the critically interacting Gross-Neveu $O(N)$ model of N Fermionic spinors [64].

We have only looked at Vasiliev theory with even spin fields. Vasiliev theory can be extended to include fields of odd spin. One can for instance extend the type A theory with $\Delta = 1$ boundary conditions such that there are now gauge fields for every integer spin. The CFT dual to this is the free $U(N)$ model, obtained by replacing the real scalars of the $O(N)$ model by complex scalars,

$$S = \int d^3x\sqrt{h}\delta_{ij}\partial_\mu\overline{\phi}^i\partial^\mu\phi^j \,. \tag{4.2.12}$$

The conserved currents,

$$J_{(s)} = J_{\mu_1..\mu_s} = \sum_{k=0}^{s} d_{(s)\,k}\delta_{ij}\partial_{\{\mu_1}..\partial_{\mu_k}\overline{\phi}^i\partial_{\mu_{k+1}}..\partial_{\mu_s\}}\phi^j \,, \tag{4.2.13}$$

[5]Which alone in this family of currents is not conserved.

are nonzero for odd spin. The other Vasiliev theories discussed can be extended to include odd spins in the same manner.

Vasiliev theories with a generic parity-breaking angle other than type A or B also have CFT duals. In this case, the CFT dual picks up a Chern-Simons term [65].

One can also extend Vasiliev theories to include half-integer fermionic fields. In this case, the higher spin gauge algebra is extended to a superalgebra.

4.3 The $Sp(N)$ model and higher spin dS/CFT

We see that the simplest higher spin theory in de Sitter space, with a conformally massive scalar field and massless fields of spin $s = 2, 4, 6, ..$ is expected to be dual to the $O(-N)$ model. Crucially, all bulk quantities are related to the singlet sector. While $O(-N)$ might in general seem ill-defined, one can show [66] that if one takes a quantity in the singlet sector of the $O(N)$ group and sends $N \to -N$, one gets the corresponding quantity in $Sp(N)$. Thus, $O(-n) \cong Sp(N)$, at least for the singlet sector[6]. We thus need a boundary theory with N scalars φ that are contracted with the totally antisymmetric matrix

$$\Omega_{ij} = \begin{bmatrix} 0 & \mathbb{I}_{N/2\times N/2} \\ -\mathbb{I}_{N/2\times N/2} & 0 \end{bmatrix} . \tag{4.3.1}$$

This is only (nontrivially) possible if the scalars are anticommuting. Such a theory was constructed in [68] and its correlators were shown to match the correlators of the corresponding bulk higher spin theory. The action of the boundary theory is then

$$S = \int d^3x \sqrt{h} \Omega_{ij} \partial_\mu \varphi^i \partial^\mu \varphi^j . \tag{4.3.2}$$

Conserved currents are defined analogously to the $O(N)$ model, only now all contractions happen with Ω_{ij}.

The bulk higher spin theory in de Sitter space is essentially the analytical continuation of the type A higher spin theory in AdS. In particular, the bulk scalar now has mass $m^2 = 2/l_{dS}^2$.

The reader may experience some discomfort at this point. If she is aware of the spin-statistics theorem [69], she will know that quantum field theories with anticommuting scalars are no good, they are non-unitary. However, as discussed in section 4.1, this is besides the point.

[6] A detailed examination of such continuations from a purely group-theoretic perspective can be found in [67].

4.4 Holographic quantum cosmology

So far, we have dealt with higher spin dS/CFT in terms of correlators of operators and fields. As we saw, Z_{CFT} is identified with Ψ_{HH}, eq. (4.1.4). This is the duality we will focus on in the remainder of this part of the thesis.

Our approach will be to compute $Z[h_{ij}, \phi_0]$ for a boundary theory whose symmetries match those of a bulk theory we wish to study. We then essentially take this to define Ψ_{HH} of the bulk theory.

It is not viable to compute Z for arbitrary boundary metrics h_{ij} and field configurations ϕ_0. We will be forced to restricts ourselves to a subset of possible configurations. Thus, we will be computing Ψ_{HH} in a minisuperspace, see section 2.3 .

We can then plot the dependence of $|\Psi_{HH}|^2$ on h_{ij} and ϕ_0. More generally, for an observable $\mathcal{O}$ we can plot $\Psi^*_{HH}\mathcal{O}\Psi_{HH}$. This situation is not ideal. One would like to compute

$$\langle \Psi_{HH}|\mathcal{O}|\Psi_{HH}\rangle = \int [\mathcal{D}h]\,[\mathcal{D}\phi_0]\,\Psi^*_{HH}\mathcal{O}\Psi_{HH}\,. \tag{4.4.1}$$

However, firstly, one can only evaluate h_{ij} and ϕ_0 on a minisuperspace. Secondly, even in this minisuperspace, the correct integration measure is not known.

Not all is lost and it seems reasonable to expect that meaningful information can be gleamed from plotting $|\Psi_{HH}|^2$. Consider for instance the quantum harmonic oscillator with Hamiltonian

$$H = -\frac{\partial^2}{(2\partial x)^2} + \frac{\omega^2}{2}x^2\,. \tag{4.4.2}$$

It is then well-known that the ground state depends on x as

$$\Psi(x) \sim e^{-\omega x^2/2}\,. \tag{4.4.3}$$

The wave function is peaked at the classical ground state $x = 0$, with Gaussian damping around it. The same behavior appears for the ground state of a free QFT. The wave function is peaked at the classical ground state of the fields, with Gaussian damping around it.

Since Ψ_{HH} is supposed to be the ground state of quantum gravity, we expect the same behavior if a stable vacuum indeed exists. Ψ_{HH} should be peaked around classical ground state values of h_{ij} and ϕ_0. If this ground state is a global de Sitter vacuum, h_{ij} should be peaked at the metric of an undeformed S^3. We will indeed see this happen in chapter 5 and chapter 6. For the fields ϕ_0,

the situation is more subtle but we will see indications that there is a ground state that is at least stable against small perturbations. [70]

In the form of dS/CFT we use, all one gets for computing the boundary partition function is a single bulk state. This is very different from AdS/CFT, where different CFT states correspond to different bulk states and there is a duality between the Hilbert spaces. One may well wonder what happened to the rest of the de Sitter quantum gravity Hilbert space. Work towards capturing a whole bulk Hilbert space holographically can be found in [70, 71].

This type of analysis of the wavefunction for higher spin dS/CFT was first performed in [72]. They studied the free $Sp(N)$ model of section 4.3 in this manner.

In particular, looking at the bulk scalar field corresponds to sourcing $J_{(0)} = \Omega_{ij}\varphi^i\varphi^j$ on the CFT side,

$$S = \int_{S^3} d^3x\sqrt{h}\Omega_{ij}\left[\partial_\mu\varphi^i\partial^\mu\varphi^j + \left(\frac{3}{4l_{dS}^2} + \sigma(x)\right)\varphi^i\varphi^j\right] . \tag{4.4.4}$$

The source $\sigma(x)$ then corresponds to the bulk scalar in the wave function $\Psi_{HH}[h, \sigma]$. Note that we have also added a term $\frac{3}{4l_{dS}^2}$ to the action. This is because we have moved from the planar patch to global coordinates. Globally the boundary of de Sitter space is an S^3 and h_{ij} is now the metric on the three-sphere. The term $\frac{3}{4l_{dS}^2}$ needs to be added to ensure the theory is conformal on S^3.

If one takes σ constant on the S^3, the partition function of this CFT can be computed exactly as a Gaussian integral since the theory is free. One finds

$$Z[\sigma] \propto \det\left(-\nabla^2 + \frac{3}{4l_{dS}^2} + \sigma\right)^{N/2} . \tag{4.4.5}$$

This can be computed in Mathematica and one can obtain an exact analytic expression. Using the holographic dictionary eq. (4.1.4), one can plot the resulting dependence of the bulk wavefunction, which can be seen in fig. 4.1 . One sees that for small σ the wavefunction is peaked around $\sigma = 0$, suggesting a stable ground state. For large fluctuations, there appear growing oscillations in the wavefunction. It was argued in [1, 70, 73] that with the correct integration measure for the wavefunction this regime of large fluctuations should not be included in the integration domain of $\langle\Psi_{HH}|\Psi_{HH}\rangle$.

This type of analysis has been extended in various ways for instance by looking at metric deformations, different boundary topologies, and models other than free $Sp(N)$ model in [72] and follow-ups such as [74–82].

In section 3.3 we discussed how supersymmetry is important for AdS/CFT. The claim of [43] that AdS must be supersymmetric to be stable explicitly excluded higher spin theories. Still, higher spin AdS/CFT can be supersymmetric and the examples of higher spin AdS/CFT that are believed to be related to string theory [83] are supersymmetric. Studying dS/CFT in a theory with a metastable de Sitter vacuum would lead to similar concerns to those raised for nonsupersymmetric AdS/CFT by [43]. For instance, one thinks of the Ψ_{HH} one computes as the state in the far future at the conformal boundary. But what is the meaning of this boundary when the de Sitter space can decay?

Clearly, it would be interesting to have an example of supersymmetric dS/CFT. In part III we will work towards de Sitter compactifications of string theory, but as we will see such de Sitter vacua must break supersymmetry. It has been shown [84, 85] that $\mathcal{N} = 2$ supergravity with unbroken supersymmetry in de Sitter space always has fields with the wrong sign in the action for the kinetic term, known as ghost fields, resulting in an instability and rendering such theories non-unitary.

In the following chapters, we will work out a supersymmetric extension of the free $Sp(N)$ model. We will argue that this is the dS/CFT dual to a supersymmetric $\mathcal{N} = 2$ de Sitter higher spin theory constructed by [86]. We holographically compute the ground state wavefunction and argue that the theory has a stable ground state. We will suggest that there may be an interpretation of such supersymmetric dS/CFT in terms a type of exotic string theory.

Higher spin theory is quite distinct from Einstein gravity. With the infinite tower of higher spin fields, unobserved in reality, it may seem that studying higher spin theory will not tell us much about the real world. We will argue that if one restricts to sourcing fields with spin $s \leq 2$ the results one get from higher spin theory might not be entirely dissimilar from the results obtained from Einstein gravity + matter.

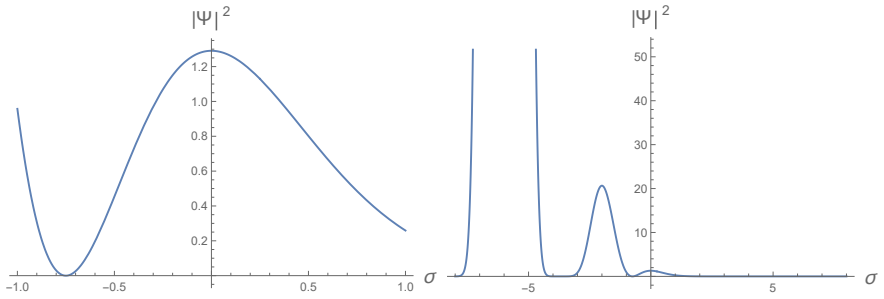


Figure 4.1: The probability distribution $|\Psi^2_{HH}|[\sigma]$ computed from eq. (4.4.5) for both a short and a long range of σ at $N = 2$.

Chapter 5

Supersymmetric dS/CFT

This chapter is a reprint of [1] where we constructed a supersymmetric extension of the $Sp(N)$ model. The formalism of superfields provides a natural geometric formulation of supersymmetry. We extend this formalism to describe superfields with flipped spin-statistics. We then use this formalism to construct the action for the free and critically interacting $\mathcal{N} = 2$ supersymmetric extensions of the $Sp(N)$ model. We argue that this extension of the $Sp(N)$ model should be the CFT dual of an $\mathcal{N} = 2$ supersymmetric higher spin theory with higher spin symmetry preserving boundary conditions. We compute Ψ_{HH} for a configuration space of supersymmetry preserving configurations of bosonic fields. We claim that the resulting Ψ_{HH} is indicative of the bulk higher spin theory having a perturbatively stable de Sitter ground state. We speculate that this model may point towards a route for embedding dS/CFT in string theory.

5.1 Introduction

As discussed in part II, gauge-gravity duality with de Sitter (dS) boundary conditions [44, 87, 88] has proved to be a fruitful route to put cosmology on firm theoretical ground. Recasting the dS/CFT duality eq. (4.1.4) in a notation more useful for this chapter,

$$\Psi_{HH}[h_{ij}, A_s] = Z_{QFT}[\tilde{h}_{ij}, J_s] \exp(\mathrm{i}S_{st}[h_{ij}, A_s]/\hbar) \ . \tag{5.1.1}$$

Here A_s stands for the matter configurations of spin s and h_{ij} is the three-geometry of the spacelike surface Σ on which Ψ is evaluated. In this paper we take the latter to be topologically a three-sphere. The sources $(\tilde{h}_{ij}, J_s)$ in

(5.1.1) are conformally related to the argument (h_{ij}, A_s) of the wave function, and S_{st} are the usual surface terms.

It is a central question in holographic cosmology what class of deformed CFTs in (5.1.1) specifies a well-defined, normalizable wave function. Euclidean AdS/CFT provides a starting point to study this since its generalization to complex relevant deformations of CFTs implies a realisation of dS/CFT that is valid in the semiclassical approximation in Einstein gravity [45, 89–91] and possibly exact in Vasiliev gravity in dS [68]. It has been suggested indeed that Euclidean AdS and Lorentzian dS, and their duals, can be viewed as two real domains of a single complexified theory [45, 87, 92–95].

In these examples of dS/CFT the partition functions featuring in (5.1.1) are the inverse of those of the original AdS/CFT duals [91]. The case of higher-spin gravity is particularly illuminating because the duals are vector models for which the partition function can be evaluated explicitly for a range of deformations[1] [72, 74, 81, 82]. The Vasiliev higher-spin (HS) theory has massive scalars and an infinite tower of massless gauge fields of increasing spin [50]. The duals have conserved currents for the same symmetries [58, 96]. Deforming the boundary theory action with a conserved current J_s corresponds to turning on the spin-s field A_s in the bulk.[2] Explicit calculations of the partition function as a function of homogeneous scalar and spin-2 deformations in the $Sp(N)$ vector model, which is dual to the minimal Vasiliev theory in dS [68], have provided some evidence that dS/CFT yields a well-defined wave function and in particular one which is better behaved than the usual semiclassical Hartle-Hawking wave function in Einstein gravity [73].

So far attention has focussed on the duality between the minimal Vasiliev theory in de Sitter and the dual $Sp(N)$ models consisting of anti-commuting scalars. In this case the dS duality follows relatively directly from the higher-spin duality in AdS [96]. However the connection between the AdS and the dS domain of the theories appears more general and profound [87, 92, 94, 95], which suggests there may well be a broader set of realizations of dS/CFT in the same spirit. Here we show that the supersymmetric extension of the higher-spin duality in AdS [83] also carries over to dS. This is particularly interesting because a supersymmetric realisation of dS/CFT has a better chance of specifying a well-defined wave function that predicts stable asymptotic de Sitter space. In fact Ooguri and Vafa [43] have recently advocated that supersymmetry is essential even for AdS

[1] This amounts to a minisuperspace approximation in cosmology. Note that we will be using the term superspace in two different contexts: on the one hand there is the minisuperspace of quantum cosmology, on the other hand we have the superspace used for supersymmetry.

[2] The spin-0 field carries no indices and is not really a current. Still, it is usually referred to as such.

holography,[3] although they note higher-spin gravity may evade their arguments.

It is usually argued that unbroken supersymmetry and dS space do not go together (see e. g. [18]).[4] This is because in dS space there is no positive conserved quantity, whereas supersymmetry would allow one to construct one. Indeed if there were a nonzero supercharge Q then either $(Q + Q^\dagger)$ or $\mathrm{i}(Q - Q^\dagger)$ would be Hermitian. Redefining Q to be the Hermitian supercharge would then imply that Q^2 were a positive conserved quantity, which does not exist in dS.

The fact that the known dS vacua in supergravity have ghosts [84,85], indicating they are perturbatively unstable, is a manifestation of this general argument. Supersymmetric HS gravity theories in de Sitter may however circumvent this problem. This is because in the higher-spin theories in dS constructed in [86] the Hermitian conjugate is an anti-involution, defined as $(Q^\dagger)^\dagger = -Q$. With this definition one cannot construct a Hermitian quantity from a supercharge Q. In fact, the $\mathcal{N} = 2$ dS_4 supersymmetry algebra is realized in terms of oscillators of Vasiliev theory, comprising the bosonic fields y_α and $\bar{y}_{\dot{\alpha}} = (y_\alpha)^\dagger$, such that the momentum operator $P_a = \mathrm{i}/4(\sigma_a)^{\alpha\dot{\beta}} y_\alpha \bar{y}_{\dot{\beta}}$ is anti-Hermitian. Hence, from the irrepresentations of the higher-spin superalgebra there does not seem to exist an operator associated to a globally defined Hamiltonian in the first place.

In this paper we provide evidence, using holography, that supersymmetric de Sitter space is stable and has no ghosts in higher-spin gravity. We first propose a supersymmetric generalization of the higher-spin dualities in de Sitter. The bulk theories involved are the supersymmetric extensions of Vasiliev theory described in [86]. On the boundary side we construct, in Section 6.3, new $\mathcal{N} = 2$ supersymmetric extensions of the three-dimensional $Sp(N)$ models. We then relate these to the theories of Sezgin and Sundel in Section 5.2.3, thereby establishing a supersymmetric gauge-gravity duality with de Sitter boundary conditions. We also briefly discuss the fermions in the theory. In Section 5.3 we evaluate the partition function of the supersymmetric extension of the free $Sp(N)$ model as a function of homogeneous scalar, vector and spin-2 deformations that preserve supersymmetry. The duality put forward in Section 6.3 conjectures that the partition function specifies the Hartle-Hawking wave function in a supersymmetric minisuperspace consisting of anisotropic deformations of de Sitter space with scalar and vector matter. We find the wave function is globally peaked at the undeformed de Sitter space, with a low amplitude for strong deformations. This indicates that supersymmetric de Sitter space is stable in higher-spin gravity.

[3] Related to this, Danielsson et al. [97] have conjectured that all non-supersymmetric AdS vacua are even perturbatively unstable, and in [98] it was argued that all non-supersymmetric vacua – be it AdS, Minkowski or dS – must eventually decay.

[4] Note that superconformal theories on a fixed de Sitter background do exist [99]. They avoid the argument of [18] by the existence of an everywhere timelike conformal killing vector.

It is tempting to speculate that our findings are connected to the supersymmetric dS constructions in exotic string theories [87]. The latter have vector ghosts in their supergravity limits related to the existence of non-compact R-symmetry groups in their representation of the algebra.[5] However Hull has argued that the massive string states in exotic string theories may well render the de Sitter vacua ghost-free and unitary. In Section 5.4 we conjecture that the supersymmetric higher-spin theories in dS that we construct are related indeed to the tensionless limit of these exotic string theories.

5.2 Supersymmetric Vector Models and Duality

5.2.1 Free chiral superfields and higher-spin deformations

To construct Euclidian supersymmetric vector models we must consider complex fields and $U(N)$ global symmetry instead of $O(N)$ or $Sp(N)$. The free $U(N)$ vector model is a theory of N commuting complex scalar fields ϕ^i and $\tilde{\phi}_i$, with $i = 1, \ldots, N$, transforming in the fundamental and anti-fundamental representations of $U(N)$ with Lagrangian $\mathcal{L} = \partial^\mu \tilde{\phi}_i \partial_\mu \phi^i$. The $Sp(N)$ model or more precisely the anti-commuting $U(N)$ model, also referred to as the $U(-N)$ model, is defined by changing the statistics of the fields. This yields a set of complex anti-commuting scalar fields φ^i and $\tilde{\varphi}_i$ governed by the Lagrangian $\mathcal{L} = \partial^\mu \tilde{\varphi}_i \partial_\mu \varphi^i$.

Here we are interested in the three-dimensional, Euclidean $\mathcal{N} = 2$ supersymmetric extension of these vector models.[6] Three dimensional $\mathcal{N} = 2$ supersymmetry has four real supercharges. The supersymmetric extension of a 3D scalar field can be described off-shell by using chiral and anti-chiral superfields Φ^i and $\tilde{\Phi}_i$, which we assume to be commuting for now. These satisfy the constraints $\tilde{D}_\alpha \Phi^i = 0$ and $D_\alpha \tilde{\Phi}_i = 0$ where the $\mathcal{N} = 2$ superspace coordinates and spinor covariant derivatives are denoted $z^M = (x^\mu, \theta^\alpha, \tilde{\theta}^\alpha)$ and $(D_\alpha, \tilde{D}_\alpha)$.[7] Note that in Euclidian signature $\tilde{\Phi}_i$ is not necessarily the complex conjugate of Φ^i. The off-shell components of the chiral superfield Φ^i are $(\phi^i, \psi^i_\alpha, F^i)$ with $\phi^i(x) = \Phi^i|_{\theta=0}$ the physical complex scalar fields, $\psi^i_\alpha(x) = 1/\sqrt{2} D_\alpha \Phi^i|_{\theta=0}$ Dirac spinors and $F^i(x) = -1/4 D^2 \Phi^i|_{\theta=0}$ complex auxiliary fields (and analogously for the anti-chiral multiplet). Below we give the supersymmetry transformations for the

[5] This differs from the $\mathcal{N} = 2$ Vasiliev case in [86] where the dS R-symmetry group is $SO(2)_R \simeq U(1)_R$. The $\mathcal{N} = 2$ case is actually the only dS supergroup possessing compact R-symmetry [84, 85].

[6] See e.g. [83, 100–102] for an analysis of the analogous extension in the context of the AdS-Vasiliev/CFT duality.

[7] We adopt the notation of [103, 104] except for their supersymmetry parameters $(\zeta, \tilde{\zeta})$ which we will name $(\epsilon, \tilde{\epsilon})$.

fields in a chiral multiplet. The $\mathcal{N}=2$ supersymmetric $U(N)$ model is then

$$S=\int \mathrm{d}^3x\mathrm{d}^4\theta\,\tilde{\Phi}_i\Phi^i=\int \mathrm{d}^3x\left[\partial_\mu\tilde{\phi}_i\partial^\mu\phi^i-\mathrm{i}\tilde{\psi}_i\not{\partial}\psi^i-\tilde{F}_iF^i\right], \tag{5.2.1}$$

where the first term is the free bosonic $U(N)$ model, the second term describes the free massless Gross-Neveu model and the last term is non-dynamical. If the $U(1)_R$ charge and the conformal dimension of Φ^i are chosen to be $1/2$, this action is invariant under the 3D Euclidian superconformal group $OSp(2|2,2)$.[8] Moreover, the action possesses an infinite set of singlet higher-spin supercurrents $J^{(s)}$ [106]. The scalar supercurrent of the free theory is given by $J^{(0)}:=\tilde{\Phi}_i\Phi^i$ and satisfies on-shell the conservation equation $D^2J^{(0)}=\tilde{D}^2J^{(0)}=0$.

The superspace approach straightforwardly allows us to define an $\mathcal{N}=2$ extension of the $U(-N)$ model: It suffices to replace the commuting chiral and anti-chiral superfields Φ^i and $\tilde{\Phi}_i$, with a set of anti-commuting fields Υ^i and $\tilde{\Upsilon}_i$. The change of statistics is such that[9] $\Upsilon^i\Upsilon^j=-\Upsilon^j\Upsilon^i$, $\tilde{\Upsilon}_i\tilde{\Upsilon}_j=-\tilde{\Upsilon}_j\tilde{\Upsilon}_i$, $\Upsilon^i\tilde{\Upsilon}_j=-\tilde{\Upsilon}_j\Upsilon^i$. The anti-commuting chiral superfields have components given by the physical anti-commuting complex scalars $\varphi^i(x)$, commuting fermions $\chi^i_\alpha(x)$ and anti-commuting auxiliary scalar fields $G^i(x)$. The supersymmetry transformations of Υ^i and $\tilde{\Upsilon}_i$ are the same as those of Φ^i and $\tilde{\Phi}_i$. The only difference is that the component fields in the off-shell multiplet now have reversed statistics. The full superspace Lagrangian for the $U(-N)$ model is then simply given by $\tilde{\Upsilon}_i\Upsilon^i$, as in (5.2.1). For dimension half anti-commuting chiral multiplets this has the same $OSp(2|2,2)$ symmetry and higher-spin currents as the $\mathcal{N}=2$ $U(N)$ theory.

One can extend the free $U(-N)$ model to include interactions by considering an arbitrary full superspace Lagrangian[10] $K(\Upsilon^i,\tilde{\Upsilon}_i)$, and chiral and anti-chiral superpotentials $W(\Upsilon^i)$ and $\tilde{W}(\tilde{\Upsilon}_i)$. For instance an extension we will consider is the free $\mathcal{N}=4$ hypermultiplet $U(-N)$ model. This is defined by a full superspace Lagrangian $(\tilde{\Upsilon}_{+i}\Upsilon^i_+ + \tilde{\Upsilon}^i_-\Upsilon_{-i})$ involving two sets of free anti-commuting chiral superfields Υ^i_+ and Υ_{-i} and the antichiral cousins. By using the chiral composite $C=\Upsilon^i_+\Upsilon_{-i}$ one can construct $U(N)$ invariant superpotentials $W(C)$. The quartic superpotential $(\Upsilon^i_+\Upsilon_{-i})^2$ provides an example and represents a classically marginal deformation of the $U(-N)$ model

[8] We use the $OSp(2|2,2)$ supergroup notation employed in [105] which differs from the unitary quaternionic one used in [84, 85]. In the latter case the minimal 3D Euclidian superconformal group, equivalent to the minimal 4D dS supergroup, is denoted as $UU_\alpha(1,1;1;\mathbb{H})$.

[9] Note also that the commutation rules with the spinor derivatives are somewhat subtle, e.g. $D_\alpha\Upsilon_i\Upsilon_j=(D_\alpha\Upsilon_i)\Upsilon_j-\Upsilon_iD_\alpha\Upsilon_j$.

[10] This is the analogue of the Kähler potential for a standard supersymmetric sigma-model.

preserving the full $\mathcal{N}=2$ superconformal group, but not the higher-spin (HS) symmetry.

We do not elaborate further on interacting theories but concentrate first on the free $\mathcal{N}=2$ $U(-N)$ model defined on the three sphere and deformed by background boundary sources for higher-spin supercurrents. This can be used to realize the simplest example of a supersymmetric extension of dS/CFT. In this context, the boundary sources are related to the argument of the bulk wave function in the large volume limit (cf. (5.1.1)).

Consider background $\mathcal{N}=2$ conformal higher-spin superfields [107] $H^{(s)} := H_{\alpha_1\alpha_2\cdots\alpha_{2s}}(x,\theta)$, together with the primary supercurrents of the free chiral multiplets [106], $J^{(s)} := J_{\alpha_1\alpha_2\cdots\alpha_{2s}}(x,\theta)$. These are both completely symmetric tensors in the $2s$ spinor indices. Then consider a linearly deformed full superspace Lagrangian of the form

$$\mathcal{L}[H^{(s)}] = \tilde{\Upsilon}_i\Upsilon^i + \sum_{s=0}^{+\infty} H^{\alpha_1\alpha_2\cdots\alpha_{2s}} J_{\alpha_1\alpha_2\cdots\alpha_{2s}} \ . \tag{5.2.2}$$

The deformed model is also off-shell $OSp(2|2,2)$ invariant as can be proven by using the results of [107] (see also the recent 4D analysis of [108]). The partition function of the Lagrangian (5.2.2) leads to a natural superfield description of the Hartle-Hawking wave function (5.1.1). The spin zero supercurrent is $J^{(0)} := \tilde{\Upsilon}_i\Upsilon^i$. For $s \geq 1$ the supercurrents satisfy on-shell the conservation equations

$$D^{\alpha_1} J_{\alpha_1\alpha_2\cdots\alpha_{2s}} = 0 \ , \tag{5.2.3}$$

$$\tilde{D}^{\alpha_1} J_{\alpha_1\alpha_2\cdots\alpha_{2s}} = 0 \ , \tag{5.2.4}$$

$$\Longrightarrow \quad (\gamma^\mu)^{\alpha_1\alpha_2}\partial_\mu J_{\alpha_1\alpha_2\alpha_3\cdots\alpha_{2s}} = 0 \ . \tag{5.2.5}$$

Note that the spin-1 supercurrent [109, 110]

$$J_{\alpha\beta} = -2(\tilde{D}_{(\alpha}\tilde{\Upsilon}_i)D_{\beta)}\Upsilon^i + 2\mathrm{i}\tilde{\Upsilon}_i(\gamma^\mu)_{\alpha\beta}\partial_\mu\Upsilon^i - 2\mathrm{i}\Upsilon^i(\gamma^\mu)_{\alpha\beta}\partial_\mu\tilde{\Upsilon}_i \tag{5.2.6}$$

contains the stress-energy tensor, the supersymmetry current and the current for the $U(1)_R$ symmetry. Similarly, the currents $J_{\alpha_1\cdots\alpha_{2s}}$ comprise several component HS currents including two series of integer and half-integer ones [106]. From here onwards we concentrate on deformations involving only the spin zero, $J^{(0)}$, and spin one, $J_{\alpha\beta}$, supercurrents.

The scalar superfield $V \equiv H^{(0)}$ describes an $\mathcal{N}=2$ vector multiplet associated with the gauging of the diagonal $U(1)$ symmetry within the flavour $U(N)$ group.

In the context of dS/CFT the scalar components in this multiplet will be dual to the scalar and pseudo-scalar currents of the bulk Vasiliev theory. On the other hand the background superfield $H_{\alpha\beta}$ describes the gravitational superfield for 3D $\mathcal{N}=2$ supergravity [111]. This is the supersymmetric extension of a background metric and it contains as components the dual of the spin-two and spin-one currents in the bulk. The following action

$$S=\int \mathrm{d}^3x\mathrm{d}^4\theta\, E\,\tilde{\Upsilon}_i \mathrm{e}^V \Upsilon^i \ , \tag{5.2.7}$$

describes the nonlinear $s=0,1$ deformations of the $U(-N)$ model in a simple manner as a system of anti-commuting scalar superfields in an arbitrary off-shell vector multiplet and conformal supergravity background [112, 113]. Here ${E_M}^A$ is the supervielbein of the 3D $\mathcal{N}=2$ background supergeometry and $E\equiv \mathrm{Sdet}{E_M}^A$ is its superdeterminant. The linearization of (5.2.7) leads to the Lagrangian in (5.2.2) with $s=0,1$.

5.2.2 Spin $0\leq s\leq 2$ deformations in components

The above superspace description is potentially abstract for many readers. We therefore give here the action in components as well as the details of the supersymmetric backgrounds used in the rest of the paper.

We consider the free chiral multiplets associated to the anti-commuting (anti-)chiral superfields Υ and $\tilde{\Upsilon}$. In order to study spin 0, 1 and 2 deformations that preserve supersymmetry, it suffices to consider the coupling to a general background of 3D new minimal supergravity [113] in the presence of a background vector field (see also [103]). The explicit ingredients of the action (5.2.7) in terms of component fields are presented using the notations of [103,104].

The field content of off-shell new minimal supergravity in three dimensions comprises as component fields the metric $g_{\mu\nu}$, the gravitini ψ_μ and $\tilde{\psi}_\mu$, the $U(1)_R$ symmetry gauge field $A_\mu^{(R)}$, a 2-form gauge field $B_{\mu\nu}$ together with its field strength $H=\frac{\mathrm{i}}{2}\epsilon^{\mu\nu\rho}\partial_\mu B_{\nu\rho}$, a vector auxiliary field C_μ and its dual $V^\mu=-\mathrm{i}\epsilon^{\mu\nu\rho}\partial_\nu C_\rho$. The background Abelian vector multiplet comprise a gauge field A_μ together with its field strength $F_{\mu\nu}=\partial_{[\mu}A_{\nu]}$, a scalar field σ, the gaugini λ and $\tilde{\lambda}$, and a second scalar field D. Note that in Euclidian signature the fields do not need to be real.

With the background fields that we consider, the off-shell supersymmetry transformations of the component fields of a chiral multiplet Υ of R-charge q

are:

$$\delta\varphi = \sqrt{2}\varepsilon\chi \ , \tag{5.2.8a}$$

$$\delta\chi = \sqrt{2}\varepsilon G - \sqrt{2}\mathrm{i}\gamma^{\mu}\tilde{\varepsilon}D_{\mu}\varphi + \sqrt{2}\mathrm{i}\sigma\tilde{\varepsilon}\varphi + q\sqrt{2}\mathrm{i}H\tilde{\varepsilon}\varphi \ , \tag{5.2.8b}$$

$$\delta G = -\sqrt{2}\mathrm{i}D_{\mu}(\tilde{\varepsilon}\gamma^{\mu}\chi) - \sqrt{2}\mathrm{i}\sigma\tilde{\varepsilon}\chi + 2\mathrm{i}\tilde{\varepsilon}\tilde{\lambda}\varphi - \sqrt{2}\mathrm{i}(q-2)H\tilde{\varepsilon}\chi \ , \tag{5.2.8c}$$

where the ε and $\tilde{\varepsilon}$ are the supersymmetry parameters with $U(1)_R$ charge 1 and -1, respectively. The covariant derivative D_{μ} are gauge and $U(1)_R$ covariant besides including the spin connection.

We consider the case where the $U(1)_R$ charge for the chiral supermultiplet is $q = 1/2$. This leads to the superconformal theory with Lagrangian

$$\mathcal{L} = D_{\mu}\tilde{\varphi}D^{\mu}\varphi + \tilde{\varphi}\Big(\frac{1}{8}R + \sigma^2 + D\Big)\varphi + \mathrm{i}\tilde{\chi}\gamma^{\mu}D_{\mu}\chi + \mathrm{i}\sigma\tilde{\chi}\chi - \tilde{G}G \tag{5.2.9}$$

where R is the scalar curvature of the background manifold. This is the component version of the superspace action (5.2.7). It is worth mentioning that the quadratic terms in the bosonic spinors χ and $\tilde{\chi}$ acquire an overall minus sign compared to the case of commuting chiral multiplets due to the unusual statistics (compare e.g. with (5.2.1)). Including in addition the chiral integral of a superpotential $W(\Upsilon^I)$ (where we now consider multiple chiral multiplets labelled by the index I), its component Lagrangian is

$$\mathcal{L}_W = \frac{\partial W(\varphi)}{\partial\Upsilon^J}G^J + \frac{\partial^2 W(\varphi)}{\partial\Upsilon^J\partial\Upsilon^I}\chi^I\chi^J \ , \tag{5.2.10}$$

together with its conjugate. For the interesting case with $\Upsilon_{\pm}$ multiplets mentioned earlier, the classically marginal deformation $W_{\text{marginal}}(\Upsilon_{\pm}) = (\Upsilon^i_+\Upsilon_{-i})^2$ leads to the component action

$$\begin{aligned}\mathcal{L}_{W_{\text{marginal}}} &= 2(\varphi^k_+\varphi_{-k})\big(\varphi^i_+G_{-i} - \varphi_{-i}G_+{}^i\big) \\ &\quad +2\big((\varphi^i_+\chi_{-i} + \varphi_{-i}\chi_+{}^i)^2 - 2\varphi^i_+\varphi_{-i}\chi^j_+\chi_{-j}\big) \ .\end{aligned} \tag{5.2.11}$$

Integrating out the auxiliary field $G_{\pm}$ from the sum of (5.2.9) and (5.2.11) we obtain the classically marginal deformation V_{marginal}

$$\begin{aligned}V_{\text{marginal}} &= 8(\varphi^i_+\varphi_{-i})(\tilde{\varphi}_{+j}\tilde{\varphi}_-{}^j)\big(\varphi^k_+\tilde{\varphi}_{+k} + \varphi_{-k}\tilde{\varphi}_-{}^k\big) \\ &\quad +2\big((\varphi^i_+\chi_{-i} + \varphi_{-i}\chi_+{}^i)^2 - 2\varphi^i_+\varphi_{-i}\chi^j_+\chi_{-j}\big) \ ,\end{aligned} \tag{5.2.12}$$

where all the interaction terms are double and triple trace operators.

So far we have neglected the gaugini and gravitini which would couple to spin $1/2$ and $3/2$ currents in the actions. By truncating these modes, the invariance of the action require the supersymmetry variation of the background fermions to be zero. By imposing the variation of the gravitini to be zero, $\delta\psi_\mu = \delta\tilde{\psi}_\mu = 0$, one finds the Killing spinor equations:

$$(\nabla_\mu - \mathrm{i}A_\mu^{(R)})\varepsilon = -\frac{1}{2}H\gamma_\mu\varepsilon - \mathrm{i}V_\mu\varepsilon - \frac{1}{2}\epsilon_{\mu\nu\rho}V^\nu\gamma^\rho\varepsilon \ , \tag{5.2.13a}$$

$$(\nabla_\mu + \mathrm{i}A_\mu^{(R)})\tilde{\varepsilon} = -\frac{1}{2}H\gamma_\mu\tilde{\varepsilon} + \mathrm{i}V_\mu\tilde{\varepsilon} + \frac{1}{2}\epsilon_{\mu\nu\rho}V^\nu\gamma^\rho\tilde{\varepsilon} \ . \tag{5.2.13b}$$

Requiring the variation of the gaugini to be zero, $\delta\lambda = \delta\tilde{\lambda} = 0$, one finds the following additional constraints on the background fields

$$0 = \Big(\mathrm{i}(D + \sigma H) - \frac{\mathrm{i}}{2}\epsilon^{\mu\nu\rho}\gamma_\rho F_{\mu\nu} - \mathrm{i}\gamma^\mu(\partial_\mu\sigma + \mathrm{i}V_\mu\sigma)\Big)\varepsilon \ , \tag{5.2.14a}$$

$$0 = \Big(-\mathrm{i}(D + \sigma H) - \frac{\mathrm{i}}{2}\epsilon^{\mu\nu\rho}\gamma_\rho F_{\mu\nu} + \mathrm{i}\gamma^\mu(\partial_\mu\sigma - \mathrm{i}V_\mu\sigma)\Big)\tilde{\varepsilon} \ . \tag{5.2.14b}$$

We refer to [104] and references therein for details about the geometrical constraints imposed on a 3-manifold admitting some residual rigid supersymmetry. The sphere S^3 of radius l is a maximally supersymmetric background. In this case, besides the metric the only nontrivial field turned on is a constant H-flux of the form $H = \mathrm{i}/l$.

5.2.3 Duality

We conjecture that the $\mathcal{N} = 2$ supersymmetric extensions of the Euclidean $Sp(N)$ model that we constructed above are dual to the $\mathcal{N} = 2$ supersymmetric higher-spin gravity theories in de Sitter space found in [86].[11]

There is a continuous family of Vasiliev theories in de Sitter that depend on the parity-breaking angle θ. In the CFT dual, θ corresponds to a Chern-Simons coupling constant k, i.e. $\theta = \frac{\pi N}{2k}$ [83]. We first consider Vasiliev theories with $\theta = 0$. This corresponds to the $k \to \infty$ limit such that $N/k \to 0$. In this limit we can neglect the Chern-Simons terms since our boundary is simply connected. The dual field theory is then free and determined by the symmetry group,

[11] We refer in particular to Section 4.4 of [86] for a detailed discussion of these HS theories.

i.e. the amount of supersymmetry and the choice of internal gauge symmetry. Its Lagrangian is given in full generality in (5.2.2), and in (5.2.9) when restricted to low spin deformations. The symmetry groups of the bulk and boundary theories are the same: both theories have $OSp(2|2,2)$ invariance possessing 8 real supercharges. They also have the same higher-spin symmetry,[12] which brings the full symmetry generated by spacetime, supersymmetry and HS of both theories to $ho(1,1|4,1)$.[13]

Sezgin and Sundell [86] note that the counting of dynamical fields of $\mathcal{N}=2$ dS_4 higher spin theory matches that of $\mathcal{N}=2$ AdS_4 higher spin theory. Since the counting of $\mathcal{N}=2$ $U(N)$ currents matches the $\mathcal{N}=2$ AdS_4 bulk fields, mutatis mutandis the counting of $\mathcal{N}=2$ $U(-N)$ currents matches the $\mathcal{N}=2$ dS_4 bulk fields.

The bulk theory contains in particular a scalar and a pseudo-scalar with mass $m^2 = 2/R^2$. This mass allows for two different boundary conditions which in the boundary theory correspond to either a free or an interacting CFT [68, 83]. Here we consider the free boundary theory in which the $U(-N)$ singlet scalar operators $\tilde{\varphi}_i\varphi^i$ have dimension $\Delta = 1$ and $\tilde{\chi}_i\chi^i$ have $\Delta = 2$. They correspond respectively to the bulk scalar and the pseudo-scalar. This means the former must obey alternative boundary conditions for which the modes asymptotically behave as $\sim z$ whereas the latter obeys standard boundary (or quantization) conditions, with modes behaving as $\sim z^2$ asymptotically.[14]

There are also two towers of integer spin bulk fields or equivalently two towers of boundary integer spin conserved currents. One tower is constructed by applying derivatives to $\tilde{\phi}_i\phi^i$ to make traceless spin-s currents. The other tower is made analogously from $\tilde{\psi}_i\psi^i$.

Finally there is one tower of strictly half-integer spin fields built by taking combinations of derivatives of ψ^i_α and $\tilde{\phi}_i$ and the conjugate. Fermionic half-integer higher-spin fields in the bulk enter as Grassmann valued arguments in the wave function[15] (5.1.1). The general framework of supersymmetric

[12] The higher spin symmetry of free vector models follows from the properties of the Dirac and Laplace operators and hence does not depend on the spin-statistics of the field on which these act, see e.g. [114] and [115] for the super-Laplacian case.

[13] By extending the analysis of [116, 117] (see also [118, 119]) to the supersymmetric case it may be possible to prove directly that the partition function of (5.2.2) has $ho(1,1|4,1)$ symmetry. Along this direction, regarding the use of Noether method in superspace see the recent paper [120] for the 4D $\mathcal{N}=1$ case.

[14] The dimension $\Delta_\pm$ of operators in the boundary theory is related to the mass m^2 of dual bulk scalars as $\Delta_\pm = \frac{3}{2} \pm \sqrt{\frac{9}{4} - m^2R^2}$ where R is the dS radius.

[15] To avoid confusion we note that, unlike in ordinary first quantised mechanics, the wave function in quantum cosmology does not explicitly depend on position, or momenta. Rather, the arguments of wave functions in quantum cosmology are configurations of matter fields and three-geometries which themselves are in general of course functions of position.

quantum cosmology was developed long ago, see e.g. [121–123]. One might be concerned about the physical interpretation of wave functions that include Grassmann valued arguments. However, the wave function is not directly a physical observable. Probabilities for physical observables $\mathcal{O}$ are computed as

$$< \Psi, \mathcal{O}\Psi >= \int \mathcal{D}h\mathcal{D}A_{s\in\mathbb{N}}\mathcal{D}A_{s\in\mathbb{N}+1/2}\overline{\Psi}(h, A_s)\mathcal{O}(h, A_s)\Psi(h, A_s)\Xi(h, A_s) \ . \quad (5.2.15)$$

Here we have denoted separately and abstractly the integer and half-integer matter field configurations in the functional integral and we have inserted a density measure Ξ. This shows that for bosonic operators $\mathcal{O}$, since one integrates over the Grassmann valued $A_{s\in\mathbb{N}+1/2}$ fields, physically meaningful expectation values are ordinary numbers.

We conclude with a remark on interacting theories. We have constructed interacting supersymmetric vector models, with a potential of the form (5.2.12). It is natural to conjecture these play a role in a duality with supersymmetric higher-spin gravity theories in dS with the parity-breaking θ angle turned on. The analogous AdS/CFT discussion [83] shows that marginal double and triple trace interactions correspond to generalized 'designer gravity' boundary conditions [124] in the bulk. Interaction terms involving both φ and χ will imply a relation between the asymptotic profiles of the bulk scalar and pseudo-scalar. In the context of dS/CFT this means the partition function computes the wave function in an unusual basis [72]. With θ turned on, however, also the Chern-Simons terms are important [83, 102]. In this regard we expect extended $\mathcal{N} > 2$ supersymmetric models to play an important and interesting role, in which the interplay between Chern-Simons and matter dynamics can render the marginal deformations exactly marginal. We leave a more precise and complete formulation of dS/CFT for interacting duals to future work and turn now to a first exploration of the physics of the duality relating the free theories.

5.3 Supersymmetric Minisuperspace

We now evaluate the partition function of the free $U(-N)$ model (5.2.9) for a range of supersymmetry preserving bosonic deformations of the theory. We first consider homogeneous mass deformations of the theory. These correspond to turning on a supersymmetric vector multiplet background. Next we compute the theory on squashed three-sphere boundaries where we will see supersymmetry requires an additional spin-1 deformation be turned on.

Eq. (5.1.1) shows that dS/CFT relates the sources of the deformations in the partition function to the argument of the bulk wave function in the large three-volume limit. The dependence of the partition function on the values of

the above sources therefore yields a cosmological measure on a minisuperspace of asymptotically dS configurations. From this measure predictions for semi-local observables in cosmology can be derived through further coarse-graining. The deformations we consider here specify a minisuperspace that comprises a class of anisotropic deformations of de Sitter space (which have squashed sphere future boundaries) and asymptotic dS universes with an early phase of scalar field inflation.

We consider free dual theories only in this section. Their partition functions are inversely related to the partition functions of the corresponding $U(N)$ theories with ordinary spin-statistics. By Gaussian integration the latter take the form

$$Z \propto \frac{\det(\text{half-integer spin field eigenvalues})}{\det(\text{integer spin field eigenvalues})}, \tag{5.3.1}$$

whereas for the theories with opposite spin statistics we have,

$$Z \propto \frac{\det(\text{integer spin field eigenvalues})}{\det(\text{half-integer spin field eigenvalues})}. \tag{5.3.2}$$

This relation is an exact example of the more general close connection between (Euclidean) AdS/CFT and dS/CFT evoked in the Introduction. It means that for the purpose of this section we can either work with the original free chiral multiplets (see e.g. [104]) and use (5.3.1) or evaluate directly (5.3.2). We illustrate both methods below.

Before we proceed with our analysis we should comment on two subtleties. First, in the presence of background fields contact terms can appear that render the value of the partition function unphysical [125]. A meaningful dS/CFT duality of the form (5.1.1) requires at least the norm of the partition function be physical, which is guaranteed if the duals are reflection positive. However we lack a proof of this in the present context[16]. A second subtlety concerns the regularisation of the determinants. On the $U(N)$ side the regularisation is fixed by $\mathcal{N} = 2$ localisation [103]. We assume the same regularisation holds in the $U(-N)$ case. Localisation for the $U(-N)$ theory should parallel the result of the $U(N)$ theory because the off-shell supersymmetry on the three manifolds is the same in the two cases.

[16] With more supersymmetry fewer contact terms are possible. For instance the Chern-Simons term $A \wedge dA$ is absent whenever $\mathcal{N} > 2$.

5.3.1 Scalar deformations

The action of the undeformed supersymmetric $U(-N)$ model (5.2.9) on a round S^3 boundary is

$$S_{chi} = \int d^3x\sqrt{h}\left[\partial_\mu\tilde{\varphi}_i\partial^\mu\varphi^i + \frac{3}{4l^2}\tilde{\varphi}_i\varphi^i + \mathrm{i}\tilde{\chi}_i\not{\nabla}\chi^i - \tilde{G}_iG^i\right]. \tag{5.3.3}$$

We consider mass deformations of this theory which break conformal symmetry, but preserve supersymmetry[17] by a coupling to the vector multiplet V. First we set the gauge multiplet to a BPS configuration in which only the scalar fields σ and D are nonzero and constant. The constraints (5.2.14) then imply that a supersymmetric vector multiplet background has $D = -\mathrm{i}\sigma/l$. Substituting this condition in (5.3.3) yields the following mass deformation,

$$\mathcal{L}_{mass} = \left(\sigma^2 - \mathrm{i}\frac{\sigma}{l}\right)\tilde{\varphi}_i\varphi^i + \mathrm{i}\sigma\tilde{\chi}_i\chi^i\ . \tag{5.3.4}$$

where l is the radius of the sphere.

We see that the deformation gives masses to both the scalars and the spinors in the boundary theory. As discussed above the component scalar current $j_+^{(0)} = \tilde{\varphi}_i\varphi^i$ is dual to a bulk scalar whereas the current $j_-^{(0)} = \tilde{\chi}_i\chi^i$ constructed out of the spinor fields is dual to the bulk pseudoscalar. Supersymmetry implies both fields are coupled.

The independent complex scalar field σ plays the role of an external, constant mass parameter in the deformed theory [104]. It therefore enters as a source in the partition function. In the bulk σ corresponds to the coefficient of the subleading term $\sim z^2$ in the asymptotic profile of the scalar. The phase of σ is determined from the requirement that the theory must have a well-defined, asymptotically classical, real de Sitter structure [95][18]. For $m^2 = 2/R^2$ scalars this implies in particular we must take σ imaginary [126, 127]. Hence the asymptotic profile of the bulk scalar has a real leading term and an imaginary subleading term, precisely what one expects for scalar fields in the Euclidean Hartle-Hawking vacuum. For the susy deformations we consider, the boundary conditions on the pseudo-scalar are completely fixed by those on the scalar. We are thus led to consider the following mass deformation $\sigma = \mathrm{i}m$, with m real,

$$\mathcal{L}_{mass} = \left[-m^2 + \frac{m}{l}\right]\tilde{\varphi}_i\varphi^i - m\tilde{\chi}_i\chi^i\ . \tag{5.3.5}$$

[17] One can also consider supersymmetry breaking mass deformations by leaving σ and D in (5.2.9) independent. Taking $\sigma = 0$ for instance yields the setup of [72] multiplied by an overall factor coming from the spinors.

[18] A different but equivalent way to see this is that any other 'reality' condition on σ would imply the bulk scalar to behave as a ghost, and the resulting wave function to be ill-defined.

To evaluate the partition function as a function of m we must first find the eigenvalues of the Laplace and Dirac operators on S^3. These are well-known and can for instance be found in [104] (see [128] for a derivation in superspace). Setting $l = 1$ the scalar eigenvalues are given by

$$\lambda_n = n(n+2) + 3/4 - m^2 + m, \tag{5.3.6}$$

with $n = 0, 1, 2, ..$ and degeneracy $(n+1)^2$. Note that the first eigenvalue is negative when $m < -1/2$. The eigenvalues for the spinor are

$$\lambda_n^{\pm} = \pm\Big(n + \frac{1}{2}\Big) - m, \tag{5.3.7}$$

with $n = 1, 2, 3, ..$ and degeneracy $n(n+1)$. There are negative eigenvalues for all values of m. However, each eigenvalue has an even degeneracy. Since eigenvalues are raised to the power of their degeneracy in the partition function this means the spinor contribution need not necessarily lead to divergences.

With the eigenvalues at our disposal it is straightforward to evaluate the Gaussian integrals defining the partition function $Z[m]$. See [104] and references therein for a pedagogical description of the case with standard statistic. For a single multiplet in the theory with reversed spin-statistics we get

$$Z[m] = \prod_{j=0}^{\infty} \left[\frac{j+m+1/2}{j-m+3/2}\right]^{j+1}. \tag{5.3.8}$$

To numerically perform the product in (5.3.8) we found it convenient to first calculate the second derivative of the free energy. Fig 5.1(a) shows the resulting distribution ZZ^* as a function of the mass parameter m. One sees it has a local maximum at zero deformation, corresponding to the amplitude of pure dS space in higher-spin gravity. This is in accordance with general field theory results such as the F–theorem.[19] For negative masses the distribution closely resembles that found in [72]. In particular it goes to zero as $m \to -1/2$. For more negative masses the holographic wave function is zero. This is because the path integral defining the partition function diverges in deformed (free) theories for which the scalar operator in the action has a negative eigenvalue. Hence the form (5.3.8) no longer holds. Therefore we show the behavior following from (5.3.8) with dotted curves in Fig 5.1(a).

For positive masses the distribution diverges as $m \to 3/2$. Whether or not this renders the wave function non-normalizable and ill-defined may depend on the

[19] Had we instead taken σ real we would have found a pathological wave function even for small deformations. In particular the distribution would have had a $\cosh(m)$ behavior. Hence pure dS would have been a local minimum of the distribution, leading to two-point functions characteristic of an unstable ghostlike theory.

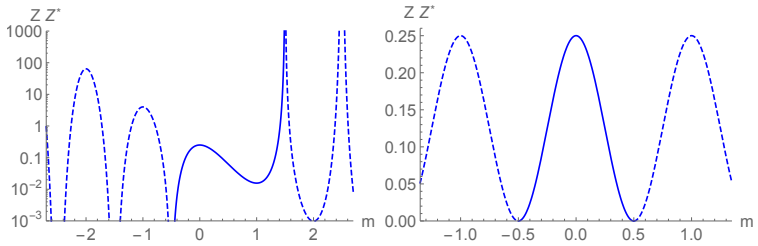


Figure 5.1: The response of ZZ^* to various mass deformations for two chiral multiplets with reversed spin-statistics. We conjecture this corresponds to the Hartle-Hawking wave function in a homogeneous isotropic minisuperspace model in HS gravity. **Left:** Both multiplets are in the same representation of the gauge multiplets inducing the mass deformation eq. (5.3.5) with parameter m. **Right:** the two multiplets are in opposite representations; one has mass parameter m and the other $-m$ under eq. (5.3.5).

integration measure. However this behavior can be improved by adding a flavor symmetry. This can be done by placing N chiral multiplets in the fundamental and N in the anti-fundamental of the gauge multiplet, as for the case with Υ^i_+ and Υ_{-i} described in section 6.3. In the BPS configuration of the gauge multiplet, this means half of the fields have opposite mass m. Fig 5.1(b) shows the resulting distribution $ZZ^*[m]$ which is of the form of a cosine squared. The wave function now has support on the interval $|m| \leq 1/2$ only. For larger values it is zero. The distribution is well-behaved over the entire configuration space and has a global maximum at $m = 0$.

The addition of a flavor symmetry may appear ad hoc, but it is not. In [83], it is precisely the addition of such flavor symmetry that relates the model to string theory. Imposing this constraint from string theory in our setup appears to enhance the stability of de Sitter space.

The distribution $ZZ^*[m]$ specifies the no-boundary wave function $\Psi[A^{(0)}_+, A^{(0)}_-]$ in a supersymmetric minisuperspace model consisting of homogeneous, asymptotically de Sitter universes in HS gravity with a scalar $A^{(0)}_+$ and a pseudo-scalar $A^{(0)}_-$ turned on. The histories therefore exhibit an interior region with scalar field driven inflation. The upper bound on the deformation m might mean the bulk potential is such that there is a maximum number of efolds of scalar field inflation.

5.3.2 Squashings and Vector Fields

Next we consider deformations of the background metric which preserve four real supercharges. In particular we consider the partition function of the free $U(-N)$ model (5.2.9) on homogeneous squashings of the three-sphere that are characterized by a squashing parameter $v > 0$. The metric can be written as

$$ds^2 = d\theta^2 + \sin^2\theta d\phi^2 + \frac{1}{v^2}(d\psi + \cos\theta d\phi)^2 \tag{5.3.9}$$

where θ, ϕ, ψ are the Euler angles on S^3 such that $\theta \in [0, \pi]$, $\phi \in [0, 2\pi]$ and $\psi \in [0, 4\pi]$. Supersymmetry requires that we turn on a background $U(1)_R$ symmetry gauge field on squashed backgrounds. In the $U(N)$ model with standard spin-statistics, this is given by the one-form [129, 130]

$$A^{(R)} = -\frac{1}{2v^2}\sqrt{1 - v^2}(d\psi + \cos\theta d\phi). \tag{5.3.10}$$

In this section we evaluate the wave function with asymptotic dS boundary conditions using the inverse of the partition function of deformations of the $U(N)$ theory. As before the phases of the sources are specified by the condition that the wave function is defined on a real configuration space that is asymptotically dS [91]. The gauge field $A^{(R)}$ is the boundary value of a bulk gauge field $\vec{A}$ in the dual AdS theory. To evaluate the wave function in the dS domain $\vec{A}$ must be asymptotically imaginary, because the original AdS scale factor a is also imaginary in the dS domain. Taken together this yields asymptotically real frame fields $\vec{A}/a$ on the dS side [75]. Hence we ought to compute the partition function for purely imaginary values of the source $A^{(R)}$. This selects the range $v \geq 1$ in (5.3.10). Note that the round three-sphere corresponds to $v = 1$. On this background, the gauge field is appropriately turned off.

The free energy of AdS dual $U(N)$ theories on squashed boundaries was computed in [104, 129–131]. From the relations (5.3.1) and (5.3.2) it follows that it suffices to change its overall sign to find the free energy of the dS dual theories with reversed spin-statistics. In the $v \geq 1$ domain this yields the following partition function (for a single multiplet),

$$Z = \mathrm{Exp}\left[-\int_0^\infty \frac{dx}{2x}\mathscr{V}\right], \tag{5.3.11}$$

$$\mathscr{V} = \frac{\sinh(x/v)}{\sinh(x(v^{-1} \pm \sqrt{v^{-2} - 1}))\sinh(x/(v^{-1} \pm \sqrt{v^{-2} - 1}))} - \frac{1}{vx}\,. \tag{5.3.12}$$

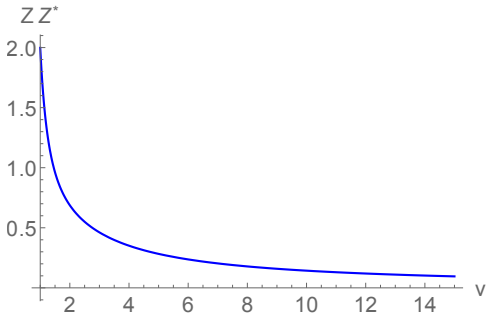


Figure 5.2: The holographic Hartle-Hawking wave function in higher-spin gravity in the one-dimensional minisuperspace of anisotropic but homogeneous deformations of de Sitter coupled to a gauge field, parameterized by v, for a single multiplet.

This specifies the large three-volume limit of the no-boundary wave function in higher-spin gravity in the one-dimensional minisuperspace of anisotropic deformations of dS coupled to a gauge field. Fig. 5.2 shows the resulting distribution is well-behaved and normalizable with a global maximum at pure dS space. In particular the constraints implied by supersymmetry – together with a careful analysis of the asymptotic structure – appear to eliminate the usual problem that gauge fields in supergravity theories on de Sitter backgrounds are ghosts.

To conclude we comment on deformations that are non-supersymmetric squashings of chiral multiplets where one does not turn on a background gauge field. The free energy of such deformed theories was computed in [80] in the context of AdS duals. For free chiral multiplets this can be interpreted in the context of dS/CFT by inverting the partition function as we discussed. This yields a well-behaved distribution in which undeformed dS space seems stable, at least against small deformations. This indicates that for $v \geq 1$ the contributions to Z due to the chiral multiplet and the background gauge field are separately stable under squashings. On the other hand, for $v < 1$ there is a clear difference. The translation to dS of the results of [80] shows that $v = 1$ is a local maximum for non-supersymmetric squashings even when one includes the $v < 1$ regime. By contrast this would not be the case if one were to extend our result for supersymmetric squashings to $v < 1$, which would yield a distribution that diverges as $v \to 0$. This is precisely what one expects because under the boundary conditions corresponding to $v < 1$, which we did not take, the vector field is ghost-like [75]. It is interesting that the $v < 1$ regime covers the entire region of configuration space where the Yamabe invariant of the

conformal boundary is negative. The fact that the holographic wave function has no support in this region once gauge fields are taken in account lends further support to the conjecture [73] that the holographic measure strongly suppresses conformal boundaries far from the round conformal structure.

5.4 Speculations on a triality with exotic string theories

In the context of ordinary AdS/CFT it has been argued [83] that that a triality relates type IIA string theory on $\mathrm{AdS}_4 \times \mathrm{CP}_3$ (possibly with some B-field flux) to specific Vasiliev theories in AdS_4 and Chern-Simons vector models. These type IIA backgrounds lift to 11-dimensional supergravity on $\mathrm{AdS}_4 \times \mathrm{S}^7/\mathrm{Z}_k$ (possibly with torsional flux added).

Our results suggest this triality can be 'Wick rotated' and thus generalized to a dS context where the third party would be the exotic M-theory $\mathrm{M}^{-}_{9,2}$ in a $\mathrm{dS}_4 \times \mathrm{AdS}_7/\mathrm{Z}_k$ background constructed by Hull [87].[20] This exotic M-theory has two time directions and the M2 brane has a Euclidean world volume. In flat space its near-horizon geometry is $\mathrm{dS}_4 \times \mathrm{AdS}_7$. The Vasiliev theories in dS would then be the tensionless limit of this theory.

A triality of this kind would resonate with the intuition recalled in the Introduction that exotic string theories in dS can be well-behaved despite having ghosts in the supergravity limit. Hull has argued that the massive string states render the starred string theories ghost-free [87]. The Vasiliev limit which features in the triality precisely corresponds to the limit where the massive string tower becomes massless. It is therefore plausible that Hull's 'ghost-exorcism' shows up at the classical higher-spin level. The fact that the minisuperspace wave function computed in Section 5.3 is well behaved provides some holographic support for this.

We conclude with an illustration of how this triality might be realized more explicitly. For this purpose let us regard the above setup from a type IIB viewpoint. It is well known that ordinary ABJ(M)-theory emerges in the IR from specific brane configurations (cf. Table 5.1(a)). In Table 5.1(b) we configure branes in an analogous manner in the exotic $\mathrm{IIB}^{-+}_{(9,1)}$ theory. The $D3$-branes have become Euclidean while the $D5$-branes are Lorentzian. If one

[20] See [92] for this notation, where the possible existence of exotic string theories was also connected to supergroup-gaugings and negative branes. Negative branes naturally lead to signature changes between different regions of spacetime and thereby connect AdS and dS. Hence they may play a role in a microscopic description of the no-boundary wave function.

	0	1	2	3	4	5	6	7	8	9
$NS5$	•	•	•	•	•	•				
$D3$	•	•	•				•			
$D5$	•	•	•	•	•					•

↓

	0	1	2	3	4	5	6	7	8	9
$ENS5$		•	•	•	•	•				•
$ED3$		•	•	•			•			
$D5$	•	•	•	•	•	•				

Table 5.1: **Top table**: ABJ(M) setup in IIB string theory. The Lorentzian 3D CFT is in the 012 direction. **Bottom table**: Analogous setup to ABJ(M) in $\text{IIB}^{-+}_{(9,1)}$ with Euclidean $D3$ branes. The Euclidean 3D CFT is in the 123 direction.

can argue that strings between the Euclidean $D3$ and the ordinary $D5$ branes have reversed spin-statistics, then the resulting 3D field theories have limits in which they turn into the Euclidean $U(-N)$ field theories we have constructed. For example, separating the Euclidean $D3$ and ordinary $D5$ branes would then correspond exactly to the mass deformation in Section 5.3.1 with an additional flavor symmetry that gives half the fields opposite masses.[21]

5.5 Discussion

We have constructed supersymmetric Euclidean vector models in three dimensions with reversed spin-statistics. We conjecture these are holographically dual to specific supersymmetric Vasiliev theories in four-dimensional de Sitter space.

We have begun to explore this duality by computing the partition function for a range of scalar, vector and tensor deformations that preserve supersymmetry. The duality asserts this specifies the Hartle-Hawking wave function in a supersymmetric minisuperspace consisting of anisotropic bosonic deformations of dS with scalar and vector matter. We found the wave function is globally peaked at de Sitter, with a low amplitude for strong deformations. This suggests that

[21] As an aside we note that it was argued [48] that the near-horizon geometry of the Euclidean branes in exotic string theories does not involve de Sitter space but rather elliptic de Sitter space (where the antipodal points are identified). This in turn appears to resonate with [132] where it was shown that the physical arrow of time in asymptotic dS histories predicted by the Hartle-Hawking wave function reverses near the dS throat.

supersymmetric de Sitter space is stable in higher-spin gravity and in particular free from the usual ghosts.

An important generalization of our analysis concerns the calculation of the CFT partition function for deformations sourcing half-integer spin fields. Free spin-1/2 and spin-3/2 fields in de Sitter space have been studied in [133], which also derives a number of general properties of dual CFTs. In particular, [133] identifies the general form of the boundary-boundary two-point function of a bulk spinor, for an arbitrary spacetime dimension and mass, and up to a constant factor. At the time, however, no CFT dual was known to compare this with. Our model provides a concrete setup in which this can be done. The supersymmetric HS bulk theory we consider includes a specific nonlinear, interacting extension of the free spinors considered in [133]. Specializing the general result for the two-point function in [133] to the massless case in four bulk dimensions yields the following structure for the 3D boundary-boundary two-point function,

$$< \mathcal{O}_{1/2}(x)\tilde{\mathcal{O}}_{1/2}(x') >= \text{const}\, \frac{\gamma \cdot (x-x')}{(x-x')^4} \ . \tag{5.5.1}$$

This can be compared with our CFT result. On the CFT side we have, $\mathcal{O}_{1/2} \propto \tilde{\varphi}\chi$ and analogously for $\tilde{\mathcal{O}}_{1/2}$. The form of the $\mathcal{O}_{1/2}$ two-point function is determined up to a constant factor by conformal symmetry. Since $\mathcal{O}_{1/2}$ has conformal dimension $\Delta = 3/2$, one exactly obtains (5.5.1).[22] Therefore our CFT result matches the general structure of the fermionic two-point functions derived from a bulk analysis in [133]. This provided evidence that our proposed dS/CFT duality holds in the half-integer spin sector.

Deformations of the boundary theory that correspond to turning on spinors in the bulk are couplings of $\mathcal{O}_{1/2}$ and $\tilde{\mathcal{O}}_{1/2}$ to the background gaugini λ and $\tilde{\lambda}$ as described in section 5.2.2, although we chose to set these terms to zero. Note that the reality properties we have imposed on the background scalars σ and D imply specific reality properties on the background gaugini λ and $\tilde{\lambda}$ through the supersymmetry variation equations. Supersymmetry thus relates the two-point function (5.5.1) to other two-point functions containing integer spin currents. We leave a more detailed analysis of the complete partition function in supersymmetric minisuperspace beyond the level of the two-point function, which one would expect also determines the sign of the latter, to future work.

Our results open up new ways to develop dS/CFT further. First, it would be very interesting to demonstrate the absence of ghosts directly in Vasiliev gravity in de Sitter. We have mainly focused on the duality for free theories,

[22] The analogous bulk and boundary computations in the AdS/CFT case are given in [134].

but it would also be interesting to formulate the duality for supersymmetric interacting theories and to clarify in particular how the Chern-Simons terms and extended $\mathcal{N} > 2$ supersymmetry enter. Supersymmetry will also enable one to introduce in dS/CFT new and powerful calculational techniques, such as localisation, that have led to important advances in the context of AdS/CFT in recent years.

Finally, it would be especially interesting to understand whether the supersymmetric higher-spin theories featuring in our duality are indeed the tensionless limit of Hull's exotic string theories in their de Sitter vacua. This would establish a triality similar to the one put forward in [83], which in turn may lend support to the conjecture [87] that the massive string states render the starred string theories ghost-free. This would constitute a first step toward the generalization of the HS realisations of dS/CFT to other, more realistic theories in dS.

Chapter 6

Spinors in Supersymmetric dS/CFT

This chapter is a reprint of [2]. We continue the analysis of the model constructed in chapter 5. As we already have the model in hand, the focus in this chapter is much more on computing Ψ_{HH} for a variety of field configurations. In particular, we compute Ψ_{HH} for supersymmetry breaking field configurations and with the inclusion of fermionic bulk fields. The state Ψ_{HH} continues to look like a perturbatively stable de Sitter ground state with these new configurations included. The inclusion of the fermionic bulk degrees of freedom causes an interesting transition in the behaviour of Ψ_{HH} between large and small N.

Although the bulk higher spin theory intrinsically only contains massless fields and scalars with mass $m^2 = 2/l_{dS}^2$, we argue that one can mimic a fermionic field with a tunable mass. The behaviour of the mimic we construct qualitatively matches a massive fermion coupled to Einstein gravity. This suggests that higher spin dS/CFT with sources whose spin is limited to $s \leq 2$ and with such mimics can indeed be informative of quantum gravity theories in de Sitter space that are Einstein gravity + matter at low energy.

6.1 Introduction

In chapter 5 we put forward a supersymmetric generalization of higher spin dS/CFT. In this paper we initiate the study of fermions in dS/CFT. In particular we compute the contribution from bulk fermions to the Hartle-Hawking ground

state wave function Ψ_{HH} in these supersymmetric HS theories. The properties of superpartner fermions are especially interesting since in higher spin theory, fermionic fields can essentially only arise as superpartners to bosonic fields. We find that, within the analysis performed so far, the existence of a stable supersymmetric de Sitter vacuum - our fundamental conclusion in [1] - remains unchanged with the inclusion of these fermionic fields. As before we perform all calculations in the conjectured dual and use the duality (5.1.1) to obtain the Hartle-Hawking wave function. Purely computationally, the dS/CFT dictionary (5.1.1) applies directly to fermionic fields. However, the interpretation of Ψ_{HH} in the presence of fermionic fields requires a certain care as detailed long ago in [121, 135].

The paper is organised as follows. In section 6.2 we begin with a general discussion of fermionic bulk fields in the wave function of the universe and in the context of dS/CFT. This discussion is independent of the specifics of our model and the presence of supersymmetry. In section 6.3, we summarise the model of supersymmetric higher spin dS/CFT we constructed in [1], focussing especially on how the fermionic bulk fields enter. In section 6.4, we deform the boundary theory to turn on a spin-1/2 fermion in the bulk, while keeping all other bulk fields turned off. We study how the bulk wave function and observables respond to this fermionic field. In section 6.5, we repeat this process, now simultaneously turning on both a fermionic and a bosonic bulk field. We consider scalar, pseudoscalar and metric bulk fields. This allows us to study the interplay between fermionic and bosonic bulk fields. We see no sign of instability of the bulk vacuum coming from this interplay. In section 6.6, we first discuss the behaviour of Ψ_{HH} in the presence of bulk fermions in our model and we then conclude with a more general discussion of dS/CFT in string theory.

6.2 Bulk fermions and the wave function

As we have described in [1], half-integer spin CFT sources enter into the Hartle-Hawking wave function and the duality eq. (5.1.1) in a way that is formally completely analogous to the bulk bosons. However, on a conceptual level, the interpretation of these fields in the wave function deserves some extra attention. Here we discuss in general terms how spinorial bulk de Sitter fields enter into the dS/CFT correspondence, not limiting ourselves to our particular model.

In the action of a QFT, fermionic fields are often described using Grassmann variables. But, it is important to remember that, just as quantised bosons are not simply commuting variables, quantised fermions are not Grassmann variables. The Grassmann variables simply provide a convenient way to describe

the fermions pre-quantisation. Let us briefly review the canonical quantisation of fermions. Our treatment here will be rather brief and qualitative. A more detailed discussion of the quantization of a free fermion in de Sitter space and the properties of the $\breve{\psi}^{A'}$ conjugate spinors can be found in appendix A. To keep our presentation concise, we will sometimes suppress constant factors and indices when not relevant. A more complete discussion can be found in [121,135]. Consider the bulk spinor $\psi(x)$ in four-dimensional de Sitter space at some time-slice. The spatial geometry on this time-slice is an S^3 and the spinors can be expanded in spherical harmonic (commuting) spinors.[1] The three-sphere harmonics are ρ_A^{np}, $\sigma_{A'}^{np}$ with positive frequencies and their conjugates $\overline{\rho}_{A'}^{np}$, $\overline{\sigma}_A^{np}$ with negative frequencies. Here, A is the bulk spinor index, $n = 0, 1, 2, ..$ tracks the eigenvalue under the harmonic equation and $p = 1, 2, .., (n+1)(n+2)$ tracks the degeneracy for given n. Ignoring the degeneracy (which will be broken anyway when we deform the geometry), one can expand as

$$\psi_A \sim \sum_n \left(s_n \rho_A^n + \breve{t}_n \overline{\sigma}_A^n \right), \tag{6.2.1a}$$

$$\breve{\psi}_{A'} \sim \sum_n \left(\breve{s}_n \overline{\rho}_{A'}^n + t_n \sigma_{A'}^n \right). \tag{6.2.1b}$$

If one is describing a classical and commuting spinor function; s, t and their breves should be constant coefficients.[2] However, we wish to quantise ψ as a fermion. Then as usual, we want these coefficients to essentially play the role of creation and annihilation operators. This means that we should impose the anticommutation relations

$$\{s_n, t_m\} = \{s_n, \breve{t}_m\} = \{\breve{s}_n, t_m\} = \{\breve{s}_n, \breve{t}_m\} = 0\,, \tag{6.2.2a}$$

$$\{s_n, \breve{s}_m\} = \{t_n, \breve{t}_m\} = \delta_{nm}\,. \tag{6.2.2b}$$

As pointed out in [121, 135, 136], one should take the wave function on the future boundary surface to depend only on the positive frequency modes. This is convenient as the fermionic dependence of the wave function is then only a dependence on anticommuting Grassmann variables s and t. If the negative frequency modes had entered as well, we would have got variables with nontrivial anticommutation relations.

It is then clear how, for example, we should interpret the $J_{1/2}$ source of the boundary CFT related to a spin-1/2 bulk field, which enters into the wave

[1] On a deformed geometry the same line of reasoning goes through, with the harmonics of the deformed geometry.

[2] If one wants to describe how ψ evolves in time rather than restricting to one time-slice, these coefficients would become time-dependent.

function as Grassmann valued spinors, rather than fully quantised fermionic fields. In the boundary CFT, we can turn on a separate source term

$$J_{1/2\,,n}\mathcal{O}_{1/2} \tag{6.2.3}$$

for each three-sphere harmonic. In our specific model, $J_{1/2}$ is related to a background gaugino field λ in the CFT and $\mathcal{O}_{1/2}$ to a combination of dynamical CFT scalar and spinor, $\overline{\chi}\varphi$ and we will adopt this notation from here on for future convenience.

We then have a separate source term

$$\lambda_n(x)\overline{\chi}\varphi + \chi\breve{\lambda}_n(x)\overline{\varphi} \tag{6.2.4}$$

for each three-sphere harmonic. We associate λ_n in the boundary theory with $s_n\rho_A^n$ (no Einstein summation) in the bulk wave function and we associate $\tilde{\lambda}_n$ with $t_n\sigma^n_{A'}$.[3]

The purpose of the CFT in dS/CFT is to compute the bulk wave function Ψ. So, our sources are just Grassmann variables and from the path integral CFT perspective, we need not concern ourselves with the nontrivial commutation relations 6.2.2b. Of course, once we have our bulk wave function, to be able to interpret it and study its properties, we need a notion of conjugate wave function and a definition of inner product between two wave functions with fermionic fields. Here, $\tilde{s}$, $\tilde{t}$ and the nontrivial commutation relations 6.2.2b will come into play. The correct description of a second quantised fermionic field in a wave function formalism has long been known. The classic references are [137,138] and a discussion in the context of quantum cosmology can be found in [121,135]. The point is that the inner product should be defined such that wave functions with different fermion states occupied should be orthonormal. This is what one naturally expects and how things also work in the, for QFT more conventional, description in terms of state vectors in a Hilbert space. One can then expand the wave function in terms of the Grassmann variables, where due to their Grassmann nature, each λ_n, $\tilde{\lambda}_n$ can appear only twice at most in one term, but one can of course have many different λ_n with different n in one term[4].

$$\begin{aligned}\Psi =&\Psi_{\text{Bosonic}} + \Psi_0(\tilde{\lambda}_0\lambda_0) + \frac{1}{2}\Psi_{00}(\tilde{\lambda}_0\lambda_0)^2 + \Psi_1(\tilde{\lambda}_1\lambda_1)\\ &+ \Psi_{01}(\tilde{\lambda}_1\lambda_1)\ (\tilde{\lambda}_0\lambda_0) + \frac{1}{2}\Psi_{001}(\tilde{\lambda}_1\lambda_1)\ (\tilde{\lambda}_0\lambda_0)^2 + ..\end{aligned} \tag{6.2.5}$$

[3] Or the association is the other way, that is arbitrary.

[4] The wave function could in principle also include terms for instance of the type $(\lambda\lambda)$. The dynamics of our specific theory will ensure we only have $(\tilde{\lambda}\lambda)$ terms appearing. The physical picture here is that $\tilde{\lambda}$ is associated to particle creation and λ to antiparticle creation. If the dynamics of the theory produces particles and antiparticles in pairs, one then expects only terms of the form $(\tilde{\lambda}\lambda)$.

The $\Psi_{i_1\cdots i_k}$ only depend on the bosonic fields and the inner product between two bulk wave functions is given by

$$\begin{aligned}(\Psi,\Phi) =&(\Psi_{\text{Bosonic}},\Phi_{\text{Bosonic}})_B + (\Psi_0,\Phi_0)_B + (\Psi_{00},\Phi_{00})_B + (\Psi_1,\Phi_1)_B \\ &+ (\Psi_{01},\Phi_{01})_B + (\Psi_{001},\Phi_{001})_B + .. \,,\end{aligned} \tag{6.2.6}$$

where $(\,,\,)_B$ refers to the inner product over the bosonic bulk fields. For simplicity, we will only study the constant spinor λ_0 harmonic in this paper and will refer to it simply as λ in the future, but all other harmonics enter in an analogous way.

An interesting quantity to compute is the number operator for the fermion zero mode, N_F. When this operator acts on a state, it gives the number of $\tilde{\lambda}\lambda$ in that state. So, it acts as

$$N_F\Psi = N_F(\Psi_B + \Psi_0 + \Psi_{00}) = 0\Psi_B + 1\Psi_0 + 2\Psi_{00} \tag{6.2.7}$$

on a state Ψ. Its expectation value is given by

$$< N_F >= \frac{(\Psi, N_F\Psi)}{(\Psi,\Psi)} = \frac{(\Psi_0,\Psi_0)_B + 2(\Psi_{00},\Psi_{00})_B}{(\Psi_{\text{bosonic}},\Psi_{\text{bosonic}})_B + (\Psi_0,\Psi_0)_B + (\Psi_{00},\Psi_{00})_B}\,. \tag{6.2.8}$$

This takes a value between zero and two. The denominator in this expression serves to normalize the state. When its value is low, empty de Sitter space is preferred, for a high value, a state with bulk fermions present is preferred. For a massless free fermion in de Sitter space in the Hartle-Hawking state, a bulk computation by [121, 135] has shown that $< N_F >= 0$. In principle, this is a perfectly valid observable to consider. But, when we consider a minisuperspace that involves both bosonic and fermionic deformations, we run into a practical issue. It is in general unclear what the precise definition of the bosonic inner product $(\,,\,)_B$ should be, preventing us from actually evaluating $< N_F >$. To deal with this, we can instead compute the local value of $< N_F >$ at a given boundary value B of the bosonic bulk fields

$$< N_F > [B] = \frac{\Psi_0^*[B]\Psi_0[B] + 2\Psi_{00}^*[B]\Psi_{00}[B]}{\Psi_{\text{bosonic}}^*[B]\Psi_{\text{bosonic}}[B] + \Psi_0^*[B]\Psi_0[B] + \Psi_{00}^*[B]\Psi_{00}[B]}\,, \tag{6.2.9}$$

where for the wave functions that depend only on the bosonic fields, the conjugate wave function is simply the complex conjugate. Note that we have done something unusual in the denominator of this expression. The denominator serves to normalise our expression and normally we should normalise against the full value of the wave function (Ψ,Ψ). However, here we are normalising against the value of the wave function at the given bosonic deformation B. This means

that we effectively consider the wavefunction of a state where we have already fixed bosonic fields to a certain value. This ensures we have values between zero and two, which we will see leads to a clearer and more interesting analysis. We should keep in mind that we are no longer truly computing something which when we integrate it over B with the correct measure will give us $< N_F >$ for the holographic no-boundary state. Instead, we are saying: suppose we have already imposed that the bosonic bulk fields take value B on the future boundary. After making this imposition on our state, how do the fermionic spinor bulk fields respond to the bosonic bulk fields as an imposed background? Because of this, we need to normalise the wave function at given B. As we will see, this will lead to an interesting analysis in our model.

Another interesting observable to consider is $< \tilde{\lambda}\lambda >$. Clearly, its value is given by

$$\begin{aligned} < \tilde{\lambda}\lambda > =& (\Psi_{\text{bosonic}}, \Psi_0)_B + (\Psi_0, \Psi_{\text{bosonic}})_B \\ &+ (\Psi_0, \Psi_{00})_B + (\Psi_{00}, \Psi_0)_B \,. \end{aligned} \tag{6.2.10}$$

Again, it would be problematic to evaluate $(\ ,\ )_B$ in a minisuperspace where we also consider the bosonic bulk fields. Instead, we can again consider it at a given value of the bosonic bulk fields B,

$$\begin{aligned} < \tilde{\lambda}\lambda > [B] =& \Psi^*_{\text{bosonic}}[B]\Psi_0[B] + \Psi^*_0[B]\Psi_{\text{bosonic}}[B] \\ &+ \Psi^*_0[B]\Psi_{00}[B] + \Psi^*_{00}[B]\Psi_0[B] \,. \end{aligned} \tag{6.2.11}$$

Note that in this case we are not imposing normalisation against the wave function at B. This is because we want to keep the expression in a form where we could in principle integrate over B with the correct measure to obtain $< \tilde{\lambda}\lambda >$ for the holographic no-boundary state. We will see that in our model, despite not knowing the exact measure to use, we will still be able to draw some interesting conclusions for $< \tilde{\lambda}\lambda >$ of the no-boundary state from the results we obtain. As the wavefunction is not normalized, we can interpret how $< \tilde{\lambda}\lambda >$ changes with B, but the actual numerical value of $< \tilde{\lambda}\lambda >$ cannot be trusted as there might be a rescaling by a constant factor. In particular, wavefunctions computed at different N in the CFT might have different normalization rescalings, resulting in different rescalings of $< \tilde{\lambda}\lambda >$.

One can construct other observables related to fermionic bulk fields analogously to the ones we have discussed. It is clear from the form of eq. (6.2.5) that e.g. $< \lambda >= 0$.

6.3 Supersymmetric Vector Models and Duality

First, let us review the work done in [1], focussing especially on the specific results we need here. The simplest bulk de Sitter higher spin theory consists of a conformally massive scalar of mass $m^2 = 2/l^2$ and an infinite tower of massless fields of every even spin starting at two. The CFT dual to the simplest de Sitter higher spin theory is the free $Sp(N)$ vector model, consisting of anticommuting scalars [68]. In [1], an $\mathcal{N} = 2$ supersymmetric extension of the $Sp(N)$ model, which we called the $\mathcal{N} = 2$ $U(-N)$ model, was constructed and argued to be dual to the $\mathcal{N} = 2$ supersymmetric de Sitter higher spin theory constructed in [86]. The $\mathcal{N} = 2$ supersymmetry extends the spectrum of the bulk higher spin theory such that there are no longer just even spins present, instead fields of every possible spin arise. That is, there are a scalar and pseudoscalar, conformally massive $m^2 = 2/l^2$, followed by massless fields of every spin $s = 1/2, 1, 3/2, 2, 5/2, .., \infty$. This entire tower belong to a representation of the $ho(1, 1|4, 1)$ higher spin symmetry. Restricted to the sector of spin up to and including two, which we will be especially interested in, this yields an $OSp(2|2, 2)$ supersymmetry subgroup with eight real supercharges. The exact full bulk spectrum is in one-to-one correspondence with boundary single trace conserved supercurrents and can in this way be elegantly captured in the language of $3D$ $\mathcal{N} = 2$ superfields as described in [1].

The action of the undeformed supersymmetric $U(-N)$ model on a round S^3 boundary sphere of radius l is

$$S_0 = \int d^3x\sqrt{h}\left[\partial_\mu\tilde{\varphi}_i\partial^\mu\varphi^i + \frac{3}{4l^2}\tilde{\varphi}_i\varphi^i + \mathrm{i}\tilde{\chi}_i\nabla\!\!\!/\chi^i - \tilde{G}_iG^i\right]. \tag{6.3.1}$$

Here, all fields have had their spin-statistics flipped. That is, φ^i and G^i are anticommuting scalars, χ^i is a commuting spinor, analogously for the "tilde" fields and $i = 1, \ldots, N$. We follow the same notation as [1], convenient for Euclidean $3D$ $\mathcal{N} = 2$ supersymmetric theories. Importantly, this means that the "tilde" fields are the "would-be" conjugate fields. These are fields that are allowed to be taken independent from the fields without tilde while still preserving supersymmetry. However, were we working in Lorentzian signature instead, they would be required to be hermitian conjugates to the fields without tilde. For the dynamical fields appearing in eq. (6.3.1), we will indeed take the fields with and without tilde to be independent. On the background field which we will introduce in the following and that couple to the higher-spin currents constructed out of the dynamical fields in (6.3.1), on the other hand, we will impose additional reality conditions stemming from the reality conditions of their bulk dual fields. We will comment more on this point at the end of the section.

The action (6.3.1) is invariant under the supersymmetry transformations of the matter chiral multiplet

$$\delta\varphi^i = \sqrt{2}\epsilon\chi^i \,, \tag{6.3.2a}$$

$$\delta\chi^i = \sqrt{2}\epsilon G^i - \sqrt{2}\mathrm{i}\gamma^\mu\tilde{\epsilon}\partial_\mu\varphi^i - \frac{1}{\sqrt{2}l}\tilde{\epsilon}\varphi^i \,, \tag{6.3.2b}$$

$$\delta G^i = -\sqrt{2}\mathrm{i}\tilde{\epsilon}\gamma^\mu\nabla_\mu\chi^i \,, \tag{6.3.2c}$$

and similarly for the "tilde" fields, where $(\epsilon, \tilde{\epsilon})$ and ∂_μ are respectively Killing spinors and covariant derivatives on the three-sphere S^3, see e.g. [104] for more detail.[5]

In [1] we considered a class of scalar deformations of the previous action that arise from the supersymmetric coupling of the $U(-N)$ model to source spin-zero currents. In practice, these couplings, that amount to particular mass deformations once one restricts to constant deformations, arise as a subsector of the $U(-N)$ model coupled to a background Abelian supersymmetric vector multiplet. The main scope of this paper is to analyse deformations induced by fermionic sources that typically manifestly break supersymmetry and the isometries of S^3. The simplest of these deformations arise from the coupling of the spin-1/2 current of the supersymmetric $U(-N)$ model to a background gaugino. Let us show explicitly how this term arises.

First of all, we remind the reader that an Abelian vector multiplet on the sphere is described by the following fields: the gauge connection A_μ with field strength $F_{\mu\nu} = \partial_{[\mu}A_{\nu]}$; two gaugini fermionic fields λ and $\tilde{\lambda}$; a scalar and pseudoscalar fields ς and D. Their supersymmetry transformations on the sphere are given

[5] As in [1] we use the notation and conventions of [104] up to name redefinition of the fields of the matter sector and of the Killing spinors. Note also that, compared to the result of [104], there are some different signs due to the reversed statistic of the fields in the chiral multiplet of the $U(-N)$ model.

by

$$\begin{aligned}
\delta A_\mu &= -\mathrm{i}(\epsilon\gamma_\mu\tilde{\lambda} + \tilde{\epsilon}\gamma_\mu\lambda) \ , && (6.3.3a)\\
\delta\varsigma &= -\epsilon\tilde{\lambda} + \tilde{\epsilon}\lambda \ , && (6.3.3b)\\
\delta\lambda &= \Big(\mathrm{i}D - \frac{\mathrm{i}}{2}\epsilon^{\mu\nu\rho}\gamma_\rho F_{\mu\nu} - \mathrm{i}\gamma^\mu\partial_\mu\varsigma - \frac{1}{l}\varsigma\Big)\epsilon \ , && (6.3.3c)\\
\delta\tilde{\lambda} &= \Big(-\mathrm{i}D - \frac{\mathrm{i}}{2}\epsilon^{\mu\nu\rho}\gamma_\rho F_{\mu\nu} + \mathrm{i}\gamma^\mu\partial_\mu\varsigma + \frac{1}{l}\varsigma\Big)\tilde{\epsilon} \ , && (6.3.3d)\\
\delta D &= \epsilon\gamma^\mu\partial_\mu\tilde{\lambda} - \tilde{\epsilon}\gamma^\mu\partial_\mu\lambda + \frac{1}{3}(\nabla_\mu\epsilon\gamma^\mu\tilde{\lambda} - \nabla_\mu\tilde{\epsilon}\gamma^\mu\lambda) \ . && (6.3.3e)
\end{aligned}$$

Before continuing it is important to underline that these fields have the standard spin-statistics.

The coupling of the $U(-N)$ model to a background gauge supermultiplet amounts to adding to the action (6.3.1) the following interaction terms

$$S_{gauge-int} = \int d^3x\sqrt{h}\Big[\tilde{\varphi}_i(\varsigma^2 + D)\varphi^i - \mathrm{i}\varsigma\tilde{\chi}_i\chi^i + \sqrt{2}\mathrm{i}(\tilde{\varphi}_i\lambda\chi^i - \tilde{\chi}_i\tilde{\lambda}\varphi^i)\Big] \quad (6.3.4)$$

The action $S_0 + S_{gauge-int}$ is invariant under supersymmetry transformations provided that the transformation for the matter multiplet is modified to

$$\begin{aligned}
\delta\varphi^i &= \sqrt{2}\epsilon\chi^i \ , && (6.3.5a)\\
\delta\chi^i &= \sqrt{2}\epsilon G^i - \sqrt{2}\mathrm{i}\gamma^\mu\tilde{\epsilon}D_\mu\varphi^i + \sqrt{2}\mathrm{i}\varsigma\tilde{\epsilon}\varphi^i - \frac{1}{\sqrt{2}l}\tilde{\epsilon}\varphi^i \ , && (6.3.5b)\\
\delta G^i &= -\sqrt{2}\mathrm{i}\tilde{\epsilon}\gamma^\mu D_\mu\chi^i - \sqrt{2}\mathrm{i}\varsigma\tilde{\epsilon}\chi^i + 2\mathrm{i}\tilde{\epsilon}\tilde{\lambda}\varphi^i \ , && (6.3.5c)
\end{aligned}$$

where the D_μ derivatives are gauge covariant.

In [1] we noticed that an interesting deformation that amount to giving a supersymmetric mass to the chiral multiplets is based on choosing a constant BPS configuration for the scalar fields ς and D in the vector multiplet where $D = -\mathrm{i}\varsigma/l = const.$ Substituting this condition in (6.3.4) yields the following mass deformation,

$$\mathcal{L}_{mass} = \Big(\varsigma^2 - \mathrm{i}\frac{\varsigma}{l}\Big)\,\tilde{\varphi}_i\varphi^i - \mathrm{i}\varsigma\tilde{\chi}_i\chi^i \ , \quad (6.3.6)$$

where l is the radius of the sphere. Further, we also studied a BPS combination of a spin-1 and spin-2 deformation in our earlier work. Here, results from

supersymmetric localisation allowed us to compute the partition function exactly. Since we constructed a supersymmetric dS/CFT correspondence in our earlier work, it seemed natural to study BPS configurations there; both to show the simplifying power of supersymmetry to our wave function computations and because we believed that if supersymmetry would introduce an instability in our theory, supersymmetric deformations would be the most probable culprit. We found in [1] that our vacuum is stable under BPS deformations.

If we aim at turning on a spin-1/2 source on the boundary we should turn on the gauginos in the last two terms of (6.3.4). This will be the main class of deformation we will study in this paper. Note that such a deformation will generically break all supersymmetries. We will compute the partition function and interpret it as the Hartle-Hawking wave function both for the spin-1/2 source by itself and for the combination of the spin-1/2 with a number of integer spin sources. Since the spin-1/2 source breaks supersymmetry, there is no longer a reason to consider the integer spin sources in would-be BPS configurations. It is also no longer possible to use results from localisation techniques to e.g. compute the spin-1 + spin-2 'BPS' deformation partition function exactly.

We want to make an important remark about what we (do not) mean by a source breaking supersymmetry. We mean that the CFT action with that source turned on is no longer supersymmetric. We do not mean that the bulk Hartle-Hawking state loses its supersymmetry. In fact, one should expect the full wave function, assuming one hypothetically fully computes it over all possible bulk field configuration, to be a supersymmetric ground state of the supersymmetric bulk theory. When one evaluates the inner product of the bulk wave function, one integrates over all bulk field configurations. From the CFT perspective, one then integrates over all configurations of the sources. These sources are described by background fields, which enter as supermultiplets. The integration over all source configurations then becomes a path integral over the 'background fields', which have now become dynamical[6] by this procedure. The background fields for all spins enter in supermultiplets, which can be described in superspace using techniques of [110, 113] as has been done in [1], making it manifest that supersymmetry is preserved. So, the full bulk wave function is manifestly supersymmetric. In practice, it is only possible to compute the wave function for a subset of all field configurations. The wave function evaluated over this subset might fail to be supersymmetric but this is merely an unfortunate side effect of restricting to the calculations one can in practice do. When one goes to the full configuration space, supersymmetry is always restored. This is entirely analogous to how a particular configuration that enters into the

[6] In the sense that they enter into the path integral, there are no kinetic terms for these fields in the action.

Hartle-Hawking wave function by itself does not preserve diffeomorphisms, but the wave function as a whole has a diffeomorphism-invariant inner product.

We still need to address the scaling of our sources with N. We can redefine our sources such that different factors of N appear in front of them. This results in the corresponding bulk quantities having different scaling with N in their n-point functions. When one discusses the bulk wave function for bosonic fields, this only results in a different overall factor of N in the wave function and the issue need not be explicitly addressed. We shall see that with half-integer bulk fields, there is a qualitatively different behaviour for different values of N and we need to make a physically sensible choice of scaling with N for our sources. We demand that the bulk two-point functions produced by our rescaled sources have unit strength with regard to (do not scale with) N. In higher spin theories, this is achieved by scaling them such that there is a factor $1/\sqrt{(N)}$ in the boundary action in front of the source terms [58, 59, 64, 100]. This is the scaling we will choose in the rest of the paper.

Loosely speaking, half-integer spin fields enter into higher spin theories 'as superpartners' of the integer spin fields. While it is permissible to have more integer spin fields than half-integer spin fields, the reverse is not permitted [86]. From the CFT perspective, this is intuitively clear by combinatorics. Conserved currents whose terms have either two boundary scalars or two boundary spinors yield integer spin fields in the bulk; whereas conserved currents whose terms have one boundary scalar and one boundary spinor produce half-integer spin bulk fields. Then, when there are as many boundary scalars as spinors, there are as many half-integer conserved currents as integer conserved currents, while in all other cases there are less half-integer conserved currents. So, our investigation here should be suggestive for spinors in generic, non necessarily supersymmetric, higher spin theories. If issues arise in our restricted case, one expects half-integer spin fields in de Sitter higher spin theories to be generically pathological.

In the following sections, we will consider turning on sources that can be written in new variables as

$$S = \int d^3x \sqrt{h_{\rm def}} \left[\begin{array}{c|c} \tilde{\chi} & \tilde{\varphi} \end{array} \right] \left[\begin{array}{c|c} \mathrm{i}\nabla\!\!\!/ + \frac{m}{\sqrt{N}} & -\sqrt{\frac{2}{N}}\mathrm{i}\tilde{\lambda} \\ \hline \sqrt{\frac{2}{N}}\mathrm{i}\lambda & -\partial^2 + \frac{R}{8} + \frac{\sigma}{\sqrt{N}} \end{array} \right] \left[\begin{array}{c} \chi \\ \hline \varphi \end{array} \right], \quad (6.3.7)$$

where m and σ must be real, as discussed in [1]. Here, σ sources a scalar bulk field, m a pseudoscalar bulk field, and we will also consider a squashing $h_{\rm def}$ of the boundary metric. Imposing that m and σ must be real imposes in terms of the variables used in (6.3.4) that D must be real and ς must be imaginary. This in turn imposes reality conditions on the other background fields through the supersymmetry variations eq. (6.3.3). In particular, one can see that $\tilde{\lambda}\lambda$ must be real. When we compute $< \tilde{\lambda}\lambda >$ later, we shall see that it is indeed

real, consistent with the reality conditions coming from supersymmetry. This is nontrivial as eq. (6.2.11) is not by definition real for an arbitrary theory.

6.4 Spin-1/2 deformation

As discussed in section 6.3, the spin-1/2 current deformation is given by adding

$$\sqrt{\frac{2}{N}}\mathrm{i}(\tilde{\varphi}_i\lambda\chi^i - \tilde{\chi}_i\tilde{\lambda}\varphi^i) \tag{6.4.1}$$

to the Lagrangian. Constant spinors[7] exist on S^3 and in what follows, we will take λ and $\tilde{\lambda}$ to be constant as discussed in section 6.2.

For the spin-1/2 deformation, the path-integral of the partition function is Gaussian, so an exact calculation of the partition function is in principle possible if we know the spectrum of the operators whose determinants we need. The action can be schematically written in the form

$$S = \int d^3x\sqrt{h}\,\left[\ \tilde{\chi}\ \middle|\ \tilde{\varphi}\ \right]\left[\begin{array}{c|c} \mathrm{i}\not{\nabla} & -\sqrt{\frac{2}{N}}\mathrm{i}\tilde{\lambda} \\ \hline \sqrt{\frac{2}{N}}\mathrm{i}\lambda & -\partial^2 + 3/4 \end{array}\right]\left[\begin{array}{c} \chi \\ \hline \varphi \end{array}\right], \tag{6.4.2}$$

where one has supervectors and a supermatrix. Note that we have set $l = 1$ in our action, which we will continue to do throughout the rest of the paper. Performing the Gaussian integral yields the partition function

$$Z \propto \left(Sdet\left[\begin{array}{c|c} \mathrm{i}\not{\nabla} & -\sqrt{\frac{2}{N}}\mathrm{i}\tilde{\lambda} \\ \hline \sqrt{\frac{2}{N}}\mathrm{i}\lambda & -\partial^2 + 3/4 \end{array}\right]\right)^{-N}. \tag{6.4.3}$$

A convenient property of superdeterminants is that

$$Sdet\left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] = det(A - BD^{-1}C)det(D)^{-1}. \tag{6.4.4}$$

Using this, the $det(-\partial^2 + 3/4)$ provides an uninteresting overall factor and we are left with computing

$$Z \propto \left[det\Big(\mathrm{i}\not{\nabla} - \frac{\frac{2}{N}\tilde{\lambda}\lambda}{-\partial^2 + 3/4}\Big)\right]^{-N}. \tag{6.4.5}$$

[7] Constant in the basis naturally constructed by recalling that S^3 is the $SU(2)$ group manifold, as e.g. in [104].

Let us calculate the eigenvalues of

$$\mathrm{i}\not{\nabla} - \frac{\frac{2}{N}\tilde{\lambda}\lambda}{-\partial^2 + 3/4}\,. \tag{6.4.6}$$

We are looking for eigenspinors χ_n with eigenvalues l_n such that

$$\left(\mathrm{i}\not{\nabla} - \frac{\frac{2}{N}\tilde{\lambda}\lambda}{-\partial^2 + 3/4}\right)\chi_n = l_n\chi_n\,. \tag{6.4.7}$$

We could write χ_n in a basis of eigenspinors of the Dirac operator. Instead, let us assume that χ_n is simply an eigenspinor of the Dirac operator with eigenvalue Λ_n and we will see that this is the correct solution. Note that eigenspinors of the Dirac operator are also eigenspinors of $-\partial^2 + 3/4$ with eigenvalue $(\Lambda_n)^2$. One then has

$$(\Lambda_n)^3\chi_n^\alpha - \frac{2}{N}\tilde{\lambda}^\alpha\lambda_\beta\chi_n^\beta = l_n(\Lambda_n)^2\chi_n^\alpha\,. \tag{6.4.8}$$

Acting with $\tilde{\lambda}^\gamma\lambda_\alpha$ and then dividing out $\lambda_\beta\chi_n^\beta$ yields

$$(\Lambda_n)^3\tilde{\lambda}^\gamma - \frac{2}{N}\tilde{\lambda}^\gamma\lambda_\alpha\tilde{\lambda}^\alpha = l_n(\Lambda_n)^2\tilde{\lambda}^\gamma\,, \tag{6.4.9}$$

which is solved by

$$l_n = \frac{(\Lambda_n)^3 - \frac{2}{N}\lambda_\alpha\tilde{\lambda}^\alpha}{(\Lambda_n)^2} = \frac{(\Lambda_n)^3 - \frac{2}{N}\tilde{\lambda}\lambda}{(\Lambda_n)^2}\,. \tag{6.4.10}$$

Sensibly, this gives Λ_n when the deformation is turned off.

One can compute an explicit expression for Z by first treating $\tilde{\lambda}\lambda$ as a scalar and computing the free energy as a sum using zeta function regularization and then exponentiating. This yields an impressive-looking expression which when taking into account the supernumber nature of $\tilde{\lambda}\lambda$ reduces to

$$Z = -\frac{\left(\pi^4\left(\pi^2-10\right)\left(\tilde{\lambda}\lambda\right)^2 - 15N\right)e^{\frac{9N\zeta(3)}{8\pi^2}}}{15N}. \tag{6.4.11}$$

Using dS/CFT, this is interpreted as the bulk wave function.

The value of N_F, which is calculated from (6.2.9), is given by

$$N_F = \frac{8\pi^8\left(\pi^2-10\right)^2}{225N^2 + 4\pi^8\left(\pi^2-10\right)^2}\,. \tag{6.4.12}$$

We see that for very low values of N, fermionic states are highly occupied in the bulk wave function. With increased N, N_F quickly goes to zero. Since,

$N = (G_N\Lambda)^{-1}$, with G_N Newton's constant and Λ the cosmological constant in the bulk, and we are at constant Λ, the large N limit corresponds to the free limit. Thus, we find that for a free massless spinor field in de Sitter space, there is no pair production in the Hartle-Hawking ground state. The same result was obtained from a bulk computation of the Hartle-Hawking wave function for a free massless fermion in [121,135]. This result provides a nontrivial confirmation of the agreement between bulk and boundary computations of the Hartle-Hawking state. Further, $< \lambda\tilde{\lambda} >= 0$ as there is no first order term in $\lambda\tilde{\lambda}$ in the wave function.

6.5 Interplay between bulk gauginos and bosons

So far, we have looked at the bulk spin-1/2 field as the only source turned on and we have seen that it behaves sensibly. Now we will look at the impact that the spin-1/2 field has on a few bosonic bulk deformations. Exact expressions for the eigenvalues of the Laplace and Dirac operators can still be obtained in this case. The complexity of the expressions for these grows and in some cases we were unable to analytically compute the partition function and relied on numerics instead. A subtlety here is that since we need to isolate each order in $\tilde{\lambda}\lambda$ of our result, we cannot rely on numerics so long as there still is $\tilde{\lambda}\lambda$ dependence present in our expressions.

Consider first the case $N = 1$. Generically, we will have eigenvalues $\alpha_n(\tilde{\lambda}\lambda)$ and $Z = \prod_n \alpha_n[\tilde{\lambda}\lambda]$. First, we rewrite the eigenvalues as $\alpha_n[\tilde{\lambda}\lambda] = \beta_n[\tilde{\lambda}\lambda]\gamma_n$, where $\beta_n[0] = 1$. We obtain $Z = \prod_n \beta_n[\tilde{\lambda}\lambda] \prod_m \gamma_m$. We define $\beta[\tilde{\lambda}\lambda] \equiv \prod_n \beta_n[\tilde{\lambda}\lambda]$ and $\gamma \equiv \prod_m \gamma_m$ such that $Z = \beta[\tilde{\lambda}\lambda]\gamma$. We now expand β_n in powers of $\tilde{\lambda}\lambda$ as $\beta_n = 1 + a_n\tilde{\lambda}\lambda + b_n(\tilde{\lambda}\lambda)^2$. Now note that

$$\beta = \prod_n \beta_n = 1 + \left(\sum_n a_n\right)\tilde{\lambda}\lambda + \sum_n \left(b_n + a_n \sum_m a_m\right)(\tilde{\lambda}\lambda)^2. \qquad (6.5.1)$$

Within the approximations made, the properties of Grassmann numbers have turned an infinite product into an infinite sum, which is easier to evaluate. These sums and the $\prod_n \gamma_n$ can now be evaluated numerically for a given configuration of the bosonic bulk fields. In what follows we will do this concretely for a few examples

Now, let us use the previous result to obtain a result for arbitrary N in such a way that we will clearly see the N dependence. Firsly, we should remember to divide all the sources by factors of $\sqrt{N}$ as discussed in section 6.3. Crucially, this means that all factors of $\tilde{\lambda}\lambda$ will be divided by N. Suppose we followed the $N = 1$ analysis to compute the partition function Z[sources] for the $U(-1)$ model. The

partition function of the $U(-N)$ model is then given by $Z[\text{sources}/\sqrt{N}]^N$. The overall factor $\prod_m \gamma_m$ in front of every order of $\tilde{\lambda}\lambda$ simply becomes $(\prod_m \gamma_m)^N$. However, when considering the β_n sector, we should keep the properties of the Grassmann variables in mind. We see that we get[8]

$$\begin{aligned}(\prod_n \beta_n)^N =& 1 + \left(\sum_n a_n\right)\tilde{\lambda}\lambda + \left(\frac{1}{N}\sum_n \left(b_n + a_n \sum_m^{n-1} a_m\right)\right. \\ &\left. + \frac{(N^2-N)}{N^2}\frac{\left(\sum a_n\right)^2}{2}\right)(\tilde{\lambda}\lambda)^2.\end{aligned} \tag{6.5.2}$$

We see that there is an interesting transition in the behaviour of the $(\tilde{\lambda}\lambda)^2$ term between small and large N. Note that if $\sum_n a_n = 0$, the gaugino contribution to the wave function is supressed at large N. This is precisely what happened when we considered turning on only the gaugino deformation without any other deformations.

6.5.1 Bulk pseudoscalar

The simplest deformation to combine with turning on the gauginos consists of giving a mass to the χ spinors. This corresponds to turning on the pseudoscalar field in the bulk. The action for $N = 1$ is then of the form

$$S = \int d^3x\sqrt{h}\,\left[\ \tilde{\chi}\ \middle|\ \tilde{\varphi}\ \right]\left[\begin{array}{c|c} \mathrm{i}\not{\nabla} + m & -\sqrt{2}\mathrm{i}\tilde{\lambda} \\ \hline \sqrt{2}\mathrm{i}\lambda & -\partial^2 + 3/4 \end{array}\right]\left[\begin{array}{c} \chi \\ \hline \varphi \end{array}\right]. \tag{6.5.3}$$

Proceeding in the same way as the computation with only the gauginos, we see that the partition function is of the form

$$Z \propto \left[det\Big(\mathrm{i}\not{\nabla} + m - \frac{\lambda\tilde{\lambda}}{\partial^2 + 3/4}\Big)\right]^{-1}, \tag{6.5.4}$$

which means we need to know the eigenvalues of

$$\mathrm{i}\not{\nabla} + m - \frac{\tilde{\lambda}\lambda}{\partial^2 + 3/4}\,. \tag{6.5.5}$$

By the same procedure as used for just the gauginos in section 6.4, we find for these eigenvalues

$$l_n = \frac{(\Lambda_n)^3 + m(\Lambda_n)^2 - 2\tilde{\lambda}\lambda}{(\Lambda_n)^2}\,. \tag{6.5.6}$$

[8] Although we have supressed this dependence, remember that a_n, b_n and γ_n still depend on the bosonic sources, as well as that this dependence will pick up factors of $1/\sqrt{N}$.

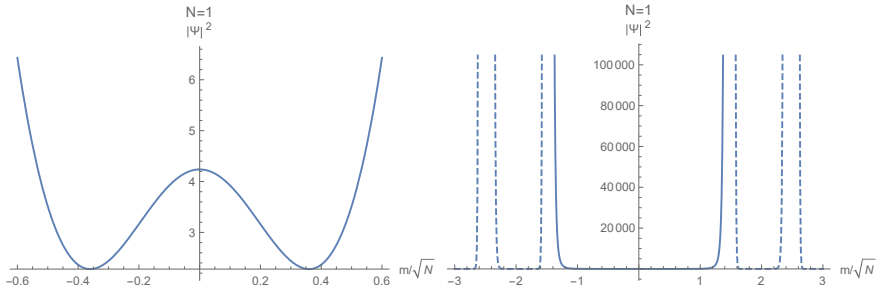


Figure 6.1: For a deformation corresponding to turning on a bulk pseudoscalar field, $|\Psi^2(m)|$ over a short and a long range at $N = 1$. The point $m = 0$ has been excluded from the short range plot due to problems with the numerical evaluation of very small numbers.

Then, Z is given by

$$Z \propto \prod_n a_n^{-1} = \beta\,\gamma\ , \tag{6.5.7}$$

$$\beta := \prod_n \frac{(\Lambda_n)^3 + m(\Lambda_n)^2}{(\Lambda_n)^3 + m(\Lambda_n)^2 - 2\tilde{\lambda}\lambda}\ , \tag{6.5.8}$$

$$\gamma := \prod_n \frac{(\Lambda_n)^2}{(\Lambda_n)^3 + m\,(\Lambda_n)^2}\ . \tag{6.5.9}$$

We evaluated β numerically and obtained the following analytic expression for γ

$$\begin{aligned}\gamma \propto \sqrt[4]{\cos(\pi m)}\exp\Bigg(&\frac{\mathrm{i}m}{\pi}\mathrm{Li}_2\left(-e^{2\mathrm{i}\pi m}\right) - \frac{1}{2\pi^2}\mathrm{Li}_3\left(-e^{2\mathrm{i}\pi m}\right)\\ &+ \frac{1}{3}\mathrm{i}\pi m^3 - m^2\log\left(1+e^{2\mathrm{i}\pi m}\right)\Bigg)\,.\end{aligned} \tag{6.5.10}$$

The behaviour of $|\Psi|^2$ can be seen in figs. 6.1 and 6.2. We see that undeformed de Sitter space is a local maximum of $|\Psi|^2$, indicating the perturbative stability of the supersymmetric de Sitter vacuum. For very large deformations, we see divergences in $|\Psi|^2$. These divergences do not come as a surprise. They are not a result of the inclusion of the gauginos, they are already present when one considers the bulk pseudoscalar field by itself, as we already discussed in [1]. In

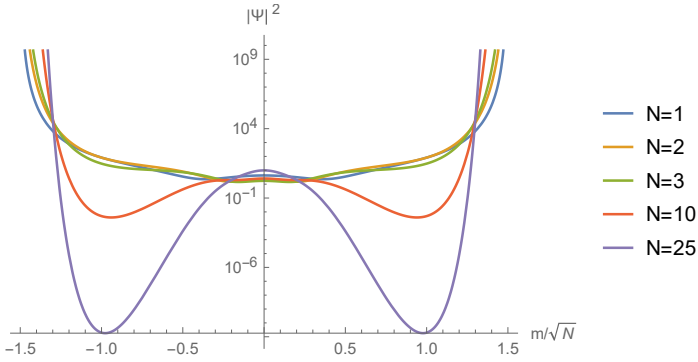


Figure 6.2: For a deformation corresponding to turning on a bulk pseudoscalar field, $|\Psi^2(m)|$.

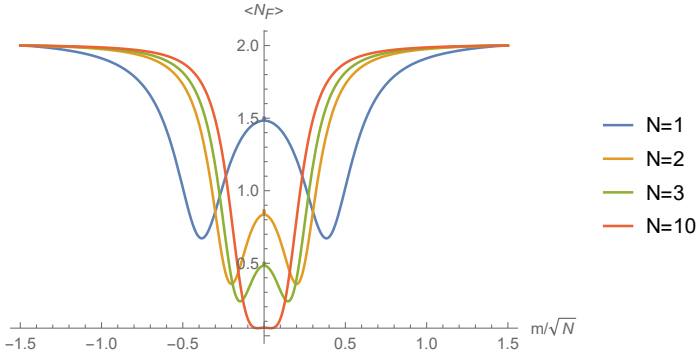


Figure 6.3: For a deformation corresponding to turning on a bulk pseudoscalar field, $N_F(m)$.

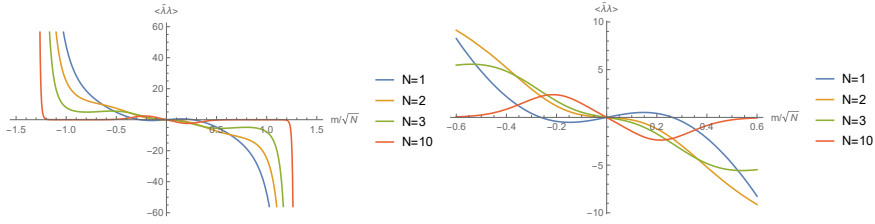


Figure 6.4: For a deformation corresponding to turning on a bulk pseudoscalar field, $< \tilde{\lambda}\lambda > [m]$.

fact, we should restrict the outermost edge of the configuration space at which our results can be hoped to be trusted to $|m/\sqrt{N}| < 3/2$ and we restrict our analysis of observables to this range. We defer the discussion of the reason for this restriction to section 6.5.4

In fig. 6.3 the value of $< N_F > [m]$ can be seen. We shall interpret its behaviour and compare it to earlier bulk results in section 6.5.4.

In fig. 6.4, $< \tilde{\lambda}\lambda > [m]$ can be seen. We see that it increases as we increase $|m|$. We also see that for $m/\sqrt{N}$ between $-3/2$ and $3/2$, it is antisymmetric, while $|\Psi|^2$ is symmetric in m. While we do not know the exact measure against which we should integrate the bosonic fields in the wave function, it seems reasonable that it should respect such symmetries. If this is the case, $< \tilde{\lambda}\lambda >= 0$ for the holographic no-boundary wave function after integration over m.

6.5.2 Bulk scalar

For a deformation based on turning on a bulk scalar, the action at $N = 1$ takes the form

$$S = \int d^3x\sqrt{h}\,\left[\ \tilde{\chi}\ \middle|\ \tilde{\varphi}\ \right]\left[\begin{array}{c|c} \mathrm{i}\not{\nabla} & -\sqrt{2}\mathrm{i}\tilde{\lambda} \\ \hline \sqrt{2}\mathrm{i}\lambda & -\partial^2 + 3/4 + \sigma \end{array}\right]\left[\begin{array}{c} \chi \\ \hline \varphi \end{array}\right], \tag{6.5.11}$$

To compute the partition function

$$Z \propto \prod_n a_n^{-1} = \beta\,\gamma\ , \tag{6.5.12}$$

$$\beta := \prod_n \frac{(\Lambda_n)^3 + \sigma\Lambda_n}{(\Lambda_n)^3 + \sigma\Lambda_n - 2\tilde{\lambda}\lambda}\ , \tag{6.5.13}$$

$$\gamma := \det(-\partial^2 + 3/4 + \sigma)\prod_n \frac{1}{(\Lambda_n)}\ , \tag{6.5.14}$$

we proceed in the same way as the previous cases. Note that in this case, the $det(D)^{-1}$ term in eq. (6.4.4) is not constant, but depends on σ, so we must take it into account. We find for $N = 1$

$$\begin{aligned}\beta =& 1 - \frac{\pi\mathrm{sech}^2\left(\pi\sqrt{\sigma}\right)}{32\sigma^{5/2}}\Big(8\pi\sigma^{3/2} + 4\pi\sqrt{\sigma} - (4\sigma+3)\sinh\left(2\pi\sqrt{\sigma}\right) \\ &+ 2\pi\sqrt{\sigma}\cosh\left(2\pi\sqrt{\sigma}\right)\Big)(2\tilde{\lambda}\lambda)^2\,,\end{aligned} \tag{6.5.15}$$

and

$$\gamma \propto \exp\Bigg\{ -\frac{1}{24\pi^2}\Bigg[\pi^2\left(\mathrm{i}\pi\left(4\sigma\sqrt{1-4\sigma}-\sqrt{1-4\sigma}+1\right)+6(4\sigma-1)\log\left(1-e^{-\mathrm{i}\pi\sqrt{1-4\sigma}}\right)\right)$$
$$-12\mathrm{i}\pi\sqrt{1-4\sigma}\mathrm{Li}_2\left(e^{-\mathrm{i}\sqrt{1-4\sigma}\pi}\right)-12\mathrm{Li}_3\left(e^{-\mathrm{i}\sqrt{1-4\sigma}\pi}\right)\Bigg]\Bigg\}. \tag{6.5.16}$$

We again expand β via the methods described previously. In this case, it was possible to find β analytically. It is worth noting that $\sum_n a_n = 0$. Looking at eq. (6.5.2), this ensures that in the large N limit, the parts of the wave function with bulk fermions excited are supressed and the local fermion excitation number tends to zero. Of course, any actual divergences introduced by the gauginos remain at any finite N.

Without the gauginos but with only the scalar mass deformation, our deformation would be essentially equivalent to the scalar deformation considered at the start of [72]. As such, it makes sense to compare this wave function to the one discussed there. For $N = 1$, $|\Psi|^2$ can be seen in fig. 6.5. We see that for small deformation, there is a local maximum for $|\Psi|^2$ at undeformed de Sitter space. For positive deformations, this maximum is global. For negative deformations, we see large fluctuations, as were also found in [72]. Here the problem seems to be exacerbated. Rather than large finite fluctuations, $|\Psi|^2$ now only diverges for sufficiently large negative deformations, with the first divergency at $\sigma/\sqrt{N} = -9/4$. As we shall discuss in section 6.5.4, we should restrict the configuration space to $\sigma/\sqrt{N} > -3/4$.

The fermion number can be seen in fig. 6.6, we postpone its analysis to section 6.5.4. We see that $< \tilde{\lambda}\lambda >= 0$ as there is no first order term in $\tilde{\lambda}\lambda$ in the wave function.

6.5.3 Squashing

Finally, we combine the spin-1/2 deformation with a squashing of the three-sphere. The action takes the form

$$S = \int d^3x\sqrt{h_{\mathrm{def}}}\begin{bmatrix}\tilde{\chi} & \tilde{\varphi}\end{bmatrix}\left[\begin{array}{c|c} \mathrm{i}\not{\nabla} & -\sqrt{\frac{2}{N}}\mathrm{i}\tilde{\lambda} \\ \hline \sqrt{\frac{2}{N}}\mathrm{i}\lambda & -\partial^2+\frac{R}{8}\end{array}\right]\begin{bmatrix}\chi \\ \varphi\end{bmatrix}. \tag{6.5.17}$$

Turning on the squashing changes the eigenvalues of the Dirac operator and Laplacian. We consider a squashing preserving $SU(2)\times U(1)$ symmetry described

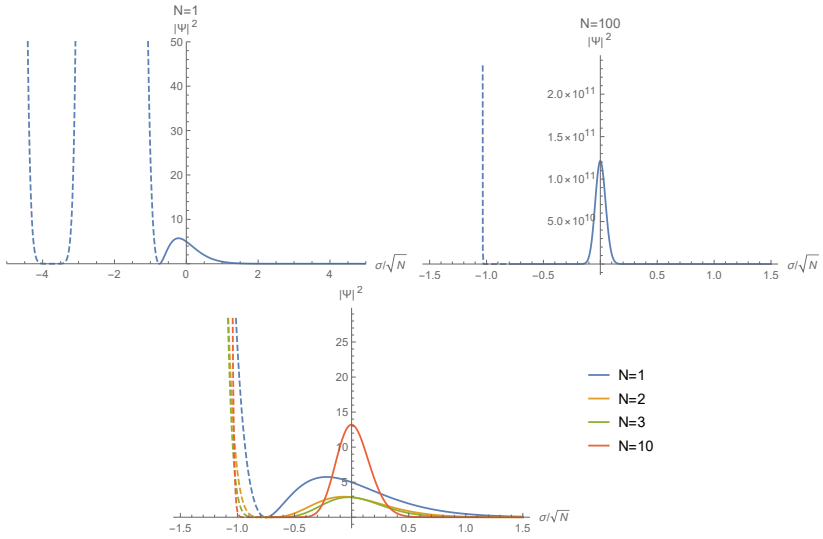


Figure 6.5: For a deformation corresponding to turning on a bulk scalar field, $|\Psi^2(\sigma)|$.

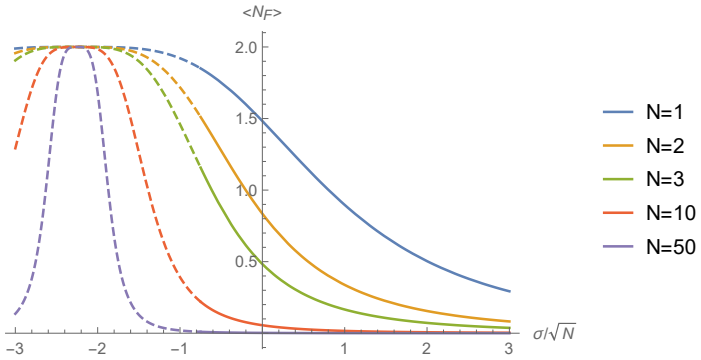


Figure 6.6: For a deformation corresponding to turning on a bulk scalar field, $N_F(\sigma)$.

by the metric

$$ds^2_{S^3_\alpha} = d\theta^2 + \sin^2\theta d\phi^2 + \frac{1}{1+\alpha}(d\tilde{\psi} + \cos\theta d\phi)^2\,, \tag{6.5.18}$$

where $\theta \in [0,\pi]$, $\phi \in [0,2\pi]$, $\tilde{\psi} \in [0,4\pi]$, and the real deformation parameter α is such that $\alpha > -1$.

The eigenvalues of the Dirac operator are of the form [80, 139–141]

$$\Lambda_{n,q,\pm} = \frac{1}{2\sqrt{1+\alpha}} \pm 2\sqrt{\frac{n^2(1+\alpha)}{4} - \alpha q(n-q)}\,, \tag{6.5.19}$$

with degeneracy n. Here the eigenvalues run over two branches. For the positive branch, $n = 1,..,\infty$ and $q = 0,..,n$ while for the negative branch, $n = 2,..,\infty$ and $q = 1,..,n-1$. To compute the partition function, the same logic as in the previous sections applies, except that we use the eigenvalues of the Dirac operator on the squashed three-sphere. We find for $N = 1$

$$\beta = \prod_{n,q} \frac{\Lambda^3_{n,q}}{\Lambda^3_{n,q} - 2\tilde{\lambda}\lambda}\,. \tag{6.5.20}$$

The gaugino independent part of the partition function, γ, is the partition function for a free spinor and complex scalar on a squashed three sphere. This was computed numerically in [80] where a good agreement was found between the numerical result and the following analytic expression which we will use for γ,

$$\gamma = -(-1)^{23/24}\sqrt[4]{2}e^{\frac{3\zeta(3)}{4\pi^2} - \frac{\pi^2\alpha^2}{16(\alpha+1)^2}}\,. \tag{6.5.21}$$

We compute β numerically, analogously to the previous sections, in this way obtain $Z = \beta\gamma$ and identify it with the wave function under dS/CFT.

The behaviour of $|\Psi|^2$ under squashing can be seen in fig. 6.7. We find an interesting dependence on N. At small N, $|\Psi|^2$ is peaked for a squashed three-sphere. As N increases, we see that the peak of $|\Psi|^2$ quickly moves back to te undeformed three-sphere. At large[9] N, $|\Psi|^2$ has a global maximum at undeformed de Sitter space, indicating the stability of the de Sitter vacuum. At small N, the stable vacuum around which one should consider perturbation theory seems to be squashed away from the three-sphere. For reasons discussed in section 6.5.4, we should restrict the configuration space to $\alpha > -3/4$

The fermion occupation number, whose properties we will discuss further in section 6.5.4, can be seen in fig. 6.8. In fig. 6.9, $< \tilde{\lambda}\lambda > [B]$ can be seen. We see that it is negative and reaches an extremum at finite squashing.

[9] We see that at, for instance, $N = 10$, large N effects are totally dominant. The traditional wisdom that "three is a large number" when it comes to large N expansion seems to apply here too.

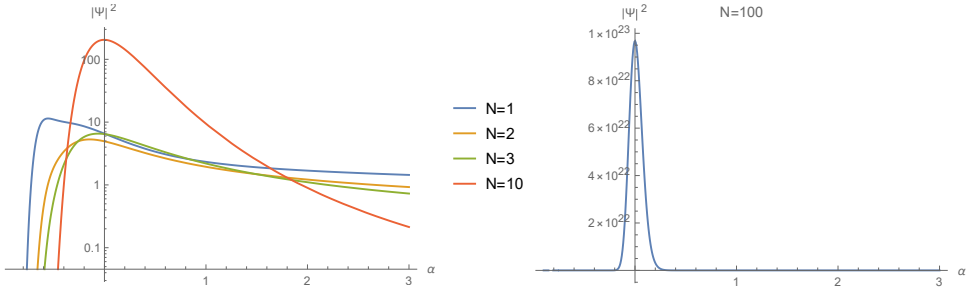


Figure 6.7: For a squashing, $|\Psi(\alpha)|^2$.

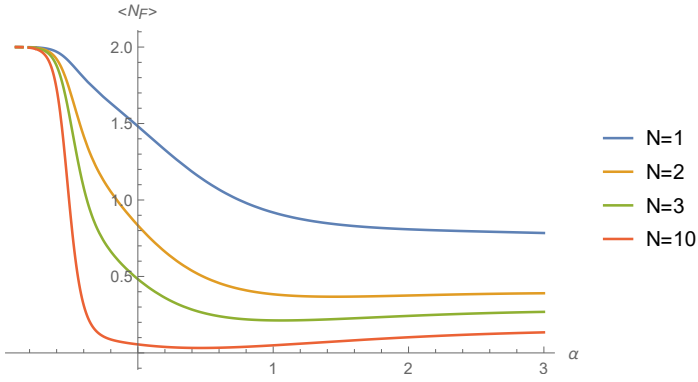


Figure 6.8: For a squashing, $N_F(\alpha)$.

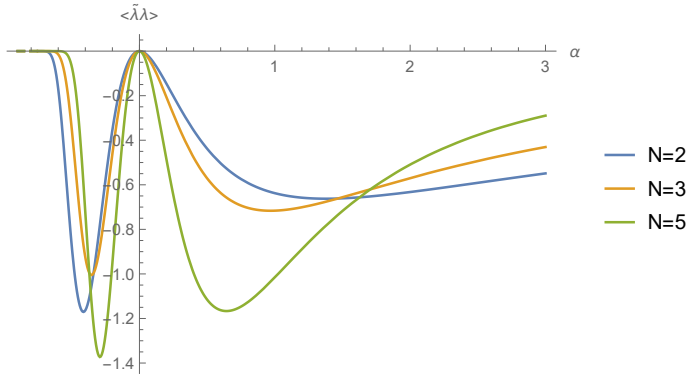


Figure 6.9: For a squashing, $< \tilde{\lambda}\lambda >$.

6.5.4 Interpretation and Comparison

We have computed the Hartle - Hawking wave function for an asymptotically nearly de Sitter universe in a 'minisuperspace' model consisting of the zero mode of a spin-1/2 field, a pseudoscalar, a scalar and a metric deformation as excitations around dS space. A summary of the deformations studied in this paper and the earlier literature can be seen in table 6.1. The wave function predicts probabilities for different asymptotic configurations (or more precisely, different spacetime histories). The figs. 6.1, 6.2, 6.5 and 6.7 show that the probability distributions exhibit a local maximum for reasonably small deformations in all directions of the minisuperspace. At small N, this maximum can be shifted somewhat away from the undeformed dS configuration. At large N, however, it always lies perfectly at the empty, undeformed de Sitter space. In this semiclassical limit the wave function becomes increasingly sharply concentrated around this configuration. The presence of a local maximum at empty dS also indicates that dS is perturbatively stable. The behaviour of the wave function for small fluctuations around dS describes correlators of fields in dS, and a maximum ensures these behave correctly.

For integer-spin sources, the fact that the probabilities are peaked at empty undeformed de Sitter space follows basically from an analytic continuation of the F-theorem [142–145]. This states that for unitary QFTs on S^3, the free energy is maximized by a UV CFT and any deformation - including complex deformations - decreases it. This theorem has been proven and extended to a variety of CFT deformations, e.g. the extension in [80] is especially relevant for us. With the spin-1/2 source deformation included, it is at small N no longer true that the wave function squared is peaked at empty undeformed de Sitter space. It is also not clear whether the wave function has a meaningful interpretation in terms of probabilities in this limit. In fact, with a spin-1/2 source, the partition function is supernumber-valued and there is no longer an unambiguous notion of maximum for Z. First, Z must be mapped to the real numbers to speak of maximization. The inner product eq. (6.2.6) provides such a map, but there is no reason why an F-theorem should imply maximization at the undeformed CFT with respect to this specific inner product. Still, there is a notion of Z being extremized with respect to the spin-1/2 deformation at the undeformed CFT. Clearly the first derivative of eq. (6.4.11) with respect to λ, $\tilde{\lambda}$, or both, is zero at $\lambda = 0$ or $\tilde{\lambda} = 0$. This implies that $< \tilde{\lambda}\lambda >= 0$.

For large scalar or pseudoscalar deformations our results may give the impression that the probabilities exhibit divergences. However, as we already mentioned in section 6.5, dS/CFT comes with a significant restriction on the configuration space of deformations which in particular excludes the regime where we find the wave function diverges. These restrictions are indicated with a transition to

Deformation	**Effect on wavefunction**	**Studied in**
Scalar	Perturbatively stable around empty de Sitter	[72]
BPS scalar + pseudoscalar	Perturbatively stable around empty de Sitter	section 5.3.1
BPS vector + metric squashing	Perturbatively stable around empty de Sitter	section 5.3.2
Spinor	Fermion states occupied at small N, unoccupied at large N	section 6.4
Spinor + pseudoscalar	Perturbatively stable around empty de Sitter	section 6.5.1
Spinor + scalar	Perturbatively stable, peak shifted from empty dS at small N	section 6.5.2
Spinor + metric squashing	Perturbatively stable, peak shifted from empty dS at small N	section 6.5.3

Table 6.1: Summary of for which bulk fields the wavefunction of the $\mathcal{N} = 2$ supersymmetric higher spin theory has been computed using dS/CFT. The pseudoscalar field or squashing by itself in the SUSY model have not been given a detailed discussion, but its behaviour can easily be seen by setting $\beta = 1$ in the relevant section.

dotted lines in the figures above.

For the scalar deformation, we can identify the boundaries of the configuration space with a good degree of confidence. Here, the completion of dS/CFT of [70] which provides not just a wave function but also a Hilbert space and an inner product, directly applies to our model. It implies a measure that restricts the configuration space of the wave function to deformations with $\sigma/\sqrt{N} > -3/4$. In [1] we gave an independent argument for the same bound based on the origin of dS/CFT as a continuation of Euclidean AdS/CFT. At $\sigma/\sqrt{N} = -3/4$ one of the eigenvalues of the dynamical CFT scalar becomes zero, and below this it becomes negative. As a result, the (super)gaussian formula for the partition function path integral no longer applies. Instead the EAdS dual partition function is manifestly divergent for $\sigma/\sqrt{N} < -3/4$ and thus predicts a vanishing wave function in this domain [82, 91]. Similar arguments apply to the metric deformations we considered; the dynamical CFT scalar has zero or negative eigenvalues for squashings $\alpha \leq -3/4$ leading to a vanishing wave function in this regime [82].

Finally, for the pseudoscalar deformation, the situation is less clear. The pseudoscalar deformation only affects the eigenvalues of the dynamical CFT spinor, not those of the scalar. Spinors always have negative eigenvalues and these do not pose a problem for the path integral. However, trouble does appear when there are spinor zero eigenmodes resulting in a divergent wave function. The first of these appear when $|m/\sqrt{N}| = 3/2$ and thus our results should at most be trusted for $|m/\sqrt{N}| < 3/2$. It would be very interesting to obtain a better understanding of the measure against which the wave function should be evaluated for pseudoscalar bulk fields. To give a spark of hope that the zero-eigenmode divergencies can in due time be dealt with, we point out that an analogous problem appears in the RNS quantization of the superstring. Here, commuting spinor ghosts β and γ are introduced [37]. These have a number of zero eigenmodes depending on the genus of the worldsheet one consider which produce formal divergencies in the computations, but it is understood how the theory must be modified to resolve these.

For a pseudoscalar bulk deformation, the value of $< N_F > [m]$ can be seen in fig. 6.3. We stress again that, as discussed in section 6.2, this object has been constructed to compute the fermion occupation number at a given value of the bulk scalar, with the latter treated as an external condensate. At large N, we see that N_F is zero at $m = 0$, that it increases with increasing $|m|$, saturating at complete occupation at $|m/\sqrt{N}| = 3/2$. This agrees beautifully with the earlier bulk computations of [121, 135] as follows: We do not know the bulk action when one integrates out all degrees of freedom, retaining only the bulk pseudoscalar and spin-1/2 field. Still, generically, the leading coupling between a scalar and a spinor is of the Yukawa-type. In the presence of a non-dynamical

external scalar field, the Yukawa coupling effectively acts as a mass term for the spinor. We expect the large N limit in the CFT to correspond to the free limit since $N = (G_N\Lambda)^{-1}$ and we work at constant Λ. One then expects that for a boundary source $m/\sqrt{N}$ and at large N, the bulk fermion will effectively behave as a free massive spin-1/2 fermion of mass $m_F = m/\sqrt{N}$. In [121, 135] the Hartle-Hawking wave function, together with $< N_F > [m_F]$, was computed for a free fermion in Einstein gravity in de Sitter space. For a massless bulk fermion, they found that N_F should be zero. Next, N_F should increase with increasing $|m_F|$. For modes with $n + 3/2 \ll |m_F|$ at $l = 1$, they found that N_F should saturate at maximum occupation. We are looking at constant spinors with $n = 0$, so this completely agrees with our boundary result, within the range of $|m_F|$ where our analysis is to be trusted.

The interplay of the spin-1/2 field with the bulk scalar differs from that with the pseudoscalar, especially in the large N limit. Since $\sum_n a_n = 0$, in the scalar case the part of the wave function with spin-1/2 states excited totally disappears here[10]; it appears the scalar and spin-1/2 field do not interact in the large N limit. This may be a consequence of the boundary conditions we employ. The deformations of the boundary CFT correspond to the standard quantisation for the bulk pseudoscalar and to the alternate quantization for the bulk scalar [1]. It would be interesting to explore whether the interplay between the bulk bosons and the spin-1/2 field changes for different boundary conditions.

6.6 Outlook

We have initiated the study of fermionic bulk fields in dS/CFT, working with the dualities relating $\mathcal{N} = 2$ supersymmetric Euclidean vector models with reversed spin-statistics in three dimensions to supersymmetric Vasiliev theories in four-dimensional de Sitter space. Specifically we have holographically evaluated the Hartle - Hawking wave function in the bulk in a number of homogeneous minisuperspace models consisting of a half-integer spin field coupled to a scalar, a pseudoscalar or a metric squashing. With a well-motivated measure we have found the wave function to be normalizable and globally peaked at or near the supersymmetric de Sitter vacuum, with a low amplitude for large deformations. A detailed discussion of its behavior and a comparison with earlier bulk computations in the semiclassical limit is given in Section 6.5.4 above.

[10] Except at points where there is a competing divergence in $|\Psi|^2$, such as at $\sigma/\sqrt{N} = -9/4$. However we argued these should be at or beyond the edge of the configuration space.

We have considered the lowest, homogeneous mode of a spin-1/2 bulk field only. It would be natural to look at higher modes, and to study bulk fermions of higher spin. Gravitinos have been studied in a supergravity context in AdS/CFT and in supersymmetric quantum cosmology [122, 123, 146–148]. Beyond this, higher spin theories have an infinite tower of fermionic higher spin fields. The expressions for the conserved CFT currents related to these fields are known exactly [106][11]. Since it is relatively straightforward to compute the wave function for fermionic bulk fields (compared to bosonic fields) one might hope to compute the wave function for the lowest modes of these fermionic higher spin fields exactly.

We have seen that the fermionic contributions introduce an interesting N-dependence in the theory. dS/CFT at finite N is largely unexplored territory, but recently the Q-model [70,152] has been put forward as a possible completion of higher-spin dS/CFT. We made use of the measure implied by the Q-model to identify the domain of the wave function in some of the models we considered in this paper. An interesting avenue for future research would be to examine whether fermionic bulk fields can be implemented in the Q-model and to explore how various small N effects play out against each other.

Other possible generalizations in the HS context include the study of fermions in the causal patch version of dS/CFT, currently formulated in terms of a boundary particle mechanics [153], and in FLRW-like cosmologies in higher spin theory [154] for which currently a dual description has yet to be found.

Undoubtedly the most challenging open question concerns the formulation of a precise dS/CFT duality in string theory. One important difference with HS theory, when it comes to dS/CFT, is that string theory has towers of fields of arbitrarily high mass whereas HS theory contains only massless and very light fields. It is well known that light scalar fields in dS with masses below the dS analogue $m^2_{BF} = 9/4H^2$ of the Breitenlohner - Freedman (BF) bound behave very differently from more massive fields [44, 155–158]. It is worth noting in this context that the semiclassical Hartle - Hawking wave function vanishes in dynamical models of de Sitter gravity coupled to massive scalars with masses $m^2 > m^2_{BF}$, at least for reasonably small deformations of de Sitter [159]. This suggests that if it computes the Hartle - Hawking wave function, dS/CFT duality should incorporate a final condition on such massive scalars that sets these to zero[12]. This is effectively what we have seen in the minisuperspace models we have analysed, as discussed in Section 6.5.4 above, and this would

[11] We refer to [149–151] for a recent analysis in $\mathcal{N} = 2$ supersymmetric AdS_3 which might be Wick-rotated to supersymmetric S^3.

[12] We note, however, that light scalar fields in de Sitter coupled to gravity give rise to the stochastic dynamics associated with eternal inflation and thus come with their own set of challenges [73].

resolve the tension between dS/CFT and the swampland arguments [160–165], some of which suggesting dS with $m^2 > m^2_{BF}$ scalars is unstable[13].

Finally, it is tempting to speculate that our findings are connected to the supersymmetric dS constructions in exotic string theories [87]. The latter have vector ghosts in their supergravity limits related to the existence of non-compact R-symmetry groups in their representation of the algebra. However Hull has argued that the massive string states in exotic string theories may well render the de Sitter vacua ghost-free and unitary. In [1] we conjectured that the supersymmetric higher-spin theories in dS that we construct are related indeed to the tensionless limit of these exotic string theories. It would be very interesting to explore the extension to dS/CFT of the ABJ triality [83] linking higher spin theory and string theory in AdS/CFT from a complementary angle and in different regimes.

[13]Whilst the discussion of instabilities has mostly focussed on scalar fields we note that our study of massive spin-1/2 fields implies a similar limit $|m_F| \leq 3/2$, which seems in line with the reasoning above.

Part III

String Compactifications

Chapter 7

Introduction to string compactifications

As we discussed in chapter 3, critical string theory is ten-dimensional. If we wish to start from a semiclassical description of ten-dimensional string theory and say something about four-dimensional physics, we must be able to hide away six dimensions. To do this, we will attempt to compactify these extra, internal dimensions so they are too small to be directly experienced by a macroscopic observer in the four-dimensional space. At the same time, we must compactify in such that our ten-dimensional semiclassical description does not break down in an uncontrolled way.

These internal dimensions are more than merely a burden. The goal is not just to kill them and then hide the evidence. The structure of the internal dimensions will determine the particles and coupling that exist in an effictive four-dimensional description after compactification.The many possible choices for the internal geometry allow a theory with a limited number of dynamical objects like string theory to produce a wide variety of four-dimensional theories with different interactions and particle spectra.

Before we start the daunting task of studying string compactifications, we will first study some simpler and highly instructive toy models in section 7.1 . We will present the story of compactifying extra dimensions as a series of failures, with each example adding a new ingedient to resolve some issue of the previous example, while still remaining imperfect in some way. In addition to the references appearing in the text, we found [40, 166, 167] very useful throughout this chapter.

7.1 Extra dimensions

7.1.1 Kaluza-Klein

One of the simplest examples of compactifying extra dimensions is the five-dimensional theory of Kaluza and Klein. This will illustrate some basic features, as well as issues, of compactifying extra dimensions. Kaluza-Klein theory is reviewed in detail in [168], on which we will rely.

First, let us consider a fixed five-dimensional flat geometry with coordinates X^M with $M = 0, .., 4$. We periodically identify the fifth dimension as

$$X^4 \sim +X^4 L_4 \,. \tag{7.1.1}$$

The fifth dimension is then a circle with radius $l_4/2\pi$. Consider a massless scalar field ϕ in this background. Its equation of motion is then given by

$$\partial^M \partial_M \phi = 0 \,. \tag{7.1.2}$$

Denoting the four external coordinatis with the index $\mu = 0, .., 3$, this is given by

$$\partial^\mu \partial_\mu \phi + (\partial_4)^2 \phi = 0 \,. \tag{7.1.3}$$

Since the fifth dimension is periodic, one can fourier expand

$$\phi(X^\mu, X^4) = \sum_n \phi_i(X^\mu) e^{i \frac{2\pi n X_4}{L_4}} \,. \tag{7.1.4}$$

Filling this into the equation of motion yields

$$\sum_n \left(\partial^\mu \partial_\mu \phi_n - \left(\frac{2\pi n}{L_4} \right) \right)^2 e^{i \frac{2\pi n X_4}{L_4}} = 0 \tag{7.1.5}$$

One can now re-interpret this from an effective four-dimensional perspective as an infinite number of scalar fields $\phi(X^\mu)_n$. Of these ϕ_0 is massless, while ϕ_n has mass $\frac{2\pi n}{L_4}$. One calls this infinite series of massive fields the Kaluza-Klein (KK) tower of the scalar field. Note that once a four-dimensional observer starts probing energies of the scale $1/R$, she becomes sensitive to the presense of the fifth dimension and the effective four-dimensional description begins to break down.

We have considered a dynamical field on a fixed background geometry. Now let us consider how to describe a dynamical five-dimensional metric when compactifying to four dimensions.

We denote to five-dimensional metric as G_{MN} and use indices μ, ν for the four external dimensions. One can then suggestively write

$$G_{MN} = \begin{bmatrix} g_{\mu\nu} + \kappa^2 \phi^2 A_\mu A_\nu & \kappa A_\mu \phi^2 \\ \kappa A_\nu \phi^2 & \phi^2 \end{bmatrix} \tag{7.1.6}$$

The five-dimensional diffeomorphism invariance of G_{MN} will ensure four-dimensional diffeomorphism invariance for $g_{\mu\nu}$ and four-dimensional gauge invariance for A^μ. Consider now the five-dimensional Einstein-Hilbert action

$$S = M_{p5}^3 \int d^4 X^\mu \int dX^4 \sqrt{-G} \mathcal{R}_5 \,, \tag{7.1.7}$$

with M_{p5} the five-dimensional Planck mass and $\mathcal{R}_5$ the five-dimensional Ricci scalar. One can then fill in eq. (7.1.6) and perform the integral over X^4, taking it compact to obtain an action in four-dimensional terms. We work at first order in X^4, that is, ignoring the massive KK modes. We identify $X^4 \sim X^4 + L_4$. The physical length of the fifth dimension is then ϕL_4 because of the prefactor appearing in eq. (7.1.6) . This yields after setting $\kappa = \frac{1}{2M_{p5}^3 L_4}$

$$S = \int d^4 X \sqrt{-g} \phi \left[M_{p5}^3 L_4 \mathcal{R} - \frac{\phi^2}{4} F_{\mu\nu} F^{\mu\nu} + \frac{2}{3\phi^2} \partial_\mu \phi \partial^\mu \phi \right] . \tag{7.1.8}$$

And so we obtain a four-dimensional theory of gravity, a gauge field A_μ with field strength $F_{\mu\nu}$, and a scalar ϕ. We identify the four-dimensional Planck mass as $M_{p4} = M_{p5}^3 L_4$. We see that the strength of the lower dimensional gravitation coupling depends on the size of the internal dimensions. The reason for this is that gravity interacts with all dimensions equally. The larger L_4 is, the more gravity will leak into the internal dimension and the weaker the four-dimensional gravitational interaction will be.

The action we obtain does not give is the familiar Einstein-Hilbert action for the gravity sector. Let us reparametrize the action so this does happen in the case $A_\mu = 0$. We rescale the metric as $\tilde{g} = \phi^2 g$ and redefine the scalar as $\Phi = \frac{2 \log \phi}{\sqrt{3}}$ to obtain

$$S = \int d^4 X \sqrt{-\tilde{g}} \left[M_{p4}^2 \tilde{\mathcal{R}}_4 + \frac{1}{2} \tilde{\partial}_\mu \Phi \tilde{\partial}^\mu \Phi \right] . \tag{7.1.9}$$

There is something unpleasant happening here, however. The size of the internal space is determined by the dynamical parameter Φ. Such a parameter that sets the properties of the internal manifold is called a modulus[1]. Note that Φ is a

[1] In a narrow sense, only parameters without a potential are called moduli. Sometimes, in a broader sense, even parameters with a potential will be called moduli.

massless scalar and has no preferred ground state value. As a result, R will stochastically wander over time and will eventually become small enough that a semiclassical geometric picture breaks down or become so large that the fifth dimension decompactifies. So, we have not truly achieved a compactification. We must give Φ a potential so that it can stabilize at some local minimum of that potential. This process is referred to as stabilizing a modulus. We will turn to this issue in the next section.

When Φ is massless, it will act as a new long-range interaction, a "fifth force". As Φ comes from the five-dimensional graviton, it couples universally and will modify the long-range interaction for all particles.

7.1.2 Freund-Rubin

Moduli can be stabilized by performing a Freund-Rubin type compactification [169, 170]. We follow along the lines of the review [171]. We will consider Einstein gravity with a background manifold $M_D = X_d \times Y_{D-d}$ with M the entire $D-$dimensional manifold, X_d the d external dimensions, and Y_{D-d} the $D-d$ internal dimensions we wish to compactify. In addition, we assume there is a $(D-d)-$form gauge strength F present in the theory. The idea now is to take Y_{D-d} to be a manifold with internal curvature. This curvature will generate a potential in terms of the scalars in the compactified theory. However, this potential will generically be runaway, sending the extra dimensions to zero size (and a breakdown of the theory) for positive curvature internal manifolds, and to infinite size for negative curvature manifolds. One then turns on the gauge field on the internal manifold. This too provides a contribution to the scalar potential. The intent is to have this provide a contribution in the opposite direction to the curvature. Then, the scalar potential can possibly have a stable minimum at a finite value, thereby stabilizing this modulus.

We will look at this in more detail in the case $M_6 = X_4 \times Y_2$. The gauge field now is a two-form F_{MN}. Topologically, the internal manifold can be either S^2, T^2, or a higher genus manifold which will have negative curvature.

The full action is given by

$$S = \int d^6x\sqrt{-G}\left(M_{p6}^4\mathcal{R}_6 - M_{p6}^2|F|^2\right) . \tag{7.1.10}$$

Here, M_{p6} is the six-dimensional planck scale and $\mathcal{R}_6$ the six-dimensional Ricci scalar. We now follow an analogous procedure as for the Kaluza-Klein reduction earlier. We consider a metric of the form

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu + R^2 v_{mn}dy^n dy^m \tag{7.1.11}$$

with $g_{\mu\nu}$ the metric on the four external dimensions, v_{mn} a unit volume metric on the two internal dimensions, and R setting the length scale of the internal dimensions.

We now consider how this effectively looks to the four external dimensions, focussing first on solely the gravitational dynamics. One writes $\mathcal{R}_6$ in terms of the external and internal components. To leading order in derivatives, this produces a term in the action of the form

$$S_{\mathcal{R}} = M_{p6}^4 R^2 \int d^4x\sqrt{-g}\left(R^2\mathcal{R}_4 + \left[\int d^2y\sqrt{v}\mathcal{R}_2\right]\right) . \tag{7.1.12}$$

The curvature of the unit volume term of the two internal dimensions is given by

$$\int d^2y\sqrt{v}\mathcal{R}_2 = 2 - 2\,\text{genus}\,. \tag{7.1.13}$$

We must reparametrize the action to obtain the four-dimensional Einstein frame. To this end, we define the four-dimensional planck mass as $M_{p4} = M_{p6}^2 R$ and rescale the metric as $h_{\mu\nu} = R^2 g_{\mu\nu}$. We then obtain an action of the form

$$S_{\mathcal{R}} = M_{p4}^2 \int d^4x\sqrt{-h}\left[\mathcal{R}_{4h} - V(R)\right] . \tag{7.1.14}$$

Crucially, the internal curvature effectively provides a potential from the four-dimensional perspective,

$$V(R) = \frac{1}{R^4}(2\,\text{genus} - 2)\,. \tag{7.1.15}$$

The physics is then set by the genus of the internal manifold. One can then think of finding a stable vacuum as finding a minimum of this potential. In case one has a torus, the potential is flat and R has no preferred value. This is the higher-dimensional analogue of the Kaluza-Klein circle reduction. In case of a sphere, rolling down the potential means going to $R \to 0$ and the internal dimensions shrink until we reach a point where our approximations are no longer valid. For surfaces of genus higher than the torus, the potential is runaway towards $R \to$ and the extra dimensions spontaneously decompactify.

So far, we have have seen that adding curvature to the internal dimensions makes the internal moduli no longer flat directions. However, we have failed to stabilize them at a finite value. To achieve this, we will need the gauge field.

We take there to be some background magnetic gauge field wrapping the internal manifold and no background gauge field on the external directions. A gauge field in the external directions would generically spoil the isometries of the

external space. That is, we have

$$\int_{Y_2} F = n \,. \tag{7.1.16}$$

In principle, in classical Einstein-Maxwell theory, n can be a real number. After charge quantization, it will be become integer. Filling this in for the gauge field in eq. (7.1.12) and reducing to four dimensions analogously to the gravity sector produces a term in the action of the form

$$S_F = \int d^6x\sqrt{-G}\left(M_{p6}^2|F|^2\right) = \int d^4x\sqrt{-h}\left(-\frac{n^2}{R^6}\right) . \tag{7.1.17}$$

We can incorporate this term in the potential $V(R)$, yielding

$$V(R) = \frac{1}{R^4}(2\,\text{genus} - 2) + \frac{n^2}{R^6} \,. \tag{7.1.18}$$

We see that the magnetic field by itself causes a runaway of R towards decompactification. If we now take the internal space to be spherical, the potential has a minimum at finite R, the R modulus can be stabilized at that finite value. This magnetic field is one example of what we will generically refer to as a flux in the future.

This provides the simplest example of 'moduli being stabilized by flux'. In this simple example with two internal dimensions, there is a clearly physical picture. The sphere wishes to shrink down to a point to minimize its action, but this squeezes the magnetic field lines together, at some point these two effects cancel out. Later set-ups may be more convoluted, but this is a nice picture to keep in mind for flux compactifications.

It now seems that we have compactified down from six to four dimensions at some finite size of the internal dimensions, and with a correct choice of parameters, this finite size could be set to a scale where our approximations are valid. However, there is still something unsatisfactory about what we have done. The minimum value of $V(R)$ will set the cosmological constant for the four external dimensions. This is given by

$$V_{min} = -\frac{5}{3R} \,. \tag{7.1.19}$$

So, one gets an AdS_4 vacuum whose cosmological scale is set by an order one factor times the scale of the internal manifold. There is no separation of scales between the internal and external space. Choosing a cosmological constant of the magnitude (but not sign) that we observe today, earth-scale physics would still be a very six-dimensional experience. As such, this is arguably not a true compactification.

7.2 Special manifolds

When compactifying superstring theory to four dimensions, we need to compactify not one or two, but six extra dimensions.

We assume the reader is familiar with the notions of homology and cohomology. The p-th cohomology of a manifold $\mathcal{M}$ is given by

$$H^p(\mathcal{M}) = \frac{\text{closed } p\text{-forms on } \mathcal{M}}{\text{exact } p\text{-forms on } \mathcal{M}} \,. \tag{7.2.1}$$

The dimension of $H^p(\mathcal{M})$ is the p-th Betti number b_p, which depends only on the topology of the space.

There exists a second order differential operator Δ which reduces to the Laplacian in flat space

$$\Delta = \star d \star d + d \star d \star \,. \tag{7.2.2}$$

A form H which obeys $\Delta H = 0$ is said to be harmonic. The harmonic forms are in one-to-one correspondence with the cohomology classes. Recall how in the KK reduction of a scalar field in section 7.1.1 the massless four-dimensional scalar emerged from the five-dimensional scalar field mode which obeys $(\partial_4)^2\phi = 0$. The harmonic forms are the higher-dimensional analogues to this. How many and what type of lightest four-dimensional fields will appear from compactifying the fields in the ten- or eleven-dimensional supergravity action will then be determined by H^p of the internal manifold[2].

Compactifying a generic six-dimensional manifold (or seven-dimensional for M-theory) would be a very difficult task. Certain manifolds have special properties that make them easier to deal with. See [37,172–175] for reviews of the material that follows.

7.2.1 Special structure manifolds

For what follows we will need the formalism of fibre bundles. We will not be too rigorous and assume the geometries we discuss to be well-behaved. Consider a d-dimensional manifold $\mathcal{M}$. A bundle $\mathcal{E}$ is a manifold which consists of $\mathcal{M}$ plus some additional structure at each point p of $\mathcal{M}$. This additional structure is called the fibre $\mathcal{F}$ of the bundle. Locally near a point p of $\mathcal{M}$ or on a patch U_α of $\mathcal{M}$ the bundle looks like a direct product manifold $\mathcal{E} = \mathcal{M} \times \mathcal{F}$ called

[2]We are talking here of a purely geometric compactification. Additional elements such as fluxes or higher-order correction can render these four-dimensional fields massive. In addition, string theory also has other sources of massless four-dimensional fields such as those from open string degrees of freedom on branes.

the local trivialisation of the bundle. The bundle need not look like a direct product globally.

Let x^i, $i = 1, .., d$, be the coordinates on a patch U_α of $\mathcal{M}$. At a point $p \in U_\alpha$ the d-dimensional space of tangent vectors is called $T_p\mathcal{M}$ and is spanned by the $\frac{\partial}{\partial x^i}$ vectors. The tangent bundle $T\mathcal{M}$ is formed by taking the fibre at each point p to be $T_p\mathcal{M}$. By selecting an element of the fibre of $T\mathcal{M}$ for each point p, called taking a section of $T\mathcal{M}$, one obtains a vector field on $\mathcal{M}$. The dual bundle to $T\mathcal{M}$ is the cotangent bundle $T * \mathcal{M}$ whose sections are one-forms.

Let $e_a = e_a^i \frac{\partial}{\partial x^i}$, $a = 1, .., d$, be a basis for $T_p\mathcal{M}$ of the points in a patch U_α. Their dual one-forms are $e^a = e_i^a dx^i$. The frame bundle $F\mathcal{M}$ has as fibre the set of ordered bases of $T_p\mathcal{M}$ at each point p. Suppose a point $p \in \mathcal{M}$ is in the intersection of a patch U_α with coordinates x^i and local trivialisation (p, e_a) of $F\mathcal{M}$ and a patch U_β with coordinates x'^i and local trivialisation (p, e'_a) of $F\mathcal{M}$. The bases are then related as

$$e'^i_a = \frac{\partial x'^i}{\partial x^j} e^j_a \,. \tag{7.2.3}$$

This can be writen as the action of a group element $t_{\beta\alpha}$

$$e'^i_a = e^j_b (t_{\beta\alpha})^b{}_a \,. \tag{7.2.4}$$

For consistency this should obey

$$t_{\alpha\beta} t_{\beta\alpha} = 1 \tag{7.2.5}$$

and on the triple overlap between U_α, U_β and U_γ

$$t_{\alpha\beta} t_{\beta\gamma} = t_{\alpha\gamma} \,. \tag{7.2.6}$$

One calls these $t_{\alpha\beta}(p)$ the transition functions and their group the structure group. In general, the structure group is $GL(d, \mathbb{R})$.

For some manifolds it is possible that through a judicious choice of local frame in the patches the structure group is restricted to a subgroup $g \subset GL(d, \mathbb{R})$. In this case, the manifold is said to have G-structure.

In the most extreme case the structure group is the identity. Such a manifold is said to be parallelizable. One set of e_a suffices for an entire parallelizable manifold. Alternatively, there are d globally well-defined one-forms e^a. For instance $\mathbb{R}^d$ is parallelizable. Other parallelizable manifolds that we will encounter in chapter 9 are group manifolds. For these, the basis one-forms obey a group structure

$$de^a = \frac{1}{2} f^a_{bc} e^b \wedge e^c \,, \tag{7.2.7}$$

with f^a_{bc} the structure constant of the group. If the group is nilpotent the manifold is a nilmanifold, if the group is solvable the manifold is a solvmanifold. An overview of these in a physics context can be found in [176].

As an example, consider the three-dimensional nilmanifold with

$$de^1 = 0\,, \qquad de^2 = 0\,, \qquad de^3 = Ne^1 \wedge e^2\,. \tag{7.2.8}$$

This can be implemented in coordinates as

$$e^1 = dx^1\,, \qquad e^2 = dx^2\,, \qquad e^3 = dx^3 + Nx^1e^2\,. \tag{7.2.9}$$

So far we have not worried about compactness. Clearly we can identify $x^2 \sim x^2 + R_2$ and $x^3 \sim x^3 + R_3$ analogously to section 7.1.1. If one tries to identify $x^1 \sim x^1 + R_1$, then e^3 is no longer single-valued. Instead one can identify $(x^1, x^2, x^3) \sim (x^1 + R_1, x^2, x^3 - R_1x^2)$. Because of this modification needed in the identification such compact group manifolds are called twisted tori. There is no guarantee that an arbitrary group manifold can be compactified.

We saw that a manifold being parallelizable is equivalent to the presence of d nondegenerate globally defined one-forms e^a. More generally, a manifold having G-structure will be equivalent to the existence of some globally defined nondegenerate objects on the manifolds.

If there is a globally defined symmetric tensor g_{ij}, the metric, the manifold has $O(d)$-structure[3]. If there is a globally defined volume element and the manifold is orientable it has $SO(d)$ structure.

Let us focus for the moment on even-dimensional manifold. A manifold of d real dimensional can *locally* always be thought of as a complex manifold of dimension $d/2$ by the introduction of holomorphic coordinates $z^i = x^i + ix^{d/2+i}$ and antiholomorphic coordinates $\overline{z^i} = x^i - ix^{d/2+i}$. A form with p holomorphic dz pieces and q antiholomorphic $d\overline{z}$ pieces is said to be a (p, q) form. A two-form for instance generically contains a $(2, 0)$, $(1, 1)$, and $(0, 2)$ piece.

If a manifold has a globally defined real (1,1) form J and complex decomposable[4] (d,0) form Ω which is everywhere nondegenerate

$$\Omega \wedge \overline{\Omega} \neq 0\,, \tag{7.2.10}$$

it is an $SU(d/2)$ structure manifold[5].

[3]Since we are concerned with the internal dimensions we restrict to Riemannian geometry.

[4]A decomposable form is a form that can be written as a product of one-forms. In particular, $\Omega = f(z^i, \overline{z^i})dz^1 \wedge dz^2 \wedge dz^3$.

[5]For our choice of normailzation of these forms and other conventions, see appendix E.

For string compactifications to four dimensions we are especially interested in six-dimensional manifolds with $SU(3)$ structure. In this case

$$d\Omega = W_1 J^2 + W_2 \wedge J + \bar{W}_5 \wedge \Omega \tag{7.2.11}$$

$$dJ = \frac{3}{2}\mathrm{Im}\,(\bar{W}_1 \Omega) + W_4 \wedge J + W_3 \tag{7.2.12}$$

where the W_i are the torsion classes. This is a decomposition of dJ and $d\Omega$ in different $SU(3)$ representations, we refer to [172] for a detailed discussion.

Another case of a G-structure manifold is a $G2$-structure manifold. The authoritative text on $G2$-manifolds is [177], a physics review is [178]. These only appear in seven dimension and have a globally defined positive[6] three-form Φ. We refer to [178] for the properties of the torsion classes appearing the expansion of $d\Phi$. The three-form Φ defines a globally defined four-form $\star\Phi$ and a unique metric.

7.2.2 Special holonomy manifolds

We now consider some of the classes of manifolds we encountered in the previous section in a different light.

Let V be a vector or tensor or spinor on the manifold. Parallel transport V around a closed loop γ. Then V transforms as

$$V \to U(\gamma)V\,, \tag{7.2.13}$$

with $U(\gamma)$ a group element. The group made up of $U(\gamma)$ for all possible closed paths on the manifold is the holonomy group of the manifold.

This certainly sounds related to the structure group. In fact, a manifold with G-holonomy can be thought of as an enhancement of a manifold with G-structure.

An $SU(d/2)$-structure manifold for which all torsion classes vanish such that

$$dJ = 0\,, \quad d\Omega = 0\,, \tag{7.2.14}$$

is an $SU(d/2)$-holonomy manifold. In this case, Ω is known as the holomorphic $d/2$-form, $\overline{\Omega}$ as the antiholomorphic $d/2$-form, and J as the kähler form. A d-dimensional manifold with $SU(d/2)$-holonomy is known as a Calabi-Yau $d/2$-fold ($CY_{d/2}$).

[6] A form Φ is positive if locally there is always a frame such that $\Phi = e^{123} + e^{145} + e^{167} + e^{246} - e^{257} - e^{347} - e^{356}$ where e^{ij} is shorthand for $e^i \wedge e^j$ et cetera.

Yau's theorem [179, 180] guarantess the existence of a unique metric $g_{i\bar{j}}$ defined by the kähler form as

$$J = g_{i\bar{j}} dz^i d\bar{z}^{\bar{j}}\,, \tag{7.2.15}$$

for any Calabi-Yau manifold.

Such manifolds are Ricci flat $\mathcal{R}_{ij} = 0$. They then automatically solve the Einstein equation eq. (1.1.4) on the compact dimensions without matter. The resulting spacetime in string theory is then flat space on the external dimensions times the Calabi-Yau[7].

One can introduce $\partial = dz^i \partial_i$ and $\partial = d\bar{z}^{\bar{i}} \partial_{\bar{i}}$. Then

$$d = \partial + \overline{\partial}\,, \tag{7.2.16}$$

and one can construct cohomologies of (p, q) with respect to ∂ and $\overline{\partial}$ analogously to eq. (7.2.1). For a Calabi-Yau $H^{p,q}_{\partial} = H^{p,q}_{\overline{\partial}} \equiv H^{p,q}$. One denotes the dimension of these cohomology classes as $h^{p,q}$.

For CY_3, $h^{2,1} = h^{1,2}$ and $h^{1,1} = h^{2,2}$. Further, $h^{0,0} = h^{3,3} = h^{0,3} = h^{3,0} = 1$ and all other cohomology classes are zero.

A seven-dimensional manifold has $G2$-holonomy when it has $G2$-structure and

$$d\Phi = d \star \Phi = 0\,. \tag{7.2.17}$$

Manifolds with $G2$-holonomy are Ricci-flat. The only independent Betti numbers on a $G2$-holonomy manifold are b^2 and b^3 [177].

7.2.3 Physical implications

So far, we have treated these special geometries purely geometrically. The goal is to use them as the internal dimensions for string compactifications.

Superstring theory is supersymmetric, but a generic internal manifold and background field configuration will break supersymmetry.The special manifolds we discussed may result in the four-dimensional theory preserving some supersymmetry.

Maintaining supersymmetry when compactifying is useful. Under assumptions which hold throughout this part of the thesis, maintaining supersymmetry and solving the Bianchi identities guarantees that one solves the equations of motion.

[7] Assuming one only solves the Einstein equation and there is no matter contribution or cosmological constant in the higher dimension.

We then simplify from solving second-order differential equations to solving first-order differential equations.

Further, supersymmetry ensures stability[8] allowing us to avoid doing a difficult stability analysis to ensure we really have a vacuum.

Compactifying all the degrees of freedom of superstring theory on a generic six-dimensional manifold will result in a very complex and compuationally intractable spectrum of four-dimensional fields and interactions. Maintaining supersymmetry at least ensures that the fields must appear in supermultiplets and that they must couple in a way that preserves supersymmetry, greatly constraining the allowed structure.

The simplifications and cancellations that supersymmetry provides will further aid when doing computations in the four-dimensional effective theory. For example, we will later have need of gaugino condenstaion. Supersymmetry will be of use for computing such a strongly coupled gauge theory effect.

To preserve supersymmetry, the supersymmetry variations of the fields must vanish. In the absence of any background fields (other than the graviton), this is equivalent to the existence of a globablly defined nonvanishing spinor η on the internal manifold that is covariantly constant

$$\nabla_i \eta = 0\,. \tag{7.2.18}$$

The existence of such a covariantly constant is equivalent to the internal manifold having certain special holonomy.

Compactifying type II string theory on a CY_3 will result in the four-dimensional theory having $\mathcal{N} = 2$ SUSY. Recall that the cohomology determines the light field content of the four-dimensional theory. In IIA string theory the four diimensional theory will have $h^{1,1}$ Abelian $\mathcal{N} = 2$ vector multiplets and $h^{2,1} + 1$ hypermultiplets. Compactifying IIB yields $h^{2,1}$ Abelian $\mathcal{N} = 2$ vector multiplets and $h^{1,1} + 1$ hypermultiplets. The extra hypermultiplet in both cases is the universal multiplet containing the dilaton. Finally, the four-dimensional theory contains an $\mathcal{N} = 2$ gravity multiplet.

The massless scalars appearing in these multiplets form a moduli space. The moduli associated to $h^{1,1}$ are known as the kähler moduli, those associated to $h^{2,1}$ are known as the complex structure moduli.

These light fields capture only the low energy closed string degrees of freedom on a smooth manifold. In addition, there can be light degrees of freedom associated to open strings on D-branes and to singularities on the manifold.

[8] In the sense that scalar potential has no unstable tachyonic directions. Flat moduli directions are still possible and common.

When compactifying eleven-dimensional supergravity on a manifold with $G2$-holonomy, one preserves $\mathcal{N}=1$ SUSY. The four-dimensional theory has b_2 Abelian $\mathcal{N}=1$ vector multiplets, b_3 chiral multiplets, and $\mathcal{N}=1$ gravity multiplet [181].

7.2.4 Pure spinors

When one turn on background field fluxes on the CY_3, which will backreact on the geometry, it is not at all guaranteed that supersymmetry will be preserved. A necessary condition to preserve $\mathcal{N}=1$ SUSY is that the internal manifold has $SU(3)$ structure as this guarantees the existence of a globally defined nonvanishing spinor η. With background fields present, η no longer needs to be covariantly constant to preserve SUSY. Instead, it must satisfy certain differential equations.

The reader may be familiar with the fact that in four Lorentzian dimensions a vector V^μ can be written as a bispinor $V^{\alpha\dot{\alpha}}$, with α and $\dot{\alpha}$ two-component spinor indices, and vice versa through the gamma matrices

$$V_{\alpha\dot{\alpha}} = (\sigma^\mu)_{\alpha\dot{\alpha}}\, V_\mu\,, \quad V_\mu = -\frac{1}{2}\,(\overline{\sigma}_\mu)^{\dot{\alpha}\alpha}\, V_{\alpha\dot{\alpha}}\,. \tag{7.2.19}$$

Analogously, the SUSY conditions on the internal spinor η can be recast as a conditions on polyforms $\Phi_\pm$ with

$$\Phi_+ = \frac{1}{8} e^{-iJ}\,, \tag{7.2.20}$$

$$\Phi_- = -\frac{i}{8}\Omega\,. \tag{7.2.21}$$

The correct formalism to develop this polyform construction is known as generalized complex geometry, for which we refer to [172, 174].

Consider a ten-dimensional metric of the form

$$ds^2 = e^{2W(y)} g_{\mu\nu} dx^\mu dx^\nu + g_{ij} dy^i dy^j\,, \tag{7.2.22}$$

with $g_{\mu\nu}$ the metric on the four external dimensionsions, which is Minkowski or (A)dS and g_{ij} the metric on the six internal dimensions.

The ten-dimensional spinors ϵ appearing in the SUSY variation are split in a four-dimensional part ξ and a six-dimensional part η as

$$\epsilon^1_{IIA} = \xi_+ \otimes (a\eta_+) + \xi_- \otimes (\overline{a}\eta_-)\,, \quad \epsilon^2_{IIA} = \xi_+ \otimes (\overline{b}\eta_-) + \xi_- \otimes (b\eta_+) \tag{7.2.23}$$

$$\epsilon^1_{IIB} = \xi_+ \otimes (a\eta_+) + \xi_- \otimes (\overline{a}\eta_-)\,, \quad \epsilon^2_{IIB} = \xi_+ \otimes (b\eta_+) + \xi_- \otimes (\overline{b}\eta_-) \tag{7.2.24}$$

where $\pm$ denotes chirality. There are two spinors for SUSY variations and hence $\mathcal{N}=2$ SUSY. Here $a(y)$ and $b(y)$ are complex functions. When a and b are related, only one independent spinor remains and the background is broken to $\mathcal{N}=1$ SUSY.

One can show that in vacua with $\mathcal{N}=1$ SUSY they are related as

$$d|a|^2 = |b|^2 dW\,, \quad d|b|^2 = |a|^2 dW\,. \tag{7.2.25}$$

The pure spinor equations to preserve $\mathcal{N}=1$ SUSY are then for IIA string theory[9]

$$e^{-2W+\phi}(d+H\wedge)(e^{2W-\phi}\tilde{\Phi}_+) = 0\,, \tag{7.2.26}$$

$$\begin{aligned} e^{-2W+\phi}(d+H\wedge)(e^{2W-\phi}\tilde{\Phi}_-) =& dW\wedge\overline{\tilde{\Phi}}_- - \frac{1}{16}e^{\phi}\big[(|a|^2-|b|^2)F_{IIA-} \\ & - i(|a|^2+|b|^2)\star F_{IIA+}\big]\,, \end{aligned} \tag{7.2.27}$$

and for IIB string theory

$$\begin{aligned} e^{-2W+\phi}(d+H\wedge)(e^{2W-\phi}\tilde{\Phi}_+) =& dW\wedge\overline{\tilde{\Phi}}_+ - \frac{1}{16}e^{\phi}\big[(|a|^2-|b|^2)F_{IIB+} \\ & - i(|a|^2+|b|^2)\star F_{IIB-}\big]\,, \end{aligned} \tag{7.2.28}$$

$$e^{-2W+\phi}(d+H\wedge)(e^{2W-\phi}\tilde{\Phi}_-) = 0\,. \tag{7.2.29}$$

Here,

$$F_{IIA\pm} = F_0 \pm F_2 + F_4 \pm F_6\,, \quad F_{IIB\pm} = F_1 \pm F_3 + F_5\,, \tag{7.2.30}$$

and

$$\tilde{\Phi}_+ = a\bar{b}\Phi_+\,, \quad \tilde{\Phi}_- = ab\Phi_-\,. \tag{7.2.31}$$

7.3 Extended objects

In string theory we have extended objects at our disposal, such as the branes that we enountered in section 3.2. We wish to compactify such that the four external spacetime dimensions are homogeneous and isotropic, either Minkowski space or (A)dS. If we place extended objects in our spacetime, they must then fill the four external dimensions entirely as otherwise their position break the isometry.

[9] The type II string theory fields appearing here were introduced in section 3.1.

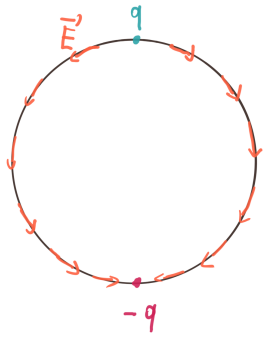


Figure 7.1: In Maxwell theory, the total charge on a compact spatial manifold must be zero.

D-branes will contribute gravitationally via their tension T_p and to the flux via their charge Q. Imagine placing a particle with electrical charge q on a compact spatial manifold, say a circle. Clearly, the electrical field vectors must meet up again at the antipode to q, see fig. 7.1 . This implies that there must be a charge $-q$ at the antipode. In general, in Maxwell theory the total charge on a compact manifold without boundary must be zero as a consequence of Stokes' theorem. As the branes we will consider fill the four external spacetime directions, their transverse directions are compact and they must obey an analogous but slightly modified condition known as tadpole cancellation, which we will encounter in section 7.5 .

There are open string degrees of freedom living on the D-branes. As the branes fill the external spacetime directions, so do the open string degrees of freedom. After compactification, the open string degrees of freedom then contribute fields and interactions to the low-energy EFT. A stack of N D-branes will for instance include $U(N)$ gauge theory in its contribution to the four-dimensional theory.

One can orbifold the internal manifold: define a discrete group Γ that acts on the internal manifold. Then identify points that can be mapped onto each other by the action of Γ with each other. The resulting quotient space $\mathcal{M}/\Gamma$ is said to be an orbifold of $\mathcal{M}$. For example, imagine two-dimensional flat space with cartesian coordinates x and y. Identify $x \sim -x$. This identification has a fixed line at $x = 0$ where the space becomes singular. One may imagine this as a giant mirror floating in space at $x = 0$. This is a $\mathbb{Z}_2$ orbifold. Depending on how Γ acts, orbifold may either produce fixed surfaces or they may leave the quotient geometry smooth. For example, identifying $x \sim x + 1$ is also a $\mathcal{Z}_2$ orbifold, but the resulting geometry is smooth.

In string theory, one can consider a $\mathbb{Z}_2$ which acts as $x \sim -x$ for some spatial directions while at the same time changing the orientation of the string worldsheet. Quotienting by this $\mathbb{Z}_2$ acts as an orbifold on the spacetime, but it clearly does something more. Such a $\mathbb{Z}_2$ quotient is known as an orientifold. The action of the orientifold on the string worldsheet will project out some string states. As a result, the $\mathcal{N} = 2$ supersymmetry of our compactification will be broken to at most $\mathcal{N} = 1$. Fixed planes of the orientifold action are known as orientifold planes or Op-planes for a fixed plane extended in time and p spatial directions. Orientifold planes have negative tension and are charged.

7.4 Warping and hierarchy

In eq. (7.2.22) there was a dependence $W(y)$ of the external dimensions of the metric on the internal dimensions y. Such a dependence is known as a warping. What warping gives us is that it allows a large hierarchy of energy scales to appear in the compactified theory. We will exemplify this by sketching a model due to Randall and Sundrum (RS) [182]. Consider the five-dimensional metric

$$ds^2 = e^{-2kr_c|\theta|}\eta_{\mu\nu}dx^\mu dx^\nu + r_c^2 d\theta^2 \tag{7.4.1}$$

The fifth dimension θ is an interval running from 0 to π after orbifolding a circle with $\theta \in (-\pi, \pi]$ as $\theta \sim -\theta$ and we intend to compactify to four dimensions. The constant r_c is the compactification scale and k an energy scale whose value we will set soon.

Now insert three-branes filling the four external spacetime directions and pointlike in the internal dimension. Place a brane corresponding to a visible matter sector at $\theta = \pi$ and a brane corresponding to a hidden matter sector at $\theta = 0$ (the names of these sectors will make sense at the end). Associate potential energies V_{vis} and V_{hid} to these branes in their ground state. Let the five-dimensional theory have a cosmological constant Λ_5 and Planck mass M_{p5}. One may then show that the Einstein equations are solved when

$$V_{hid} = -V_{vis} = 24M_{p5}^3 k\,, \quad \Lambda_5 = -24M_{p5}^3 k^3\,, \tag{7.4.2}$$

which fixes k in terms of the other parameters in the theory.

The degrees of freedom on the three-branes will provide the matter fields in the four-dimensional theory after compactification. Suppose these matter fields obtain a mass through the Higgs mechanism. Let m_0 be the mass scale associated to the Higgsing absent any warping. The four-dimensional action coming from the branes after compactification is

$$S = \int d^4x \left[\sqrt{-g_{hid}}\mathcal{L}_{hid} + \sqrt{-g_{vis}}\mathcal{L}_{vis}\right]\,. \tag{7.4.3}$$

As there is no warping at $\theta = 0$, for the hidden sector $\sqrt{-g_{hid}} = \sqrt{-g}$, with $\sqrt{-g}$ denoting the four-dimensional metric without any warping prefactor. The mass scale is then unchanged,

$$m_{hid} = m_0 \,. \tag{7.4.4}$$

For the visible sector at $\theta = \pi$ there is warping and so $\sqrt{-g_{vis}} = e^{-2kr_c\pi}\sqrt{-g}$. One may show that as a result of this factor appearing in the action, the mass scale is shifted as

$$m_{vis} = e^{-kr_c\pi} m_0 \,. \tag{7.4.5}$$

The visible sector fields are then exponentially in term of kr_c lighter than the hidden sector fields. For large enough kr_c at low energy in the four-dimensional theory one can the integrate out the hidden sector and consider a dynamical theory with only the visible sector (hence the names). As kr_c decrease the visble sector mass exponentially its value does not need to be very big at all for this to happen.

Because if this exponential dependence, once one has warping, one can achieve very large hierarchies between the energy scales of different sectors in the theory while needing only small values of the parameters in the compactification to accomplish this. Phenomenologically, one can imagine this leading to a very light standard model of particle physics sector while all the other ingredients necessary for the compactification remain very heavy.

Finally, one can show that the four-dimensional Planck mass is given by

$$M_{p4}^2 = \frac{M_{p5}^3}{k}\left(1 - e^{-2kr_c\pi}\right) \,. \tag{7.4.6}$$

7.5 Flux vacua

We now turn to compactifications with flux in superstring theory. We will see that such vacua can support warping and that the flux can be used to stabilize moduli [183]. The IIB string theory action in the supergravity limit has for its Bosonic terms

$$\begin{aligned} S_{IIB} =& \frac{1}{2\kappa}\int d^{10}x\sqrt{-G}\left[\mathcal{R} - \frac{|d\tau|^2}{2\left[\mathrm{Im}\tau\right]^2} - \frac{|G_3|^2}{2\mathrm{Im}\tau} - \frac{|\tilde{F}_5|^2}{4}\right] \\ & - \frac{i}{8\kappa^2}\int \frac{C_4 \wedge G_3 \wedge \overline{G}_3}{\mathrm{Im}\tau} \,. \end{aligned} \tag{7.5.1}$$

The fields in this action are convenient combinations of the fields that appeared in chapter 3

$$\tau = C_0 + ie^{-\phi}\,, \quad G_3 = F_3 - \tau H_3\,, \quad \tilde{F}_5 = F_5 - \frac{1}{2}C_2 \wedge H_3 + \frac{1}{2}B_2 \wedge F_3\,. \tag{7.5.2}$$

In addition, $G_{MN} \equiv e^{-\Phi/2}G_{MN}$ compared to eq. (3.1.4) and one must impose by hand the equation of motion

$$\tilde{F}_5 = \star_{10}\tilde{F}_5\,. \tag{7.5.3}$$

The field τ is known as the axio-dilaton.

we will go over some results due to Giddings, Kachru, and Polchinski (GKP) [184] in IIB string theory. Consider the metric

$$ds^2 = e^{2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu + e^{-2A(y)}g_{ij}dy^i dy^j\,. \tag{7.5.4}$$

The external spacetime is flat Minkowski space. The internal manifold is a Calabi-Yau with metric g_{ij}. The facter $A(y)$ dependent on the internal coordinates is the warp factor.

We consider turning on G_3 and $\tilde{F}_5$ flux. In order to preserve Lorentz invariance in the four external dimensions we must impose that only the internal components of G_3 are nonzero. Since $\tilde{F}_5$ is self-dual, we cannot only turn on its internal components, it then preserves four-dimensional Lorentz invariance when

$$\tilde{F}_5 = (1 + \star_{10})\, d\alpha(y) \wedge dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3\,. \tag{7.5.5}$$

By tracing the ten-dimensional Einstein equations over the four external dimensions one obtains

$$\nabla^2 e^{4A(y)} = \frac{e^{8A}}{2\mathrm{Im}\tau}|G_3|^2 + e^{-4A}\left(|\partial\alpha|^2 + |\partial e^{4A}|^2\right)\,, \tag{7.5.6}$$

where the indices are with respect to g_{ij}. One can integrate this over the compact internal manifold. By Stokes' theorem, the left hand side is zero. The right hand side must then also be zero. We see that all terms on the RHS are nonnegative. The only possible solution is then $G_3 = 0$ and constant A and α.

It is then not possible to produced a warped internal manifold with just flux. We saw in section 3.2 that in string theory we have localized objects such as D-branes at our disposal. These produce a local contribution to the energy-momentum tensor $(T_{MN})^{\mathrm{loc}}$. Adding these modifies eq. (7.5.6) into

$$\nabla^2 e^{4A} = \frac{e^{8A}}{2\mathrm{Im}\tau}|G_3|^2 + e^{-4A}\left(|\partial\alpha|^2 + |\partial e^{4A}|^2\right) + \frac{\kappa^2}{2}e^{2A}\mathcal{T}^{\mathrm{loc}}\,, \tag{7.5.7}$$

with

$$\mathcal{T}^{\rm loc} = \left(g^{ij}T_{ij} - \eta^{\mu\nu}T_{\mu\nu}\right)^{\rm loc} \tag{7.5.8}$$

When this extra contribution is positive, warping is still impossible. This is a specialized version of the Maldacena Nuñez no-go theorem against warped compactifications [185] in type II string theory with only fluxes and local sources that contribute positively. This no-go further forbids de Sitter vacua when one only considers these contributions.

When this extra contribution is negative, warping is possible. D-branes that fully cover the external dimensions will not provide a negative contribution, but O-planes will.

Our solution must satisfy the Bianchi identity

$$d\tilde{F}_5 = H \wedge F_3 + 2\kappa^2 T_3 \rho_3^{\rm loc}\,, \tag{7.5.9}$$

with $\rho_3^{\rm loc}$ the locaclized sources of $\tilde{F}_5$-flux. Clearly D3-branes and O3-planes filling the four external dimensions and pointlike on the internal manifold serve as sources, but also for instance D7-branes filling the four external dimensions and wrapping an internal four-cycle Σ_4 can serve as sources at higher α' corrections. Integrating eq. (7.5.9) over $\mathcal{M}_6$ and applying Stokes' theorem yields

$$Q_3^{\rm loc} + \frac{1}{2\kappa^2 T_3}\int_{\mathcal{M}_6} H_3 \wedge F_3 = 0, \tag{7.5.10}$$

with $Q_3^{\rm loc}$ the total three-brane charge. This is known as the tadpole condition.

One can combine eqs. (7.5.5), (7.5.7) and (7.5.9) to derive

$$\begin{aligned}\nabla^2\left(e^{4A}-\alpha\right) =& \frac{e^{8A}}{24{\rm Im}\tau}\left|iG_3 - \star_6 G_3\right|^2 + e^{-4A}\left|\partial\left(e^{4A}-\alpha\right)\right|^2 \\ &+ 2\kappa^2 e^{2A}\left(\mathcal{T}^{\rm loc} - Q_3^{\rm loc}\right)\,.\end{aligned} \tag{7.5.11}$$

This can be integrated over the internal manifold. The LHS must vanish by Stokes' theorem. For the RHS to vanish, each term is again nonnegative and must vanish seperately.

The G_3 flux must be imaginary self-dual (ISD)[10]

$$G_3 = i \star G_3\,, \tag{7.5.12}$$

for the solution to preserve $\mathcal{N}=1$ SUSY, one can show that G_3 has to be $(2,1)$ and primitive[11].

[10] We will denote the ISD flux as G_3^+ and the imaginary anti-self-dual (IASD) flux, $G_3 = -\star G_3$ as G_3^-.

[11] A form is primitive when its contraction with J vanishes.

The sources of three-brane flux must obey

$$\mathcal{T}^{\rm loc} = Q_3^{\rm loc} \,. \tag{7.5.13}$$

The warping must obey

$$e^{4A} = \alpha \,. \tag{7.5.14}$$

A solution to eq. (7.5.11) with D3- and D7-branes, O3- and O7-planes will solve the equations of motion. In fact, it is a solution to the classical supergravity equations of motion even when supersymmetry is broken. We refer to [184] for further details and will now look at the effective four-dimensional theory that emerges from such a compactification.

Computing the four-dimensional theory is greatly complicated by the warping $e^{-4A(y)}$ appearing in front of the four-dimensional metric in eq. (7.5.4). Computing the four-dimensional effective theory with warping present is the subject of warped effective field theory. We will not do this and instead assume the warping is constant so the metric is of the form

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu + e^{2u(y)} \tilde{g}_{ij} dy^i dy^j \,, \tag{7.5.15}$$

with $\tilde{g}_{ij} dy^i dy^j$ unit volume. But wasn't the point of our construction to achieve nontrivial warping? The idea here is to have most of the Calabi-Yau essentially unwarped and then insert in one place in the Calabi-Yau a region of very high warping, known as a warped throat[12]. As long as the circumference of the throat where it is glued into the Calabi Yau is small enough and the throat is far removed from D-branes and O-planes in the Calabi-Yau, it is then hoped that one can treat the geometry as essentially unwarped as far as the Calabi-Yau moduli and brane dynamics are concerned.

Still, it seems unsatisfying to go to the trouble of constructing a warped compactification only to give up on an EFT description of warping at the end. Clearly giving a proper description of how the warping appears in the EFT would be desirable. In chapter 9 we will study warped IIA compactifications. We will develop a method to uplift a class of warped compactifications with $SU(3)$-structure to M-theory compactifications with G_2-holonomy. We will see that after the uplift to eleven dimensions the warping vanishes. Such warped compactifications can then be understood from an unwarped eleven-dimensional perspective without simplifying assumptions on the warping.

Returning to our IIB compactification, we will focus on the case with a single Kähler modulus ρ. The real part of ρ is an axion while the imaginary part

[12]The traditional example of a warped throat is the Klebanov-Strassler (KS) throat [186].

depends on the volume of the internal manifold $\rho \propto ie^{4u}$. We denote this volume-dependent part as

$$\sigma \equiv \mathrm{Im}\,\rho\,, \tag{7.5.16}$$

and denote the volume of the internal dimensions as $\mathcal{V}_6$.

The resulting four-dimensional effective theory after compactification is an $\mathcal{N}=1$ supergravity theory. The sector of this theory that we are interested in are the scalar moduli fields. We collectively denote the scalar moduli as z^m. Their action is given by [184]

$$S_m = \int d^4x\sqrt{-g}\left[K_{m\overline{n}}\partial_\mu z^m \partial^\mu \overline{z}^{\overline{n}} - V\right]\,, \tag{7.5.17}$$

with

$$V = e^{K_0}\left(K^{m\overline{n}} D_m W_0 D_{\overline{n}}\overline{W}_0 - 3|W_0|^2\right)\,. \tag{7.5.18}$$

The metric and covariant derivative on the moduli space are

$$K_{m\overline{n}} = \frac{\partial^2 K_0}{\partial z^m \partial \overline{z}^{\overline{n}}}\,, \quad D_m W_0 = \left[\partial_m + (\partial_m K_0)\right] W_0\,. \tag{7.5.19}$$

The kähler potential is

$$K_0 = -3\log\left[-i\,(\rho-\overline{\rho})\right] - i\log\left[-i\,(\tau-\overline{\tau})\right] - \log\left[-i\int_{\mathcal{M}_6}\Omega\wedge\overline{\Omega}\right]\,. \tag{7.5.20}$$

The superpotential is the Gukov-Vafa-Witten superpotential [187]

$$W_0 = c\int_{\mathcal{M}_6}\Omega\wedge G_3\,, \tag{7.5.21}$$

with $c = g_s^{3/2}/\sqrt{4\pi l_s^4}$ [188].

In order for a solution to preserve $\mathcal{N}=1$ supersymmetry, one must have

$$D_m W_0 = 0\,, \tag{7.5.22}$$

for the value of the moduli at the solution. One can show that

$$K^{\rho\overline{\rho}} D_\rho W_0 D_{\overline{\rho}}\overline{W}_0 = 3|W_0|^2\,. \tag{7.5.23}$$

Using this, the scalar potential can be rewritten with α and $\overline{\beta}$ indices running over all moduli except ρ

$$V = e^{K_0}\left(K^{\alpha\overline{\beta}} D_\alpha W_0 D_{\overline{\beta}} W_0\right)\,. \tag{7.5.24}$$

This potential has a minimum at $V = 0$ when $D_\alpha W_0 = 0$. This condition is independent of ρ and so there is no preferred value where $\mathcal{V}_6$ is stabilized. The potential is said to be of *no-scale* type. We see that this potential stabilizes all moduli except ρ when $G_3 \neq 0$.

Clearly this minimum satisfies the SUSY conditions eq. (7.5.22) for all moduli except ρ. The SUSY condition for ρ gives

$$D_\rho W_0 = -\frac{3W_0}{\rho - \overline{\rho}} = 0\,. \tag{7.5.25}$$

This condition is only satisfied when $W_0 = 0$. Generic minima will have $W_0 \neq 0$ and break supersymmetry. Higher-order corrections will modify the nonsupersymmetric theory and one expects that points with $W_0 \neq 0$ will no generically longer be minima after these corrections.

7.6 KKLT

7.6.1 A stabilized AdS vacuum

In the GKP solution the complex structure moduli have been stabilized. However, the kähler moduli are still massless. Kachru, Kallosh, Linde and Trivedi (KKLT) [189] proposed a method to stabilize the kähler moduli.

The idea is to stabilize the kähler moduli by non-perturbative corrections. Both instantonic Euclidean D3-branes and gaugino condensates have been considered for this. We will focus on gaugino condensates and again restrict to internal geometries with a single kähler modulus ρ.

Place a stack of N D7-branes such that they fill the four external spacetime dimensions $\mathcal{M}_4$ and wrap a four-cycle Σ_4 on the internal manifold. The open string degrees of freedom on the D7 branes then contain a $U(N)$ gauge group. After compactification these give rise to an $\mathcal{N} = 1$ super-Yang-Mills (SYM) theory in the four-dimensional low-energy effective description. The SYM theory contains gauginos λ which can potentially form a condensate $\langle\lambda\lambda\rangle$. This will contribute to the superpotential as [190][13]

$$W_{\langle\lambda\lambda\rangle} = -\frac{N}{32\pi^2}\langle\lambda\lambda\rangle\,. \tag{7.6.1}$$

The value of $\langle\lambda\lambda\rangle$ will depend on the properties of the four-dimensional gauge theory. The coupling constant g_{ym} of the four-dimensional theory depends on

[13] One approximates the four-dimensional gauge theory as $\mathcal{N} = 1$ SQCD on Minkowski space.

the coupling of the seven-brane g_7, eq. (3.2.3), and the volume of the internal cycle $\mathcal{V}_{\Sigma_4}$ as [191]

$$\frac{1}{g_{ym}^2} = \frac{\mathcal{V}_{\Sigma_4}}{g_7^2} \,. \tag{7.6.2}$$

On a manifold with a single Kähler modulus, the volume of Σ_4 is directly determined by the total volume of the internal manifold. The gaugino contribution to the superpotential then depends on σ. One may show that the contribution is given by

$$W_{\langle\lambda\lambda\rangle} = \mathcal{A} e^{-2\pi\sigma/N} \,, \tag{7.6.3}$$

where $\mathcal{A}$ is a parameter dependent on the dynamical scale of the four-dimensional gauge theory.

Assuming one can just sum the contributions eq. (7.6.3) to the superpotential without any mixing or corrections, the total superpotential is

$$W = W_0 + \mathcal{A} e^{-2\pi\sigma/N} \,. \tag{7.6.4}$$

The resulting scalar potential is

$$V(\sigma) = \frac{a\mathcal{A} e^{-a\sigma}}{2\sigma^2} \left[W_0 + \left(1 + \frac{a\sigma}{3}\right) \mathcal{A} e^{-a\sigma} \right] , \tag{7.6.5}$$

with $a = 2\pi/N$. This has a supersymmetric minimum at $\sigma = \sigma_\star$,

$$D_\rho W|_{\sigma=\sigma_\star} = 0 \quad \rightarrow \quad W_0 = -\mathcal{A} e^{-a\sigma_\star} \left(1 + \frac{2}{3} a\sigma_\star\right) . \tag{7.6.6}$$

The scalar potential at this minimum is

$$V(\sigma_\star) = -\frac{a^2 \mathcal{A}^2 e^{-2a\sigma_\star}}{6\sigma_\star} \,, \tag{7.6.7}$$

this is negative and so we obtain a supersymmetric AdS_4 vacuum with all moduli stabilized. Because of the appearance of an exponential damping, the cosmological constant of the AdS_4 can be much smaller than the scale set by the size of the internal geometry $\sigma_\star$. We have conquered the issue with Freund-Rubin compactifications in section 7.1.2 that the cosmological scale is the same as the scale of the internal geometry. Still, the AdS_4 cosmological constant cannot be made arbitrarily small and we still do not have a de Sitter vacuum.

Before we continue, let us make a few comments on the parameter range where we can reasonable expect our computations to be valid. We work in units where the ten-dimensional Planck mass is set to one. We need $\sigma_\star \gg 1$, the volume of our internal geometry to be large, to ensure stringy corrections do

not become important and our supergravity approximations remain valid. We require $|W_0| \ll 1$ to be confident in the results of the gauge theory calculation. In particular, from eq. (7.6.6) this ensures that $\mathcal{A}e^{-a\sigma_\star}$ is small, allowing us to keep only the leading nonperturbative correction.

7.6.2 Towards de Sitter vacua in string theory

We now have a vacuum with all moduli stabilized in IIB string theory. This is already a great achievement. However, the cosmological constant is still negative. KKLT went further and also proposed a method to obtain AdS vacua with an arbitrarily small Λ and even de Sitter vacua.

Since our own work is focused on the preceding step of gaugino condensation, we will be brief here.

The idea is to add supersymmetry-breakining anti-D3 branes ($\overline{D3}$-branes) that fill the external dimensions and are pointlike in the internal dimensions. The antibrane contributions to the potential is [192]

$$V_{\overline{D3}} = D\frac{e^{4A(p)}}{\sigma^2}\,, \tag{7.6.8}$$

with D a positive constant dependent on the number of antibranes. If the antibranes are placed at a generic point p in the internal manifold they will provide a large supersymmetry-breaking contribution to the potential.

If the antibrane contribution is the dominant term in the potential, the potential goes as $V \propto \sigma^{-2}$. There is then no (meta)stable potential minimum, instead the potential is runaway towards decompactification $\sigma \to \infty$. A second issue with a large supersymmetry breaking is that we required supersymmetry to be certain that we solve the equations of motion. When supersymmetry-breaking is small, it seems reasonable that we will be "slightly off" from a solution and by making small modifications to the geometry and fluxes we can obtain a genuine solution to the equations of motion. For large supersymmetry breaking no such logic exists and we may not have a solution at all.

These issues are avoided if one embeds in the internal geometry a highly warped down region, the warped throat. Antibranes are dynamically drawn to the tip of this throat [193] where their contribution to the potential will be exponentially supressed by eq. (7.6.8). A cartoon of the internal geometry in KKLT after these steps can be seen in fig. 7.2.

The full KKLT potential is now given by

$$V_{KKLT}(\sigma) = \frac{a\mathcal{A}e^{-a\sigma}}{2\sigma^2}\left[W_0 + \left(1+\frac{a\sigma}{3}\right)\mathcal{A}e^{-a\sigma}\right] + D\frac{e^{4A(p)}}{\sigma^2}\,. \tag{7.6.9}$$

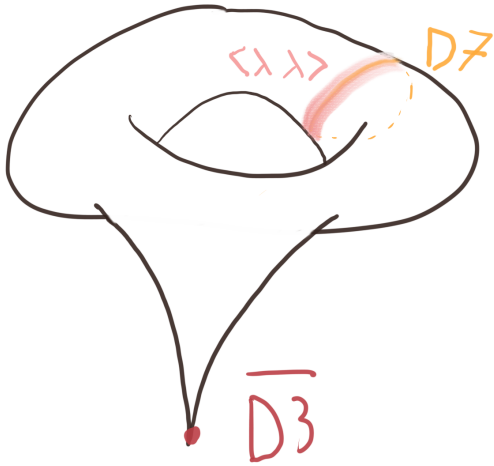


Figure 7.2: A two-dimensional cartoon of the internal dimension of the KKLT set-up.

It is now possible by a careful choice of the parameters in this potential to increase the value of the potential at the minimum without greatly affecting the value σ_0 where the minimum occurs. This raising of the minimum is known as the uplift of the KKLT vacuum. This and the whole process of the KKLT construction are captured in fig. 7.3 .

It is already remarkable that we can now obtain AdS vacua with an arbitrarily small cosmological constant independent of the four-cycle volume σ. Think back to how in the Freund-Rubin compactifications in section 7.1.2 the cosmological constant was always of the order of the scale of the internal manifold.

Even better is that it is now possible to realize de Sitter vacua. These are metastable and can quantum tunnel to $\sigma \to \infty$. This is only one channel of metastability. Not pictured by plotting $V(\sigma)$ is the fact that the antibranes can nonperturbatively annihilate with flux, providing an alternate channel for metastability that already renders the nonsupersymmetric AdS vacua metastable. This process was first discovered in [193].

In the preceding we acted as if the parameters in eq. (7.6.9) are ours to freely choose. This is not the case, these values are fixed by geometric properties of the CY_3 one chooses as a starting point and the details of the physical processes involving the objects in the spacetime such as gaugino condensation and antibrane backreaction. It could be that for a specific implementation of the KKLT scenario the parameters play out such that the minimum at σ_0 is lost

and one has a potential that is runaway to $\sigma \to \infty$ instead of a compactification.

7.6.3 Possible concerns

So far, we went through the steps of the KKLT construction without questioning their validity. In fact, the consistency of the KKLT vacua is hotly debated. See e.g. [161] for an overview of concerns about two years ago. We will go over some concerns that we feel are most pertinent, including some that are new since then.

The supersymmetric KKLT vacuum has $W_0 \neq 0$. It then starts nonsupersymmetric and requires the nonperturbative effects of $\langle\lambda\lambda\rangle$ to restore supersymmetry. In [194] it was argued that a point with $W_0 \neq 0$ is not suitable as a background to compute nonperturbative effects as after corrections it will generically result in a rolling solution.

It is worth stressing again that the parameters in eq. (7.6.9) are not ours to choose. The parameters depend on the details of the physics of the compactification. Unless explicit examples are worked out, there is no guarantee that any de Sitter vacua or AdS vacua with a parametrically small cosmological constant exist. The compactification one considers should further be in a reliable regime. If for instance the scale of a cycle in the internal manifold is Planckian, one cannot be confident in an effective supergravity description.

The $\overline{D3}$ branes break SUSY. One could worry about their backreaction. This issue seems to have been resolved with the conclusion that the $\overline{D3}$ branes polarize the flux near them without producing an instability [195–202].

One needs to show that a throat geometry which generates sufficient warping can be embedded into the Calabi-Yau. This throat needs to be stable against backreaction from $\langle\lambda\lambda\rangle$ and the $\overline{D3}$ branes. See for instance [203–209] for recent work on this topic.

The issue we will focus on in our own work in chapter 8 is the description of the gaugino condensate. In section 7.6.1 we relied entirely on a four-dimensional description. Ideally, one would have an entriely ten-dimensional description and show that the gaugino condensate behaves sensibly from the ten-dimensional perspective.

- One should compute the value of $\langle\lambda\lambda\rangle$ in ten dimensions. At leading order, $\langle\lambda\lambda\rangle$ should be localized entirely on $(A)dS_4 \times \Sigma_4$ but one expects higher-order quantum corrections to cause some smearing over the entire ten-dimensional spacetime.

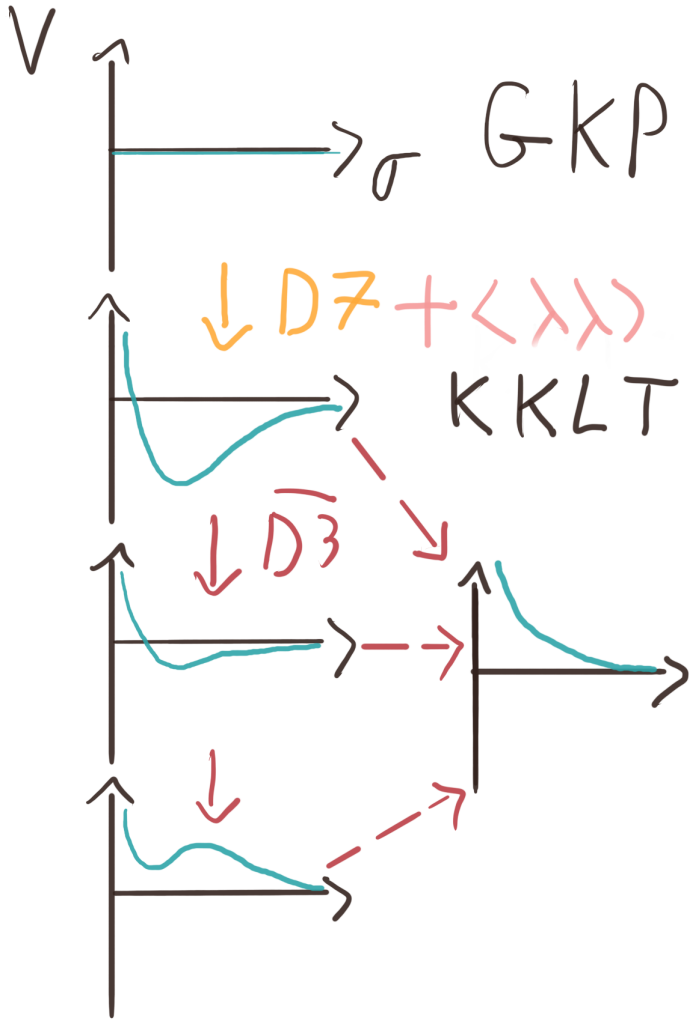


Figure 7.3: An outline of the $V(\sigma)$ potential in the KKLT construction. One starts with the complex strucure moduli stabilized, but not the Kahler modulus σ. The introduction of a $D7$ brane stack (yellow) and the quantum effects of the gaugino condensate stabilize σ at an AdS vacuum. Uplifting with $\overline{D3}$ branes (red) uplifts the potential minimum but possibly causes a runaway (dotted).

- One should compute how $\langle\lambda\lambda\rangle$ backreacts on the ten-dimensional geometry.

The first point is unfortunately not within our grasp, but we will be able to make progress on the second point, taking $\langle\lambda\lambda\rangle$ to be an undetermined parameter. This program of providing a ten-dimensional description of SUSY KKLT vacuum was initiated in [203, 210]. Unfortunately, they ran into a problem. The total action S contains a contribution from the ten-dimensional degrees of freedom S_{10D} and a part from the degrees of freedom on the D7-brane stack S_{D7}. This can be described as one ten-dimensional action by introducing a Dirac-delta function $\delta_\perp(y)$ on the directions perpendicular to the D7 branes. The total action is then

$$S = \int d^{10}x \left[\mathcal{L}_{10D} + \delta_\perp(y)\mathcal{L}_{D7}\right] . \tag{7.6.10}$$

The gaugino condensate couples to the G_3 flux [211]. Ignoring for the moment all other fields in the action, this and the G_3 kinetic term lead to an action schematically of the form

$$S = \int d^{10}x \left[\frac{1}{2}|G_3|^2 + \delta_\perp(y)G_3\lambda\lambda\right] . \tag{7.6.11}$$

Solving the G_3 equation of motion then leads to a contribution to G_3 that diverges as $\delta_\perp(y)$. This is not yet necessarily concerning. However, computing the curvature of the four external dimensions, one finds [210]

$$\mathcal{R}_4 = -\frac{1}{\mathcal{V}_6}\int \star_6 \frac{e^{2A}}{\mathrm{Im}\tau}\left(|G_3^-|^2 - \frac{3}{2}|G_3^+|^2\right) . \tag{7.6.12}$$

Filling in the solution to the G_3 equation of motion to obtain the on-shell value for $\mathcal{R}_4$, there appears a nonvanishing $\delta_\perp^2(y)$ contribution. As a result, there is a singular contribution to $\mathcal{R}_4$. This singularity was believed to be unphysical and [212] proposed a method to remove it. In supersymmetric theories, the action tends to decompose into a sum of squares. Then, [212] argued that the higher-order terms in S_{D7} should contain quartic in the gauginos so that the total action is

$$S = \int d^{10}x \left[\frac{1}{2}|G_3|^2 + \delta_\perp(y)G_3\lambda\lambda + \frac{1}{2}\left(\delta_\perp(y)\lambda\lambda\right)^2\right] \tag{7.6.13}$$

$$= \int d^{10}x \frac{1}{2}\left|G_3 + \delta_\perp(y)\lambda\lambda\right|^2 , \tag{7.6.14}$$

where [212] claimed the quartic term should resolve spurious singularities. In chapter 8 we will give a more detailed discussion and investigate how this quartic term affects the ten-dimensional description of the KKLT vacuum.

Chapter 8

A 10d view on the KKLT AdS vacuum and uplifting

This chapter is a reprint of [3]. We analyse the ten-dimensional Einstein equations in the KKLT setting. We verify that the quartic gaugino term is needed to remove singularities in the on-shell action as suggested by Hamada et. al. We contrast two approaches that have been taken in the literature when employing the effect of gaugino condensation in the ten-dimensional equations of motion. Here we follow the proposal to insert explicit non-zero fermion bilinar vev into the localised energy-momentum tensor of the 7-branes obtained from varying the 10d off-shell action with respect to the 10d metric. Our procedure is common in semi-classical physics and is manifestly local in 10d. However, it does not lead to the KKLT effective field theory. The alternative procedure of deriving the energy momentum tensor after replacing fermion bilinears by the gaugino vev, might be less well motivated in 10d, but reproduces the results of the KKLT effective field theory.

8.1 Introduction

As we reviewed in sections 7.5 and 7.6 , moduli stabilisation in string theory has a long history and its understanding is crucial for the study of top-down phenomenology. One of the main breakthroughs came in 2003 with the KKLT model [189] and in 2005 with the Large Volume Scenario [213]. Both methods to stabilise moduli rely on computing leading-order corrections to ten-dimensional

supergravity solutions with orientifold sources and three-form fluxes in type IIB supergravity [183, 184, 214]. Despite the long history of this field there has never been any genuine top-down understanding of these mechanisms to stabilise moduli. The arguments always involved a mixture between top-down and bottom-up viewpoints. This is not a problem per se, but it can lead to a false sense of freedom to tune parameters in the bottom-up effective field theory. It is therefore desirable to find at least one concrete top-down description of moduli-stabilisation.

Recently the field of moduli stabilisation has regained attention due to a growing suspicion that string theory may fail to accommodate any de Sitter (dS) vacuum, see section 7.6.3 .

This revival of interest has provided a healthy spark of ideas to construct new possible dS vacua (or more general accelerating cosmologies) [215–220] or revisit existing ones [203, 206, 210, 221]. In this paper we direct our attention to the consistency of the KKLT model; both the construction of the AdS vacuum and its potential uplift to a meta-stable dS vacuum. In particular the uplift procedure has been heavily criticised on two fronts. First the stability of the anti-branes themselves came under intense scrutiny in [222–224]. It is our opinion that the arguments pointing towards the instability of anti-branes have been to a large extent addressed and refuted in [196, 197, 225, 226] (see however [227, 228]). The second concern, raised in [203] (see also [229]), has to do with the interaction between the open string degrees of freedom of the anti-brane and closed string degrees of freedom of the background. This interaction is only possible in a compact model where the two effects cannot be separated indefinitely.

The authors of [203] suggested that it should be possible to study the interaction using classical ten-dimensional supergravity including the energy-momentum of a gaugino condensate on D7-branes that live in the UV.[1] However, there is some confusion regarding the gravitational backreaction of the gaugino condensate in ten-dimensional supergravity. This was discussed in [203, 210] and we refer the reader to those references for further elucidations. The main observation of [203] was that the ten-dimensional energy-momentum tensor behaves such that a dS vacuum is not attainable with a single gaugino condensate. From a four-dimensional viewpoint this effect can be understood as a flattening of the potential caused by a backreaction of the supersymmetry-breaking ingredients on the volume modulus.

Later, it was shown in [210] that there is a loophole in the argument of [203] due to a subtlety in the computation. Unfortunately the issue could not be settled in [210] since some of the terms in the energy-momentum tensor were

[1] Other somewhat orthogonal worries about the consistency of uplifting were expressed in [194, 206].

divergent and a regularisation is needed that could influence the fate of the flattening effects. Recently Hamada *et al* [212] (see also [230]) proposed how to perform this regularisation by adding a quartic fermion term to the action. In this note we redo the computations in [203, 210] including this new term. We find that indeed the four-dimensional curvature is expressed entirely in terms of regular *background* fluxes and the gaugino condensate, but we fail to find the KKLT effective theory and explain why.

Simultaneously with this work two other works appeared [231, 232] about the same problem and one of these works [232] comes to a very different conclusion then us. We discuss in section 8.6 on the relation between our work and these two other papers. The difference relates to manner the energy-momentum tensor is computed and the alternative method of [232] and [231] does reproduce qualitatively the KKLT effective field theory and finds no dangerous flattening effects when anti-branes are added.

8.2 The framework

The KKLT proposal [189] to stabilize moduli in a supersymmetric AdS vacuum rests on reasonable assumptions about quantum corrections to the classical flux compactifications of [183, 184]. These corrections have been argued for using four-dimensional effective field theories described within $\mathcal{N} = 1$ supergravity as reviewed in section 7.6 . The quantum correction used by KKLT is the leading non-perturbative quantum effect to the superpotential W. Perturbative corrections to W are absent because of non-renormalisation theorems whereas it is argued that (non-)perturbative corrections in the Kähler potential K can sometimes be self-consistently ignored.[2] The non-perturbative quantum correction arises as a result of gauginos living on a stack of D7-branes condensing in a confining vacuum of the *four-dimensional* gauge theory. In what follows we assume, like KKLT, that the underlying Calabi-Yau manifold only admits a single Kähler deformation. It then follows that there is only one (holomorphic) 4-cycle wrapped by the D7-branes.

At first sight the gaugino condensation obscures a dimensional oxidation of the KKLT model. However, it has been suggested in [203] that it should nonetheless be possible using ten-dimensional supergravity with D7 probe actions included. The way the gaugino condensate couples to gravity and other closed strings could then be understood simply by keeping explicit non-zero gaugino bilinears

[2]The Large Volume Scenario [213] uses perturbative corrections to the Kähler potential to stabilize the volume modulus at exponentially large value in a non-supersymmetric vacuum.

of the fermionic part of the D7 action and compute its contribution to the ten-dimensional energy-momentum tensor [233–237].

Like [203] we compute the trace over the four-dimensional part of the (trace-reversed) Einstein equation. We will perform this computation in detail in next section but here we will discuss the form of the resulting expression,

$$R_4 = \int_6 \sqrt{g_6}\, \mathcal{E}[F^2, \lambda^2]\,, \tag{8.2.1}$$

where R_4 is the curvature scalar of the four large dimensions, $\int \sqrt{g_6}$ integration over the compact dimensions and $\mathcal{E}[F^2, \lambda^2]$ represents some function of the background fluxes (F^2) and the gaugino bilinear (λ^2). We find that our approach is consistent with an AdS spacetime but further relations between the fields are required to make a quantitative comparison with the KKLT result. Strictly speaking the method of the trace-reversed Einstein equation allows to compute only the curvature of a vacuum solution which is related to the on-shell potential. The relation between fields required to make a comparison to KKLT is only obtained from the off-shell potential which we do not derive here.

Adding anti-branes to this computation is straightforward as explained in [203, 210]. One can show it simply adds the following term to the right hand side of equation (8.2.1):

$$R_4 = \int_6 \sqrt{g_6}\, \left(\mathcal{E}[F^2, \lambda^2] - 2nT\delta_6\right)\,, \tag{8.2.2}$$

where T is the warped tension of a stack of anti-D3 branes and n the number of anti-D3's. This seems a contradiction since this term contributes negatively compared to the rest, so how can it uplift? In fact [203] realised that, in the right approximation scheme, the anti-brane term is completely subdominant compared to the bulk piece $\mathcal{E}$ and should be ignored.

This naive "paradox" arises because (8.2.2) computes an *on-shell* four-dimensional potential instead of a full off-shell expression[3]. The way uplifting should work from this viewpoint is rather different. Uplifting works by a shift of the volume modulus ρ induced by the supersymmetry-breaking, altering the value of $\mathcal{E}$ and potentially flipping its sign. To make that point very explicit let us for simplicity assume that the term $\mathcal{E}$ has two pieces

$$\mathcal{E}[F^2, \lambda^2; \rho] = \mathcal{E}_+[F^2, \lambda^2; \rho] + \mathcal{E}_-[F^2, \lambda^2; \rho], \tag{8.2.3}$$

where $\mathcal{E}_-$ is negative definite and $\mathcal{E}_+$ positive definite and we introduced the symbolic dependence on ρ. In the supersymmetric vacuum we obviously have

[3] See [203, 210] for a more elaborate explanation.

$\mathcal{E}_+[\rho] < -\mathcal{E}_-[\rho]$. But when ρ shifts to $\rho + \delta\rho$ due to supersymmetry breaking we expect that the value of R_4 and therefore $\mathcal{E}[\rho + \delta\rho]$ is less negative. A dS is reached by making sure that $\mathcal{E}_+[\rho + \delta\rho] > -\mathcal{E}_-[\rho + \delta\rho]$.

Apart from the ρ-dependence the only quantities appearing in $\mathcal{E}$ are the *background* fluxes and gaugino vev. We can assume that their values have not altered when breaking supersymmetry. This is a crucial assumption that is at the core of the KKLT model. In other words the fluxes F^2 are the ones of the GKP solution in the background. One could worry about this assumption since there are two sources for deviations from the GKP solution: 1) near the anti-D3 branes non-ISD fluxes are sourced whereas the original GKP construction only allowed for ISD fluxes [184]. But as we emphasised earlier this is a local backreaction effect, and any extra terms in the Einstein equation that are otherwise missed in the approximation will only contribute negatively. 2) Near the 7-branes also non-ISD fluxes are sourced. Even worse, these fluxes are divergent, similar to what was believed to be the case for anti-D3 branes [222,224] but later shown not too happen because of brane polarisation [196, 197]. In fact the discussion of anti-D3 flux singularities parallels that of D7 gaugino-induced flux singularities. Whereas brane polarisation cures the singularities for anti-D3 branes, it was shown in [212] that quartic gaugino-terms, previously ignored, do the same for D7-branes.

The essence of this paper is the computation of the right hand side of equation (8.2.2) incorporating the quartic gaugino term of [212] and is outlined in the next section.

8.3 A subtlety in the semi-classical Einstein equations

At this point a note of caution is required. While this paper was finalised we learned of two other papers [231, 232] that appeared about the same problem. Their results differ from ours and the reason is rather straightforward. The difference relies in how one computes the semi-classical Einstein equation in 10-dimensions:

$$G_{\mu\nu} = \langle T_{\mu\nu} \rangle. \tag{8.3.1}$$

Our method relies upon the following procedure. We first compute the form of the tensor $T_{\mu\nu}$ in 10 dimensions by varying the action with respect to the 10d metric *keeping all other fields fixed* and keep the fermion bilinears arbitrary. Subsequently we use the 4d gauge theory results for the vevs of the fermion bilinears to fix the expression of $T_{\mu\nu}$. Notice that since the 4d vevs of the fermion bilinears have an explicit dependence on the volume of the compact

space, there is an implicit dependence on the 10d metric. Hence this procedure will lead to different result than if one instead first replaces the fermion bilinears in the 10d action with the 4d vevs and *then* compute the variation of the action with respect to the volume. In Section 8.6.2, we demonstrate that this difference in the order of variations leads to the results of Hamada et al [232]. Note that our method gives a manifest local description in 10d whereas there seems to be no ten-dimensional, manifestly local, covariant description in the approach of [231, 232].

The main advantage of the approach followed by [232] is that it seems to directly reproduces the 4d computation done by KKLT. In this work we prefer to remain agnostic about the correctness of the 4d result for multiple reasons which we now explain. First of all the motivation of our paper is to establish (or refute) the KKLT model from a 10d perspective, before and after uplift. Hence we cannot assume the 4d EFT as a prior. Alternatively one can regard the spirit of this work to be a test for the proposal that a 10d derivation of KKLT can be found using fermion bilinear vevs in 10D as the only relevant extra contribution to the "GKP" background. Whether semi-classical 10d supergravity with localised 7-brane sources is the appropriate description of the four-dimensional gauge theory in its confining phase is an open question. In this paper we report our results assuming that for the sake of determining the vacuum, it is adequate. Secondly, the precise way in which gaugino condensates couple to gravity is not understood, even in four dimensions. KKLT makes a proposal to simply sum the background GVW superpotential with the field theory superpotential. This is reasonable, but we are not aware of a derivation of this and caveats can be around the corner. For instance, since gaugino condensation is a consequence of long-wavelength fluctuations of fields, one expects it to be sensitive to the background it is supposed to live on. Hence gaugino condensation can look different in AdS or some other curved space. In other words: how exactly gaugino condensation couples to gravity can be more complicated than simply adding it into the supergravity F-term. Furthermore, the classical gravity background breaks supersymmetry via its fluxes. This means that there is an explicit SUSY breaking occuring in theory once we incorporate the gravitational degrees of freedom and hence the assumptions of having SUSY of the off-shell Lagrangian, required for computing gaugino condensation, is not obvious. In what follows we lay out the details of the computation of the 4d piece in the 10d Einstein equation (8.3.1). As we will show, our results are not incompatible with an AdS solution but we are unable to determine a quantitative value for the vacuum energy that can be compared with the KKLT results. This will be discussed further in Section 8.6.

8.4 The four-dimensional cosmological constant

In this section we utilise the approach of [184] to compute the four-dimensional curvature when compactifying ten-dimensional type IIB supergravity with fluxes and D7-brane sources with non-vanishing gaugino bilinear. In doing so, we assume a maximally symmetric four-dimensional spacetime. The ten-dimensional metric reads [4]

$$ds_{10}^2 = \epsilon^{2A} ds_4^2 + \epsilon^{-2A} ds_6^2 \ , \tag{8.4.1}$$

where ds_4^2 is a maximally symmetric on a four-dimensional spacetime, with a constant curvature scalar R_4. The warp factor A only depends on the coordinates of the transverse manifold M_6 equipped with the metric $\epsilon^{-2A} ds_6^2$. Maximal symmetry demands that the ten-dimensional axion-dilaton τ and 3-form field strength G_3 only have legs and dependence along M_6. The type IIB 5-form F_5 is self-dual and given by

$$F_5 = (1 + \star_{10}) dC_4 \ , \quad C_4 = \alpha \, \mathrm{vol}_4 \ , \tag{8.4.2}$$

where α is a function on M_6. The inclusion of three-form fluxes induces three-brane charge that contributes to the tadpole which must be cancelled by the inclusion of local sources such as O3/O7 planes. Taking these ingredients into account, [184] found a consistency condition demanded by the trace-reversed Einstein equation that takes the form[5]

$$\triangle_6 \Phi^- = R_4 + \frac{\epsilon^{2A}}{\mathrm{Im}\, \tau} |G_3^-|^2 + \epsilon^{-6A} |d\Phi^-|^2 \ , \tag{8.4.3}$$

where

$$\Phi^\pm = \epsilon^{4A} \pm \alpha \ , \quad G_3^\pm = \phi 12 (\star_6 \pm i) G_3 \ . \tag{8.4.4}$$

This equation can also be derived directly from the type IIB action (together with O3/O7 source terms) upon varying with respect to the warp factor A (see appendix C). The power of this equation becomes apparent when one integrates (8.4.3) over M_6. The left-hand-side integrates to zero which implies that the four-dimensional curvature scalar R_4 is non-positive. The GKP vacua are the ones with $R_4 = G_3^- = \Phi^- = 0$.

[4] Note that the four-dimensional metric ds_4^2 is not in four-dimensional Einstein frame because the volume modulus has not been properly factored out. This does not cause any issue with our computation in this section since we assume that all moduli are stabilised by the gaugino condensate. We will return to this issue when we make an explicit comparison to KKLT in section 8.5.

[5] Here $\triangle_6 \Phi^- = - \star_6 \dot{\delta} \star_6 \dot{\delta} \Phi^-$ and all inner products of forms are ten-dimensional, i.e. they include warp factors.

We now derive the equation corresponding to (8.4.3) when $N > 1$ D7-branes are included with non-vanishing fermion bilinear. At this stage we will be agnostic about the value of the fermion-bilinear, and assume that it will take some non-vanishing value at low energies where the gauge group condenses. In the procedure outlined we combine the ten-dimensional type IIB action with the effective action of D7-branes. The bosonic D7-brane action

$$S_{\rm D7} = -2\pi N \int_{\Sigma_8} d^8x ({\rm Im}\tau)^{-1} \sqrt{|P[g]|} + 2\pi N \int_{\Sigma_8} P[C_8] \ , \tag{8.4.5}$$

does not contribute to (8.4.3) since "it is BPS" with respect to the background.[6] One can verify this claim by using the ansatz (F.0.7) and noticing that the warp factor drops out. Even though the bosonic action does not play an important role in our discussion, the fermion terms do. Indeed the gauge theory living on the D7-brane world volume descends to an $\mathcal{N} = 1$ gauge theory in four dimensions with various matter couplings. In particular the eight-dimensional world-volume fermions give rise to the $\mathcal{N} = 1$ gaugini in four dimensions.[7] The fermionic D7-brane action contains an interaction term beween the gaugino bilinear $\lambda\lambda \equiv {\rm tr}(\lambda^{\dot\alpha}\lambda_{\dot\alpha})$ and the three-form G_3 [211] (see [238] for an early discussion)

$$S^{\rm ferm}_{\rm D7} = \pi \int \star_{10} \left(G_3 \cdot I + \overline{G_3} \cdot \bar I \right) \ , \qquad I = \frac{e^{-4A}}{\sqrt{{\rm Im}\,\tau}} \frac{\bar\lambda\bar\lambda}{16\pi^2} \Omega \ \delta(\Sigma_8) \, . \tag{8.4.6}$$

The contribution of the fermionic terms to (8.4.3) turns out to be singular due to the backreaction of the fermion bilinears on the three-form fluxes [203, 210]

$$G_3 \sim ({\rm Im}\,\tau) \bar I \ , \tag{8.4.7}$$

where I is the source appearing in (8.4.6) and carries an explicit delta function. In fact, even the on-shell action is UV divergent due to this singular backreaction. In [212] it was suggested that the singular backreaction is ultimately a result of omitting a four-fermion term in the D7-brane action. Currently the four-fermion terms are not known for the D-brane action and so [212] suggested to fix their form such that the singularities would be cancelled. The suggestion of [212] was inspired by a similar problem appearing in Hořava-Witten theory [239–241] which was resolved by a four-fermion term that appears together with the M-theory 4-form in a perfect square. This was in direct analogy with a similar square structure that appears in heterotic supergravity [230, 242]. It is therefore reasonable to assume that the fermionic D7-brane coupling (8.4.6) together with

[6] BPS is used in a loose form since fluxes can already break supersymmetry of the GKP background.

[7] We refer to [203, 210] for an explicit map beween the eight-dimensional fermions and the gaugino.

the kinetic terms for G_3 should be combined with the four-fermion terms to form a perfect square [212, 230]. Indeed this goes a long way to prevent singular on-shell action as a result of backreaction the fermion bilinear.

The perfect square replaces the original action for G_3 and the fermionic coupling by

$$S_3 = -\pi \int \star_{10} \frac{\left|G_3 - (\mathrm{Im}\,\tau)\mathcal{P}(\bar{I})\right|^2}{(\mathrm{Im}\,\tau)}\,, \tag{8.4.8}$$

The projector $\mathcal{P}$ is defined to eliminate the coexact piece of the form it acts on. It does not affect the G_3 equation of motion but is the final piece of the puzzle to ensure that the on-shell action is regular [212]. Upon expanding the square the action consists of three terms:

$$S_3 = -\pi \int \star_{10} \frac{|G_3|^2}{(\mathrm{Im}\,\tau)} + \pi \int \star_{10} \left(G_3 \cdot \mathcal{P}(I) + \overline{G_3} \cdot \mathcal{P}(\bar{I})\right) - \pi \int \star_{10} (\mathrm{Im}\,\tau)|\mathcal{P}(I)|^2\,. \tag{8.4.9}$$

The first term is the standard bulk term for G_3, the second term is the fermionic action of the D7-brane considered before in [203, 210, 235], with the exception that the three-form I is now projected to the set of closed forms. The last term is the new quartic fermion term. We are now ready to re-derive the equation (8.4.3) with the D7-brane fermion contribution. The explicit computation is carried out in appendix C and the result is

$$\triangle_6 \Phi^- = R_4 + e^{-6A}|d\Phi^-|^2 + e^{2A}\left(\frac{|G_3^-|^2}{\mathrm{Im}\,\tau} - \frac{3}{4}\mathrm{Re}\left(G_3 \cdot \mathcal{P}(I)\right) + \frac{\mathrm{Im}\,\tau}{4}|\mathcal{P}(I)|^2\right)\,. \tag{8.4.10}$$

This expression is exact and despite its appearance is completely regular.[8] In order to demonstrate this and further analyse equation (8.4.10) we must now make simplifying approximations.

First we rewrite the delta function appearing in the source term I in terms of a Green's function. Schematically we write

$$\frac{\partial^2 G}{\partial z \partial \bar{z}} = \delta(\Sigma_8) - \frac{1}{\mathcal{V}_2}\,, \tag{8.4.11}$$

where z and $\bar{z}$ are complex coordinates on the transverse two-cyle and $\mathcal{V}_2$, a volume factor of that two-cycle, is introduced to make this equation consistent. We see that the 3-form $I \sim \delta(\Sigma_8)\Omega$ is not closed and so the role of the projector

[8] Equation (8.4.10) does have delta-function sources on the right-hand-side but is regular upon integration. This should be compared to the situation without quartic fermion terms where even the integrated expression was singular due to δ^2 terms appearing.

$\mathcal{P}$ in this case is to add terms of the form $(\partial_z\partial_z G)\Omega$ such that the resulting expression is closed. Explicitly we have [212]

$$\mathcal{P}(\bar{I}) = \frac{e^{-4A}}{\sqrt{\text{Im}\,\tau}}\frac{\lambda\lambda}{16\pi^2}\left(d\frac{\partial G}{\partial z}\wedge\bar{\Omega}_2 + \frac{1}{\mathcal{V}_2}\bar{\Omega}\right), \tag{8.4.12}$$

where Ω_2 is the holomorphic two-form on the holomorphic four-cycle. Notice that we have made a crucial assumption here which we will continue making in the following. Namely we have assumed that close to the seven-branes where most of our analysis takes place, we can treat the warp factor as constant. Furthermore we assume the so-called Sen limit [243] where the stack of 7-branes consists of an O7 with 4 parallel D7-branes on top, in this case the 7-brane stack does not source a gradient for τ and so it can be taken to be a constant.[9] These assumptions allow us to determine the backreacted three-forms in the same spirit as [235]. For notational simplicity we write

$$\mathcal{P}(I) = I^{\text{sing}} + I^{\text{reg}}, \tag{8.4.13}$$

where I^{sing} is the exact part of $\mathcal{P}(I)$ in (8.4.12) which is also singular, and I^{reg} is the harmonic part which is regular. Finally in order to solve for G_3 we assume that the background close to the seven-branes is only slightly perturbed from the standard GKP background for which $\Phi^- = e^{4A} - \alpha = 0$. By our previous assumption that ϵ^{4A} is constant, this implies that the Chern-Simons terms in the action does not enter in the equations of motion for G_3. With these approximations, the solution to the equation of motion for G_3 derived from action eq. (8.4.8) takes a simple form[10]

$$G_3 = (\text{Im}\,\tau)\overline{I^{\text{sing}}} + G_3^0, \tag{8.4.14}$$

where G_3^0 is harmonic and quantised. We can now evaluate the expression (8.4.10) which constrains the four-dimensional cosmological constant. Substituting the solution of the G_3 equation of motion (8.4.14), and after some algebra outlined in appendix C we find

$$0 = \int \star_6 \left(R_4 + \frac{e^{2A}}{\text{Im}\,\tau}|G_3^{0,-}|^2 - \frac{3e^{2A}}{4}\text{Re}\,\left(G_3^0\cdot I^{\text{reg}}\right) + \frac{e^{2A}}{4}(\text{Im}\,\tau)|I^{\text{reg}}|^2\right), \tag{8.4.15}$$

[9] One could wonder whether in this case the orientifold projection does not eliminate the backreaction of the gaugino bilinears on the three-forms. We do not address this question here, but argue that our setup is sufficiently close to the Sen limit such that the gradients of τ do not enter without eliminating the three-form backreaction altogether.

[10] Under these assumptions, the Chern-Simons action does not contribute to the G_3 equations of motion. The details of the fate of the Chern-Simons action in our set-up are discussed in appendix D. Note that since I^{reg} is harmonic with these assumptions, it can always be combined with an arbitrary harmonic contribution that can be added to a solution to the equations of motion to produce another solution to the equations of motion and we denote the entire harmonic contribution as G_3^0.

The equation only contains the background flux G_3^0 and the fermion condensate through the regular part $I^{\rm reg}$. We notice that all singular terms have been eliminated from the expression as a result of our approximations. We expect that more generally the expression (8.4.10) will be regular also when the approximations are relaxed.

Equation (8.4.15) is the main result of our paper. From it we can extract the four-dimensional curvature R_4. However we notice that as it stands the curvature is not restricted to be negative for all possible configurations of the fields. In the next section we discuss a schematic comparison to the four-dimensional KKLT vacuum before any anti-D3 branes are included.

8.5 Comparing to KKLT

We now compare our results with the four-dimensional effective theory considered by KKLT [189]. This theory is specified in terms of a Kähler potential K and superpotential W, given by

$$K = -3\log(2{\rm Im}\,\rho) - \log(2{\rm Im}\,\tau)\ , \qquad W = W_0 + \mathcal{A}\,\exp(2\pi i\rho/N)\ , \quad (8.5.1)$$

where ρ is the complex volume modulus, related to the volume of the internal manifold by ${\rm Im}\,\rho \sim \mathcal{V}_6^{2/3}$ and W_0 is the constant Gukov-Vafa-Witten superpotential (after integrating out the complex-structure moduli). In order to compare our ten-dimensional result (8.4.15) with the effective theory (8.5.1) we translate two results from the four-dimensional effective field theory back to ten dimensions. In particular we view the fluxes G_3^0 as the ISD ones, before the inclusion of the non-perturbative effects. Therefore $G_3^{0,-}$ vanishes and $G_3^0\cdot\Omega$ can be related to W_0[11]

$$\int \star_6(G_3^0\cdot\Omega) = W_0\ . \quad (8.5.2)$$

In KKLT the gaugino condensate $\lambda\lambda$ is related to the non-perturbative effects appearing in the four-dimensional superpotential (8.5.1). We use the standard gauge theory expression for the fermion bilinear in the confining vacuum (here in string units) [244]

$$\langle\lambda\lambda\rangle = 16\pi i\ e^{K/2}\ \partial_\rho W = -\frac{32\pi^2}{N}\frac{\mathcal{A}\epsilon^{2\pi i\rho/N}}{(2\ {\rm Im}\,\rho)^{\frac{3}{2}}}({\rm Im}\,\tau)^{-\frac{1}{2}}\ , \quad (8.5.3)$$

where, like KKLT [189] we used that the complex volume modulus ρ equals the complex gauge coupling of the four-dimensional gauge theory. We also

[11] We would like to thank Pablo Soler for correcting a phase factor convention.

rescaled with the string coupling as in [232]. We note that $\mathcal{A}$ is determined by the dynamically generated scale of the IR theory and is left unspecified.

We emphasise here that the exponential dependence of the volume modulus is crucial in the 4d EFT and its off-shell scalar potential. When balanced with the small flux contribution W_0, this exponential stabilises the Kähler modulus at a somewhat large volume and exponentially small values of the 4d scalar potential. The 10d supergravity analysis performed here, where we opted for a local description of the gaugino condensate, does not reproduce this exponential behaviour. This is reminiscent from the fact that this procedure requires to plug in the vev after varying the action with the metric (which only renders polynomial dependences). Nevertheless we proceed in further rewriting eq. (8.4.15).[12] Using the map between the four-dimensional theory and the ten-dimensional quantities we find the following result

$$\int \star_6 \mathrm{Re}\left(G_3^0 \cdot I^{\mathrm{reg}}\right) = \frac{e^{-4A}\mathrm{Re}\left(W_0\bar{\lambda}\bar{\lambda}(\mathrm{Im}\,\rho)^{\frac{3}{2}}\right)}{16\pi^2\mathcal{V}_2\sqrt{\mathrm{Im}\,\tau}}, \tag{8.5.4}$$

where we made use of eq. (8.5.2).

More explicitly, we can insert the expression for I^{reg} in (8.4.12) and make use of the rescalings of R_4 and $\lambda\lambda$ as above to find

$$R_4 = -\frac{2}{\mathcal{V}_2^2}(\mathrm{Im}\,\rho)^3\left|\frac{\lambda\lambda}{16\pi^2}\right|^2 + \frac{3}{4}\frac{1}{\mathcal{V}_2\sqrt{\mathrm{Im}\,\tau}}\mathrm{Re}\left(\overline{W_0}\frac{\lambda\lambda}{16\pi^2}\right). \tag{8.5.5}$$

We will also extract the volume dependence out of $\mathcal{V}_2 = \tilde{\mathcal{V}}_2(\mathrm{Im}\,\rho)^{\frac{1}{2}}$ so that we can rewrite eq. (8.5.5) with the explicit volume dependence

$$R_4 = -\frac{2}{\tilde{\mathcal{V}}_2^2}(\mathrm{Im}\,\rho)^2\left|\frac{\lambda\lambda}{16\pi^2}\right|^2 + \frac{3}{4}\frac{1}{\tilde{\mathcal{V}}_2\sqrt{(\mathrm{Im}\,\rho)(\mathrm{Im}\,\tau)}}\mathrm{Re}\left(\overline{W_0}\frac{\lambda\lambda}{16\pi^2}\right). \tag{8.5.6}$$

Without further information we are unable to relate the two terms in (8.5.6). Based on the four-dimensional effective theory, we expect the two terms to be related to each other on shell but without solving the equations of motion we are unable to verify this. In particular, this equation does not directly reveal the sign of the curvature. A negatively curved AdS solution is however certainly

[12] It is important to note the λ appearing in equation (8.5.3) is a canonically normalised gaugino in Einstein frame. As mentioned in footnote 4, the 4d metric must be rescaled in order to go to proper 4d Einstein frame. This amounts to a rescaling of the metric $g_4 \to (\mathrm{Im}\,\rho)^{-3/2}g_4$ which leads to the rescaling of the curvature $R_4 \to (\mathrm{Im}\,\rho)^{3/2}R_4$. In this new frame the fermion gaugino term is not canonical and so we must also rescale the gaugino $\lambda \to (\mathrm{Im}\,\rho)^{9/8}\lambda$ such that $\lambda\lambda \to (\mathrm{Im}\,\rho)^{9/4}\lambda\lambda$. It also amounts to rescaling $\Omega \to (\mathrm{Im}\,\rho)^{\frac{3}{4}}\Omega$ and $A \cdot B \to (\mathrm{Im}\,\rho)^{-\frac{3}{2}}A \cdot B$ where A, B are three-forms.

not ruled out. One way to progress would be to perform the supersymmetry analysis directly in ten dimensions but this is beyond the scope of our work. For comparison we give here the EFT value of the curvature obtained from the four-dimensional KKLT model before anti-D3 branes are included (in Planck units):

$$R_4^{\rm KKLT} = 4V_{\rm KKLT} = -12\epsilon^K|W|^2 = -\frac{16\pi^2}{3}(\mathrm{Im}\,\rho)^2\left|\frac{\lambda\lambda}{16\pi^2}\right|^2 . \tag{8.5.7}$$

Finally, let us recall that the anti-D3 brane tension contributes only negatively to R_4. An uplifting to dS is supposed to occur through the mediation of gaugino terms that contribute positively to R_4. Such positive terms can only arise from the term $\mathrm{Re}\left(G_3^0\cdot I^{\rm reg}\right)$ being non-zero in a supersymmetry breaking vacuum.

8.6 Discussion

8.6.1 Summary and discussion of results

In this paper, we have revised the computations of [210] taking into account the flux renormalisation mechanism provided by the quartic gaugino term on D7-branes, as proposed in [245]. We must rely on a number of assumptions necessary to directly use the techniques of [245], namely constant axion-dilaton, warping, and C_4-potential, and most importantly we used that the harmonic 3-form flux is the GKP flux. Consistent with the result of [210] we are unable to establish the sign of the four-dimensional curvature from ten-dimensional analysis alone. We explicitly showed that within our assumptions the quartic gaugino terms proposed by [212] are sufficient to regularize the singularities encountered in [203, 210].

A worthwile direction for future research would be to see if one can still obtain a definite result after when it comes to de Sitter solution using anti-D3 brane uplift terms. It is tempting to speculate what happens. For that we go back to equation (8.4.10). Note that this equation was derived without any of the simplifying approximations and is hence valid in general circumstances. We have argued that the renormalisation due to the four-fermion term is such that the only 3-form fluxes appearing in (8.4.10) are the *background* fluxes, ie those of the classical GKP solution. This means we can put $|G_3^{0,-}|^2 = 0$ since the flux appearing will be ISD. This is expected to be broken by the gaugino condensate but the deviations from the ISD background are mostly "self-energy" that has been removed by the renormalisation procedure. Once anti-branes are added they will also create some local non-ISD fluxes down the throat. But we will

neglect these since they anyway come with a negative contribution to R_4 and so will not help in getting de Sitter. The main question then is whether the only positive contribution to the vacuum energy

$$\int \star_6 \mathrm{Re}\big(G_3 \cdot \mathcal{P}(I)\big)\,, \tag{8.6.1}$$

can dominate over the rest. This question seems to be very difficult to answer, and most likely requires a full solution the entire system of the ten-dimensional equations of motion which surely is out of reach.

One way to approach this problem is make a precise analysis of the term (8.6.1) for a supersymmetric configuration. This could be done by analysing the ten-dimensional supersymmetry variations in presence of the gaugino bilinear. This is a very interesting exercise which we plan to carry out in the future.

Finally we wish to iterate the statements in [203, 210]: to have certainty that uplifting AdS vacua with small supersymmetry-breaking ingredients does not lead to runaways instead of meta-stable dS vacua, one should achieve the following parametric scaling:

$$m^2 L^2 \gg 1\,. \tag{8.6.2}$$

In this equation m^2 is the mass squared of the lightest modulus in the AdS vacuum and L the AdS length. Interestingly not a single clear top-down AdS vacuum in string theory achieves this. The only cases known to us are "stringy-inspired" like racetrack fine-tuning [246] or non-geometric fluxes [247,248]. It was conjectured in [221] that racetrack fine-tuning is in the Swampland[13] whereas reference [210] conjectured this to be true for all vacua obeying (8.6.2). Since the conjecture of [210] forbidding (8.6.2) is a statement about AdS vacua and not dS vacua we believe it should be easier to verify than the actual no-dS conjecture of [160, 161, 163], and it would count as non-trivial evidence in support of the no-dS conjecture.

8.6.2 Comparison with recent papers

Simultaneous with the appearance of this work, two other papers appeared discussing the same problem [231, 232]. Reference [232] set out to do similar computations as done here but came to a different conclusion. We differ from [232] in the way the energy-momentum tensor in ten dimensions is computed. This is also discussed in reference [231].

We demonstrate the difference in the language of [232]. There the trace reversed Einstein equations are computed with variations w.r.t. the volume instead of

[13] See [249, 250] for some criticism on this.

the warp factor. Below we mention why this choice is important to reach the results presented in [232]. Concretely, the 4d curvature can be computed by using the following equation [232][14]:

$$2\pi\mathcal{V}_6\mathcal{V}_4 R_4 = -\left((\mathrm{Im}\,\rho)\frac{\delta}{\delta(\mathrm{Im}\,\rho)} - 1\right) S. \tag{8.6.3}$$

We only look at the terms in the action involving the gaugino condensate where we integrated over 6D space:

$$S_{\lambda\lambda} = 2\pi \int \star_4 \frac{1}{2}\left[-8(\mathrm{Im}\,\rho)^{\frac{1}{2}}\left|\frac{\lambda\lambda}{16\pi^2}\right|^2 + (\mathrm{Im}\,\tau)^{-\frac{1}{2}}\left((\mathrm{Im}\,\rho)^{\frac{1}{4}}\frac{\bar{\lambda}\bar{\lambda}}{16\pi^2}W_0 + c.c.\right)\right] \tag{8.6.4}$$

Here we have written the volume dependence coming from the 6D metric explicitly and ignored the warping dependence.

As we have mentioned, our approach is as follows: we vary the D7 brane action to obtain its energy-momentum tensor. *Subsequently* we fill in the vev of the gaugino condensate $\langle\lambda\lambda\rangle$, whose value is set by non-perturbative physics. This is how the semi-classical limit is usually defined: one inserts the quantum vevs of operators into the classical EM tensor. Following this approach, one finds by evaluating (8.6.3) and (8.6.3) that the 4d curvature satisfies:

$$\mathcal{V}_6 R_4 = -2(\mathrm{Im}\,\rho)^{\frac{1}{2}}\left|\frac{\lambda\lambda}{16\pi^2}\right|^2 + \frac{3}{4}(\mathrm{Im}\,\tau)^{-\frac{1}{2}}\ \mathrm{Re}\left((\mathrm{Im}\,\rho)^{\frac{1}{4}}\frac{\bar{\lambda}\bar{\lambda}}{16\pi^2}W_0\right) \tag{8.6.5}$$

This result agrees with (8.4.15).

However a subtlety arises which relates to the dependence of the gaugino vev on the gauge coupling (8.5.3). Since the coupling equals the inverse volume of the 4-cycle wrapped by the D7 branes, a dependence of the vev $\langle\lambda\lambda\rangle$ on the ten-dimensional metric is induced. This dependence cannot be understood classically in four or ten dimensions. One could imagine reversing the order and plug the metric dependence of the gaugino condensate into the ten-dimensional action *before* computing the energy-momentum tensor. This introduces extra terms due to derivatives on the condensate, i.e. this approach would result in a 4d curvature of the form.

$$\begin{aligned}\mathcal{V}_6 R_4 = &-2(\mathrm{Im}\,\rho)^{\frac{1}{2}}\left|\frac{\lambda\lambda}{16\pi^2}\right|^2 + \frac{3}{4}(\mathrm{Im}\,\tau)^{-\frac{1}{2}}\ \mathrm{Re}\left((\mathrm{Im}\,\rho)^{\frac{1}{4}}\frac{\bar{\lambda}\bar{\lambda}}{16\pi^2}W_0\right)\\ &+ 4(\mathrm{Im}\,\rho)^{\frac{3}{2}}\frac{\partial}{\partial\mathrm{Im}\,\rho}\left|\frac{\lambda\lambda}{16\pi^2}\right|^2 - (\mathrm{Im}\,\tau)^{-\frac{1}{2}}\ \mathrm{Re}\left((\mathrm{Im}\,\rho)^{\frac{5}{4}}\left(\frac{\partial}{\partial\mathrm{Im}\,\rho}\frac{\bar{\lambda}\bar{\lambda}}{16\pi^2}\right)W_0\right)\end{aligned} \tag{8.6.6}$$

[14] We use a different convention for the Einstein-Hilbert action than [232]. This amounts for a difference of a factor 2 in the RHS of (8.6.3).

Working with this equation, reference [232] finds a qualitative match with the four-dimensional KKLT potential and further concludes there is no obstruction to uplifting to meta-stable dS vacua.

We want to emphasize that that it is not clear how the semi-classical method really works in this case. In particular, since replacing $\langle\lambda\lambda\rangle$ with the gaugino vev effectively eliminates 10d covariance, the method of [232] is sensitive how exactly the variation of the action is performed so that the result is (8.6.6). A straightforward method that manifestly respects ten-dimensional covariance is to have no metric dependence in $\langle\lambda\lambda\rangle$ at the level of the 10d action.

In a rather different wording reference [231] warned for this sublety and concludes that perhaps the ten-dimensional approach is not attainable after all. We remain agnostic about that point but emphasize the following; *if* a ten-dimensional viewpoint is to make sense then we want to emphasize the following three things: 1) Inserting the four-dimensional information about the gauge coupling dependence into the ten-dimensional action implies one is not following a genuine ten-dimensional approachafter all. 2) A semi-classical limit should be something universal. In other words when one couples classical gravity to quantum matter then the approach is that the quantum vevs are inserted into the classical EM tensor. The latter tensor is a universal object in the theory and how to compute the tensor does not depend on the background neither on the quantum effects of the matter sector. 3) It is not clear to us how ten-dimensional covariance is preserved in the approach of [232].

This does not imply we disagree with the outcome of the results in [232], but we want to emphasize our motivations for following the other path and what the consequences are.

Finally note that, if the second part of reference [231] turns out to be correct, then dS uplifts are questionable already for a simpler reason related to having controlled uplifts and throat volumes that fit into the compact CY space.

Chapter 9

Compact G2 holonomy spaces from SU(3) structures

This chapter is a reprint of [4]. We construct novel classes of compact G2 spaces from lifting type IIA flux backgrounds with O6 planes. There exists an extension of IIA Calabi-Yau orientifolds for which some of the D6 branes (required to solve the RR tadpole) are dissolved in F_2 fluxes. The backreaction of these fluxes deforms the Calabi-Yau manifold into a specific class of SU(3)-structure manifolds. The lift to M-theory again defines compact G2 manifolds, which in case of toroidal orbifolds are a twisted generalisation of the Joyce construction. This observation also allows a clear identification of the moduli space of a warped compactification with fluxes. We provide a few explicit examples, of which some can be constructed from T-dualising known IIB orientifolds with fluxes. Finally we discuss supersymmetry breaking in this context and suggest that the purely geometric picture in M-theory could provide a simpler setting to address some of the consistency issues of moduli stabilisation and de Sitter uplifting.

9.1 Introduction

Manifolds with compact G2 holonomy have been discussed before in several works, most notably the original work of Joyce [177, 251] and more recently many more examples were found by using twisted connected sums [252]. The reader can consult [178] for a fairly-recent review and [253–255] for some most

recent state-of-the art work on constructing these manifolds and their relation with various 10-dimensional string theories using dualities. The construction method presented in this paper is however different.

Before we discuss the actual details of our construction, we present the main idea underlying our construction. For that we first use the observation of Kachru and McGreevy [256] that compact G2 spaces obtained from desingularising certain orbifolds of $\mathbb{T}^7$ have a simple IIA dual description in terms of IIA Calabi-Yau (CY) orientifolds. Compact G2 spaces also admit Type IIA Calabi-Yau orientifold descriptions where the Calabi-Yau manifold is smooth but the branes are intersecting [257–259]. In fact, these constructions [257, 258] are prototypes of the intersecting brane constructions of realistic particle physics models, whose M-theory lift to G2 spaces was discussed in [259]. More generally, one can discuss Calabi-Yau 3-folds X that allow discrete involution symmetries that leave special Lagrangian 3-cycles invariant such that they can be promoted to Calabi-Yau orientifold solutions of IIA string theory. The compactness of the CY orientifold enforces a RR tadpole condition which implies the addition of D6 branes wrapping cycles in the same homology class as the O6 planes such that the total 6-brane charges vanishes. Such manifolds should lift to compact 7-dimensional manifolds Y with G2 holonomy of the form

$$Y = \frac{X \times S^1}{\mathbb{Z}_2}\,, \tag{9.1.1}$$

where the involution acts both on the M-theory circle and the Calabi-Yau X, and the singularities of Y are resolved after the $\mathbb{Z}_2$ orbifolding. Explicit realisations of this are discussed in [256].

The main point of our construction is the observation that the IIA background X can be deformed with fluxes while preserving supersymmetry and the flatness of the 4D vacuum (Minkowski), in such a way that the lift to 11 dimensions provides again a compact G2 space. For that to be possible the only allowed fluxes are the RR two-form flux F_2 since any other flux would switch on the M-theory 4-form destroying the flatness of the 7d compact space. More loosely speaking one could say that the D6 branes in the Calabi-Yau constructions get dissolved in F_2 fluxes, which then backreact the CY geometry X into a specific (non-Ricci flat) SU(3)-structure $\tilde{X}$ in such a way that the lift to 11d provides a Ricci flat compact G2 space $\tilde{Y}$. Hence the 7d space $\tilde{Y}$ can be seen as a twisted product (non-trivial fibre product) of a SU(3)-structure $\tilde{X}$ with a circle, where the twist is such that the 7d geometry $\tilde{Y}$ remains Ricci flat:

$$\tilde{Y} = \frac{\tilde{X} \ltimes S^1}{\mathbb{Z}_2}\,. \tag{9.1.2}$$

To our knowledge the first examples of these "generalised Calabi-Yau" compactifications of IIA on $\tilde{X}$ were found in [260] by T-dualising type IIB

orientifolds with 3-form fluxes and a general discussion of such solutions appeared in [176]. An important difference between (9.1.2) and (9.1.1) is that in (9.1.2) the $\mathbb{Z}_2$ involution may by construction have less fixed points or even acts freely, depending on the number of D6-branes. Each singularity introduces degrees of freedom that can be associated with their resolution. The non-trivial information about RR two-form flux and (some of the) O6-planes is completely encoded in the non-trivial fibration of S^1 over $\tilde{X}$. This fibration is not unique and in one-to-one correspondence with the possible set of consistent, supersymmetry-preserving F_2 fluxes one can put over $\tilde{X}$. In principle this method works for any BPS combination of sources. For simplicity however we restrict to parallel D6/O6 sources.

In the course of this work we learned that this research direction is not entirely uncharted, as some initial forays in the case of non-compact G2 spaces can be found in the references [261–264]. However, to our knowledge, we are the first to point out how this method allows the construction of new *compact* spaces of G2 holonomy, provide an investigation of the resulting moduli spaces, and suggest how to break supersymmetry.

The phenomenologically interesting properties of our new examples is that the G2 spaces $\tilde{Y}$ have smaller moduli spaces than their counterparts Y, since in the IIA picture we obviously stabilise some moduli using the F_2 flux on $\tilde{X}$. However, concerning interesting matter-couplings, we have co-dimension 4 singularities that can accommodate non-Abelian gauge groups, but not chiral matter. This is because in this paper, we consider M-theory lift of IIA backgrounds with parallel O6/D6 sources. It would be interesting to extend our analysis include backgrounds with both fluxes and non-parallel sources, like the ones in [265,266] as in this case the corresponding G2 lifts would have desirable phenomenological features. However, we will not attempt to do so here because of technical complications with the supergravity solutions. What we find especially appealing in our constructions is that our G2 moduli spaces should be identical to the IIA(IIB) moduli spaces of warped compactifications with flux, after using the M-IIA duality (and IIB T-duality in cases it is known). The latter is notoriously complicated [267–269] due to fluxes and the loss of a purely geometric picture. What we show here is that for a certain set of type II flux vacua a pure geometric picture in terms of G2 holonomy spaces exists.[1] This might lead to a deeper understanding of warped effective field theories.

[1] See [270] for similar lines of reasoning.

9.2 SU(3) structures from IIA strings

9.2.1 General discussion

We start from the general metric Ansatz in 10D string frame

$$\mathrm{d}s_{10}^2 = \epsilon^{2W}\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + \mathrm{d}s_6^2\,. \tag{9.2.1}$$

Aside from the metric, the possible non-zero supergravity fields are the 2-form fieldstrength F_2 and the dilaton Φ that is allowed to depend on the 6D coordinates. Supersymmetry of flux backgrounds is best understood using the pure spinor formulation which we discussed in section 7.2.4. In this context the pure spinor equations boil down to the following two poly-form equations

$$\begin{aligned}
&\mathrm{d}(e^{2W-\phi}\Phi_+) = 0\,,\\
&\mathrm{d}(e^{2W-\phi}\Phi_-) = e^{2W-\phi}dW\wedge\bar{\Phi}_- - \frac{i}{8}e^{3W}\star_6 F_2\,.
\end{aligned} \tag{9.2.2}$$

To obtain compactifications to 4d Minkowski space orientifolds are unavoidable. From here onwards we assume as only sources parallel O6 planes and possibly D6 branes. Our conventions for SU(3)-structures are spelled out in Appendix E.

The polyform equations (9.2.2) are solved when:

$$e^{\phi-3W} = e^{\phi_0} = \text{constant} \equiv g_s\,, \tag{9.2.3}$$

$$\mathrm{d}J = 0\,, \tag{9.2.4}$$

$$\mathrm{d}(e^{-W}\Omega_R) = 0\,, \tag{9.2.5}$$

$$\mathrm{d}(e^{W}\Omega_I) = -g_s e^{4W}\star_6 F_2\,. \tag{9.2.6}$$

The last equation should be interpreted as the one that defines F_2 in terms of the $SU(3)$ structure and the warping. To promote this to a solution of the 10D EOM, it is sufficient to enforce the Bianchi identity [271]

$$\mathrm{d}F_2 = \sum_i Q_i\delta_i\,, \tag{9.2.7}$$

where the right hand side symbolises all localised sources carrying 6-brane charges. Using the pure spinor equations the Bianchi identity amounts to

$$\mathrm{d}\Big(e^{-4W}\star_6\,\mathrm{d}(e^W\Omega_I)\Big) = -g_s\sum_i Q_i\delta_i\,. \tag{9.2.8}$$

The F_2 field strength is obtained from (9.2.6) as

$$g_s F_2 = -e^{-4W} \star_6 \mathrm{d}(e^W \Omega_I)\,. \tag{9.2.9}$$

We can split F_2 into the sum of a "background part" and a "sourced part" $F_2 = F_2^B + F_2^S$, where the background part is defined as the value of F_2 in the constant warp factor limit (or smeared limit), that is

$$F_2^B \sim \star_6 \mathrm{d}\Omega_I^{smeared}\ . \tag{9.2.10}$$

The remaining part of F_2 is accounted by F_2^S.

The only non-zero torsion classes for these orientifold solutions with just F_2 flux are W_2 and W_5

$$W_2 = ie^{\phi} F^{1,1}\,, \qquad \mathrm{d}W = W_5 + \bar{W}_5\,. \tag{9.2.11}$$

The computation of the torsion can be found in Appendix G.

From (9.2.11) one can deduce that F_2^B has to be $(1,1)$ and primitive. This is indeed the supersymmetry condition on the fluxes in case the orientifold is smeared and this can be explicitly verified using 4d supergravity.[2] On the other hand F_2^S can have in general $(1,1)$, $(2,0)$ and $(0,2)$ pieces, but the $(1,1)$ part has to be primitive.

9.2.2 An example from its type IIB mirror

An explicit IIA flux backgrounds of the above kind was found in [176, 260] by T-dualising three times a simple IIB orientifold with 3-form fluxes. The triple T-duality is such that the H_3 flux is gone and replaced by "'metric flux" whereas F_3 is T-dualised to F_2 and the O3/D3 sources are mapped to parallel O6/D6 sources.

Let us follow the example in [176] (see also [260]) and strip off all details, such as moduli dependence and flux/charge quantisation, and warping since we simply want to illustrate the essence. The internal space is $\mathbb{T}^6/\mathbb{Z}_2$ with $\mathbb{Z}_2$ the orientifold involution. The pure spinors are

$$e^{-iJ} = (1 + ie^6 \wedge e^1) \wedge (1 - ie^5 \wedge e^2) \wedge (1 - 2ie^4 \wedge e^3)\,, \tag{9.2.12}$$

$$\Omega = (e^6 - ie^1) \wedge (e^5 + ie^2) \wedge (e^4 + 2ie^3) \equiv dz_1 \wedge dz_2 \wedge dz_3\,, \tag{9.2.13}$$

[2] Note that the standard equations for the 4d supergravity ignore the warping.

where e^i are the Cartesian one-forms dx^i. The fluxes are given by

$$g_s G_3 = g_s F_3 - iH_3 = \frac{i}{2}\left[d\bar{z}_1 \wedge dz_2 \wedge dz_3 + dz_1 \wedge d\bar{z}_2 \wedge dz_3 - 2dz_1 \wedge dz_2 \wedge d\bar{z}_3\right], \tag{9.2.14}$$

such that $G_3 \in H^{(2,1)}(\mathbb{T}^6)$. Therefore the solution preserves at least $\mathcal{N} = 1$ supersymmetry. It is possible to show that this solution preserves not more supersymmetry, but in absence of further orbifolding this vacuum can be described as an $\mathcal{N} = 1$ state in an $\mathcal{N} = 4$ gauged supergravity.

A triple T-duality along x_4, x_5, x_6 gives the nilmanifold defined by the following Maurer-Cartan equations:

$$\mathrm{d}e^4 = e^{12}\,, \qquad \mathrm{d}e^5 = e^{13}\,, \qquad \mathrm{d}e^6 = e^{23}\,, \tag{9.2.15}$$

whereas e^1, e^2 and e^3 are closed (and simply given by dx^i). The flux and pure spinors become

$$\Omega = (e^1 + ie^6) \wedge (e^2 - ie^5) \wedge (e^3 - 2ie^4)\,, \tag{9.2.16}$$

$$J = e^1 \wedge e^6 - e^2 \wedge e^5 - 2e^3 \wedge e^4\,, \tag{9.2.17}$$

$$g_s F_2 = -4e^{34} + e^{25} - e^{16}\,. \tag{9.2.18}$$

This solution solves the equations of motion with smeared O6 sources. The solution with localised sources is derived in the Appendix F.

This example of T-dualising O3 backgrounds to O6 backgrounds with just F_2 flux can be generalised, and we give more examples in section 9.4. But the general picture is rather straightforward: the mirror of the D3/O3 sources leads to D6/O6 sources (possibly non-aligned), wrapping torsional cycles (see below). The mirror of H-flux leads to an SU(3)-structure with torsion. Whereas the mirror of the F_3-flux is the background (non-sourced part of the) F_2 flux. The mirror of the F_5 flux is the sourced part of the F_2 flux. The mirror of the ISD condition is then the Hodge duality between the torsion $\mathrm{d}\Omega_I$ and F_2.

9.2.3 Comments on RR tadpoles

Tadpole cancellation conditions are crucial for achieving compactness of the flux solutions. RR tadpoles are simply the statement that compact spaces allow no net charges. From a mathematical viewpoint these conditions ensure that the PDE's describing the G2 space (after lift to 11D, see next section), allow solutions describing compact spaces. In practice tadpole cancellation conditions

may be understood as necessary conditions to have a well-defined solution to the Bianchi identities of the NSNS and RR fields. In the more familiar case of type IIB compactifications with O3/D3 sources the tadpole cancellation condition comes from the Bianchi identity of the RR five-form of type IIB which has the form[3]

$$d\tilde{F}_5 = H_3 \wedge F_3 + 2\kappa_{10}^2 \, T_3 \, \rho_{\rm D3} \,, \tag{9.2.19}$$

where $\rho_{\rm D3}$ contains the contributions to the D3-brane charge coming from various sources in the compactification (that is O3/D3 and/or D7-branes with magnetic fluxes or wrapping curved 4-cycles).[4] Given that the internal space has a non-trivial topology it is necessary to impose some conditions to ensure that a solution to (9.2.19) exists with a well defined profile for $\tilde{F}_5$. For the case at hand this is equivalent to asking that the right hand side of the Bianchi identity (9.2.19) is zero in cohomology, a condition which translates into the familiar equation

$$\frac{1}{2\kappa_{10}^2 T_3} \int H_3 \wedge F_3 + Q_{\rm D3} = 0 \,, \tag{9.2.20}$$

where $Q_{\rm D3}$ is the integral of the D3-brane charge density.

We can now follow this discussion and translate it to type IIA compactifications with O6/D6 sources, where sources couple magnetically to the RR two form $\tilde{F}_2$ so there is only one Bianchi identity that we need to check. Moreover it is important to note that in a twisted torus differential forms can be non-closed even when they have constant coefficients. Let us review that: using the Maurer-Cartan forms e^i we can express any differential p-form Υ with constant coefficients as

$$\Upsilon = \frac{1}{p!} \Upsilon_{i_1 \dots i_p} \, e^{i_1} \wedge \cdots \wedge e^{i_p} \,. \tag{9.2.21}$$

However given that the Maurer-Cartan forms are not closed the differential form written above is not automatically closed even if the coefficients of the tensor are. In fact taking the exterior derivative we have[5]

$$\mathrm{d}\Upsilon = -\frac{1}{2(p-1)!} f^l_{jk} \Upsilon_{l i_1 \dots i_{p-1}} \, e^j \wedge e^k \wedge e^{i_1} \wedge \cdots \wedge e^{i_{p-1}} \,. \tag{9.2.22}$$

[3] We define $\tilde{F}_p = dC_{p-1} - H \wedge C_{p-3} + m e^B$ where m is the Romans mass.

[4] We choose the convention of measuring the charge of the orientifold plane in the space quotientied by the orientifold involution, which leads to the relation $Q(\mathrm{Op}) = -2^{p-5} Q(\mathrm{Dp})$.

[5] The covering space of a twisted torus is a group manifold and in our conventions the Maurer–Cartan forms on group manifolds obey $\mathrm{d}e^i = -\frac{1}{2} f^i_{jk} e^j \wedge e^k$.

In the following we employ a shorthand notation writing the right hand side of the previous equation as $f \cdot \Upsilon$, giving therefore that $\mathrm{d}\Upsilon = f \cdot \Upsilon$. Using this we can easily obtain the Bianchi identity for $\tilde{F}_2$

$$\mathrm{d}\tilde{F}_2^S = -f \cdot \tilde{F}_2^B + 2\kappa_{10}^2 \, T_6 \, \rho_{\mathrm{D6}} \,, \tag{9.2.23}$$

where we have split $\tilde{F}_2$ in its background $\tilde{F}_2^B$ and sourced part $\tilde{F}_2^S$ as discussed below equation (9.2.9). To make a comparison with (9.2.19) we have that under T-duality $\tilde{F}_5$ maps to $\tilde{F}_2^S$, F_3 maps to $\tilde{F}_2^B$ and ρ_{D3} maps to ρ_{D6}. Again we can pass this equation and convert it into a tadpole cancellation condition by asking that the right hand side is a trivial class in cohomology. However it is not sufficient to ask that the right hand side vanishes in de Rham cohomology (that is in $H^3_{dR}(\tilde{X}) \simeq H^3(\tilde{X}, \mathbb{R})$) but it is necessary to impose this condition in integer cohomology (that is in $H^3(\tilde{X}, \mathbb{Z})$). The difference between the two is relevant for us in the following because in our examples all sources are parallel and wrap torsional cycles and therefore their contribution to the Bianchi identity would be invisible in de Rham cohomology. The Poincaré dual Π of this homology class will therefore be a torsion element of $H^3(\tilde{X}, \mathbb{Z})$ and we take it to be of order N, that is $[N\Pi] = 0$.[6] Moreover to obtain the tadpole we need to consider what happens to the term $f \cdot \tilde{F}_2$. However given the way we constructed it, $f \cdot \Upsilon$ is always an exact form, and therefore it disappears when taking (9.2.23) in cohomology. Putting all of this together we find that the tadpole condition is [273]

$$N_{\mathrm{D6}} + rN = 2N_{\mathrm{O6}} \,, \qquad \exists\, r \in \mathbb{Z} \,. \tag{9.2.24}$$

The meaning of this equation is that the total charge carried by the sources, that is the D6-branes and the O6-planes need to be a multiple of N to ensure that it is trivial in cohomology. This implies that the total charge D6-brane charge is a well defined object only modulo N, meaning that there can be different solutions satisfying the RR tadpole condition with different number of D6-branes. In particular it is possible to have transitions between different solutions via some domain-wall where N D6-branes dissolve into flux [274,275],[7] a phenomenon observed also in GKP solutions for the D3-brane charge. This seems to give a large freedom in building the solutions, specially given the fact that the number r is not fixed in (9.2.24). However there is still one piece missing: while imposing the cancellation of the RR tadpole in the compact space ensures vanishing of the RR charge, it is necessary to check that the

[6] Here we used standard Poincaré duality, i.e. $H^p(\mathcal{M}, \mathbb{Z}) \simeq H_{d-p}(\mathcal{M}, \mathbb{Z})$ for $\dim(\mathcal{M}) = d$. In addition to this it is sometimes useful to recall that by using the universal coefficient theorem it is possible to prove that $\mathrm{Tor}\, H_p(\mathcal{M}, \mathbb{Z}) \simeq \mathrm{Tor}\, H^{p+1}(\mathcal{M}, \mathbb{Z})$. For the precise statements and proofs one may look at Chapter 3 of [272].

[7] Even if these domain-walls are present it may be impossible to nucleate them, for instance in supersymmetric cases, thus effectively making the number of D6-branes stable.

tension carried by the fluxes and the sources cancels to find a supersymmetric solution. This additional constraint fixes the integer r to be the amount of F_2 flux present in the compact space [273].

9.3 Lift to G2 compactifications

9.3.1 General discussion

Since the IIA flux vacua of concern feature O6/D6 sources, a non-constant dilaton, and F_2 flux they lift to pure geometry in 11d. The preservation of minimal supersymmetry further restricts the 7d compactification manifold to be of G2 holonomy. We now flesh out the details of that, similar to the analysis done in [261–263].

The 11-dimensional reduction Ansatz to IIA supergravity, in string frame, is given by

$$\mathrm{d}s_{11}^2 = e^{-\frac{2}{3}\phi}\mathrm{d}s_{10}^2 + e^{\frac{4}{3}\phi}(\mathrm{d}z + A)^2\,, \tag{9.3.1}$$

where z runs over the compact M-theory circle: $z = (0, 1)$ and locally $F_2 = \mathrm{d}A$. Using equations (9.2.1) and (9.2.3) the 11-dimensional metric (9.3.1) becomes *unwarped*:

$$\mathrm{d}s_{11}^2 = g_s^{-2/3}\eta_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu + \mathrm{d}s_7^2\,. \tag{9.3.2}$$

with $\mathrm{d}s_7^2$ the G2-holonomy metric

$$\mathrm{d}s_7^2 = e^{-2\phi/3}\mathrm{d}s_6^2 + e^{4\phi/3}(\mathrm{d}z + A)^2\,. \tag{9.3.3}$$

The lift of the SU(3)-structure to a G2 structure can be found in various papers [261, 263, 264, 270] and is completely specified by the G2-invariant 3-form Φ[8]

$$\Phi = e^{-\phi}\Omega_I - J \wedge (\mathrm{d}z + A)\,, \tag{9.3.4}$$

We demonstrate the equivalence of the pure spinor equations (9.2.2) with Φ's closure ($\mathrm{d}\Phi = 0$) and co-closure ($\mathrm{d} \star_7 \Phi = 0$) in Appendix H.

Now let us discuss sources. In the case of vanishing two-form flux, each O6-plane comes with two D6-branes that cancel its charge. The uplift of this setup to the seven-dimensional manifold in M-theory naturally corresponds to a $(\mathbb{R}^3 \times S^1)/\mathbb{Z}_2$ singularity, whose appearance suggests non-Abelian gauge theories.

[8] The normalisation here is chosen to ensure the usual normalisation $\Phi \wedge \star_7\Phi = 7d\mathrm{vol}_{\tilde{Y}}$. This can be easily checked noting that (9.3.3) implies that $d\mathrm{vol}_{\tilde{Y}} = e^{-4\phi/3}d\mathrm{vol}_{\tilde{X}}$ together with the usual normalisations $\frac{i}{8}\Omega \wedge \overline{\Omega} = \frac{1}{3!}J^3 = d\mathrm{vol}_{\tilde{X}}$.

Such singularities can be resolved by introducing new cycles, which correspond to the degrees of freedom coming from the D6-branes. The resulting M-theory geometry is known to be a composition of KK-monopoles and Atiyah-Hitchin spaces [276–280]. In case we have F_2 flux the M-theory uplift is not supposed to introduce such degrees of freedom or any singularities. Moreover, F_2 flux corresponds to a non-trivial fibration before applying the $\mathbb{Z}_2$ involution on the M-theory side. For this we have to replace the position of the O6-plane by a double cover of Atiyah-Hitchin space. Indeed Atiyah-Hitchin space admits double covers, and we can glue such double covers into the O6-positions. Since the $\mathbb{Z}_2$ involution acts freely on that double cover of Atiyah-Hitchin space, no new singularities and degrees of freedom are introduced. For this construction to be consistent, we have to ensure that gluing in such a double cover of Atiyah-Hitchin space is consistent with the non-trivial SU(3) structure obeying (9.2.6).

Note that this construction of G2 spaces is closely related to the construction obtained by lifting Calabi-Yau orientifolds in which the 7-dimensional manifold has a covering space that corresponds to a direct product of a circle with the Calabi-Yau 3-fold. The essential difference with the construction in this paper is that the covering space is a non-trivial fibre bundle of a SU(3)-structure manifold over a circle, with the twists that defines the fibration encoded in the O6 and D6 sources. In the case of the CY orientifold the net twist vanishes since the net charges in the sources cancel out. In the case at hand this is not true since the RR tadpole is satisfied by non-trivial flux such that there is a non-trivial twist of the circle over the base. It then turns out that with a non-trivial twist of the circle a G2 space can still be defined on the condition that the SU(3) holonomy gets relaxed into a more general SU(3) structure.

9.3.2 Moduli spaces

In general it is a difficult problem to count the moduli of a warped flux compactification since the torsion (or non-closure of Ω and J) complicates the search for fluctuations that leave the pure spinor equations invariant. On top of that the metric on moduli space is not well understood. The "on-shell" expression for this metric contains the warp factor, which itself depends on the moduli in a complicated way. The study of these issues is known as *warped effective field theory*, see for instance [267–269,281]. For now we observe that our 11d picture at least bypasses the warping issue at first sight. It also facilitates the counting of the moduli since the moduli of G2 compactifications are counted by the third homology class of the G2 space.

The reduction of the C_3 potential over the 3-cycles completes the real moduli

into complex fields. The complex moduli are then constructed by expanding Φ and C_3 in basis of harmonic 3-forms α_i

$$\Phi + \mathrm{i}C_3 = \sum_{i=1}^{b^3} \phi_i \alpha^i \,. \tag{9.3.5}$$

The moduli space metric, determining the kinetic term for the ϕ^i's is then derivable from the following Kähler potential (see for instance [282]):

$$\mathcal{K} = -3 \log \left(\frac{1}{14\pi^2} \int_{\tilde{Y}} \Phi \wedge \star_7 \Phi \right). \tag{9.3.6}$$

It is instructive to write the Kähler potential in terms of IIA data

$$\mathcal{K} = -3 \log \left(\frac{1}{12\pi^2} \right) - \log \left(\int_{\tilde{X}} e^{-4\phi/3} J^3 \right) - 2 \log \left(\frac{3i}{4} \int_{\tilde{X}} e^{-4\phi/3} \Omega \wedge \bar{\Omega} \right), \tag{9.3.7}$$

where we used (E.0.3) in the last step (see also [237]). Using (9.2.3), it is possible to trade the dilaton ϕ for the warped factor W and obtain an expression for the Kähler potential which is consistent with the ones suggested in warped effective field theory, that is [9]

$$\mathcal{K} = -3 \log \left(\frac{g_s^{-4/3}}{12\pi^2} \right) - \log \left(\int_{\tilde{X}} e^{-4W} J^3 \right) - 2 \log \left(\frac{3i}{4} \int_{\tilde{X}} e^{-4W} \Omega \wedge \bar{\Omega} \right). \tag{9.3.8}$$

Closed string sector

We first ignore the presence of D6 branes and their associated moduli. This can be achieved by solving the tadpole condition purely with fluxes canceling orientifold charges. We will discuss the effect of D6 branes in the next section.

Using the fibre bundle structure, the most general Ansatz for α^i can be written as

$$\alpha^i = \Sigma_3^i + \Sigma_2^i \wedge (\mathrm{d}z + A_1) \,, \tag{9.3.9}$$

where Σ_2 and Σ_3 are forms on the 6D base $\tilde{X}$. Note that the α_i must be even under the $\mathbb{Z}_2$ action present in the 7d construction (9.1.2). Since $\mathrm{d}z + A_1$ is odd

[9] For instance, by redefining $\Phi \to g_2^{2/3} \Phi$ we find that (9.3.8) matches 3.38 in [237], where $\mathcal{Z} \sim e^{i\alpha} e^{iJ}$, for a generic phase α. We do not care about numerical factors here, since they can be further accommodated by redefinitions of Φ here or $\mathcal{Z}$ there. Moreover, in the unwarped limit $W \to 0$, (9.3.8) matches (again, up to some numerical factors) eq. 3.48 in [283], which has to be slightly modified by assuming our normalisation (E.0.3).

we have that Σ_3^i is even and Σ_2^i is odd. Closure of α^i implies

$$\mathrm{d}\Sigma_3^i = -\Sigma_2^i \wedge F_2\,, \quad \text{and} \quad \mathrm{d}\Sigma_2^i = 0\,. \tag{9.3.10}$$

For practical purposes it is easier to phrase the moduli problem in terms of cohomology. The equivalence relation that defines equivalence classes in cohomology consistent with (9.3.10) are

$$\Sigma_3^i \sim \Sigma_3^i + \mathrm{d}\lambda_2^i - \lambda_1^i \wedge F_2 \quad \text{and} \quad \Sigma_2^i \sim \Sigma_2^i + \mathrm{d}\lambda_1^i\,, \tag{9.3.11}$$

for any pair of one- and two-forms λ_1 and λ_2. From (9.3.10) we see in particular that all even harmonic 3-forms on $\tilde{X}$ provide moduli of the G2 space. It is not difficult to see that in case a mirror IIB flux solutions exists, the Kähler moduli of the IIB (conformal) Calabi-Yau space map to the moduli defined by these harmonic 3-forms on the SU(3)-structure base.

We interpret the first equation in (9.3.10) as a differential equation for Σ_3: If for a given Σ_2 we can integrate equation (9.3.10) to find a Σ_3 then we have found a G2 modulus. For *generic* F_2 flux one expects this not to be possible.[10] This is of course a manifestation of moduli-stabilisation due to fluxes and this lift of moduli is T-dual to the lift of complex structure moduli. Similar to the IIB solution it is of course possible that not all such moduli are lifted.

Aside the chiral field moduli that are counted by $b_3(\tilde{Y})$, there can be vector multiplet moduli and they are counted by $b_2(\tilde{Y})$. To compute $b_2(\tilde{Y})$ in terms of data coming from the SU(3)-structure base and data from the fibration (encoded in F_2) is completely analogues to the method we used for computing $b_3(\tilde{Y})$. Any even two-form γ^α can be decomposed as

$$\gamma^\alpha = \Psi_2^\alpha + \Psi_1^\alpha \wedge (\mathrm{d}z + A_1)\,, \tag{9.3.12}$$

with Ψ_2^α an even two-form on the base and Ψ_1^α an odd one-form on the base. Any even $\Psi_2^\alpha \in H^2(\tilde{X})$ will lift to an element of $H^2(\tilde{Y})$, but any odd $\Psi_1^\alpha \in H^1(\tilde{X})$ can only lift to an element of $H^2(\tilde{Y})$ if there exists an even two-form Ψ_2^α such that

$$\mathrm{d}\Psi_2^\alpha = \Psi_1^\alpha \wedge F_2\,, \quad \text{and} \quad \mathrm{d}\Psi_1^\alpha = 0\,. \tag{9.3.13}$$

These forms again satisfy the equivalence relationships

$$\Psi_2^\alpha \sim \Psi_2^\alpha + \mathrm{d}\kappa_1^\alpha - k^\alpha F_2 \quad \text{and} \quad \Psi_1^\alpha \sim \Psi_1^\alpha + \mathrm{d}k^\alpha\,, \tag{9.3.14}$$

for any one-form κ_1^α and function k^α.

[10] One can integrate (9.3.10) over a closed 4-surface $\mathcal{C}_4$ that contains a closed 2-surface $\mathcal{C}_2$ encapsulating an O6/D6 source, such as $\mathcal{C}_4 = \mathcal{C}_2 \times \mathcal{D}_2$ then $\int_{\mathcal{C}_4} \mathrm{d}\Sigma_3 = 0 = -Q_{O6/D6} \int_{\mathcal{D}_2} \Sigma_2$, if we take $\mathcal{D}_2$ the cycle Poincaré dual to Σ_2 then the integral on the RHS can be normalised to $+1$ such that one indeed finds an obstacle to define Σ_3.

To summarise, we have found that the moduli space of chiral multiplets of $\tilde{Y}$ is given by $H_3(\tilde{X})$ and possibly a subset of $H_2(\tilde{X})$ if equation (9.3.10) can be solved. For the sake of phenomenology one is interested in F_2 fluxes that do not allow a solution to (9.3.10). Analogously, the vector multiplets of the moduli space is given by $H_2(\tilde{X})$ and possibly a subset of $H_1(\tilde{X})$ if equation (9.3.13) can be solved.

Open string sector

The discussion so far allows us to neatly identify the geometric moduli of type IIA models. However this cannot be the whole story for in generic compactifications there will be additional moduli coming from the open string sector, specifically from D6-branes. Since the fibre becomes singular at the position of D6 sources, we cannot simply use the formalism above to count open string moduli and one should be more careful. We have decided not to present a detailed analysis of this since it would obscure the main point of this paper. Furthermore, once fluxes are present one can eliminate open string modes by tuning the amount of flux appropriately. We therefore settle with a brief qualitative discussion.

Two kind of sources will play a role in the following: the first is the Atiyah-Hitchin manifold and the second is the Taub-NUT manifold. It is well known that these two manifolds are uplifts of sources appearing in type IIA, specifically at weak coupling the former becomes an O6-plane and the latter a D6-brane [276–280]. The presence of these objects will distort the six-dimensional geometry and have an impact on the cohomology of the 7-manifold that we are considering.

In type IIA the open string moduli can still be accommodated for in a simple way as long as we only deal with non-intersecting D6 branes, as is the case here. To preserve supersymmetry all sources need to wrap calibrated cycles, and the calibration condition for 3-cycles in Calabi-Yau threefolds gives special Lagrangian cycles [284]. The Lagrangian condition amounts to asking that the pullback of the symplectic two form J is zero on the cycles, that is

$$\imath^*(J) = 0\,, \tag{9.3.15}$$

where $\imath$ is the embedding of the brane locus $\tilde{S}$ into $\tilde{X}$, and $\imath^*$ is the corresponding pullback to $\tilde{S}$. While in principle for generic sources this condition might lead to stabilisation of some of the Kähler moduli [285] it is always automatically satisfied if all sources are parallel because of the parity of J under the orientifold involution. The degrees of freedom on the brane are deformations of $\tilde{S}$, given by sections in the normal bundle of $\tilde{S}$, and the worldvolume gauge field A_1. On special-Lagrangian cycles J defines a map from the normal bundle to the cotangent bundle, mapping a vector field ξ to $\imath^*(J(\xi, \cdot))$ [284]. This one-form

combines with A_1 into a complex field that describes the open string moduli. This completes the story for open string moduli if all sources are parallel because further stabilisation mechanism [285] are absent for the same reason as above. While we recalled the calibration condition for the case of branes in Calabi-Yau manifolds it is actually possible to generalise this discussion to the case of manifolds with $SU(3)$ structure that are our case of study. In particular in a supersymmetric background the forms J and $e^{-W}\Omega_R$ are still closed and can be used to calibrate the cycle similarly to the Calabi-Yau case [286–288]. Finally one may additionally study the supersymmetric deformations of the calibrated cycles and find that the picture described above does not change when the background geometry is not Calabi-Yau [289].

Now let us understand the counting of open string moduli for $\tilde{Y}$. The locus $\tilde{S}$ will be the location of Taub-NUT centres in $\tilde{Y}$, i.e. the locus where the fibre circle degenerates. We can see that (9.3.9) only remains well-defined in the presence of D6-branes if Σ_2 vanishes on $\tilde{S}$. With parallel sources all forms Σ_2 odd under the orientifold involution satisfy this criterion and therefore lift to well defined 3-forms in $\tilde{Y}$. Let us now discuss one-cycles on $\tilde{S}$. If that one-cycle is trivial in $\tilde{X}$, it is the boundary of a two-chain in the orientifold. The M-theory circle just pinches off at the position of the D6 so that the two-chain times the circle fibration gives a closed three-cycle. The size of this cycle controls the position of the D6 in M-theory. The Wilson line of C_3 on this cycle gives the Wilson line degree of freedom on the brane. Thus, it gives a complex modulus of the G2 compactification. If the one-cycle on $\tilde{S}$ is not trivial in $\tilde{X}$, it won't give rise to a two-chain in $\tilde{X}$, and it does not lead to a non-trivial three-cycle in $\tilde{Y}$, therefore not adding moduli. Thus, we find that

$$H^3(\tilde{Y}) = H^3(\tilde{X})_+ \oplus H^2_{F_2}(\tilde{X})_- \,, \tag{9.3.16}$$

where

$$H^2_{F_2}(\tilde{X}) = \left\{ \Sigma_2 \in H^2(\tilde{X}) \mid \Sigma_2 \wedge F_2 \text{ is exact} \right\} . \tag{9.3.17}$$

We therefore find agreement between the type IIA and M-theory counting of moduli.

9.4 Examples

In what follows we provide explicit examples. We start by treating one example in detail, including a study of its moduli space, and then we describe a generalisation of this example as a proof of principle that various examples can be found. However, in our generalisation we simply content ourselves with demonstrating the existence, rather than providing a full discussion about

features of the models, like moduli spaces, tadpoles et cetera. We proceed by working in the IIA frame and give explicit examples of the supersymmetric Minkowski vacua with just F_2 flux and parallel O6/D6 sources since the lift to the G2 manifold was described in detail above.

9.4.1 A simple example without flux

Consider a 6-torus with Cartesian coordinates x^i, taking values in the interval $[0, 1[$. We assume the following orientifold action:

$$\sigma(x) = (-x^1, -x^2, -x^3, x^4, x^5, x^6)\,. \tag{9.4.1}$$

We have 8 O6 planes along directions x^a, $a = 4, 5, 6$, and localised in the x^1, x^2, x^3. In order to find genuine $\mathcal{N} = 1$ supersymmetry (off-shell) we need to further orbifold. The difficulty of adding orbifold involutions is that one typically creates multiple intersecting images of the O6 planes. But as we argued, we do not have a full supergravity description of those and we therefore should make sure the orbifold images create no further intersecting O6 planes. The following $\mathbb{Z}_2^2$ actions

$$\Theta_1(x) = (-x^1, -x^2, x^3, x^4 + 1/2, -x^5, -x^6 + 1/2)\,, \tag{9.4.2}$$

$$\Theta_2(x) = (x^1, -x^2, -x^3, -x^4, -x^5 + 1/2, x^6 + 1/2)\,, \tag{9.4.3}$$

do the job since each O6 plane is mapped into itself. The resulting space is not an actual orbifold since the discrete symmetries have no fixed points. As a consequence one can show that the holonomy group is not the full SU(3) but a restricted $\mathbb{Z}_2^2$ subgroup.

We now follow the method of Kachru-McGreevy to uplift the CY orientifold without fluxes [256] and find the moduli spectrum. This means we add 16 D6 branes to cancel the tadpole. We can straightforwardly uplift this setup to a Joyce manifold of type $\mathbb{T}^7/\mathbb{Z}_2^3$, where the $\mathbb{Z}_2^3$ orbifold is generated by

$$\alpha(x) = (\Theta_1(x), x^7)\,, \qquad \beta(x) = (\Theta_2(x), x^7)\,, \qquad \gamma(x) = (\sigma(x), -x^7)\,. \tag{9.4.4}$$

with x^7 the coordinate on the M-theory circle. Note that the resulting 7d space does not have full G2 holonomy but rather a restricted $\mathrm{SU}(2) \times \mathbb{Z}_2^2$. This is a consequence of the freely-acting $\mathbb{Z}_2^2$ involutions (9.4.2, 9.4.3). If we lift the 6-torus with the O6 involution (9.4.1) and blow up the resulting 7d orbifold we obtain $K3 \times \mathbb{T}^3$. If we then add the first $\mathbb{Z}_2$ involution (9.4.2) it is freely acting on $\mathbb{T}^3$ and is Enriques involution on $K3$. This effectively lifts to the Enriques CY $\times S^1$ [290]. The second $\mathbb{Z}_2$ involution (9.4.3) then ensures one ends up with a G2 manifold in 11d but with restricted holonomy $\mathrm{SU}(2) \times \mathbb{Z}_2^2$.

We now compute the spectrum from a 10d and an 11d viewpoint. In 10d, we have to compute the untwisted cohomology first. Then, additional two-/three-forms may appear from resolution of singularities. In this case, however, the orbifold has no fixed points, so there are no singularities and therefore no additional forms. Finally, we have to consider the open string spectrum given by D6's.

Out of all forms on $\mathbb{T}^6$, there are no one-forms that are invariant under $\langle\Theta_1, \Theta_2\rangle$. The invariant forms are 3 two-forms $(\mathrm{d}x^{16}, \mathrm{d}x^{25}, \mathrm{d}x^{34})$, and 8 three-forms[11]. Therefore, we have $h^{2,1} = 3, h^{1,1} = 3$, corresponding respectively to 4 $(h^{2,1} + 1)$ $\mathcal{N} = 2$ hypermultiplets (considering the dilaton), and 3 $\mathcal{N} = 2$ vector multiplets. By taking into account the orientifold action, we get a $\mathcal{N} = 1$ theory, and all 4 hypermultiplets become chiral multiplets, while the vector multiplets split following the behaviour of the $(1, 1)$-forms under σ. We only have untwisted 2-forms, and these are σ-odd, giving thus other 3 chiral multiplets. The (untwisted) cohomology therefore accounts for 7 chiral multiplets. The 16 D6 branes needed for tadpole cancellation provide 16 vector multiplets and 48 chiral multiplets. Summing up, the final spectrum is given by 16 vector multiplets and 55 chiral multiplets.

In 11d, we expect to find the same $\mathcal{N} = 1$ spectrum solely from untwisted and twisted cohomology of $\mathbb{T}^7/\mathbb{Z}_2^3$. There are no one- nor two-forms of $\mathbb{T}^7$ which are invariant under $\langle\alpha, \beta, \gamma\rangle$. The only invariant forms are 7 three-forms $(\mathrm{d}x^{124}, \mathrm{d}x^{135}, \mathrm{d}x^{456}, \mathrm{d}x^{263}, \mathrm{d}x^{167}, \mathrm{d}x^{257}, \mathrm{d}x^{347})$. Therefore, the untwisted cohomology yields to 7 chiral multiplets, as in 10d. The spectrum from singularity resolution must then match the open string spectrum found in 10d. For that we have to count the 2-/3-forms from singularity resolution. Elements $\alpha, \beta, \alpha\beta, \alpha\gamma, \beta\gamma, \alpha\beta\gamma$ have no fixed points, and the only element having fixed points is γ. Its fixed points are 16 $\mathbb{T}^3$'s, parallel and along 456, and these are all distinct from each other under the action of the remaining elements of the orbifold. The local form of the singularities at the fixed T_3's is $\mathbb{R}^4/\mathbb{Z}_2 \times \mathbb{T}^3$, and resolving each of these as in [256, 291] yields a two-form and 3 three-forms. Eventually, we have $b_2 = 16$ vector multiplets and $b_3 = 55$ chiral multiplets, as in 10d.

9.4.2 With fluxes

We expect that the presence of fluxes stabilises some moduli, so that the spectrum gets reduced. A way of thinking is via mirror symmetry: we know that the (supersymmetric) IIB dual model is of DRS (GKP) type [183] ([184]), where fluxes stabilise complex structure moduli. Since complex structure moduli

[11] These three-forms are: $\mathrm{d}x^{123}, \mathrm{d}x^{124}, \mathrm{d}x^{135}, \mathrm{d}x^{145}, \mathrm{d}x^{356}, \mathrm{d}x^{456}, \mathrm{d}x^{236}, \mathrm{d}x^{246}$.

are dual to IIA Kähler moduli, we expect some of the latter to be lifted by F_2. Also, due to fluxes, the tadpole cancellation is now satisfied with fewer D6's than in the fluxless case, so that the open string spectrum is reduced too.

We operate the moduli counting following the generic discussion developed above, see (9.3.10). For simplicity we assume F_2 is such that the tadpole cancellation is satisfied in absence of D6's. We twist the torus using the following Maurer-Cartan forms, i.e. with[12]

$$e^4 = \mathrm{d}x^4 + Nx^1\mathrm{d}x^2\,, \tag{9.4.5}$$

$$e^5 = \mathrm{d}x^5 + Nx^1\mathrm{d}x^3\,, \tag{9.4.6}$$

$$e^6 = \mathrm{d}x^6 + Nx^2\mathrm{d}x^3\,, \tag{9.4.7}$$

and $e^i = \mathrm{d}x^i$ with $i = 1, 2, 3$. The geometric flux N is quantised as $N \in \mathbb{Z}$ since our coordinates run from 0 to 1. One can check that this nilmanifold algebra is consistent under the orbifold and orientifold actions, since

$$\Theta_1(e) = (-e^1, -e^2, e^3, e^4, -e^5, -e^6)\,, \tag{9.4.8}$$

$$\Theta_2(e) = (e^1, -e^2, -e^3, -e^4, -e^5, e^6)\,, \tag{9.4.9}$$

$$\sigma(e) = (-e^1, -e^2, -e^3, e^4, e^5, e^6)\,, \tag{9.4.10}$$

consistent with the Maurer–Cartan relations. Again, we have 8 O6's, along directions $a = 4, 5, 6$, each one mapped into itself by the orbifold action. On a nilmanifold the cohomology can be computed by restricting to the left-invariant cohomology and one can then verify that the only non-trivial cohomology groups are [273]

$$H^2(\tilde{\mathbb{T}}^6/\mathbb{Z}_2^2, \mathbb{Z}) = \mathbb{Z}^2\,, \tag{9.4.11}$$

$$H^3(\tilde{\mathbb{T}}^6/\mathbb{Z}_2^2, \mathbb{Z}) = \mathbb{Z}^6 \times \mathbb{Z}_N\,, \tag{9.4.12}$$

with $\mathbb{Z}_N$ representing the torsion piece. More explicitly, the closed and non-exact 2-forms surviving the orbifolding are $v_1 = e^{16} + e^{25}$ and $v_2 = e^{16} - e^{34}$, while the closed and non-exact 3-forms are e^{124}, e^{135}, e^{145}, e^{236}, e^{246}, e^{356} (and the torsion factor is given by e^{123}, since Ne^{123} is exact). A solution to the supersymmetry equations, consistent with the orbifolding, is given by:

$$J = e^{16} - e^{25} - 2e^{34} = 2v_2 - v_1\,, \tag{9.4.13}$$

$$g_s F_2^B = N\big[-\tilde{e}^{16} + \tilde{e}^{25} - 4\tilde{e}^{34}\big]\,, \tag{9.4.14}$$

$$g_s F_2^S = e^W \star_3 \mathrm{d}e^{-4W}\,, \tag{9.4.15}$$

[12]Notice here e^a correspond to $\tilde{e}^a$ in [176].

where the warp factor is sourced by O6's only. We have split the F_2 field strength into a background and a sourced part as discussed around equation (9.2.9). As we discussed, given the Kähler deformations $\delta J = t^a \omega_a$, with $\omega_a \in H^2_-(\tilde{X})$ $(a = 1, 2)$, the Kähler modulus t^a remains massless if there exists a $\gamma_a \in \Omega^3(\tilde{X})$ such that

$$\mathrm{d}\gamma_a = \omega_a \wedge F_2 \,. \tag{9.4.16}$$

Otherwise, the modulus is lifted. Let us take only F_2^B, since one can show that $v_1 \wedge F_2^S \sim \mathrm{d}[e^{-3W}(e^{623} + e^{531})], v_2 \wedge F_2^S \sim \mathrm{d}[e^{-3W}(e^{623} - e^{412})]$. In other words, the backreaction has no effect on stabilising closed string moduli. We find that only $\omega_1 = 2v_2 - v_1 = J$ does satisfy the equation, with trivial 3-form, $\gamma_1 = 0$. The corresponding modulus, t_1, is not stabilised by F_2. Not surprisingly, this follows directly from primitivity condition $J \wedge F_2^B = 0$. On the other hand, we cannot find any other 3-form satisfying the equation for any choice of $\omega_2 \in H^2_-(\tilde{X})$, meaning t_2 is lifted. Without loss of generality, we take $\omega_2 = v_2$. One can check that this is indeed the correct scenario by computing the superpotential (off-shell quantity, here $J = t^a \omega_a$):

$$\mathcal{W} \equiv \int_{\tilde{X}} J^2 \wedge F_2^B \sim t_2^2 \int_{\tilde{X}} w_2^2 \wedge F_2^B \sim t_2^2 \tag{9.4.17}$$

where, due to primitivity $w_1 \wedge F_2^B = 0$, t_1 disappeared. So in total we find 7 chiral multiplets in our moduli space. Concerning the vector multiplets we potentially pick up a multiplet from the first cohomology class of the base $\tilde{X}$. But the way we orbifolded the twisted torus base, does not allow non-trivial one-cycles and we therefore remain with two vector multiplets.

9.4.3 A class of examples

We now describe a generalisation of the previous example but not in the same amount of detail. We do not discuss the moduli spaces, flux quantisation or the tadpole condition, but it can be done following the steps outlined in the above example.

We start again from groupmanifold covering spaces which are further orbifolded and orientifolded such that we obtain $\mathcal{N} = 1$ (off-shell) supersymmetry. In the covering space we can then rely on the existence of a global co-frame build from the Maurer-Cartan forms e^i with $i = 1, \ldots, 6$, obeying

$$\mathrm{d}e^i = -\tfrac{1}{2} f^i{}_{jk} e^j \wedge e^k, \tag{9.4.18}$$

with f traceless Lie algebra structure constants. To classify solutions that cannot be related by simple relabelling, or coordinate transformations, one

needs to fix a basis for the structure constants. For that we follow the strategy outlined in [292]: one first fixes the $\mathbb{Z}_2$ orientifold and the orbifold group and then consider the possible Lie algebras consistent with the orientifold and orbifold group. Like in our previous example the O6 plane involution $\mathcal{R}$ is fixed as follows:

$$\mathcal{R} : (e^1, e^2, e^3, e^4, e^5, e^6) \to \left(-e^1, -e^2, -e^3, e^4, e^5, e^6\right) , \tag{9.4.19}$$

and the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold action Θ_1, Θ_2 are:

$$\Theta_1 : (e^1, e^2, e^3, e^4, e^5, e^6) \to (-e^1, -e^2, e^3, e^4, -e^5, -e^6) , \tag{9.4.20}$$

$$\Theta_2 : (e^1, e^2, e^3, e^4, e^5, e^6) \to (e^1, -e^2, -e^3, -e^4, -e^5, e^6) , \tag{9.4.21}$$

Both sides of (9.4.18) must transform in the same way under the orbifold and orientifold. Demanding this sets all structure constants to zero except those gathered in the matrix

$$r = \begin{bmatrix} f^5_{31} & f^5_{46} & -f^2_{63} & -f^2_{41} \\ -f^4_{12} & f^3_{62} & -f^4_{56} & -f^3_{51} \\ -f^6_{32} & f^1_{42} & f^1_{53} & f^6_{54}. \end{bmatrix} . \tag{9.4.22}$$

To obtain an interesting solution, at least one of the structure constants must be nonzero. Imposing $f^5_{31} \neq 0$ and demanding that the structure constants obey the Jacobi identities restricts them to one of three possible classes [13]

$$\text{Class A:}\quad r = \begin{bmatrix} f^5_{31} & f^5_{46} & -f^2_{63} & -f^2_{41} \\ -f^4_{12} & \frac{f^5_{46} f^4_{12}}{f^5_{31}} & -\frac{f^4_{12} f^2_{63}}{f^5_{31}} & \frac{f^2_{41} f^4_{12}}{f^5_{31}} \\ -f^6_{32} & \frac{f^5_{46} f^6_{32}}{f^5_{31}} & \frac{f^2_{63} f^6_{32}}{f^5_{31}} & -\frac{f^6_{32} f^2_{41}}{f^5_{31}} \end{bmatrix} , \tag{9.4.23}$$

$$\text{Class B:}\quad r = \begin{bmatrix} f^5_{31} & 0 & 0 & -f^2_{41} \\ -f^4_{12} & 0 & 0 & -f^3_{51} \\ 0 & 0 & 0 & 0 \end{bmatrix} ,$$

$$\text{Class C:}\quad r = \begin{bmatrix} f^5_{31} & 0 & 0 & 0 \\ 0 & 0 & 0 & -f^3_{51} \\ 0 & 0 & f^1_{53} & 0 \end{bmatrix} .$$

We can now look at the way the conditions on J and Ω constrain our groupmanifold. To perform the classification of possible groupmanifolds, we

[13] This classification corrects a mistake in [292] where an extra class was found, labeled C, because a number of additional structure constants were erroneously not set to zero for that class C. Doing so makes that class equal to class B of this paper, after relabelling.

work in the smeared limit where the warp factors are set to constant. They can eventually be inserted for the case one wishes to study more closely after the classification has been completed. Because J and Ω_R must be odd under the orientifold and invariant under the orbifold, their form is restricted to

$$J = k^1 e^{52} + k^2 e^{43} + k^3 e^{61}, \tag{9.4.24}$$

$$\Omega_R = \mathcal{F}_1 e^{312} + \mathcal{F}_2 e^{462} + \mathcal{F}_3 e^{563} + \mathcal{F}_4 e^{541}. \tag{9.4.25}$$

One sees that (9.2.5) is immediately satisfied and Ω_R imposes no constraints on the algebra of our groupmanifold. It is still possible that Ω_I imposes constraints on the groupmanifold algebra via (9.2.6). However, rather than seeing this equation as a constraint on the groupmanifold, we can see it as telling us to which background value we should set F_2 to realise the groupmanifold we choose. This still leaves us with the constraints on the structure constants imposed by J via (9.2.4). For class A, we find

$$\begin{aligned} f^5_{46}\left[k_1 f^5_{31} - k_2 f^4_{12} + k_3 f^6_{32}\right] &= 0\,, \\ k_1 f^5_{31} + k_2 f^4_{12} - k_3 f^6_{32} &= 0\,, \\ f^2_{63}\left[k_1 f^5_{31} - k_2 f^4_{12} - k_3 f^6_{32}\right] &= 0\,, \\ f^2_{41}\left[k_1 f^5_{31} + k_2 f^4_{12} + k_3 f^6_{32}\right] &= 0\,, \end{aligned} \tag{9.4.26}$$

must be satisfied, while for class B we get

$$\begin{aligned} k_1 f^5_{13} + k_2 f^4_{12} &= 0\,, \\ k_1 f^2_{41} - k_2 f^3_{51} &= 0\,, \end{aligned} \tag{9.4.27}$$

with all structure constants not appearing in these equations being required to be zero. The constraints on class C do not allow nontrivial solutions. In the end we find four possible types of group manifolds covering spaces, shown in table 9.1, and manifolds obtained from these by the simultaneous exchange of the axes $4 \leftrightarrow 6$ and $1 \leftrightarrow 3$

Relationships between the structure constants and the factors k^i of J are determined by either (9.4.26) or (9.4.27). All these algebras can be implemented by compact manifolds as discussed for instance in in the classification of [176], where they are referred to as n4.6, n3.5, s3.3 and s2.5 respectively[14].

If we wish to make sure that no intersecting O-planes appear, we must make sure that Θ_1 and Θ_2 do not create any additional O-planes. So far, we have

[14]Strictly, our solvmanifolds are of a larger class that contain s3.3 and s2.5 within them and compactness has only been fully established for s3.3 and a subset of s2.5 [176].

Type	Nonzero f's
Nilmanifold	f^5_{13}, f^4_{12}
Nilmanifold	f^5_{13}, f^4_{12}, f^6_{32}
Solvmanifold	f^5_{13}, f^2_{63}, f^6_{32}
Solvmanifold	f^5_{13}, f^3_{51}, f^4_{12}, f^2_{41}

Table 9.1: Groupmanifolds satisfying the constraints imposed on our class of examples.

only discussed how Θ_1 and Θ_2 act on the Maurer-Cartan forms and we still have some freedom in choosing how they act on the coordinates, so long as this does not alter how they act on the Mauer-Cartan forms. By making sure Θ_1, Θ_2 and all other involutions except for $\mathcal{R}$ are freely acting on at least one direction by making them act as $x_i \to x_i + 1/2$, we ensure they do not create additional O-planes. This is straightforward to implement for the nilmanifolds as demonstrated for a specific case in section 9.4.1. However, we were unable to do this for the solvmanifolds. The problem is that if e.g.

$$e^5 = dx^5 + x^3 dx^1 \,, \tag{9.4.28}$$

then a shift of x^3 does not leave e^5 invariant; making it disallowed. Due to restrictions like this, it is impossible to implement enough shifts to make all the required involutions freely acting.

In conclusion, we find two types of nilmanifolds and two types of solvmanifolds that are compatible with our demands, which can be seen in table 9.1. However, we could not find a way to avoid the appearance of intersecting O-planes for the solvmanifolds. The presence of intersecting O-planes may be problematic. The remaining set of examples is a handful since one has some freedom to take an integer choice of metric flux and F_2 flux to cancel the RR tadpole for 8 O6 planes. Allowing D6 branes as well, the size of this set enlarges from order 10 to roughly order 100.

9.5 Comments on supersymmetry breaking

9.5.1 SUSY-breaking fluxes and G2 structures

There is a rather straightforward way to break supersymmetry of the IIA flux backgrounds and maintain reasonable control. The inspiration comes from the supersymmetry-breaking on the mirror IIB side that occurs whenever fluxes are imaginary self-dual (ISD) but with certain components that are not $(2,1)$ and

primitive [184]. There exist simple solutions of that kind that can be T-dualised to IIA Minkowski solutions with just F_2 flux. The simplest example is perhaps the following.

Consider (an orbifold of) the 6-torus with coordinates $y^1, \ldots, y^6$. Assume the following form of the ISD flux:

$$F_3 = f \mathrm{d}y^4 \wedge \mathrm{d}y^5 \wedge \mathrm{d}y^6 \,, \quad H_3 = e^{\phi_0} \star_6 F_3 \,. \tag{9.5.1}$$

This solution is such that the ISD three-form flux on the six torus has a $(0,3)$ component [293] and hence breaks SUSY. If we now T-dualise 3 times along y^1, y^5, y^6 we obtain the following IIA twisted torus background (ignoring warping)

$$\mathrm{d}s^2 = \mathrm{d}s_4^2 + (\mathrm{d}y^5)^2 + (\mathrm{d}y^6)^2 + (e^1)^2 + (\mathrm{d}y^2)^2 + (\mathrm{d}y^3)^2 + (\mathrm{d}y^4)^2 \,, \tag{9.5.2}$$

$$F_2 = -f \mathrm{d}y^4 \wedge e^1 \,, \tag{9.5.3}$$

where the one-form e^1 is twisted as follows:

$$e^1 = dy^1 + f y_2 e^{\phi_0} dy^3 \,. \tag{9.5.4}$$

The O6 stretches along the T-duality directions e^1, y^5 and y^6 such that its $\mathbb{Z}_2$ involution on the internal space works as $y^{2,3,4} \to -y^{2,3,4}$. Depending on the choice of lattice on the resulting Nilmanifold this action has a number of invariant 3-dimensional subspaces such that the number of O6 planes is fixed and so is the total tension T_6. The number f can be called metric flux and relates to the O6 tension T_6 as follows

$$T_6 = f^2 e^{-\phi_0} \,. \tag{9.5.5}$$

If this solution is lifted to 11d, one obtains again a solution to the 11D Einstein equations with an unwarped 4D Minkowski piece and a flat compact 7d internal manifold. Since supersymmetry is not preserved it cannot have G2 holonomy. Instead it is not difficult to argue one obtains a Ricci-flat G2 *structure*. So the IIB method for breaking supersymmetry and lifting some moduli by fluxes has turned into a purely geometric picture in which G2 holonomy manifolds are altered into Ricci-flat G2 *structures* that break supersymmetry and with typically less moduli than their G2 holonomy counterparts[15].

9.5.2 A non-homogenous extension of the Scherk-Schwarz mechanism

We want to emphasise that this is a very close cousin of the mechanism found by Scherk and Schwarz (SS) to stabilise part of the torus moduli in 10- or 11-

[15] This is not unrelated to the ideas presented in [294, 295].

dimensional supergravity and break supersymmetry [296, 297]. This is achieved by dimensionally reducing over a twisted torus with a flat metric. The deviations away from the flat metric are massive and together with the massless moduli the degrees of freedom are captured by a gauged supergravity of the no-scale type that has a Minkowski vacuum with all supersymmetry broken. To be more precise, the breaking of supersymmetry and the lifting of torus moduli depends in a subtle way on the quantisation rules for the structure constants [294, 298] invisible in the gauged supergravity description. The gauged supergravity itself has the same amount of supersymmetry as the higher-dimensional theory one starts off with.

If one compares the SS mechanism with the more modern approach to moduli stabilisation and supersymmetry-breaking that uses fluxes and orientifolds there are some advantages and disadvantages. An advantage is that we do not have to rely on singular objects that source warping effects whose effective field theory description is more challenging to work out. One is furthermore not constrained by tadpole conditions that bound the size of fluxes since the 'metric fluxes' in SS compactifications can be taken to be arbitrarily large (aside the quantisation subtleties previously mentioned). A disadvantage is that SS compactifications of 10/11-dimensional supergravity lead to maximal or half-maximal gauged supergravity descriptions where supersymmetry is broken spontaneously. This is related to the fact that twisted tori are locally homogeneous, which furthermore also strongly restricts the number of examples or 'metric fluxes' that can lead to flat metrics. In fact they can easily be classified into a handful of examples [298].

Our picture connects the SS mechanism for moduli stabilisation with that of orientifolds and fluxes by demonstrating that there exists a T-duality frame (IIA with O6 planes and F_2 fluxes) in which the orientifold lifts to a generalised Scherk–Schwarz space, which differs from the twisted tori by being locally non-homogeneous in 11d. The breaking of supersymmetry in the IIA side can be understood as a T-dual of supersymmetry breaking ISD three-form fluxes in IIB. This demonstrates that there exists a class of supersymmetry-breaking flat manifolds that lead to classical Minkowski vacua beyond the purely homogenous examples of Scherk and Schwarz.

9.5.3 Non-perturbative quantum effects and uplifting?

Finally we want to comment further on moduli-stabilisation. The mirror (and 11D lift) of the Kähler moduli in IIB remain massless at the classical level. Hence one could speculate on (quantum)-corrections to the 11D supergravity action that can further stabilise those. In the IIA frame some mechanism has been suggested in [299] that is supposed to mirror the Large Volume Scenario

(LVS) [213] in IIB. Here we briefly discuss possible IIA mirror of KKLT [189] and its M-theory lift. We hope that the geometric picture in M-theory could potentially elucidate some of the questions raised about the consistency of moduli stabilisation [194].

The natural quantum effects to consider are Euclidean D2 branes wrapping calibrated cycles in the IIA frame or gaugino condensation on the reduced 7D YM theory living on wrapped D6 branes. These quantum effects can potentially lift the massless modes and turn the non-SUSY Minkowski vacuum into a SUSY AdS vacuum. We expect that, in the 11D picture, these two effects are either corresponding to Euclidean M2 branes or to localised gravitino condensation in the Ricci-flat G2 structure. It would be interesting to study this in some depth.

Then one can further speculate about possible uplift sources that break supersymmetry of the resulting AdS vacuum. The natural candidates are the mirrors to anti-D3 branes, which are anti-D6 branes [299]. For them to be consistent one should be able to redshift their tensions through local throats which should be mirrors to the Klebanov-Strassler throats. They have been reported to exist in [263]. This uplift method, from a 11d viewpoint is entirely geometrical since the anti-D6 branes lift to local anti-Taub-NUT spaces. We therefore suspect that the consistency of anti-D3 uplifting to dS [203, 210] can be more easily analysed in this framework since most of the complicated flux terms in these computations can all be reshuffled into the 11d curvature term. We hope to come back to this in the future.

9.6 Discussion

In this paper we have argued that a class of supersymmetric IIA flux compactifications to 4d $\mathcal{N} = 1$ Minkowski vacua are in one-to-one correspondence with compactifications of 11-dimensional supergravity on new G2 spaces. The IIA flux compactifications for which this works are constrained by demanding that the only sources are O6 planes and D6 branes and the only fluxes are RR F_2 fluxes. Under these conditions the uplift to 11 dimensions is guaranteed to be completely geometrical, in other words IIA vacua then lift to unwarped 4d Minkowski space times a 7d Ricci flat geometry. The requirement of minimal supersymmetry then further constrains the 7d geometries to have G2 holonomy.

It was well-known that in the fluxless limit the IIA orientifolds lift to G2 spaces and in the case of toroidal orientifolds there seems to be a nice connection with the Joyce class [256]. Here we point out that the general structure of the supersymmetric compactifications of IIA with the specified restrictions are sufficiently well-understood in general terms [174, 176] in order to analyse

the uplift to 11d. Especially in the case where the sources are all parallel to each other, the resulting geometries are under control since it is known how to describe the sources in a localised manner. The geometries are specific SU(3)-structure manifolds which we denoted $\tilde{X}$. The non-Calabi-Yau nature of $\tilde{X}$ is a consequence of the F_2 flux backreaction. The F_2 flux has furthermore a non-trivial interplay with the topology such that the RR tadpole can be satisfied without having to cancel all O6 charges with D6 charges [273]. The uplifted geometry, denoted $\tilde{Y}$, is then a twisted circle fibration of $\tilde{X}$ modded out by the $\mathbb{Z}_2$ action of the O6 planes (9.1.2):

$$\tilde{Y} = \frac{\tilde{X} \ltimes S^1}{\mathbb{Z}_2} \,. \tag{9.6.1}$$

The CY limit (obtained by cancelling all O6 charges with D6 charges) leaves no room for non-trivial F_2 fluxes and we obtain an untwisted circle fibration.

We have given a general discussion of moduli counting both in IIA and in 11-dimensional supergravity and argued that the moduli counting matches. For the sake of being explicit and as a proof of existence we have presented a handful of examples by finding explicit SU(3)-structures based on orbifolds of twisted tori. The minor drawback of that approach is that the resulting holonomy is not the full G2 group but a non-trivial subgroup thereof. We expect that the recent developments in constructing non-trivial SU(3)-structures [300, 301] should be very relevant in this context. So if more progress on the construction of explicit SU(3)-structures is to occur, then our construction implies immediately a new class of G2 spaces.

Our main interest, however, was not only to enlarge the class of G2 spaces. Instead we consider it very interesting that a class of warped flux compactifications in IIA, which are mirrors of IIB flux compactifications with 3-form fluxes, are completely geometrised. As a consequence the moduli space of a warped compactification, which is the topic of warped effective field theory (WEFT), can be rephrased in terms of ordinary Kaluza-Klein theory of 11d supergravity on G2 spaces. We hope this will elucidate some of the difficulties in understanding WEFT, which would be interesting to address in the future. Finally, we argued that this geometrisation of IIA flux compactifications should be useful in investigating recent concerns[16] about non-perturbative moduli-stabilisation [194] and the supersymmetry-breaking dS uplifts on top of it [203, 210], which we consider interesting directions for future research.

[16] See references [302, 303] for complementary viewpoints that refute the concerns.

Part IV

EFT Constraints

Chapter 10

The Landscape and the Swampland

10.1 The landscape

> And since Space is divisible *in infinitum*, and Matter is not necessarily in all places, it may be also allow'd that God is able to create Particles of Matter of several Sizes and Figures, and in several Proportions to Space, and perhaps of different Densities and Forces, and thereby to vary the Laws of Nature, and make Worlds of several sorts in several Parts of the Universe. At least, I see nothing of Contradiction in all this.
>
> Isaac Newton, Opticks

The previous part of this thesis provides a nice example of how it is clearly possible to find a large number of vacua in string theory. It then looks like string theory does not produce an unique prediction for low energy physics [304–307]. This cornucopia of vacua is known as the landscape of string theory [308].

To find a low energy EFT that resembles the real world, one must then scan through the different vacua in the landscape.

This general idea is nicely illustrated in a toy model by Bousso and Polchinski [306] which we will sketch. Consider a four-dimensional theory with Einstein gravity and a four-form gauge field. One could imagine this coming for instance from M-theory. There could also be a bare cosmological constant Λ_B which might be very negative if one starts from an AdS vacuum. The four-dimensional action is then

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{16\pi G_N}\left(\mathcal{R} - 2\Lambda_B\right) - \frac{Z}{48}F_4^2\right], \quad (10.1.1)$$

where Z is a normalization constant. The four-form equation of motion is $\partial_\mu(\sqrt{-g}F_4^{\mu\nu\rho\sigma}) = 0$, which is is solved by $F_4^{\mu\nu\rho\sigma} = c\epsilon^{\mu\nu\rho\sigma}$ with c a constant and $\epsilon^{\mu\nu\rho\sigma}$ the totally antisymmetric tensor. The F_4 term in the action then effectively contributes as a cosmological constant. Once the theory and charges are quantized, the allowed values of F_4 are quantized, $F_4 = nb$ with n a natural number and b the smallest quantum. The cosmological constant Λ for a solution to the theory is then

$$\Lambda(n) = \Lambda_B + \frac{Z^2n^2b^2}{2}. \quad (10.1.2)$$

Now imagine that n is initially very large such that $\Lambda \gg \Lambda_{\text{observed universe}} > 0$. The universe is then initially in a de Sitter phase and membranes can spontaneously nucleate through the Brown-Bunster mechanism [309, 310]. In string/M-theory there will be from the four-dimensional perspective 2-branes charged under the F_4 flux. The branes are domain walls that fill two of the three spatial dimensions and the cosmological constant will decrease in the region between a nucleated brane-antibrane pair[1]. Such regions will grow in size due to the de Sitter expansion. Different regions of space will then populate all different values of $\Lambda(n) > 0$. Most of these values of the cosmological constant will not viably produce a universe with a large amount of structure and the possibility for human-like life that comes with this [16]. One needs the spacing between different cosmological constants $|\Lambda(n)-\Lambda(n-1)|$ to be fine enough such that at least some vacua will have a cosmological constant in the anthropically viable range.

In reality, there are many more properties of the low energy EFT that have to be fixed besides the cosmological constant, such as the groups and coupling constants of the gauge theories present. But full string theory also has many more elements than just an F_4 flux. If one element scans the possible EFTs with a spacing A for some parameter in the EFT and another string theory

[1] The nucleation of a region with an increased cosmological constant is also possible, but its rate is highly suppressed and we will neglect this.

element scans with a spacing B then combining the two one can scan with spacing roughly $|A - B|$. The more elements one introduces, the finer the scan. One then needs this scan to be fine enough that one expects a few string theory solutions take the right values for low energy EFT parameters to match what we measure in the real world. If the KKLT construction that we discussed in section 7.6 works, it provides the landscape framework with a more detailed proposed string theory framework to obtain a positive cosmological constant.

One should be careful in interpreting the wavefunction of the universe, especially in the kind of multiverse that the landscape of string theory produces. Were one able to compute the true ground state wavefunction of string theory, its predictions very likely place the universe somewhere deeply inhospitable to life. But one should not directly query this wavefunction, one should be asking *given all my current knowledge of the world around me* as a prior, *what is the probability that we observe X?*

The picture we paint here is only a very crude cartoon of the work to scan the possible string theory vacua. A great deal of work has gone into trying to understand the landscape of possible string vacua e.g. [311–316].

After the preceding discussion, the reader may be left with the impression that in string theory anything goes for the low energy EFT. This is not the case, string theory is constrained in what low energy theories can emerge. Some constraints are natural already from the low energy EFT perspective. Couplings and masses in the EFT should be such that the EFT is unitary for energy scales where the EFT is to be trusted. Other constraints only become clear from a string theory perspective. We saw in part III that it is not at all straightforward to achieve scale-separation between the external and compact dimension. It is then unclear how many genuinely lower-dimensional string theory vacua there are compared to string vacua that are really ten-dimensional. The possibility of achieving a positive cosmological constant is even less clear.

10.2 The swampland

> If they can get you asking the wrong questions, they don't have to worry about answers.
>
> Thomas Pynchon, Gravity's Rainbow

If one wants to understand what low energy EFTs are permitted in string theory,

an approach one could take would be to scan the entire landscape of string theory. This task would be gargantuan. More worrying, as we saw in part III elements involved in constructing phenomenologically interesting string vacua are not always completely understood. It is unclear how long it will take to fully grasp them and what the final conclusion on the (non)-existence of some string vacua will be.

A different route to understanding what low energy EFTs are allowed is to ask if there exist certain general principles of quantum gravity that forbid a theory of quantum gravity from being coupled to certain kinds of EFTs. This approach is known as the swampland program. Low energy EFTs that cannot be consistently extended to UV complete theory of quantum gravity are said to lie in the swampland as opposed to the landscape of consistent string vacua. A great deal of this is reviewed in [317], to which we also refer for refinements and variants of conjectures discussed here, and extensive bibliography.

Two paths present themselves for finding such a principle of quantum gravity. One is to look at string vacua one believes to be reliable and search for some common pattern in the low energy theories of these vacua. The other is to come up with a heuristic argument why certain EFTs might become inconsistent when coupled to quantum gravity. One can then *conjecture* or speculate that some principle places a certain class of EFTs in the swampland. After this, one can start looking for string vacua that serve as (counter)examples to the conjecture or start sharpening the heuristic argument into something approaching a proof. In this process the conjecture may need to be modified in some way or dropped entirely. Several swampland conjectures exist. Some have reached a point where many consider them essentially proven, while others remain very speculative.

Holography provides a great arena to study swampland conjectures as holography gives access to genuine quantum gravity and allows quantum gravity questions to be mapped to quantum field theory questions where a greater level of rigor can be used. The *no global symmetries in quantum gravity* swampland conjecture has received what may even be considered a holographic proof [318, 319] in the context of theories to which AdS/CFT applies.

Black holes are natural objects to study in the swampland program. Large black holes have a small curvature near their horizon and so one expects a low energy effective description to be valid. Despite this, black holes are quantum objects, even when considering only their horizon and forgetting about their interior. The evaporation of black holes is for instance driven by quantum effects.

As an example (and because they will be useful later), let us present two such conjectures here without argument.

Conjecture 1. *Electric weak gravity conjecture (WGC) [320, 321]*

In a theory of gravity coupled to a $U(1)$ gauge field F in an asymptotically flat space

$$S = \int d^d x \sqrt{-g} \left[(M_{pd})^{d-2} \frac{\mathcal{R}_d}{2} - \frac{1}{4g^2} F^2 + .. \right] , \tag{10.2.1}$$

there must exist a particle with mass m and charge q such that

$$m \leq \sqrt{\frac{d-2}{d-3}} g q (M_{pd})^{\frac{d-2}{2}} . \tag{10.2.2}$$

The bound eq. (10.2.2) by itself does not constrain low energy EFTs as the particle obeying it could be very heavy. The strong WGC demands that the lightest charged particle in the theory obeys eq. (10.2.2) and does constrain low energy EFTs. As the WGC bound demands that there are particles for which the gauge field interacts more strongly than gravity, one can think of the WGC as roughly stating that gravity is the weakest force. This is clearly true when comparing gravity and electromagnetism in our world.

This bounds is equivalent to demanding that black holes in asymptotically flat space are able to decay. The metric of an electrically charged nonrotating black hole in four-dimensional flat space is

$$ds^2 = -U(r) dt^2 + U^{-1}(r) dr^2 + r^2 d\Omega_2^2 , \tag{10.2.3}$$

with

$$U(r) = 1 - \frac{2 G_N M}{r} + \frac{G_N Q^2}{r^2} , \tag{10.2.4}$$

where M is the black hole mass and $Q^2 = g^2 Q_r^2 / 4\pi$ with Q_r the physical black hole charge measured in the gauge theory.

Black holes with $G_N M^2 > Q^2$ are subextremal. They have an outer event horizon r_+ and inner event horizon r_-. For extremal black holes with $G_N M^2 = Q^2$ the two event horizons coincide. Black holes with $G_N M^2 < Q^2$ are superextremal. They have a naked singularity not screened by an event horizon and are generally considered unphysical unless it is understood how to resolve the singularity in a full quantum gravity theory. This is overviewed in fig. 10.1 .

The decay of charged black holes can be thought of as a combination of two processes. First, there is Hawking radiation, familiar from neutral black holes, which is has a neutral net charge current and has a thermal spectrum with with the black hole temperature $T_{BH} = \frac{r_+ - r_-}{4\pi r_+^2}$. Second, there is the Schwinger process. As the black hole is electrically charged, it produces an electric field. An electric field in QFT will produce pairs of oppositely charged particles

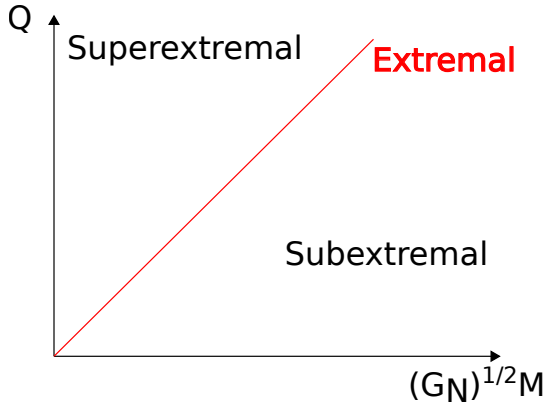


Figure 10.1: The possible charged black holes in flat space.

and send these in opposite directions to discharge itself, this is the Schwinger process [322]. As a result, the black hole will discharge [323].

Extremal black holes have $T_{BH} = 0$ and can only decay through Schwinger radiation. The near-horizon geometry of an extremal black hole is $AdS_2 \times S^2$ as can be seen by specifying our discussion in sections 3.2 and 3.3 to the black hole case. While de Sitter space causes particle production, AdS can be thought of as inhibiting particle production inside it. As a result, the Schwinger process can only take place for particles that obey eq. (10.2.2) in the higher dimensional Minkowski theory [324]. Demanding that extremal black holes decay then produces conjecture 1.

In chapter 11 we will develop a swampland conjecture of our own from studying the decay of charged black holes in de Sitter space.

Conjecture 2. *Swampland distance conjecture [325]*
Consider a theory of gravity with a moduli space $\mathcal{M}$ parametrized by the expectation values of fields ϕ^i.

- *Starting from any point P in $\mathcal{M}$, there exists an infinite distance direction in the moduli space such that if we send $Q \in \mathcal{M}$ in this direction, $d(P,Q) \to \infty$. Here d is the distance between two points in the moduli space measured using the metric G_{ij} on the moduli fields. Meaning that the moduli fields appear in the action with kinetic term $g_{\mu\nu}G_{ij}\partial^\mu\phi^i\partial^\nu\phi^j$.*
- *There exists an infinite tower of massive states with associated mass scale M such that*

$$M(Q) \sim M(P)e^{-\alpha d(P,Q)}\,. \tag{10.2.5}$$

with α a positive constant.

This conjecture implies that when one moves an infinite distance in moduli space an infinite tower of fields becomes arbitrarily light.

A nice example illustrating this conjecture is the Kaluza-Klein reduction of section 7.1.1. In this case, the modulus field is the one associated to the radius of the fifth dimension R. The field $\Phi \sim \log(R)$ appears in the action with a canonical kinetic term as we saw in section 7.1.1 . Distances are then measured as $d \sim |\Delta\Phi|$. When sending $\Phi \to \infty \cong R \to \infty$ an infinite tower of states becomes light, the KK-modes of the five-dimensional fields. Their masses go as $m \sim \frac{1}{R}$ and so decrease exponentially in terms of Φ as desired.

The limit $\Phi \to -\infty \cong R \to 0$ is also an infinite distance limit, but in our Kaluza-Klein analysis we do not see an infinite tower of states becoming light. The point is that the conjecture stems from quantum gravity and we should do our analysis in full string theory. In this case, it is possible to wrap strings around the S^1 of the fifth dimension. There is an energy needed to stretch the tensionful strings to the length of the S^1 when wrapping. This results in such wrapped strings producing massive states in an EFT description. When $R \to 0$ the mass of these states goes down with the behavior desired for the distance conjectured.

This example illustrates that the goal of the swampland program is to bound low energy EFTs from quantum gravity.

Since the focus of this thesis on de Sitter space, let us focus on the specific conjecture that claims string theory has no de Sitter vacua [161, 163, 164, 326]. The refined form of this conjecture is

Conjecture 3 (Refined no de Sitter conjecture)**.** *A potential $V(\phi)$ for a scalar field in a low-energy EFT of a theory of quantum gravity must satisfy either*

$$|\nabla V| \geq \frac{c}{M_P} V \tag{10.2.6}$$

or

$$min(\nabla_i \nabla_j V) \leq -\frac{c'}{M_P^2} V \tag{10.2.7}$$

with $c\,, c' > 0$ universal constants presumed of order one.

One argument for this conjecture is the difficulty involved in constructing explicit examples in string theory. We studied this in some detail in part III. Here, we will present a heuristic argument for this conjecture from [164].

As we discussed in section 2.2, there is an entropy associated to an observer's cosmological horizon with radius R in de Sitter space,

$$S \propto R^2 \,. \tag{10.2.8}$$

More generally, a universe that is accelerating fast enough will have an apparent horizon, which one could argue should have an entropy associated to it if the period of expansion is long-lasting. Specifically, so long as

$$|\nabla V|/V \leq \sqrt{2}\,, \tag{10.2.9}$$

for the potential $V(\phi)$ of the scalar field driving the expansion, (and $V > 0$) there will be an apparent horizon at

$$R = 1/\sqrt{V}\,. \tag{10.2.10}$$

The picture we sketch above only makes sense if we have a stable de Sitter vacuum or a slow enough rolling evolution of ϕ. If there are tachyonic modes whose fluctuations destabilize the vacuum , none of this is sensible and so we must also demand the absence of such modes, or at least that they are negligible compared to scale set by the apparent horizon,

$$\min(\nabla_i \nabla_j V) \geq -c'/R^2 = -c'V\,, \tag{10.2.11}$$

with c' some positive constant, presumed order one until some hoped-for future argument informs us of its exact value.

To continue, we will use the swampland distance conjecure. When there is a large displacement $\Delta\phi$ of the scalar field, there should be a tower of fields that become light. As these fields become light enough, they enter into the low-energy EFT. We will quantify the number of fields that enter into the EFT as

$$N(\Delta\phi) \propto n(\Delta\phi) e^{b\Delta\phi}\,. \tag{10.2.12}$$

Here, $e^{b\Delta\phi}$ is the rate at which the leading towers are becoming light with exponent $b > 0$. Also, $n(\Delta\phi)$ counts the number of towers becoming light at this rate. It is reasonable that the number of towers that become light can only go up, not down, with increased $\Delta\phi$, and so $dn/d\phi \geq 0$.

As these states become light, they contribute to the entropy of the low-energy theory. One *assumes* that contribution in an Hubble volume can be parametrized as

$$S_{\text{tower}} \propto N^\gamma R^\delta\,. \tag{10.2.13}$$

With γ and δ positive. Of course, this entropy should be no larger than the total entropy in this Hubble volume,

$$S_{\text{tower}} \propto N^\gamma R^\delta \leq R^2\,. \tag{10.2.14}$$

Once N is at least order one, this bound only makes sense if $\delta < 2$.

In the weak coupling regime, light modes dominate the entropy. One then *assumes* that this bound is saturated in the weak coupling regime. Using eq. (10.2.10), this then implies

$$V \propto R^{-2} \propto N^{-2\gamma/(2-\delta)},. \tag{10.2.15}$$

Using eq. (10.2.12), this then implies

$$\frac{|\nabla V|}{V} \geq c = \frac{2b\gamma}{2-\delta}\,. \tag{10.2.16}$$

And so, we have given an heuristic argument for the no-dS conjecture. Clearly this argument only makes sense in approximately Einstein gravity where $S \propto R^2$. It does not apply to the higher spin theories discussed in part II. The concept of a finite-dimensional Hilbert space for a hubble volume and entropy associated to it seem quite reasonable (though unproven) for an eternally stable de Sitter space. However, the more rapidly the metastable (approximate) de Sitter space decays, the less clear it is that these concepts make sense. A de Sitter space with an average lifetime of a few Hubble times then might not be excluded by such arguments. In chapter 12 we look at cosmologies that do not have a cosmological constant but instead a scalar field rolling down a potential. We show that such theories can for a range of parameters have lower-dimensional de Sitter solutions. If the refined no de Sitter conjecture is true it will bound the parameter space of such cosmologies.

If the refined no de Sitter conjecture is strictly true, there is no point in trying to understand quantum gravity in de Sitter space, only in trying to rigorously prove its inconsistency. A conjecture related to but distinct from the no de Sitter conjecture that is mindful of the caveats we presented in the previous paragraphs is the trans-Planckian censorship conjecture.

Conjecture 4. *Trans-Planckian censorship (TCC) [327, 328]*
Sub-Planckian quantum fluctuations should remain quantum.

As de Sitter space expands, the wavelength of the modes of fields in de Sitter space will redshift. Once a fluctuation grows larger than the hubble radius, it will freeze out and become a classical fluctuation. An eternal de Sitter space is then always in contradiction with the TCC. A metastable de Sitter space is permitted by the TCC so long as its lifetime T is short enough that transplanckian modes cannot classicalize. This is true when

$$T \leq \frac{1}{H}\log\left(\frac{M_P}{H}\right)\,. \tag{10.2.17}$$

For measured values of H one finds that T is bounded from above by two trillion years. Our universe with a lifetime of the order ten billion years is well within this bound.

In the asymptotic regions of field space the TCC reduces to no de Sitter conjecture with a specific value for c,

$$c_\infty = \frac{2}{\sqrt{(d-1)(d-2)}} \,. \tag{10.2.18}$$

The idea of the TCC that high energy fluctuations should be screened from the low energy EFT seems remiscent of section 2.2 where we discussed the possibility that the backreaction of quantum fluctuations of matter could destroy de Sitter space.

In fact [24] which we mentioned there proposed that the QFT vacuum of an expanding four-dimensional spacetime should be prepared such that modes with an energy above a scale Λ_{UV} do not appear. In this vacuum the cosmological constant is screened and sent to zero over time. From this, [24] found

$$c = \sqrt{\frac{2}{3}} \frac{\Lambda_{UV}}{M_P} \,, \tag{10.2.19}$$

for the no de Sitter conjecture to be in agreement with their vacuum choice. This reproduces the value of c from the TCC when one sends $\Lambda_{UV} \to M_p$.

10.3 The unity of opposites

We wish to emphasize that *What is the landscape of string theory solutions?* and *What is the swampland of situations forbidden by string theory?* are essentially the same question, but posed in the opposite manner. As such, the landscape program and the swampland program are really the same program pursued through different means.

Throughout this thesis, we have often used the terms *quantum gravity* and *string theory* in a way that appears interchangeable. Of course, most people who work on string theory labor in the assumption that string theory is the correct theory of quantum gravity. We saw in section 3.2 that all superstring theories are connected through duality. Let us end this chapter with one final swampland conjecture. In [329] it was conjectured that all consistent quantum gravity are connected. There is then only one theory of quantum gravity, string theory.

Chapter 11

Festina Lente

This chapter is a reprint of [5]. We study charged black hole evaporation in de Sitter space. We establish how the black hole mass and charge change over time due to both Hawking radiation and Schwinger pair production as a function of the masses and charges of the elementary particles in the theory. We find a lower bound on the mass of charged particles by demanding that large charged black holes evaporate back to empty de Sitter space, in accordance with the thermal picture of the de Sitter static patch. This bound is satisfied by the charged spectrum of the Standard Model. We discuss phenomenological implications for the cosmological hierarchy problem and inflation. Enforcing the thermal picture also leads to a heuristic remnant argument for the Weak Gravity Conjecture in de Sitter space, where the usual kinematic arguments do not work. All in all, charged black holes in de Sitter should make haste to evaporate, but they should not rush it. In addition to the preceding discussion in chapter 10, the discussion in section 7.1 and especially the Freund-Rubin solutions constructed in section 7.1.2 play a major role in this chapter.

11.1 Introduction

In section 10.2 we overviewed the swampland program [157, 160, 161, 163, 194, 210, 317–320, 325–327, 330–397]. The ample evidence for some swampland constraints in string theory gives us confidence they are probably correct, but it is equally important that at least in some cases we seem to have identified a physical principle underlying the conjecture. For instance, the conjecture that there are no global symmetries, which has been shown to hold in detail

in AdS/CFT [318,319], was originally motivated by UV-insensitive arguments about black holes and remnants. The fact that the conjecture seems to hold in every example could be taken as evidence that the original heuristics was probably correct. If the principle is correct, we can consider applying it in situations where it is not firmly established. For instance, consider $(B-L)$. This is a global symmetry of the SM lagrangian, and based on the above one would imagine that is gauged or broken in the real world, even if nobody has been able to get the SM or even positive vacuum energy in string theory or other would-be UV-complete framework. This is because we do have black holes and the original argument [331,336,398] which relies on getting stable remnants by throwing baryons into black holes seems to apply.

A similar story applies to the Weak Gravity Conjecture, weakgravityconjecture-conjecture. In a very handwavy way, the principle underlying WGC is that something bad happens if large extremal black holes cannot decay into slightly smaller (sub-)extremal black holes[1]. So the principle seems to be something like demanding that black holes should decay while remaining sub-extremal; this connection is made more precise in [399–401], which show that adding WGC particles to a particular theory in AdS is enough to comply with another physical principle, Weak Cosmic Censorship.

If we got the principle right, again we can hope to apply it in situations where we cannot validate it independently. In this paper we take the first steps towards generalizing WGC-like arguments to spacetimes with positive vacuum energy. Our arguments will take place in de Sitter, since this is the simplest example, but we imagine (and have checked in some cases) that they hold more generally, like in slow-roll quintessence models. We first encountered black holes in de Sitter space in section 2.1 .

The first step is to understand black hole evaporation in de Sitter. This is more complicated than in flat space since there is both a black hole and a cosmological horizon, and they exchange charge and mass. The problem was worked out early on for neutral black holes [402–405] and charged ones in the absence of charged particles [406, 407]. While there are some works that discuss emission of charged particles by black holes in de Sitter [408–412], we were unable to find a reference that described in detail how the mass and charge of the black hole deplete. We do this in the present paper, following an approach similar to [413], although the details are more complicated due to the lack of an asymptotically flat region.

[1] In the supersymmetric case, this can happen only marginally, and whether the decay actually takes place or not depends on the detailed dynamics (e.g. walls of marginal stability). What seems to matter is that the decay is kinematically allowed (a large BPS object can fragment into smaller ones).

What we will find is that, as long as the charged carrier is heavy enough in the sense that,

$$m^2 \gg qgM_P H,$$

(but still satisfying the WGC inequality) black holes in dS always evaporate all the way to empty de Sitter space. Here, m, q are the mass and charge of the elementary particle that is discharging the black hole, g is the $U(1)$ gauge coupling, M_P is Planck's mass and H is the Hubble scale. This does not come as a surprise [414], since unlike in flat space a charged particle inside the black hole can always tunnel close to or behind the cosmological horizon, therefore discharging the black hole. This result fits right in with another principle that seems to hold in de Sitter, namely that physics in the static patch behaves very much like a finite-dimensional thermal system at an equilibrium temperature of $H/(2\pi)$ [19]. Any state different from the vacuum in the static patch corresponds to taking the system out of equilibrium, and the system responds by eventually re-equilibrating. This is the case for small perturbations, small black holes, and even neutral and charged black holes in the absence of charged particles[2] [407]. We establish that this holds in the presence of heavy enough charged particles as well. This bound does not become trivial in the $M_P \to \infty$ limit. We will comment further on this in the conclusions.

However, something funny happens in the opposite limit of a very light charged particle (in the sense that $m^2 \ll qgM_P H$). In this case, we find that very large charged black holes (so large that the black hole and cosmological horizons coincide; these are called Nariai black holes [417, 418]) discharge essentially instantaneously and, due to the fact that charged Nariai solutions are more massive than neutral Nariai solutions (see Figure 11.1), they collapse into a Big Crunch, rather than returning to empty de Sitter space. Due to the quick discharge, the discharging charged Nariai solutions resemble "superextremal" Nariai solutions.

Thus, if there is at least one light charged particle in the theory, the thermodynamic picture of de Sitter in the static patch cannot hold, large charged black holes never re-equilibrate, and one gets outside of the sub-extremal region of allowed black holes in Figure 11.1. This might be a bad thing and so we entertain the possibility that $m^2 \gtrsim qgM_P H$ might be an actual constraint on the EFT. We emphasize that due to the lack of stringy examples this extrapolation is in shakier grounds than other constraints such as WGC or absence of global symmetries, since we cannot check it; all we can say is that the principles we extrapolate (thermality of the dS static patch, no

[2] We note in passing the recent interest on the Schottky anomaly [415, 416], a feature typically seen in the heat capacity of finite dimensional thermal systems which has an avatar in black holes in de Sitter space as well. This provides further evidence for the consistency of the thermal picture in dS.

superextremal black hole solutions) seem a reasonable extrapolation of what quantum mechanics and gravity in de Sitter might look like. Nonetheless the findings and computations carried out in this paper are interesting from a GR and QFT viewpoint, regardless of whether these principles survive eternally or not.

It turns out that avoiding fast discharge of Nariai solutions also leads to WGC. There are two ways to trigger a quick black hole discharge, either by having a few light particles in the above sense, or by having a huge number of heavy ones, so that the black hole discharges via their combined effect. In a theory in which the usual WGC is not satisfied, small black holes are very long-lived and play exactly this role in the limit of small weak coupling. Thus, one recovers a form of the original WGC heuristics that works in dS space.

A constraint like $m^2 \gtrsim qgM_P H$ can be checked against the real world, since it looks like de Sitter. Taking the $U(1)$ to be electromagnetism, we find it is satisfied by every charged particle in the Standard Model. One could take this as evidence for the thermal picture of the dS static patch. Since in the SM the fermions get their masses from coupling to the Higgs, the inequality alleviates (but does not solve) the cosmological hierarchy problem. It leads to no new constraints on models of milli-charged dark matter. There is an interesting interplay with inflation, where the constraint is satisfied by only some inflationary models. Possibilities include small field inflation, direct gauge kinetic term-inflaton coupling, or (tuned) models of Higgs inflation.

The paper is organized as follows:

- Section 11.2 reviews the charged black hole solutions of the Einstein-Maxwell-deSitter theory and the relevant near-horizon limits.
- Section 11.3 is the core of the paper. We analyze black hole discharge via Schwinger pair production of charged particles, both in the adiabatic and quasistatic regimes.
- Section 11.4 discusses the failure of thermality/detailed balance in the static patch in the adiabatic discharge regime. We suggest that this is evidence that the adiabatic regime could be pathological and should be avoided, discussing the phenomenological implications. We also explain how this is connected with WGC and discuss how the connection could be sharpened.
- Section 11.5 present our conclusions and afterthoughts.

Finally, some Appendices contain details of the calculations and some elaborations on the arguments we present.

11.2 Charged black holes in de Sitter space

In this Section we review the black hole solutions we will be interested in. We consider a $(3+1)$-dimensional Einstein-Maxwell-de Sitter system, with action (in $-+++$ signature)

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{16\pi G}\left(-R+\frac{6}{\ell^2}\right)+\frac{1}{4g^2}F_{\mu\nu}F^{\mu\nu}\right]. \tag{11.2.1}$$

This theory admits charged black hole solutions, the RN-dS metric, (see e.g. [418]):

$$ds^2 = -U(r)dt^2+\frac{dr^2}{U(r)}+r^2d\Omega, \tag{11.2.2}$$

with

$$U(r)\equiv 1-\frac{2GM_r}{r}+\frac{G(gQ_r)^2}{4\pi r^2}-\frac{r^2}{\ell^2}. \tag{11.2.3}$$

This metric is supported by an electric field or magnetic field. In the electric case we have

$$A=\Phi\,dt,\quad \Phi=\frac{g^2}{4\pi}\frac{Q_r}{r}. \tag{11.2.4}$$

The two parameters M_r and Q_r can be interpreted as a "mass" (more about this in Section 11.3) and the charge. In de Sitter space the notion of mass is ambiguous but, language wise we will refrain from these subtleties and will name M_r the mass from here onwards. In this normalization, a particle of charge q_r couples to the electrostatic potential via

$$q_r\int A. \tag{11.2.5}$$

If we choose g such that the particle of lowest charge in the spectrum has $q_r=1$, then charges are integer-quantized.

From now on we will work in Hubble units, $\ell=1$. Then the metric above takes the form

$$U(r)\equiv 1-\frac{2M}{r}+\frac{Q^2}{r^2}-r^2, \tag{11.2.6}$$

with the dimensionless parameters

$$M\equiv\frac{GM_r}{\ell},\quad Q^2\equiv\frac{Gg^2Q_r^2}{4\pi\ell^2}. \tag{11.2.7}$$

The RN-dS metric (11.2.3) contains two horizons accessible to an observer outside the black hole[3]: one is the usual event horizon of the charged black

[3]In addition to these, the black hole has an inner horizon r_- when subextremally charged.

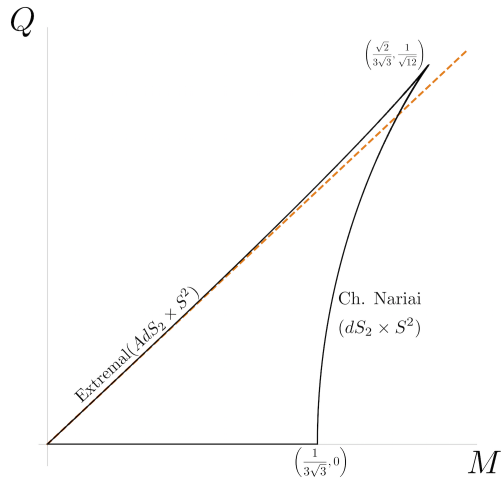


Figure 11.1: Phase diagram of Reissner-Nordstrom-de Sitter black holes, the "shark fin". Outside of the curve of extremal solutions, the metric becomes superextremal and has a naked singularity. The boundary has two branches: Extremal RN-dS black holes with an $AdS_2 \times S^2$ horizon topology and charged Nariai black holes for which the black hole and cosmological horizons coincide, with a $dS_2 \times S^2$ near-horizon geometry. The orange dashed line is the "lukewarm line" $Q = M$, where the temperatures of the black hole and cosmological horizons are identical.

hole, and the other is the cosmological horizon. In general, the black hole and the cosmological horizons will have different temperatures, and so they won't be in thermal equilibrium. When the black hole is large enough, the system will slowly drift towards equilibrium. This will always increase the entropy, whose leading contribution comes from the two horizons,

$$S = \frac{\pi}{4G}\left(r_{BH}^2 + r_{CH}^2\right), \tag{11.2.8}$$

but how this happens exactly depends on the particular details of the dynamics of the system. The solution (11.2.3) has two parameters M and Q, and the phase diagram as a function of M and Q (which we will call the "shark fin") is depicted in Figure 11.1. There is a boundary defined by the discriminant locus $\Delta = 0$ of the quartic equation $U(r) = 0$,where

$$\Delta \equiv M^2 - Q^2 - 27M^4 + 36M^2Q^2 - 8Q^4 - 16Q^6. \tag{11.2.9}$$

This boundary comprises extremal black hole solutions, which in turn split into two different branches [418]:

- The upper branch of the curve in Figure 11.1 is called the extremal branch, and parametrizes extremal black hole at zero temperature with horizons smaller than the cosmic horizon. The inner and outer black hole horizon coincide, $r_+ = r_-$. The near-horizon geometry is $AdS_2 \times S^2$.
- The lower branch of the curve in Figure 11.1 is the charged Nariai branch, and it contains subextremal charged black holes with the same area as the cosmological horizon. The outer black hole horizon and cosmic horizon are in thermal equilibrium and appear to coincide, $r_+ = r_c$, though a coordinate reparametrization will show this not to be the case. The near-horizon geometry is $dS_2 \times S^2$ and the temperature increases as one moves down the branch. We will be especially interested in this branch and discuss it in greater detail below.

These two branches meet at a point, dubbed the "ultracold" black hole [418] since semiclassically it describes an equilibrium configuration at vanishingly small temperature. Here, $r_- = r_+ = r_c$. The near horizon geometry is $\mathbb{M}_2 \times S^2$, exactly interpolating between the two branches. Outside of the extremal $\Delta = 0$ curve in the diagram one always has naked singularities, except on the $Q = 0$ axis of neutral black holes, where one has a Big Crunch singularity instead.

We want to study black holes for any values of M, Q in the shark fin. It would be convenient to have some coordinate-independent description which works smoothly even near the boundaries of the diagram. An interesting possibility is what we will call the "geodesic observer", sitting at a particular value of the radius r_g. This is the radius between the black hole and the cosmic horizon such that the pull of the black hole and cosmic expansion cancel, $U'(r_g) = 0$. An observer at $r = r_g$ can follow an orbit of the timelike Killing vector field which is at the same time a geodesic. Stuff closer to the black hole than the geodesic observer will eventually fall in, while stuff further out will keep on receding from the observer towards the cosmological horizon.

11.2.1 The charged Nariai branch

As black hole mass increases and a black hole approaches the Nariai branch, $r_+ \to r_c$. This is a near horizon limit, in which $U(r)$ develops a double zero, so as usual to properly understand this Nariai limit, we need a change of coordinate. Let

$$\rho = \frac{r - r_g}{\sqrt{|U(r_g)|}}, \quad \tau = \sqrt{|U(r_g)|}t. \tag{11.2.10}$$

The RNdS metric in terms of these radial and time coordinates is given by

$$ds^2 = -\frac{U(r)}{|U(r_g)|}d\tau^2 + \frac{|U(r_g)|}{U(r)}d\rho^2 + r^2 d\Omega^2, \tag{11.2.11}$$

The constant electric field has the same magnitude in these coordinates,

$$F = F_0\, dr \wedge dt = F_0\, d\rho \wedge d\tau, \quad F_0 = \frac{g^2}{4\pi}\frac{Q_r}{r^2}. \tag{11.2.12}$$

On this coordinate chart, the transition from subextremal to extremal black is completely smooth, and the horizons never overlap. Even though in the coordinates (11.2.2) the Nariai branch corresponds to the limit $r_+ = r_c$, we see that from the point of view of the geodesic observer the two horizons do not actually collide, although they do get closer as we approach the charged Nariai branch. Exactly on this branch, the metric components in (11.2.11) simplify to

$$\frac{U(r)}{U(r_g)} \to 1 - \frac{\rho^2}{\ell_{dS_2}^2}, \quad r^2 \to r_c^2, \tag{11.2.13}$$

which (as is well-known [19]) is exactly the $dS_2 \times S^2$ metric with a dS_2 radius

$$\ell_{dS_2}^2 = \frac{2}{U''(r_c)} = \frac{1}{6}\left(\frac{1}{\sqrt{1-12Q^2}} + 1\right) = \left(-3 + \frac{Q^2}{r_c^4}\right)^{-1}. \tag{11.2.14}$$

The radius of the S^2 on the charged Nariai branch is given by

$$r_c(Q) = \sqrt{\frac{1}{6}\left(1 + \sqrt{1-12Q^2}\right)}. \tag{11.2.15}$$

Physically, as we approach extremality, the usual behavior of near-horizon limits sets up, where the geometry is approximately $dS_2 \times S^2$. The geodesic observer happens to fall right in the middle of this tube, and the coordinates (11.2.10) are just local coordinates adapted to her.

11.3 Semiclassical evolution of black hole solutions

Classically, the black hole solutions we have just discussed are stable[4], but quantum mechanically, there is backreaction on both the metric and gauge field.

[4]Classical stability is controlled by the spectrum of quasi-normal modes on the classical solution. These have been the subject of recent study [419] in connection with variants of Strong Cosmic Censorship. The takeaway message for us is that for 4-dimensional black holes there are no instabilities, except for scalar fields of very low mass and nonvanishing charge. In this case, the instabilities only appear in a region of the (M, Q) plane far from the extremality curves, so it is not of direct relevance to us.

This is because there is Schwinger and Hawking radiation coming out of each horizon.

Let us ignore for a moment Schwinger radiation and focus on the dynamics as dictated solely by Hawking radiation. Since for generic values of M and Q the two horizons have different temperature, we expect the system to slowly drift towards equilibrium; in other words, the "mass" M will evolve towards its equilibrium value slowly [404]. Just like in flat space, the backreaction of Hawking radiation is only significant for a Planckian black hole.

Unlike in flat space, however, there is Hawking radiation coming out of both horizons, and generically, it is at different temperatures. The dashed orange line $M = Q$ in Figure 11.1 describes the line of "lukewarm" black holes [418], where the Hawking radiation coming from the black hole and cosmological horizons are at the same temperature. If we neglect charged particles, any point on this line is at thermodynamic equilibrium; we expect black holes to slowly evolve along lines of constant charge towards the lukewarm solution.

When one includes the effects of charged particles, which feel the electric field stretching between the two horizons, even this line becomes unstable. One can understand this by describing the horizons as reservoirs in the grand canonical ensemble [323]; to reach equilibrium, both the temperatures and electrostatic potentials of the two horizons must be identical. On the lukewarm line, this only happens at the lower left corner, that is, empty dS space.

The physical process by which a black hole loses charge is Schwinger radiation [322, 323, 413]. The black hole spacetime contains an electric field, and at any point, charged carriers are pair produced from the vacuum at some rate dependent on the local strength of the electric field. In flat space, this transition rate has an exponential suppression,

$$\Gamma \sim e^{-\frac{m^2}{qE}}, \tag{11.3.1}$$

controlled by the mass of the charged particle and the electric field. Our task is to understand how the black hole charge, Q, changes as a function of the Schwinger current. Unlike Hawking radiation, whose energy flux represents a small fraction of the black hole mass for any but the smallest black holes, the Schwinger current can be huge or tiny compared to the background electric field, and this is controlled roughly speaking by (11.3.1). Thus, there are two regimes, depending on whether $m^2 \lesseqgtr qE$, which we will discuss separately.

11.3.1 Quasistatic discharge: $m^2 \gg qE$

We will start with the regime in which the pair production (11.3.1) is very much suppressed. In this regime, Schwinger radiation behaves just like Hawking – the outgoing charge flux is small compared to the background charge. A similar analysis was carried out in [413] for charged black holes in flat space. We will reproduce their equations of motion in the appropriate limit.

A significant difference between the flat space and the dS cases is that in flat space one has conserved charges at infinity (total charge and mass). The problem is therefore just a matter of computing these fluxes at infinity. In de Sitter, on the other hand, there is no spatial asymptotia, and we must instead work with Einstein's equations directly. We will show that, under quasistatic evolution, the metric is very well approximated by (11.2.2) and (11.2.3) with slowly-varying (M, Q) as a function of time, just like in the flat space case.

Let us describe the slow drift of the solutions on the (M, Q) plane in more detail. This can be traced to the usual nonzero vevs $\langle T_{ab}\rangle$ and $\langle j_a\rangle$ for quantum field theory in curved spacetimes. We want to understand the dynamics of this system in the quasi-static regime. To do so, we will solve the equations of motion perturbatively,

$$\delta G_{ab} = 8\pi G\, \delta T_{ab}, \quad d\star \delta F = \star\langle \delta j\rangle, \tag{11.3.2}$$

to first order in the perturbations

$$\delta T_{ab} = \langle T_{ab}\rangle - T_{ab}^{\text{classical}}\,, \qquad \delta j = \langle j\rangle - j^{\text{classical}}\,. \tag{11.3.3}$$

For the RN-dS solutions, $j^{\text{classical}} = 0$, while $T_{ab}^{\text{classical}}$ is due to the background electromagnetic field.To ease bookkeeping, we will introduce a small dimensionless parameter ϵ

$$\epsilon \equiv G\ell^2 \sqrt{\delta T^b_a \delta T^a_b} \tag{11.3.4}$$

of the order of the largest matrix element in δT_{ab} (in the coordinate system (11.2.3)). So for instance, for thermal radiation, $\epsilon \sim T^4/(M_P H)^2$. We will also assume that the current is of order ϵ or smaller.

Then, we will solve (11.3.2) in a quasi-static approximation, using the method of separation of scales (see e.g. [420]); since there is a whole two-parameter family of solutions (11.2.3) to the unperturbed equations of motion, the perturbation will lead to slow motion in this parameter space, similarly to the computation of the perihilion precession in GR. More technically, one could work in the ϵ expansion of (11.3.2). Since the metric is t-independent, it is possible to separate variables,

$$\delta g \sim e^{i\omega t} g(r), \tag{11.3.5}$$

and then solve the linearized problems order by order to any desired accuracy in ϵ. If the spectrum of ω's is gapped, then the perturbation becomes a Fourier transform and under reasonable conditions it remains bounded (order by order in ϵ) at all times. However, if there are zero modes (secular perturbations), then they will grow linearly (as $\epsilon^n t$ at order n), spoiling agreement between the unperturbed and perturbed problems at a time ϵ^{-n}. The mass and charge parameters in (11.2.3) are examples of these zero modes.

The method of separation of scales removes the secular dependence of zero modes and works as follows. We introduce a "slow scale"

$$t_1 \equiv \epsilon\, t \tag{11.3.6}$$

on which the parameters of the background metric may depend,

$$\begin{aligned} M &\to M(t_1), \\ Q &\to Q(t_1). \end{aligned} \tag{11.3.7}$$

We also introduce an order ϵ perturbation[5],

$$\epsilon^{-1}\delta ds^2 = \delta A(r,t_1)dr^2 + \delta B(r,t_1)dt^2, \tag{11.3.8}$$

and a similar perturbation to the electric field,

$$F \to F + \epsilon g\ell\sqrt{G}\delta F(r,t_1)dr\wedge dt. \tag{11.3.9}$$

A perturbation like (11.3.7) and (11.3.8) on the metric results in a first-order perturbation to the Einstein tensor δG_{ab} which is covariantly conserved with respect to the background metric, is time independent, and respects the symmetries of the background. The most general such tensor is of the form

$$\delta G_{ab} - T^{(0)}_{ab} = \delta_1\, \xi_a\xi_b + \delta_2\, v_a v_b + \delta_3\, \xi_{(a}v_{b)} + \delta_4\, g_{ab}, \tag{11.3.10}$$

where $\xi = \partial_t$ is the future-directed time-like Killing vector field, $T^{(0)}$ is the background stress-energy tensor of the RN-dS solution, and

$$v = \frac{1}{r^2}\partial_r \qquad n = \sqrt{U}v\,, \tag{11.3.11}$$

is a harmonic vector field which satisfies $\nabla^a v_a = 0$ and which points in the outgoing radial direction. n_a is the corresponding unit spacelike vector. The expansion of the left hand side of (11.3.2) to first order in ϵ can be found in Appendix I.

[5]This is not the most general perturbation possible, but it will be enough to solve the equations of motion.

The quantum perturbation to the stress-energy tensor also has an expression in the form (11.3.10),

$$\delta T_{ab} = \eta_1\, \xi_a \xi_b + \eta_2\, v_a v_b + \eta_3\, \xi_{(a} v_{b)} + \eta_4\, g_{ab}, \tag{11.3.12}$$

with

$$\begin{aligned}
\eta_1 &= \frac{1}{2}\left(\frac{TU+3\mathcal{E}}{U^2} - r^4\mathcal{S}\right),\\
\eta_2 &= \frac{1}{2} r^4 \left(3r^4\mathcal{S}U^2 - TU - \mathcal{E}\right),\\
\eta_3 &= -2r^4\mathcal{T},\\
\eta_4 &= \frac{1}{2}\left(\frac{\mathcal{E}}{U} + r^4(-\mathcal{S})U + T\right).
\end{aligned} \tag{11.3.13}$$

where we have introduced the quantities

$$\mathcal{T} \equiv \delta T_{ab}\xi^a v^b, \quad \mathcal{E} \equiv \delta T_{ab}\xi^a\xi^b, \quad \mathcal{S} \equiv \delta T_{ab} v^a v^b. \tag{11.3.14}$$

Stress-energy conservation imposes constraints on these quantities. The linearized Einstein equations amount to $\delta_i = 8\pi G \eta_i$. For $i = 1, 2, 4$, these are a system of three coupled second-order ODE's in the radial variable that determine $\delta A, \delta B, \delta F$ to first order in the perturbation. Crucially, these equations do not involve time-derivatives; see Appendix I. We are more interested in the equation for δ_3,

$$\frac{4r(r\dot{M} - Q\dot{Q})}{-2Mr + Q^2 - r^4 + r^2} = -16\pi r^4 G\mathcal{T}. \tag{11.3.15}$$

Here, we have introduced the order ϵ parameters

$$\dot{M} \equiv \left.\frac{M(\epsilon t)}{dt}\right|_{t=0}, \quad \dot{Q} \equiv \left.\frac{Q(\epsilon t)}{dt}\right|_{t=0}. \tag{11.3.16}$$

Equation (11.3.15) is a first-order equation of motion for $(\dot{Q}, \dot{M})$. We need a second equation to fully determine the dynamics. This is obtained by a similar perturbation of the second equation in (11.3.2); we take the right hand side to be first-order in ϵ as well. One obtains from the two components of (11.3.2) a first-order ODE relating $\delta F(r)$ to the time-component of the current which is satisfied automatically due to energy-momentum conservation, and an equation of motion involving $\dot{Q}$,

$$\dot{Q} = -4\pi\mathcal{J}, \quad \mathcal{J} = Ur^4 j^a v_a. \tag{11.3.17}$$

Here, $\mathcal{J}$ is independent of the radial variable, due to current conservation[6]. We finally have the first order equations of motion for M and Q,

$$\dot{Q} = -4\pi\mathcal{J}, \quad \dot{M} = -\frac{4\pi\left(r^5 UG\mathcal{T} + Q\mathcal{J}\right)}{r}. \tag{11.3.18}$$

Here, $U(r)$ is evaluated on the background solution. The method of separation of scales provides a solution to (11.3.2) which is valid to first order in ϵ and depends on the variable t_1 only. There might be secular perturbations for this variable, so the approximation we just carried out is only valid for $\epsilon t_1 \ll 1$, or

$$t \lesssim \frac{1}{\epsilon^2}. \tag{11.3.19}$$

Hamiltonian and momentum constraints are satisfied to first order in ϵ in our solution. In fact, (11.3.18) is one of the momentum constraints; see Appendix I for the other components.

One can also write (11.3.18) in terms of parameters measured on a local inertial reference frame. Then, $\mathcal{T}$ and $\mathcal{J}$ are replaced by quantities that depend on the microphysics, M, and Q, the local charge and momentum fluxes $\mathscr{J}, \mathscr{T}$. We will do this at the location of the geodesic observer r_g, but again, physics is independent of this choice:

$$\mathcal{J} = \sqrt{U(r_g)}r_g^2 j^a n_a = \sqrt{U(r_g)}r_g^2\mathscr{J}, \quad \mathcal{T} = \frac{1}{r_g^2}T_{ab}u^a n^b = \frac{\mathscr{T}}{r_g^2}, \tag{11.3.20}$$

where u^a is a unit future-directed vector field in the same direction as ξ^a.

Finally, it is convenient to change the time variable in (11.3.18) to use the local time coordinate time coordinate t_g of the geodesic observer at some particular value of the radial coordinate, r_g. This is related to the global time coordinate in [402] simply as

$$\frac{dt_g}{dt} = \sqrt{U(r_g)}. \tag{11.3.21}$$

As a result, (11.3.18) become (from now on the dot now indicates time derivative with respect to t_g)

$$\dot{Q} = -4\pi r_g^2\mathscr{J}, \quad \dot{M} = -4\pi r_g^2\left(G\sqrt{U(r_g)}\mathscr{T} + \frac{Q}{r_g}\mathscr{J}\right). \tag{11.3.22}$$

Of course, physics should be independent of our choice of r_g.

[6]One could imagine using a similar argument to conclude that η_3 is constant. However, this does not work because there is an extra term coming from the first-order covariant derivative of the background stress-energy tensor, which is not present for the current since $j^{\text{classical}} = 0$ in the background.

A quantity of interest which is independent of the time coordinate is the rate of change of charge versus mass,

$$\frac{dM}{dQ} = \frac{\dot{M}}{\dot{Q}} = G\sqrt{U(r_g)}\frac{\mathscr{T}}{\mathscr{J}} + \frac{Q}{r_g}. \tag{11.3.23}$$

This quantity determines the direction the black hole evolves in the (M, Q) plane of Figure 11.1. The left-hand side is manifestly independent of r_g, while the right hand side is not. The way this works out is because stress-energy conservation relates $\mathscr{J}$ and $\mathscr{T}$. We will illustrate this for the particular case of a free scalar field Φ of mass m and charge q; the story generalizes to free fields of any spin in a straightforward manner. For a free field, the vector $\delta T_{ab}\xi^b$ is proportional to the current j_a. A solution to the wave equation with frequency ω (with respect to the global time coordinate t) satisfies,

$$T_{ab}\xi^b = \left(\frac{\omega}{q} + \Phi(r)\right) j_a, \tag{11.3.24}$$

where $\Phi(r) = -Q/r$ is the electrostatic potential (11.2.4). This can be checked explicitly via the expression for the Klein-Gordon conserved current. The contribution to $\mathscr{T}$ is

$$\Delta\mathscr{T} = \frac{1}{G\sqrt{U(r_g)}}\left(\frac{\omega}{q} + \Phi(r_g)\right)\Delta\mathscr{J}. \tag{11.3.25}$$

The second term in (11.3.25) cancels the r_g-dependence in (11.3.23), as it should.

We now turn to understanding the detailed expressions for $\mathscr{J}$ and $\mathscr{T}$. We will start with the former.

In de Sitter space the expansion of spacetime dilutes the particles produced by the Schwinger effect. This allows the system to have a steady current regime. This unlike Minkowski space, where current can build up *ad infinitum*. The current one-form

$$j = \frac{\mathcal{J}_r}{Ur^2}dr + \rho\, dt, \tag{11.3.26}$$

is conserved,

$$d * j = 0, \tag{11.3.27}$$

and is nonvanishing in the charged black hole background since the black hole electric field pair produces charged carriers via the Schwinger effect. In a semiclassical treatment, the Schwinger effect implies the pair production of charged carriers in the vacuum,

$$dj = 2\Gamma dr \wedge dt, \tag{11.3.28}$$

where Γ is the "Schwinger pair production rate", which can be computed via instantonic methods [324, 421] and whose value depends on the background electric field and geometry. The factor of two accounts for the fact that particles are produced in pairs. A mean field description such as (11.3.28) is good as long as the electric field varies slowly along space and time (by this we mean that the spatial gradients of the field are much smaller than the length scale associated to the field itself); we will relax this assumption later on on the Nariai branch. As long as the local electric field is much larger than the background curvature, the pair production process happens essentially in flat space; the electrons in the pair are produced at a distance much smaller than the characteristic curvature, and just accelerate from there on. As a result, we will use the flat space Γ originally computed by Schwinger [322, 421, 422],

$$\Gamma_{\text{flat-space,2d}}(m) = \frac{qE}{2\pi} e^{-\frac{\pi}{\chi}}, \quad \chi \equiv \frac{m^2}{qE}. \tag{11.3.29}$$

Notice that, for $m = 0$, equations (11.3.29) and (11.3.28) lead to $dj = \frac{q}{\pi} F$, which is precisely the chiral anomaly [423, 424].

Equation (11.3.29) is only the two-dimensional result. In the present situation we must take into account the flux on the 2-sphere, by summing over the KK modes:

$$\Gamma \approx \sum_s (2s+1)\Gamma_{\text{flat-space,2d}}\left(\sqrt{m^2 + \frac{s(s+1)}{r^2}}\right) \approx 4\pi r^2 \cdot \frac{q^2 E|E|}{8\pi^3} e^{-\frac{\pi}{\chi}}, \tag{11.3.30}$$

where we have replaced the sum by an integral; this will be a good approximation as long as $qEr^2 \gg 1$. The result (11.3.30) is the well-known four-dimensional Schwinger pair production rate[7] [421], multiplied by the area of the 2-sphere.

Following [413], we will also impose that the current is purely ingoing along the future black hole horizon, and purely outgoing on the cosmological horizon. An outgoing component of the current at the horizon would describe particles moving "against" the electric field, i.e. emission of particles with opposite charge to the black hole overcoming the huge potential barrier and escaping to the cosmological horizon, or same charge particles beating the electric field and managing to cross the black hole horizon. For a superextremal particle, these rare processes will have an additional, huge tunneling barrier (we will come back to this in Section 11.4), but they are actually important in some circumstances; for instance, any sub-extremal particle emitted by a flat-space black hole is bound to fall back. In situations like this, there will be a significant ingoing component, and the actual current will be smaller than our expressions below.

[7] The factor of two discrepancy with the classical result of Schwinger is due to the spin of a Dirac fermion as opposed to a scalar particle that we are considering here.

Mostly everywhere on the shark fin, our expressions actually constitute an upper bound on the current, which will be close to saturation for superextremal particles. However, as we will explain below, on the Nariai limit due to the additional symmetries of the problem, there is no ingoing component even for very massive particles. We should also notice that, even when there is an ingoing component, the current never vanishes, since that would conflict with (11.3.27)[8].

Neglecting their contribution, it follows that the current j must be orthogonal to the null generator of the horizon, $n_H = -\frac{dr}{U(r)} + dt$, close to the future horizon. Current conservation (11.3.27) now implies $\mathcal{J}_r$ is independent of position. These two properties together imply that

$$\mathcal{J}_r = r_+^2 \rho(r_+) = -r_c^2 \rho(r_c). \tag{11.3.31}$$

Finally, (11.3.26) becomes a first-order equation for ρ

$$\rho'(r) = 2\Gamma, \tag{11.3.32}$$

which we can integrate together with the boundary condition in (11.3.31) to obtain the expression for the current[9],

$$\mathcal{J} = \sqrt{\frac{g^2 G}{4\pi\ell^2}} \frac{r_c^2\, r_+^2}{r_c^2 + r_+^2} 2 \int_{r_+}^{r_c} dr'\, \Gamma(r'). \tag{11.3.33}$$

Using equation (11.3.20), we can write down the expression for the local current measured by an inertial observed $\mathscr{J}_r$,

$$\mathscr{J}_r = \sqrt{\frac{g^2 G}{4\pi\ell^2}} \frac{2}{\sqrt{U(r)} r^2} \frac{r_c^2\, r_+^2}{r_c^2 + r_+^2} \int_{r_+}^{r_c} dr'\, \Gamma(r'). \tag{11.3.34}$$

Equation (11.3.34), together with (11.3.30), gives a quasistatic current to use in (11.3.51) entirely as a function of M and Q. It reduces to the expression in [413] for small black holes (again, modulo a factor of two due to the electron spin degeneracy).

It is important to understand the regime of validity of (11.3.34). This is set by backreaction on the geometry. The charge density $\rho(r)$ screens the electric field

[8] In the flat space case, the statement is that anywhere around black hole even a sub-extremal particle has a (highly suppressed) chance of popping out of the vacuum. Thus, at any finite separation, the expectation value of the current is nonzero. The nucleation probability however decreases exponentially with the distance from the black hole, and therefore so does the current. There is no current flux at infinity, in accordance with the fact that a flat-space black hole cannot emit sub-extremal particles. In dS, however, this effect is cut off at the cosmological horizon, so there is always some current flowing.

[9] The prefactor accounts for the definition of Q (11.2.7) in terms of the actual integer-valued current.

of the black hole. Its overall effect should be small. From Maxwell's equation $d * F = g^2 * j$, we obtain

$$Q_r(r) = Q_r(r_+) + 4\pi \int_{r_+ + \Lambda^{-1}}^{r} dr' \, \frac{(r')^2 \rho(r')}{U(r')}. \tag{11.3.35}$$

The integral in (11.3.35) has a logarithmic divergence at the horizon, which we have regularized by introducing a regularized horizon at $r = r_+ + \Lambda^{-1}$, where Λ^{-1} is some UV cutoff. For the approximation to be trustworthy, we need

$$4\pi \int_{r_+ + \Lambda^{-1}}^{r} dr' \, \frac{(r')^2 \rho(r')}{U(r')} \ll \sqrt{\frac{4\pi\ell^2}{g^2 G}}. \tag{11.3.36}$$

This will be automatically true in the quasistatic regime due to (11.3.32) and the exponential Schwinger suppression in (11.3.29).

Notice that in this approximation, the contribution of the Schwinger current to $\mathscr{T}$ vanishes, since the same amount of left and right-moving charge carriers are produced. Thus, we can take $\mathscr{T}$ to come entirely from Hawking radiation. At the location of the geodesic observer, this is just the associated mass flux from the two horizons taking into account the redshift factors [402, 404, 425],

$$\mathscr{T} = \frac{\sigma}{(4\pi)^3} \left[r_c^2 U'(r_c) - r_+^2 U'(r_+) \right], \tag{11.3.37}$$

where r_+, r_c are the black hole and cosmological horizons, and σ is the Stefan-Botlzmann constant. Equations (11.3.34) and (11.3.37), when plugged back in (11.3.51), completely specify the dynamics of the system as a function of the mass m and charge q of the charged particle.

In equation (11.3.34) we have assumed the electric field is large compared to the background curvature, and this is a good approximation everywhere on the shark fin, except close to the neutral line, where the electric field becomes small. It turns one can work out the exact dynamics on the Nariai branch, even taking into account the curvature effects that were neglected when writing (11.3.34). The reason is the $dS_2 \times S^2$ geometry, which allows one to translate the problem to the Schwinger effect in dS_2. This problem was solved exactly in [426]; details can be found in Appendix K. The end result is that one can write an inequality for $\mathscr{J}$,

$$\mathscr{J}_{\text{Nariai}} = \sqrt{\frac{g^2 G}{4\pi\ell^2}} \mathscr{J}_r \gtrsim \sqrt{\frac{g^2 G}{4\pi\ell^2}} \frac{q^2 r_c^2 E|E|}{\pi^2 H_{dS_2}} \tanh\left(2\pi \frac{qE}{H_{dS_2}^2} \right) e^{-\frac{\pi}{\chi}} \tag{11.3.38}$$

which becomes tight in the large electric field limit $qE \gg H_{dS_2}^2$. We can compare with (11.3.34) in the Nariai limit, which is also easily computed since the two

integration limits coincide and the integral collapses. One obtains

$$\mathscr{J}_{\text{Nariai, large } E} = 2\sqrt{\frac{g^2G}{4\pi\ell^2}}\Gamma(r_c) = \sqrt{\frac{g^2G}{4\pi\ell^2}}\frac{1}{H_{dS_2}}4\pi r_c^2 \cdot \frac{q^2E|E|}{8\pi^3}e^{-\frac{\pi}{\chi}}, \quad (11.3.39)$$

which is exactly the same expression as (11.3.38) in the limit $qE \gg H_{dS_2}^2$. It is also important to notice that, on the extremal curve, the expression (11.3.38) is valid for any value of M, and it smoothly interpolates between the Hawking result for neutral particles and the Schwinger one.

An important question we should address is whether the flow (11.3.18) allows for an initially sub-extremal black hole becoming superextremal. Let us derive a simple criterion on the flow such that this is satisfied. In other words, we want to understand under which conditions the flow (11.3.18) maps the region inside the curve in Figure 11.1 to itself. Since the allowed region is defined by $\Delta > 0$ in (11.2.9), we require that the $(\dot{M}, \dot{Q})$ vector along the boundary $\Delta = 0$ points towards the gradient of $\Delta(M, Q)$, i.e.

$$\dot{Q}(\partial_Q\Delta)_{\Delta=0} + \dot{M}(\partial_M\Delta)_{\Delta=0} \geq 0. \quad (11.3.40)$$

Now, all over the subextremal region (M, Q) plane except on the $Q = 0$ axis, we expect $\dot{Q} < 0$ in a theory with charged particles, since the black hole horizon always tends to discharge and this only stops at $Q = 0$.

We separately discuss constraint (11.3.40) near the extremal branch and near the Nariai branch. A useful relation, valid all over the edge of the sharkfin ($\Delta = 0$) is

$$\left(\frac{\partial_Q\Delta}{\partial_M\Delta}\right)_{\Delta=0} = -\frac{Q}{r} \quad (11.3.41)$$

with r the horizon radius. On the Nariai branch one exactly saturates inequality (11.3.40). For that we use the vanishing of the $\sqrt{U}_g$ term in (11.3.23) together with the relation between r_g and Q as given in (11.2.15). For the extremal branch, Figure 11.1 shows that $(\partial_Q\Delta)_{\Delta=0} < 0$ whereas $(\partial_M\Delta)_{\Delta=0} > 0$. Equation (11.3.40) is satisfied because $\mathscr{T} < 0$ at the horizon. This is because the black hole is colder than the cosmological horizon, resulting in a net mass gain which dominates over Schwinger radiation in the quasi-static regime.

Figure 11.2 shows a schematic plot of the flow generated by the solutions in the shark fin, where we have taken $\mathcal{J}$ and $\mathcal{T}$ to be constant[10]. Our results are of course not exact; the detailed expressions are obtained by performing the integral in (11.3.34), which can be evaluated explicitly in terms of error functions [413]. Still, Figure 11.2 gives a roughly correct schematic picture of the flow of black hole solutions on the shark fin: Solutions flow to either the

[10]Similar plots can be found for the Kerr case in [427].

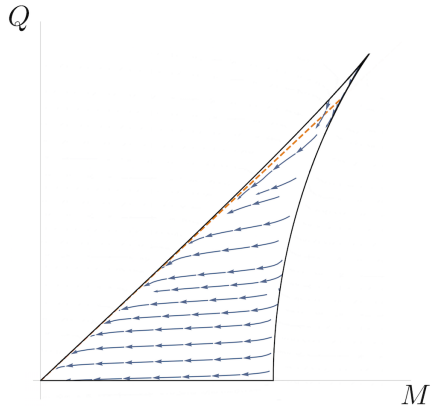


Figure 11.2: Schematic representation of the quasistatic flow on the shark fin generated by the combination of the Schwinger effect and Hawking radiation. Black hole solutions evolve towards the lukewarm or charged Nariai lines, losing charge very slowly in the meantime. Eventually, they evaporate completely. The flow stays completely within the shark fin, the black holes never turning superextremal. The flow follows exactly the charged Nariai line, so ignoring quantum fluctuations charged Nariai black holes remain Nariai while slowly discharging to the neutral solution. For illustration purposes, we took $\mathscr{J} \sim 1$ and $\mathscr{T} \sim M - Q$ to draw the plot.

lukewarm line or the Nariai branch, and then the neutral line. All solutions evaporate completely, eventually. Notice that this conclusion is insensitive to the precise values of (m, q), as long as the Schwinger effect is exponentially suppressed. This has consequences for the WGC in dS, which we discuss in Section 11.4.

Finally, a couple comments. It is worth emphasizing that we have assumed steady-state current. This is a physical assumption, which can be relaxed. One could for instance start with a black hole with no current present at all initially. But then the current will grow and, over the period of a Hubble time, such a state will develop into the steady current state.

We are looking at a quantum effects in a black hole background; we should make sure that there are no classical instabilities present. For instance, the Schwinger effect for near-extremal black holes in AdS actually takes the form of a classical instability [400], and a charged scalar field on the RN-dS is known to have classical instabilities for some values of M, Q, m, q [419]. The instability discussed in these references only takes place for very light fields, away from the

quasistatic regime, and for small values of the charge. As illustrated in Figure 1 of [419], the instability seems not to be relevant near the charged Nariai branch.

We should also point out[11] that the quasistatic computation we just described fixes the black hole at the center of the static patch – in other words, it "follows" the black hole along its worldline. This worldline is not a geodesic. The black hole receives radiation from the cosmological horizon and emits some itself, exchanging energy and momentum, so there will be some sort of Brownian motion [428, 429]. We can estimate the time that it takes for the geodesic observer to be kicked out when the black hole is much smaller than the Nariai limit[12]. Consider a small black hole on the lukewarm line. Its center of mass degrees of freedom will have thermalized and so it will have a mean kinetic energy of $3/2T$. In empty dS space, a free particle of mass M and energy E takes a time

$$t_\epsilon = \sqrt{\frac{E^2}{E^2 - M^2}} \arctan(1 - \epsilon) \tag{11.3.42}$$

to reach $r = 1 - \epsilon$ the cosmological horizon. Taking $T = H/2\pi$ to be the de Sitter temperature, one gets a timescale suppressed as $t_\epsilon \sim \sqrt{M/H}$. This is a long time since black holes are heavier than a Planck mass, but not exponentially suppressed, and so deep into the quasistatic regime a true geodesic observer will thus see the black hole kicked out of her static patch by the radiation, and will never see the evaporation come to an end. One could try to fix this by always sticking to the black hole center of mass, although this will no longer be described by the usual static patch Hamiltonian. In any case, we think these subtleties are not likely to affect our subsequent discussion, since most of our arguments in Section 11.4 will be about large black holes (close to the Nariai limit), for which the simple picture above does not hold, and also in the adiabatic regime, where the black hole discharges quickly.

We must also discuss Bousso's beading process [405–407]. These references describe a family of classical instabilities for the full four-dimensional charged Nariai geometry, coming from inhomogeneous fluctuations of size of the S^2 on the Nariai geometry. Where the radius of the S^2 becomes smaller than the Nariai value, the geometry collapses to a black hole; where it is slightly larger, it behaves like the exterior of a dS_4 geometry, with the spheres becoming asymptotically large in the asymptotic future. In short, these are fluctuations that drive the black hole towards sub-extremality. On the exact Nariai geometry, a perturbation in the n-th spherical harmonic leads to the formation of "beads", spacetimes containing n near-extremal RN-dS black holes.

[11] We thank T. Banks for bringing this up.

[12] To carry out a similar analysis for a very large black hole is beyond the scope of this paper.

Interestingly, the dynamics of this process is very different below or above the intersection of the lukewarm line with the charged Nariai line; above, the positive heat capacity of the black hole makes the Nariai geometry stable locally, so quantum fluctuations lead to an endless beading process and a fractal structure in the asymptotic future. Below the lukewarm line, at most a finite number of beads can be produced (how many exactly depends on the particular mode that dominates the perturbation), and the geometry in any given static patch goes back to a slightly sub-extremal black hole picture.

These predictions were computed with a one-loop quantum effective action which so far has always agreed with the thermodynamic results whenever available [405–407, 430]. For instance, while it seemed that neutral Nariai black holes could sometimes initially anti-evaporate [404], in the long run their behavior was always dictated by the semiclassical model. Effectively, what this means is that, below the lukewarm line, one should expect quantum effects to push the solution slightly off the Nariai branch, towards sub-extremality, where Hawking radiation drives evolution towards the lukewarm branch which then discharges quasi-statically, as in Figure 11.2. Above the lukewarm line one really stays in the Nariai branch for a long time, slowly discharging (and beading along the way) until crossing the lukewarm threshold.

11.3.2 Adiabatic discharge: $m^2 \ll qE$

We are also interested in quantifying how quickly the relaxation of the system happens when the mass of the charged carriers is below qE. Working again in the limit $qE \gg H^2$, the discharge effectively takes place in flat space, and on very short times compared to the Hubble scale. The electric field is so strong that it is immediately screened out locally by the pair-produced charge carriers, which subsequently annihilate each other, leading to radiation. As a result, one can model this kind of evaporation by replacing the electric field and charge matter at $t = 0$ by pure radiation with the same energy density.

We will only work out the detailed dynamics on the charged Nariai branch. Since the geometry is $dS_2 \times S^2$, one can work in the effective two-dimensional theory, which greatly simplifies the problem. We will start with the 4d Einstein-Maxwell action

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{16\pi G}\left[-R + \frac{6}{\ell^2}\right] + \mathcal{L}\right]. \tag{11.3.43}$$

and set $\ell = 1$. $\mathcal{L}$ describes the matter (electromagnetic fields + charged matter) we must add to the system. We will now work out the dynamics . To do this, we will work on a $S^2 \times \mathbb{R}^2$ ansatz. We will also follow [426] and resort to a FRWL open slicing which is smooth beyond the horizon, with coordinates x, τ

such that the four-dimensional metric is

$$ds^2 = e^{-\phi}(-d\tau^2 + a^2(\tau)dx^2) + e^{2\phi}d\Omega^2. \tag{11.3.44}$$

Plugging this back into (11.3.43), the kinetic term for ϕ becomes a total derivative and drops out, as usual in two-dimensional gravity models. The resulting effective action is

$$S = \frac{1}{8\pi G}\int d\Omega_2 dV_2 \left\{ e^{\phi}(\mathcal{L}+3) - \frac{1}{2}\left[2e^{-\phi} + e^{2\phi}R_{g_2}\right]\right\}. \tag{11.3.45}$$

where g_2 is the two-dimensional metric $d\tilde{s}^2 = -d\tau^2 + a(\tau)^2 dx^2$. Using $dV_2 = a(\tau)d\tau \wedge dx$, and

$$R_{g_2} = 2\frac{\ddot{a}}{a}, \tag{11.3.46}$$

one gets, after integration by parts,

$$8\pi G\, S = \int d\Omega_2 d\tau dx \left\{ (e^{\dot{2\phi}})\dot{a} - e^{-\phi}a + 3e^{\phi}a\right\} + 8\pi G\int dV_2 d\Omega_2 \sqrt{-g}\mathcal{L}. \tag{11.3.47}$$

The 4d stress-energy tensor is

$$T_{\mu\nu} \equiv \frac{2}{\sqrt{-g}}\frac{\delta S_M}{\delta g^{\mu\nu}}, \tag{11.3.48}$$

and so, the variation of the matter part of the action (the last term in (11.3.47)) is

$$8\pi G\delta S_{\text{mat.}} = 4\pi G\int d\tau dx\, a e^{\phi} T_{\mu\nu}\delta g^{\mu\nu}, \tag{11.3.49}$$

in terms of the four-dimensional stress-energy tensor $T_{\mu\nu}$. Since

$$\delta g^{\mu\nu} = -2\frac{e^{\phi}}{a^3}\delta^{\mu,x}\delta^{\nu,x}\delta a + \delta\phi\left[3e^{\phi}g_2^{\mu\nu} - 2g^{\mu\nu}\right], \tag{11.3.50}$$

the equations of motion are

$$(\ddot{r^2}) = 3r - \frac{1}{r} - 8\pi G\frac{r^2}{a^2}T_{xx}, \tag{11.3.51}$$

$$2r^2\frac{\ddot{a}}{a} = \frac{1}{r} + 3r + 4\pi G r\left[3T_2 - 2T\right], \tag{11.3.52}$$

where we have introduced the two-dimensional trace $T_2 \equiv e^{\phi}T_{\mu\nu}g_2^{\mu\nu}$. Finally, in the problem under consideration, the evolution is governed by electromagnetic stress-energy, which is traceless. We will decompose the traceless stress energy tensor of radiation as

$$T_{\mu\nu} = \rho u_\mu u_\nu + p_1\gamma_\mu\gamma_\nu + p_2 g^{\Omega}_{\mu\nu}, \tag{11.3.53}$$

where u^μ is a time-like future-pointing unit norm vector , γ_μ is the same thing on the x direction, and $g_{\mu\nu}$ is just the angular part of the metric (11.3.44). Tracelessness implies $\rho = p_1 + 2p_2$. In terms of these quantities, (11.3.51) and (11.3.52) become

$$(\ddot{r^2}) = 3r - \frac{1}{r} - 8\pi G r p_1, \tag{11.3.54}$$

$$2r^2 \frac{\ddot{a}}{a} = \frac{1}{r} + 3r + 3r\left[8\pi G \frac{p_1 - \rho}{2}\right]. \tag{11.3.55}$$

Different electromagnetic fields correspond to different ρ, p_1, p_2. For instance, in the quasistatic regime discussed in the previous subsection, the electric field is constant, which means $\rho = -p_1 = p_2$. Also, in this regime we can neglect the backreaction of the matter, the time derivatives of ϕ are small, and those of a are basically constant, so that $r\ddot{a}/a = H^2$. Then, taking

$$\mathcal{L} = \frac{1}{4g^2} F_{\mu\nu} F^{\mu\nu} \tag{11.3.56}$$

as in Section 11.2, together with the electric field

$$F = \frac{g^2}{4\pi} \frac{Q_r}{r^3} a \, dx \wedge d\tau, \tag{11.3.57}$$

one obtains (after using (11.2.7)) that

$$-8\pi G\rho = 8\pi G p_1 = 8\pi G \frac{p_1 - \rho}{2} = -\frac{Q^2}{r^4}. \tag{11.3.58}$$

Plugging back in (11.3.54) and (11.3.55), one obtains

$$(\ddot{r^2}) = 3r - \frac{1}{r} + \frac{Q^2}{r^3}, \tag{11.3.59}$$

$$2r\frac{\ddot{a}}{a} = \frac{1}{r^2} + 3\left(1 - \frac{Q^2}{r^4}\right). \tag{11.3.60}$$

Upon replacing $r\ddot{a}/a$ by H^2, and $\ddot{r}$ by 0, (11.3.59) and (11.3.60) become two algebraic equations,

$$3r^4 - r^2 + Q^2 = 0, \quad 2H^2 = \frac{1}{r^2} + 3\left(1 - \frac{Q^2}{r^4}\right). \tag{11.3.61}$$

These are the same evolution equations which one obtains from the quasistatic evolution for the full 4d geometry, particularized to the Nariai branch. In

particular, the solution to the first equation is (11.2.15) and then substituting in the second one gets (11.2.14). Then, the slow time evolution dictated by the slow Schwinger discharge as described by the first equation in (11.3.22).

As mentioned above, we will model the adiabatic regime by replacing the electric field and charge matter at $t = 0$ by pure radiation with the same energy. That is, one still takes ρ as in (11.3.58), but p_1 and p_2 are no longer related as for a constant electric field. We will parametrize

$$p_1 = \alpha\rho, p_2 = \beta\rho, \quad \alpha + 2\beta = 1. \tag{11.3.62}$$

α and β are both positive, since we are now working with radiation, and not with electrostatic fields as before. The precise relation between these two is dictated by the physics of the annihilation process and immediately after. The nucleated pairs are produced and then accelerated for a mean free path, after which they annihilate. Since the charge carrier is massless, and the electric field is very string, it mediates very efficient interactions and it is perhaps reasonable to assume that the system thermalizes locally. In that case, it seems reasonable to take $\beta = \alpha = 1/3$, corresponding to the stress-energy tensor of a four-dimensional CFT in the thermal state [13]. We will focus on this case from now on, although the precise values of α, β won't affect our conclusions. Conservation of stress-energy then fixes

$$\dot{\rho} + \rho\left(2\dot{\phi} + \frac{4}{3}\frac{\dot{a}}{a}\right) = 0, \tag{11.3.63}$$

which integrates to

$$\rho = \rho_0 e^{-2\phi} a^{-4/3}. \tag{11.3.64}$$

We also have (in Planck units already)

$$\rho_0 = \frac{Q_0^2}{r_0^2} \tag{11.3.65}$$

due to energy conservation, where Q, r_0 are the initial charge and radius (related to Q_0 as in (11.2.7)). Introducing the convenient variable $s \equiv r^2$, the equations

[13] In the opposite limit, where no thermalization occurs, one could compute α, β by integrating the angular distribution of photons produced during the scattering fo charged carriers. The perturbative QED cross section for $e^+e^- \to \gamma\gamma$ is heavily peaked as usual in the forward direction, when the t-channel goes on-shell. Hence it would seem reasonable to take the radiation to be forward-pointing, $\beta \approx 0$, but we will keep the analysis general. Both α, β are constants during time evolution if one neglects scattering of photons with the background curvature or with each other.

of motion (11.3.54), (11.3.55) become, after using (11.3.64),

$$\ddot{s} = 3\sqrt{s} - \frac{1}{\sqrt{s}}\left(1 + \frac{\rho_0}{a^{4/3}}\right), \tag{11.3.66}$$

$$\frac{\ddot{a}}{a} = \frac{1}{2s^{3/2}}\left(1 - \frac{\rho_0}{a^{4/3}} + 3s\right), \tag{11.3.67}$$

with the initial condition $a_0 = 1$.

Since $\rho_0 < 1$ (this is equivalent, due to (11.3.65), to $Q_0^2/r_0^2 < 1$, which is true everywhere on the Nariai branch) the second equation of motion tells us that the expansion is always accelerated. In fact, it is Friedmann's equation in two dimensions. The first tells us that $\dot{s} < 0$, so that the size of the two-sphere decreases. After a while, one can ignore the ρ_0 term in (11.3.66) and (11.3.67), since the redshift has diluted the radiation away. Equation (11.3.66) can then be integrated to yield an effective potential for s,

$$\frac{1}{2}\dot{s}^2 + 2\sqrt{s}(1 - s) = E \tag{11.3.68}$$

The potential is depicted in Figure 11.3. The maximum happens at $s = 1/3$, which is the size of the 2-sphere for a neutral Nariai black hole. The initial condition is therefore to the left of this point, and moving to the left. As one approaches $s \sim 0$, the scale factor blows up; the horizon size becomes tiny, and the curvatures become Planckian. Thus, this is a Big Crunch. In fact, it is a crunch of a very similar nature to the one encountered as one approaches the spacelike singularity inside a Schwarzschild-dS black hole; the 2-spheres go to zero size, and the resulting 2d cosmological constant obtained after dimensional reduction from the 4d cosmological constant becomes huge as well.

What is happening is that, by having all the electric field discharge instantaneously, we find ourselves in a situation with no charge, but a mass above that of a neutral Nariai black hole. Effectively, we have made a transition to a "superextremal" Nariai solution, as illustrated in Figure 11.4. These solutions are cosmological, describing either a Big Bang or a Big Crunch where the 2-spheres go to zero size. These correspond to the left or right branches of the potential in Figure 11.3. As we go from slightly sub-extremal to slightly super-extremal mass, the conformal diagram of the spacetime first looks like $dS_2 \times S^2$, and at super-extremality it disconnects into two pieces. The one describing what used to be the original static patch has been devoured by the black hole and in a sense describes the "black hole interior", with a crunch singularity in the future. The other one describes what used to be the region of global dS outside of the static patch of the black hole, and the two-spheres grow asymptotically to empty dS.

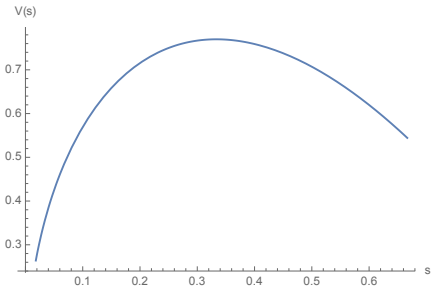


Figure 11.3: Plot of the effective potential $V(s) \equiv 2\sqrt{s}(1-s)$. The maximum at $s = 1/3$ corresponds to the neutral Nariai value. Since the value of s of the neutral Nariai black hole is higher than that of any of its charged counterparts, we are left with an initial condition to the left of $s = 1/3$ after the electric field discharges instantaneously. The equation of motion (11.3.66) then implies one is pushed even further to the left, towards $s = 0$.

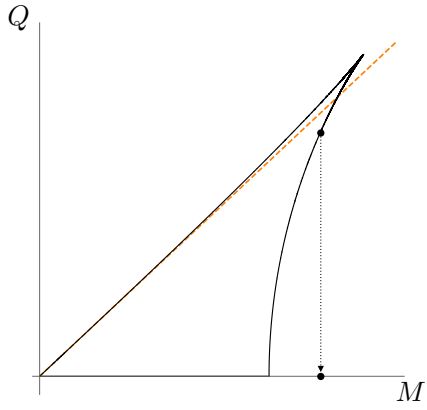


Figure 11.4: Nariai black hole evaporation in the adiabatic limit of a massless charge carrier. The electric field is screened very quickly, getting replaced by radiation, and so the black hole loses its charge without losing mass. As a result, we end in a "superextremal" neutral Nariai geometry (plus radiation), which ends in a big crunch.

Starting with an extremal charged Nariai black hole whose electric field disappears adiabatically, we find a configuration that asymptotes to the same

crunch that the super-extremal Nariai geometry. After the transition, we find ourselves in the crunching portion of the superextremal Nariai spacetime, plus some radiation that quickly redshifts. Although we have not proven this, one would expect the time for the beading process or other quantum effects to be irrelevant in this context, as the dominant source of stress-energy is the radiation, and the crunch happens quickly, within a Hubble time (see Appendix J.2).

Since this crunch will be crucial for our argument below, we must make sure it is not an artifact of the Nariai approximation[14]. Is the crunch somehow avoided if there are small inhomogeneities (that could collapse into black holes and then leave the static patch), or if we start slightly off the Nariai branch, so that the size of the two-spheres is not exactly constant? What about the choice of initial hypersurface? The choice of quantum state is also important, see e.g. [29]. Following [426], we chose an open FRW slicing, in which the dynamics is homogeneous in the Nariai limit. This is convenient for our calculations but it is important to check that the crunch is actually independent of this choice. We could have picked any other spacelike surface, like the $t = 0$ hypersurface in the static patch. This would map to inhomogeneous radiation initial data for some later $\tau = \text{const.}$ hypersurface in the FRW ansatz (11.3.44).

These concerns are averted by the fact that the crunch is somehow "local" – it happens for each observer and it is not very much affected by energy inflows or outflows, as long as these are small enough–. The precise way to state this is that the crunch is guaranteed by GR singularity theorems[15], as we explain in Appendix L.

11.4 Black hole decay in dS and an EFT constraint

We have now finished presenting the technical results in this paper. The formalism developed in these previous Sections provides a means to understand how black holes evaporate in de Sitter space. This is a calculation within in General Relativity which, to our knowledge, has not appeared in the literature before.

By contrast, in the present Section we will indulge in hopefully interesting speculation. We wish to emulate some of the logic that leads to WGC in flat space or AdS, to hopefully learn something about the properties of effective field theories in (quasi-)de Sitter spaces.

[14] We thank Grant Remmen and J. L. F. Barbón for discussions on these points.
[15] We thank Raphael Bousso for pointing us in the right direction here.

11.4.1 Strong gravity in de Sitter space

In Section 11.3, we have seen that all black holes evaporate and decay back to empty de Sitter space, which is consistent with the interpretation of de Sitter as a thermal equilibration process. There is just one exception: Black holes close to the Nariai branch, whenever there is at least one particle in the adiabatic regime $m^2 \ll qE$. In this case, the geometry evolves towards a crunch, and never returns to empty de Sitter space. The adiabatic evolution forces a black hole outside of the shark fin; since the Nariai branch tilts to the right in the (M, Q) plane, losing the charge without losing the mass leads to a neutral solution with a mass higher than that of the neutral Nariai black hole.

This is also problematic because once we land in a point outside the shark fin, we are free to deform the solution (say on a fixed Cauchy slice) away from this, towards positive charge or arbitrarily large mass. The latter is hard to reconcile with the picture of de Sitter space as having a finite-dimensional Hilbert space [18, 21, 431, 432]. Furthermore, charged superextremal solutions are continuously connected to neutral ones, since one can go around the tip of the shark fin in Figure 11.1.

These are the superextremal black holes with naked singularities that are usually forbidden by Cosmic Censorship[16], in its loose original meaning [435] that Einstein's theory should not develop arbitrarily large curvatures which are not shielded by a horizon. In flat space, existence of a weak gravity particle ensures that large extremal black holes can decay without becoming superextremal. As usual, this connection can be made far more precise in AdS, where a series of papers [399–401] have established that certain violations of Cosmic Censorship are avoided if a version of the WGC is satisfied.

There is also a lot of evidence that physics in the static patch of dS looks very much like that of an ordinary thermal system [17, 21, 431]: There is a thermal bath coming from the horizon at a temperature $(2\pi\ell)^{-1}$, and whatever initial perturbation one considers, it tends to be "smoothed out" by inflation, i.e. features fall behind the cosmological horizon and disappear. In these processes the generalized second law of thermodynamics is always satisfied [19]. So it is not unreasonable to regard dynamics in de Sitter in general as describing thermal relaxation, where perturbations equilibrate and disappear after a certain time. This picture is exactly what one gets in the quasistatic regime, where

[16] Cosmic Censorship is no fundamental principle. Solutions with unbounded curvature but with a consistent UV description are familiar in String Theory, e.g. orbifolds, KK monopoles, or all sorts of branes [433]. Gubser's criterion [434] explains that admissible singular solutions with vanishing temperature must always be cloaked behind a horizon when an arbitrarily small temperature is turned on. Superextremal RN black holes would not satisfy this.

black holes decay slowly back to empty dS; it fails spectacularly in the adiabatic regime.

So, a theory with charged extremal black holes and particles in the adiabatic regime is hard to reconcile with a self-contained description of the physics in the static patch in terms of a finite-dimensional Hilbert space. If one wants to make both compatible, we only see three possibilities:

- The Big Crunch at the end of the black hole collapse transitions back to empty dS space via some magical quantum gravity process.
- Charged black hole solutions are not allowed in a theory with particles in the adiabatic regime.
- Particles in the adiabatic regime are not allowed, and this constitutes a swampland-type constraint.

We can say little about the first possibility. The second seems difficult at least in the semiclassical regime, because again, we can smoothly deform empty de Sitter space into any other solution in the charged Nariai branch[17]. So we will entertain the third possibility.

If indeed the weird properties of the adiabatic regime – lack of thermalization, failure of detailed balance, seemingly huge Hilbert space – are a problem, perhaps we should avoid it altogether. That means $m^2 > qE$ for every particle and on every point on the charged Nariai branch. The electric field is largest at the ultracold point, where $E = \sqrt{6} M_P g H$ and we have used $M_P^2 = (8\pi G)^{-1}$. Thus, avoiding the adiabatic regime is equivalent to imposing[18]

$$m^2 \gtrsim q\, g M_P H, \quad \text{for every particle in the spectrum.} \tag{11.4.1}$$

We consider (11.4.1) a potential candidate for a swampland constraint in de Sitter. As we tried to emphasize, it is derived via similar heuristics to the ones that lead WGC in flat space. Unlike WGC, however, we lack any stringy examples to check against (11.4.1), so this result is definitely on shakier ground.

[17] In absence of charged particles, one can construct gravitational instantons that mediate transitions between empty de Sitter space and the Nariai or lukewarm, and which furthermore enforce detailed balance [407, 436, 437]. However, it is unclear whether these tunneling transitions are destabilized by the presence of light charged modes. In any case, the argument that the Nariai solutions are smoothly connected to the vacuum in configuration space still stands.

[18] We write $\gtrsim$ because we have only analyzed the two extreme regimes where m^2 is very small or very large compared to qE. Due to the exponential prefactor in (11.3.1) the quasi-static regime is reached after m^2/qE is of order 10. The precise threshold can be computed by demanding that the excess charge (11.3.35) is small (say 10^{-5}) compared to the background. We write this footnote thinking of all our friends that dislike symbols like "$\gtrsim$".

Equation (11.4.1) is just a consistency condition so that the thermal picture of de Sitter can hold.

In fact, there are good reasons to at least entertain the possibility that there are no metastable de Sitter vacua in string theory [161,163,326]. If supersymmetric string theory as we know it today really describes the real world, the logical conclusion would be we are in some sort of quintessence scenario, so it makes sense to ask what would happen to (11.4.1) in such a case. This analysis is carried out in Appendix J.2; one finds that quintessence does not change the conclusion, and that violating (11.4.1) still leads to a crunching spacetime.

We now make a couple of comments before crudely discussing the phenomenological implications. First, the constraint becomes trivial as $H \to 0$, as it should since it is a pure de Sitter phenomenon. On the other hand it is nontrivial and in particular forbids all charged particles if we take $M_P \to \infty$ at constant H and g. This suggests that it might not be possible to consistently decouple gravity in dS[19]. In a more holographic language, this means that Einstein[20] dS holographic models, if they exist, should not fall in large N families where the low-energy sector "stabilizes" and we can make gravity as weak as we want by taking large N. Rather, they should be "sporadic" or isolated theories. This point of view was also advocated in [18]. Second, we emphasize the statement holds for every particle in the theory, since just one violating the bound would be enough to cause an adiabatic discharge. This is unlike Weak Gravity, where just a single particle is enough. This difference reflects that Weak Gravity is about ensuring that black holes decay, while (11.4.1) is about ensuring they don't do it in a particular way. Extremal black holes satisfy (11.4.1) as well. Third, one should only apply this constraint in situations where a Nariai black hole makes sense. In particular it does not apply to Higgsed or confined gauge fields.

There is also an interesting interplay between (11.4.1) and magnetic/sublattice versions of the WGC which predict an upper bound on the cutoff scale of quantum gravity [378]. Given a mass for a charge carrier, (11.4.1) predicts an upper bound on the gauge coupling in terms of the mass of the lightest charged carrier,

$$g \lesssim \frac{m^2}{qM_P H}, \tag{11.4.2}$$

[19]We do not claim that QFT makes no sense in a dS background; it does, at least perturbatively. We just mean that, given a gravitational model with finite M_P, there might be no way to "deform" it, either continuously or by small discrete steps, to one where gravity is absent.

[20]In particular, higher-spin theories are not Einstein gravity and the discussion in this paper does not apply to them. Higher-spin theories in de Sitter are conjectured to have holographic duals [68] which do possess a large N limit.

which translates to an upper bound on the cutoff of the EFT.

Throughout this work, we focus on the case with a single species of charged particle. Of course, if one has multiple species of charged particles, they will all contribute to the black hole decay. This allows one to trigger the adiabatic regime, even if the individual particle species of particles are massive. However, the annihilation of the produced particles into neutral radiation may be slower than in the massless case. For instance, if one has order $\exp(m^2/qE)$ particle species of mass m and charge q, this compensates for the exponential suppression of Schwinger pair production. Demanding the adiabatic regime does not occur then bounds the number of charged particle species at certain mass in such cases. This has in fact interesting implications, which we discuss in Subsection 11.4.2.

Phenomenological implications

The nice thing about making Swamp-like statements in Sitter space is that one may compare them to the real world. Taking the $U(1)$ to be electromagnetism, the scale on the right hand side of (11.4.1) is

$$\sqrt{gM_P H} \sim 10^{-3}\, eV, \tag{11.4.3}$$

or around the vacuum energy density scale/neutrino mass scale. The Standard Model satisfies our constraint (11.4.1), since the lightest electrically charged particle is the electron, eight orders of magnitude above (11.4.3).

While at first sight it seems that (11.4.1) is satisfied with room to spare, one should remember the immense hierarchy between Planck and Hubble scales, of sixty orders of magnitude. (11.4.1) predicts a new scale which is the geometric mean of these two, and the eight orders of magnitude in the electron should be compared with these sixty, as illustrated in Figure 11.5. By contrast, the electron saturates the WGC by 19 orders of magnitude[21].

Since in the SM the fermions receive mass from their coupling to the Higgs, (11.4.1) also has implications for the cosmological hierarchy problem [438–440] (see also [360, 441] for some other proposed connections between the swampland and the EW hierarchy problem). In particular, suppose one has a fermion with

[21] While as remarked above it makes no sense to apply this constraint to massive gauge fields, we notice that the lightest fermions we know of, the neutrinos, have a mass close to (11.4.3). This could perhaps be a sign they are coupled to a hidden massless $U(1)$ with not so small gauge coupling. Since the only anomaly-free $U(1)$ with generation-independent couplings in the SM with right-handed neutrinos is $B - L$, this would suggest the presence of extra particles charged under this $U(1)$ as well.

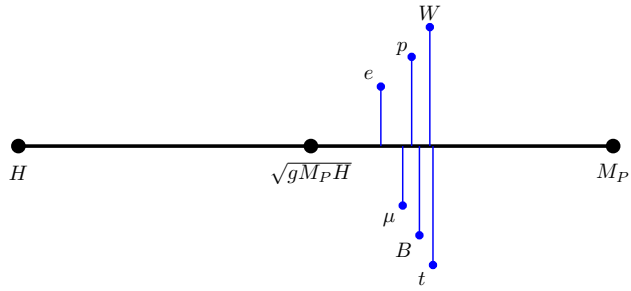


Figure 11.5: Logarithmic scale of energies displaying the Hubble and Planck scales on both ends, the masses of some SM electrically charged particles, the Higgs vev scale, and the new scale $\sqrt{gM_PH}$ in (11.4.3). The constraint (11.4.1) implies that all charged particles should lie to the right of this scale, which is satisfied.

mass $m = yv/\sqrt{2}$, which in turn satisfies (11.4.1). Then

$$\frac{y^2}{2}v^2 \gtrsim gM_PH = \frac{g}{\sqrt{3}}\rho^2, \tag{11.4.4}$$

where ρ is the gravitating vacuum energy density and we have used $3M_P^2H^2 = \rho^4$. Rearranging,

$$\rho \lesssim \frac{y}{\sqrt{g}}v, \tag{11.4.5}$$

that is, no matter what the Higgs vev v is, the gravitating vacuum energy (the one that actually enters in Einstein's equations) must be below the electroweak scale, by a factor of at least the smallest Yukawa of any charged fermion. There is a nice interplay with the arguments in [360], which introduced an additional gauge field to explain electroweak hierarchy. In this language, the electroweak hierarchy problem gets mapped to the question of why is the new gauge coupling small; (11.4.1) could be part of the reason for this.

Equation (11.4.1) does not lead to constraints on models of milli-charged dark matter, since the window excluded by the constraint has long been excluded experimentally [442].

One could also apply (11.4.1) to $(B - L)$. This is an anomaly-free global symmetry of the SM Lagrangian, which is expected to be gauged or broken in quantum gravity. For instance, in GUT models, $(B-L)$ is often a spontaneously broken global symmetry, but it is not excluded that it could be massless, as long as the gauge coupling is small enough (around $g \lesssim 10^{-24}$ [339, 360]). Our bound would mean that the lightest neutrino species cannot be massless.

Another topic we must address is inflation. During the inflationary phase the universe was approximately de Sitter, so it would be reasonable to apply (11.4.1) there. Furthermore, as explained before, replacing the cosmological constant by a slowly-rolling scalar does not change our conclusions. We do not know much about the spectrum of massless $U(1)$'s and charged particles during inflation, but a naive expectation is that if inflation happens before EWSB, we have an $\mathfrak{su}(3) \oplus \mathfrak{su}(2) \oplus \mathfrak{u}(1)$'s worth of massless gauge fields to analyze[22]. The charged spectrum of the SM remains massless at tree level (getting a mass of order Hubble scale via radiative corrections [445]), which violates (11.4.1). We now discuss some possible ways out of this conundrum[23]:

- One can take inflation to only last $N_e \sim 60$ e-folds [446]. Even if (11.4.1) is violated, we could conceivably avoid the Big Crunch if the time that it takes for the black hole to discharge adiabatically is much higher than N_e/H. For a massless carrier, this time is set by the electric field only, $t_{\text{disch.}} \sim (qE)^{-1/2}$. Rearranging, this becomes a bound on the gauge coupling,

$$g \leq \frac{H}{N_e^2 M_P}. \tag{11.4.6}$$

  Thus, we can avoid trouble, even with a large field inflation Hubble scale, by introducing a coupling between the inflaton and the gauge kinetic function of the gauge fields, in such a way that they remain small during inflation. This ensures that the gauge couplings become even more weakly coupled in the UV, while preserving gauge coupling unification as long as the coupling is taken to be same for every gauge field in the SM. This coupling would break any perturbative shift symmetry the inflaton field would have, so it is again fine-tuned unless it can be generated by non-perturbative effects. It would also lead to production of gauge fields during inflation and so, nongaussianity in the curvature or possibly curvature-magnetic field spectrum [447–449]. These effects would be suppressed by at least a factor of (11.4.6) squared, so they would be small.

- Another alternative is models of Higgs inflation [450], where electroweak symmetry is still broken even at a high scale, and the fermions are massive. In vanilla models of Higgs inflation, the Higgs vev is comparable to $H^2 M_P^2$. This means that the fermions are still massive due to their Higgs coupling, but their masses are several orders of magnitude smaller due to the small Yukawa couplings, so (11.4.1) is not satisfied. By contrast, in a more convoluted model with a complicated Higgs potential allowing for a

[22] The same analysis would apply to GUT's, although inflation is usually taken to happen after GUT breaking to avoid a monopole problem [443, 444].

[23] We thank Prateek Agrawal for illuminating discussions.

hierarchy between fermion masses and the vacuum energy, this tension could disappear.

In this scenario, one still has to think about $SU(3)$. The quarks are massive due to their coupling to the Higgs and thus pose no problem, but the gluons not commuting with the chosen $U(1)$ are charged, and naively massless. However, as long as the Hubble scale during inflation is below Λ_{QCD}, equation (11.4.1) is satisfied in the same way as it is today. This corresponds to a vacuum energy density of $\sim 10^9\, GeV$. Thus, a simple way out is a small field inflationary model. The price one pays for this is an extremely fine-tuned scalar potential, violating any reasonable form of the original no dS conjecture [163, 326]. It is worth noticing however that this is the same inflationary scale predicted by the recent TCC conjecture of [327] (the terminology "transplanckian censorship" was originally introduced in [451] to refer to a different but possibly related phenomenon).

Before wrapping up, we note that the non-abelian interaction terms in the presence of a background gauge field seem to give these gluons an extra contribution to the effective mass proportional to $\sqrt{gM_PH}$, when reduced to two dimensions to carry out the Schwinger calculation (see [452]). Still, this is at best just marginally satisfying (11.4.1), so without further analysis it is unclear whether the crunch is really avoided. Having $\Lambda_{QCD} < H$ is a sure way to avoid the trouble caused by the gluons.

11.4.2 Weak gravity in de Sitter space

We have a constraint on the EFT from black hole decay, but it would be nice if we could recover WGC as well. There are immediate obstacles to this. The usual motivation for the (mild form of the) WGC in flat space or AdS is that otherwise all extremal black holes are exactly stable or marginally unstable (as is the case for BPS black holes in a theory with BPS particles), and this is somehow undesirable. In flat space it is still unclear what precisely goes wrong if this does not happen, but in AdS a violation of the mild form of the WGC conflicts with properties of entanglement entropy in the dual CFT [397].

By contrast, we have seen in the previous Section that, the moment there is any charged particle in the spectrum, RN-dS black holes follow the fate of their uncharged flat space counterparts – they eventually evaporate completely–. This is easy to understand intuitively. In flat space or AdS , a subextremal charged particle that tunnels out of a black hole has no choice but to fall back

in, because it just does not have enough energy to escape to infinity[24]. In dS however, even a very massive particle will sometimes tunnel so far away from the black hole that it is pulled away by the cosmological expansion, being eventually eaten up by the cosmological horizon.

In fact, black holes discharge even in the absence of charged particles. In that case, they move towards the lukewarm line in Figure 11.1, or towards the charged Nariai branch if the initial charge happens to be higher than the intersection point of the charged Nariai branch with the lukewarm line. Both on the lukewarm line and on the charged Nariai branch there are gravitational instantons [407, 436] that mediate transitions from these black holes to and from empty de Sitter space. But the point of our analysis is that, even if one demands that extremal black holes must be unstable, this leads to no constraints on the spectrum of Einstein quantum gravity in dS space (or situations close to it, like slow-roll quintessence), since black holes decay automatically anyway. This was already foreseen in [414].

Although we have just argued that no charged black holes are stable in dS, small black holes have very large lifetimes if WGC is not satisfied. Charged particles must traverse an incredibly wide potential barrier, from the black hole horizon to the cosmological one, if the black hole is to discharge. The current associated to this effect is therefore exponentially exponentially suppressed in ℓ; one can see this in the exact expression for the Nariai current (11.3.38).

To sum up, in a dS-like theory which contains no WGC particles, small black holes do evaporate, but their lifetime is exponentially long in ℓ. There are so long-lived that it makes sense to include them in the computation of the Schwinger current for a large Nariai black hole. As mentioned in Subsection 11.4.1, there are two ways to trigger the adiabatic regime. One way is to have one or a few particles violating (11.4.1). Another is to have so many particles satisfying (11.4.1) that their combined current beats the Schwinger exponential suppression, resulting in an unsuppressed current. The contribution from black holes to the current is[25]

$$\mathscr{J}_r \approx \int_{\frac{\Lambda}{\sqrt{2}gM_P}}^{Q_{\max.}} dQ_r\, e^{S-\frac{M^2}{Q_r E}} \sim \left(\frac{H}{M_P}\right) \frac{H^2\Lambda^2}{g} e^{-4\sqrt{2}\pi\frac{\Lambda}{H}} \tag{11.4.7}$$

[24] In the AdS case the analysis is more subtle, since even massive superextremal particles cannot escape to infinity. Instead, they form a charged cloud which completely discharges the black hole in the extremal case [453].

[25] We have only included extremal black holes and replaced the sum over integer-quantized charges Q_r by an integral. We have neglected the S term and then taken the upper integration limit to be ∞ since the integral is dominated by small black holes as we take $Q_{\max.}$ much smaller than a cosmological size black hole. For the same reason, we have also used the flat-space extremality expression. We have taken $E = gM_P H$ to be a typical electric field in the upper part of the Nariai branch.

Here, Λ is some EFT cutoff, which cuts off the sum when black holes become so small than the semiclassical description breaks down. S is the black hole entropy counting the number of black hole states with the given charge and mass. The upper bound on the charge $Q_{\text{max.}}$ is there because here we just want to consider small black holes – the entropy contribution is then subleading and the result is dominated by the lower integration limit. Equation (11.4.7) grows without bound as $g \to 0$ which means that, as long as this limit is allowed (and it might very well not be; this is just a heuristic argument), we will enter the adiabatic regime. Thus, we obtain an argument for ordinary WGC in de Sitter, that small black holes should be allowed to decay, with a similar flavor to the original heuristics in [320]. One might also conclude that $g \to 0$ is not allowed; this is in fact the conclusion when one follows the same argument in AdS [454].

This argument is heuristic and it relies on a very unclear limit. We also tried to do better, and in the remainder of the Section we report what we got. Since WGC guarantees that black holes evaporate quickly enough, it is reasonable to imagine it is related to (an upper bound on) evaporation times of black holes. Interestingly, one can argue something like this – an upper bound on the *equilibration* time of a black hole–. The equilibration time $t^{\epsilon}_{\text{eq.}}$ of a particular quantum state is the time that it takes for the state to become close to the thermal state to any desired accuracy ϵ.

In our context, and as mentioned above, we should regard black hole evaporation as (part of) an equilibration process towards the thermal state, represented by empty dS. Once a black hole evaporates, the decay products fly away and eventually fall behind the cosmological horizon. When this happens, to any desired degree of accuracy, one can say the state has finally equilibrated.

We refer the reader to the excellent review [455] for a precise definition and more details. Interestingly, it is possible to derive a universal upper bound on the equilibration time of a generic quantum state[26] (see Theorem 1 of [455]), which grows exponentially with the system size[27]. Notice that this is still better than the quantum Poincaré recurrence time $e^{e^{S_{dS}}}$. Thus, we might want to demand that the total evaporation time of a black hole is shorter than $e^{S_{dS}}\ell$. The evaporation time for a charged black hole is controlled by the exponential factor of the Schwinger current, $e^{-m^2/(qE)}$. So a very naive realization of this bound could be to demand that $m^2/(qE) < S_{dS}$, which leads to a lower bound on the charge-to-mass ratio of the particle that mediates black hole decay,

$$\frac{gqM_P}{m} \gtrsim \frac{Hm}{M_P^2}. \tag{11.4.8}$$

[26] Assuming it thermalizes eventually.

[27] This kind of scaling is achieved by relaxation times in glassy systems.

While this bound goes on the same direction as the WGC, it is parametrically weaker and so uninteresting. It becomes trivial as $H \to 0$, so it is clearly not capturing the actual physics of WGC. We just want to illustrate that upper bounds on equlilibration times lead to lower bounds on the charge-to-mass ratio; and while the universal result in [455] does not lead to an interesting result, a stronger lower bound derived using the particular properties of the system under study might be behind the rationale for WGC in quasi-de Sitter spacetimes.

11.5 Conclusions

In flat space and AdS, the Weak Gravity Conjecture is intimately related to Cosmic Censorship and the stability of extremal black holes. In this note, we have tried to understand to what extent this kind of reasoning works in de Sitter, and more generally to spacetimes with positive vacuum energy, and to figure out if there are any new candidates for swampland constraints.

To do so, first one needs to understand how black holes evaporate in de Sitter space. Unlike their flat space counterparts, whose mass for a given charge is bounded below by the extremality curve but unbounded from above, de Sitter black holes have a largest mass for a given value of Q – the so-called Nariai solution. This is a genuinely de Sitter effect, and one can connect it to the idea that the Hilbert space in the static patch should be finite-dimensional [18, 21, 431, 432], from which it follows that there is a maximum value of any reasonably well-defined observable like the mass or the charge.

Previous studies of black hole evaporation in de Sitter [402,404,406,408–412,430] have not examined how the mass and charge of a black hole evolves in time in a theory with charged particles. We have worked out how large de Sitter black holes evaporate, depending on the spectrum of charged particles in the theory. For a single scalar particle of mass m and charge q, we find two very different regimes, controlled by the parameter $\chi \equiv m^2/(gqM_PH)$:

- When $\chi \gg 1$, the Schwinger pair production that allows black holes to lose charge is exponentially suppressed. They therefore evaporate slowly, schematically following the curves illustrated in Figure 11.2. All black holes evaporate eventually, never leaving the "shark fin" region of nice sub-extremal solutions. This is consistent with the thermodynamic interpretation of de Sitter as a thermal system at finite temperature [19, 432]. According to this idea, perturbations in de Sitter should eventually evolve back to empty de Sitter space, and this process corresponds to equilibration with the thermal bath. Field theory modes and small black holes all agree with this picture, and it was known that large neutral black

holes do so as well [404]. We have shown that this is the case for charged black holes as well, as long as $\chi \gg 1$.

- If $\chi \ll 1$ instead, a charged Nariai black hole sees its electric field immediately screened out by the very light charged carriers produced via Schwinger; the electric field is replaced by radiation, which cannot support the solution and collapses to a Big Crunch. This Big Crunch is of the same kind that the one which appears in the future of a superextremal, neutral Schwarzschild-de Sitter solution with mass above the Nariai value. Morally, the black hole loses all the charge instantaneously, while keeping its mass constant, as illustrated in Figure 11.4.

The first noteworthy (if unsurprising) consequence of these results is that, as advanced in [414], there is no kinematic barrier to black hole decay. This is unlike in flat space, where at least one particle must satisfy WGC if extremal black holes are to decay. In flat space, a subextremal charged particle can tunnel out of an extremal black hole, but it inevitably falls back because for it gravity is not the weakest force. By contrast, the same particle in dS can always tunnel far away enough that the cosmological force overcomes the gravitational attraction from the black hole. Gravity may not be the weakest force, but in any case it is always weaker than the cosmological one.

This is however not the end of the story. In the second scenario, where the black hole discharges quickly, there is a failure of the thermalization picture; there are black holes that become superextremal, crunch, and never thermalize back to de Sitter space. We lack a complete, UV description of Einstein quantum gravity in long-lived de Sitter space (if it exists at all), so we do not know if this is really an inconsistency. However, we cannot help to point out that the superextremal region in Figure 11.1 is connected, so one can continuously deform any superextremal solution into any other. A neutral superextremal black hole can be connected to a small superextremal one, which we ordinarily like to avoid based on cosmic censorship or more sophisticated ideas (like the fact that these solutions do not have any known embedding in string theory and it would be difficult to give a thermodynamic interpretation).

In any case, if one wants the thermal picture of the dS static patch to hold, then our results suggest that one should impose $\chi \gtrsim 1$ as well. This becomes a constraint on the effective field theory, that

$$m^2 \gtrsim qgM_P H,$$

for every particle in the theory. This is a constraint on the effective field theory coming from gravitational arguments, and since we do not know of any field theory arguments that would lead to it, it has the flavor of a swampland

constraint. However, since we cannot check against stringy examples and it relies on the thermal picture of the dS static patch and its finite-dimensional Hilbert space, it is on shakier ground than actual swampland constraints. It is just a consistency condition we need to enforce so that the thermal picture of dS can hold at all.

Interestingly, avoiding the adiabatic regime also leads to a new heuristic argument for the WGC from black hole decay in dS, circumventing the aforementioned lack of kinematical constraints. In a WGC-violating theory, small extremal black holes would be very long-lived, and would contribute to the discharge of Nariai ones. In the limit of small gauge coupling, there is so many of them that they can trigger the onset of the adiabatic regime by themselves. A natural way to avoid this is then the WGC.

This argument is very similar to the original WGC heuristic, and suffers from the same weaknesses. For instance, it only works in the limit $g \to 0$, and we have no idea whether this limit makes any sense in any way. Trying to address this, we have speculated that WGC might be related to an upper bound in evaporation times of black holes. In the thermal picture of dS, this would correspond to an upper bound to equilibration times; while there are universal results in this direction which lead to WGC-like bounds, they are much weaker.

Since the real world looks like de Sitter at least for now, it is reasonable to compare this constraint with it. Taking the $U(1)$ to be electromagnetism, the constraint turns out to be satisfied by every charged particle we know of. Again, this is a UV constraint, since there is nothing obviously wrong from the EFT point of view with a world where the electron was lighter than neutrinos or the cosmological constant was a few orders of magnitude higher[28].

Since the constraint ties charged fermion masses to the Hubble scale, it alleviates to some extent the cosmological hierarchy problem. It also constrains models of mili-charged dark matter, but the window where the constraint is nontrivial has been long excluded by experimental data.

The constraint has an interesting interplay with inflation. Since during inflation the universe was basically de Sitter for 60 efolds, our arguments apply, not just to electromagnetism but to whichever massless unconfined gauge fields are present during the inflationary era. This puts constraints on inflationary models. We have sketched a couple ways to comply with these constraints: Direct inflaton-gauge kinetic term couplings (which would lead to small nongaussianities), or models of Higgs inflation with a plateau potential and an inflationary scale below Λ_{QCD}.

[28] We wouldn't exist, but it is unclear that this signals an actual inconsistency of the theory.

A number of interesting research directions remain. Tracing out the boundary of Figure 11.1 by starting with an infinitesimally small extremal black hole, moving up along the extremal branch up to the ultracold point and then moving back down along the charged Nariai branch, one finds a remarkable situation. There is a continuous interpolation between $AdS_2 \times S^2$, $\text{Mink}_2 \times S^2$, and $dS_2 \times S^2$. It would interesting to understand this as a continuum of two-dimensional theories, especially in the context of the recent surge of interest in two-dimensional quantum gravity [456–461].

A natural extension of our work would be to higher dimensions. One could study both higher dimensional de Sitter spaces and black branes rather than black holes. It would be interesting to see if this produces bounds on brane tensions analogous to the bounds on particle masses obtained here. Since in the Nariai branch we get a $dS_2 \times S^2$ geometry for black holes, it is natural to guess that by looking at black branes one could get higher-dimensional de Sitter solutions supported by flux which discharge slowly, and it would possible to study their decay in a UV-regulated way.

Chapter 12

A dS obstruction and its phenomenological consequences

This chapter is a reprint of [6]. We observe that positive runaway potentials can generically be stabilized by abelian p-form fluxes, leading to parametrically controlled de Sitter solutions after compactification to a lower dimension. When compactifying from 4d to 2d the dS solutions are metastable, whereas all higher dimensional cases are unstable. The existence of these dS solutions require that a certain inequality involving the derivatives of the potential and p-form gauge coupling is satisfied. This inequality is not satisfied in simple stringy examples (outside of the scope of Maldacena-Nuñez), which unsurprisingly avoid this route to dS solutions. We can apply our techniques to construct dS_2 solutions in the Standard Model plus an additional runaway scalar such as quintessence. Demanding that these are avoided leads to (weak) phenomenological constraints on the time variation of the fine structure constant and QCD axion-photon coupling. As in the preceding chapter, the Freund-Rubin constructions of section 7.1.2 play an important role in this chapter.

12.1 Introduction

One of the most important open questions in string theory is the quest for a de Sitter solution. There is a vast literature on precise and concrete strategies to

construct a dS solution in string theory, which we discussed in some detail in part III. Our own point of view is that de Sitter constructions in string theory have never been fully explicit and trustworthy. One could therefore entertain the logical possibility that there is something about string theory that makes it incompatible with meta-stable de Sitter vacua [161, 462]. This has been turned into a conjecture, see conjecture 3 in section 10.2 which furthermore constraints the effective potential that any EFT derived from string theory should obey. There is no rigorous argument for the conjecture so far (see however [164] and [245] for connections to the Swampland Distance Conjecture [325]). As stressed in [463, 464] such a general argument might be the reason behind the failure of finding weakly coupled tree-level dS vacua in [292, 465, 466].

The inspiration for this conjecture comes from the general scaling properties of *tree-level* flux compactifications derived earlier in [462, 467, 468] which in turn took inspiration from [469, 470]. This does not constitute real evidence since there is no immediate reason why de Sitter vacua should hide in the classical corner of the landscape. However there is no obvious theorem forbidding vacua in that corner either (see [471] for recent progress in this direction). The fact that tree-level solutions, regardless of whether they are free of tachyons, have only been found at strong coupling (see [292,465,466] for a scan and overview of such solutions) might be suggestive of a deeper reason. Especially the arguments of [464] seem to hold quite generally and are discouraging to find weakly coupled solutions[1].

Of course, from these observations one cannot jump to conclusions about the non-existence of de Sitter vacua in the intermediate regime. At the same time one cannot claim that from a "potpourri" of quantum corrections to 10d supergravity one arrives at flux vacua which can be trusted sufficiently to carry the burden of proving a landscape/multiverse charachter of string theory. Even more, there are also non-trivial dS no-go's in string theory that go well beyond 10d tree-level: various papers have studied the effect of an infinite tower of higher derivative corrections, either from the worldsheet or in genuine top down effective field theory, still coming to a no-dS result [474–477]. So it seems that in all corners where reliable computations can be done de Sitter escapes existence.

In any case, a no-dS viewpoint leads one to naturally focus on runaway potentials [326] in string theory and to quintessence models in phenomenology [478, 479] (see [480] for a review). Embedding quintessence in string theory is also difficult and we will not go into any detail of this, but instead refer to [303, 481, 482].

The simple point we make on this note is that runaway potentials together with

[1] Although there is a recent revival of this discussion since classical vacua could be around at weak coupling once we allow certain singularities [472], which however have been questioned to be physical [473].

abelian $p-1$-form gauge fields (ubiquitous in string compactifications) naturally lead again to (lower-dimensional) de Sitter solutions. Flux compactifications can stabilize the runaway potential, leading to $dS_{D-p} \times S^p$ solutions. These solutions are under parametric control by taking the flux threading the S^p to be large. Hence they would be an example of explicit, controlled, simple dS saddle points, similar to the usual flux compactifications leading to AdS vacua. We will find that for $D-p > 2$ the solutions are generically unstable, but for $D-p = 2$, the instability in the homogeneous mode disappears. In fact, in the two-dimensional case, this solution arises as a near-horizon limit of charged solutions in the higher-dimensional theory, just like their AdS counterparts. In fact this work first arose as an attempt to extend charged Nariai black hole solutions which are $dS_2 \times S^2$ describing the near-horizon geometry of large black holes in de Sitter space, to slow-roll quintessence spacetimes.

In this paper we just work out the details, and find that, in order for a $dS_{D-p} \times S^p$ saddle point to exist, it is necessary and sufficient that a simple certain inequality involving the gauge kinetic function of the $p-1$-form field and the potential is satisfied. More concretely, if f is the inverse gauge coupling of the $(p-1)$-form field, dS saddle points exist provided that

$$(p-1)\left|\frac{V'}{V}\right| \leq \left|\frac{f'}{f}\right|. \tag{12.1.1}$$

We then check against simple controlled examples, including the $O(16) \times O(16)$ nonsupersymmetric string, and find that the inequality is not satisfied. If someone ever comes up with a worldsheet de Sitter vacuum, our techniques could have been used in principle be used to stabilize the dilaton, thereby turning these constructions into actual quantum gravitational dS saddle points[2]. We check explicitly that this doesn't work, either. The inequality we have is marginally violated, the amount of the violation is parametrically small and goes as g_s.

As emphasized above, for the 2-dimensional case we get stable dS_2 solutions[3]. The instabilities are frozen via the Hamiltonian constraint. If the no dS or any similar conjecture on the string potential (such as TCC [327]) are true, then the opposite of (12.1.1) is a logical consequence. Absence of dS solutions implies constraints on the couplings in a different, runaway solution.

[2]Again, by "dS saddle points" we mean that our higher dimensional dS solutions are unstable, while the 2-dimensional ones are not.

[3]As we discuss below, there is a "Bousso beading process" inhomogeneous instability, which we interpret as nucleation of two-dimensional black holes by the expanding dS_2 background. This instability can never destroy the dS_2 completely (at any given time, we expect most of the spatial volume is in a dS_2 patch).

It turns out that simple extensions of the Standard Model with scalars, such as for instance quintessence or a QCD axion, fall into our framework, and therefore the no dS conjecture constrains[4] these sectors via (the opposite of) (12.1.1). In particular, the time-variation of the fine structure constant is upper-bounded by the quintessence slow-roll parameter $|V'/V|$. Current experimental upper bounds on this, coming from the Oklo natural nuclear reactor [483], ensure that this constraint is satisfied in the real world, for $|V'/V|$ as small as 10^{-5}. We find it interesting that the no-dS conjecture, favoring quintessence, is only consistent with a weak time-variation of the fine-structure constant, which otherwise plagues quintessence models.

Another interesting consequence comes from analyzing the photon-axion coupling, which is the subject of active experimental searches [484]. In this case, avoiding lower-dimensional de Sitter solutions requires the product of the axion-photon coupling and the axion decay constant to be a not-too-large number. While this constraint is of no practical phenomenological interest, we find it interesting that it exists. It may point out the way to further, more interesting constraints.

The contents of chapter 11 and especially that chapter's discussion of the basic properties of charged black holes in de Sitter space will be relevant for what follows.. The note is organized as follows. In Section 12.2 we present our construction in any dimension, illustrating the details in the four-to-two dimensional case, write down the constraints that must be satisfied for the solution to exist, and compare to stringy wisdom. The phenomenological implications of our results + the no-dS conjecture are then analysed in Section 12.3. We conclude and discuss our assumptions in Section 12.4, leaving the rest of the technical details to Appendices.

12.2 Constructing de Sitter from runaway potentials

One can wonder whether a phase diagram similar to charged black holes in dS_4 exists for black hole solutions in quintessence spacetimes. This is much more involved questions since we are not aware of exact solutions for black holes. But for the purpose of this note we care about the boundary of the would-be

[4]We would constrain by demanding absence of $dS_2 \times S^2$ solutions, which is fishy due to the peculiarities of low-dimensional quantum gravity. For instance, [459] proposes a version of the SYK model which is dual to a patch of a (non-Einstein) dS_2 quantum gravity. On the other hand, the kind of arguments ruling out tree-level higher-dimensional dS compactifications in string theory seem to apply equally well to 2d.

phase diagram only, and more concretely the dS_2 branch. Hence our approach consists of only looking for flux compactification solutions of the form $M_2 \times S^2$ with M_2 being maximally symmetric. For related work see [403].

From here onward we work in units where $16\pi G = 1$ and consider the following action for the quintessence and electromagnetism coupled to gravity

$$S = \int \sqrt{|g|}\Big(\mathcal{R} - \tfrac{1}{2}(\partial\phi)^2 - \tfrac{1}{4}f(\phi)F_{\mu\nu}F^{\mu\nu} - V(\phi)\Big) + \frac{\theta}{8\pi^2}F\wedge F\,. \quad (12.2.1)$$

Here V is the quintessence potential and f the coupling to electromagnetism. We have also allowed for a θ angle which can depend on ϕ; this allows us to cover the case $\theta = \frac{n}{f_\phi}\phi$ when the quintessence field ϕ is itself an axion, but for the time being we keep the coupling general.

We search for solutions with a product spacetime $M_2 \times S^2$ endowed with the metric

$$ds_4^2 = h(t)[-dt^2 + dr^2] + R^2 d\Omega_2^2\,. \quad (12.2.2)$$

We now check whether we can recover the horizon geometries that exist without quintessence. For that consider the scalar equation of motion:

$$\frac{\partial_\mu\big(\sqrt{|g|}\partial^\mu\phi\big)}{\sqrt{|g|}} = V' + \tfrac{1}{4}f' F_{\mu\nu}F^{\mu\nu} - \frac{\theta'}{8\pi^2}\frac{1}{4\sqrt{|g|}}\varepsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}\,, \quad (12.2.3)$$

where a prime denotes a derivative wrt to the scalar and ε is the anti-symmetric symbol (not tensor). Let us next address Maxwell's equations

$$d\left(*fF - \frac{1}{4\pi^2}\theta F\right) = \delta_E, \quad dF = \delta_M. \quad (12.2.4)$$

Writing $F = E(r)dr\wedge dt + B(r)R^2 d\Omega$, one gets from the above, assuming the scalar has been stabilised,

$$E = \left(Q_e + \frac{\theta}{4\pi^2}Q_m\right)\frac{h}{fR^2}, \quad B = \frac{Q_m}{R^2} \quad (12.2.5)$$

Where charges were defined, via conserved currents, using the following normalisation:

$$Q_e = \frac{1}{4\pi}\int_{S^2}\left(f\star F - \frac{\theta}{4\pi^2}F\right) \qquad Q_m = \frac{1}{4\pi}\int_{S^2}F\,. \quad (12.2.6)$$

The charge Q_e is integer quantized and Q_m is quantized in multiples of 2π. For convenience we introduce the following two quantities:

$$\tilde{Q}_e = Q_e + \frac{\theta}{4\pi^2}Q_m \quad (12.2.7)$$

$$\mathcal{Q}^2 = \frac{\tilde{Q}_e^2}{f} + fQ_m^2\,, \quad (12.2.8)$$

The equations of motion for a constant scalar ϕ and radius R take then a simple form

$$V' + \frac{1}{2R^4}\mathcal{Q}^{2\prime} = 0\,, \tag{12.2.9}$$

$$0 = VR^4 - 2R^2 + \frac{1}{2}\mathcal{Q}^2\,, \tag{12.2.10}$$

$$\mathcal{R}_2 = V - \frac{1}{2R^4}\mathcal{Q}^2\,. \tag{12.2.11}$$

Where $\mathcal{R}_2$ is the 2d curvature[5]. A derivation of these equations can be found in Appendix M.

Equation (12.2.9) stabilises the scalar field for a given Q_e and Q_m and V. If that equation has a solution then the functions f and V obtain specific fixed values. One would then be tempted to think that the remaining two equations are then similar to the Einstein-Maxwell-deSitter case upon the identification: $V = 6\ell^2$ and $f = g^2$ (recall we work in Planckian units for which $16\pi G = 1$). But this turns out not to be the case, probably due to the fact that the the stabilised scalar value still depends on R.

In any case, the existence of solutions depends on the value of $\mathcal{Q}^2$ and V at the stabilised value of the scalar (if any). From equation (12.2.10) it is clear that a real solution for R implies that

$$V\mathcal{Q}^2 \leq 2 \tag{12.2.12}$$

Using (12.2.9), (12.2.11) becomes

$$\mathcal{R}_2 = V\left(1 - \frac{V'}{V}\frac{\mathcal{Q}^2}{(\mathcal{Q}^2)'}\right) \tag{12.2.13}$$

If the potential V is positive, then using (12.2.9) again to show that V'/V and $(\mathcal{Q}^2)'/\mathcal{Q}^2$ have opposite signs, we will have a dS_2 solution only if

$$\left|\frac{V'}{V}\right| \leq \left|\frac{(\mathcal{Q}^2)'}{\mathcal{Q}^2}\right|. \tag{12.2.14}$$

The right hand side becomes a sum of two terms

$$\frac{(\mathcal{Q}^2)'}{\mathcal{Q}^2} = \frac{f'}{f}\frac{f^2Q_m^2 - \tilde{Q}_e^2}{\tilde{Q}_e^2 + f^2Q_m^2} + \frac{2f\tilde{Q}_eQ_m}{\tilde{Q}_e^2 + f^2Q_m^2}\frac{\theta'}{4\pi^2 f}. \tag{12.2.15}$$

[5] $\mathcal{R}_2 = -h^{-2}\ddot{h} + h^{-3}\dot{h}^2$.

Since the dependence on the charges is homogeneous, the function above only depends on one parameter, and by explicit computation one can show that

$$\left|\frac{(\mathcal{Q}^2)'}{\mathcal{Q}^2}\right| \leq \sqrt{\left(\frac{f'}{f}\right)^2 + \left(\frac{\theta'}{4\pi^2 f}\right)^2}, \tag{12.2.16}$$

with the inequality always being achieved[6] for some value of $\tilde{Q}_e, Q_m$. So without loss of generality, to have a dS_2 solution in this way we need to satisfy the constraint

$$\left|\frac{V'}{V}\right| \leq \sqrt{\left(\frac{f'}{f}\right)^2 + \left(\frac{\theta'}{4\pi^2 f}\right)^2}. \tag{12.2.17}$$

We can immediately generalize the picture to higher dimensions. One can consider a general D-dimensional solution with an abelian p-form fieldstrength and a scalar with an action of the form

$$S = \frac{1}{2\kappa_{10}^2}\int d^{10}x\,\sqrt{|g_E^{10}|}\left[R_E - \frac{1}{2}(\partial\phi)^2 - f(\phi)|F_p|^2 - V(\phi)\right]. \tag{12.2.18}$$

Consistent truncations of this kind often show up in string compactifications after supersymmetry is broken and some of the moduli develop a potential. In this case f and V are often exponentials. The detailed analysis, which can be found in Appendix N, leads to the conclusion that there will be a $(D-p)$-dimensional dS solution obtained from flux compactification if and only if

$$(p-1)\left|\frac{V'}{V}\right| \leq \left|\frac{f'}{f}\right|. \tag{12.2.19}$$

12.2.1 Classical limit

We need to check that the dS solutions obtained in this way are in the semiclassical approximation. In the two-dimensional case, this means that R has to be large, $\mathcal{R}_2$ small, and the charges large enough such that quantization is not a worry. The detailed analysis can be found in Appendices M and N, with the conclusion being that there is always a large charge regime where the solution is trustworthy.

Quantum corrections will appear in general Casimir energy contributions to the lower-dimensional potential. Again, the Nariai solution gives an intuition as to

[6] In the large charge limit, that is, ignoring (12.2.12).

why these are small: the S^2 of a Nariai black hole is as big as the parent universe, and such backgrounds are essentially classical if the parent de Sitter solution is classical as well. More concretely, a Casimir energy in lower dimensions will go as H^{D-p}, to be compared with the classical contribution to the potential, which is of order V_0/H^p. Here we are assuming the energy density is of order V_0, since we want to stabilize the scalar at the same value that it has in the parent solution, and we are integrating over the S^p. But $H^2 \sim V_0$ due to the higher dimensional Einstein's equations, and so

$$V_{\text{Casimir}} \sim H^{D-p} \ll H^{2-p} \tag{12.2.20}$$

for small H as long as $D > 2$.

We also point out that, in contrast to the two-dimensional case, for higher dimensions our solutions are controlled against any higher-derivative corrections since we can make the charge parameter arbitrarily large, so that the solutions can be arbitrarily weakly curved.

12.2.2 Stability

Finally we address concerns about stability. Clearly stability depends on the particular form of f and V. When f and V are exponentials, with $f = f_0 e^{\delta\phi}$ and $V = V_0 e^{-\gamma\phi}$, the solution is always a minimum in the two-dimensional case, but tachyonic in the higher-dimensional case. We expect tachyonic instabilities to be rather generic in the higher-dimensional case. Hence one can contemplate to what extend the dS Swampland conjectures forbid such backgrounds. In here we take the viewpoint that is stronger than what is expressed in [164]; as long as there are no explicit examples of unstable dS solutions at the classical level that are under parametric control there might be a good reason to suspect that such backgrounds are equally forbidden. Note that all examples constructed so far of classical tachyonic dS solutions from fluxes and orientifolds [292, 464, 466] are either at strong string coupling or at strong curvature, meaning they are not even approximate solutions to string theory[7]. If string theory describes the real world then one could consider the maximum of the Higgs potential as a counterexample [486], but perhaps this solution does not live at parametric weak coupling.

Wether a dS_2 solution obtained from a flux compactification is unstable is a subtle question. As we discuss in Appendix M, stability of 4d scalars, such as ϕ, is studied via positivity of the Hessian as usual. However, stability of the scalar R^2, which parametrizes the size of the S^2, is more subtle. As explained in [487],

[7] Although these solutions have been used in [485] to argue for a no-dS conjecture that takes into account perturbative stability as in the refined no-dS conjecture of [164].

and analyzed in detail in [488], there are no instabilities in the homogeneous mode, where R^2 is perturbed in a position-dependent way on a a $t = 0$ slice of global dS_2 coordinates. This is because the Hamiltonian constraint of GR fixes R^2 as a function of the other parameters of the theory, so it cannot fluctuate on its own[8].

However, as first explained in [405] (see also [407,430] and especially [406], which discusses the charged case), at the classical level the Hamiltonian constraint allows for inhomogeneous perturbations which are unstable. These perturbations then grow and settle in a different configuration where patches of the dS_2 crunch, while others keep on expanding. Thus, the spacetime forms a series of "beads", which is why this is called the "Bousso beading process". In the charged case, the expanding patches between beads go back to the original dS_2 solution. The conformal diagram of the whole situation can be seen in [406] and looks like a chain of two-dimensional black holes (the crunching regions) with dS_2 patches in between (when the charge is large enough, which is the case we are interested in). The process repeats endlessly, with a proliferation of crunching regions among an ever increasing number of disconnected dS_2 patches.

The Bousso beading process is an universal instability of dS_2 models of quantum gravity coming from higher-dimensional compactifications. The question is whether it "counts" as an instability for Swampland purposes. That is, would we expect to have dS_2 solutions in string theory, since this instability will be generically present? Most of the no-go results we are familiar with are concerned only with stability in the homogeneous mode, and so would apply in this setup as well. But more importantly, in the beading instability, there are always ever-expanding patches of dS_2, so the original vacuum still survives (in the same sense that in eternal inflation almost every patch is in the false vacuum). Finally, to the extent that we refer to the crunching regions as black holes, a natural interpretation of Bousso beading is that it is describing pair production of 2d black holes in the background dS_2 vacuum. An expanding universe pair-produces particles of any mass due to the expansion of the universe; we should expect it to pair-produce black holes as well. If this interpretation is correct, Bousso beading is not a vacuum instability any more than particle production in any de Sitter background (also higher-dimensional ones) is.

12.2.3 Checking against string theory

The main point of this note, explained above, is that one can generically get de-Sitter solutions under parametric control in a flux compactification of an

[8]This can be happening at a maximum of the potential, which is why [487] talks about "top-of-the hill" but stable dS_2 models.

effective model with a runaway potential. These solutions are akin to the flux compactifications that give rise to AdS solutions in string compactifications, and would similarly correspond to the near-horizon limit of extended charged objects in the higher-dimensional solution with the runaway potential. In one particular case, where $p = D - 2$, the solutions we are constructing are actually Nariai-like limits of charged black hole solutions, so this picture is realized exactly.

For this construction to work, however, (12.2.17) or (12.2.19) must be satisfied. So the question is whether one can do this in controlled string compactifications.

Since the solutions we construct are classical flux compactifications, (12.2.17), or the assumptions leading to it, must be violated whenever the Maldacena-Nuñez theorem [185] (and its extensions) hold, if the single-scalar truncation is consistent. For instance, massive IIA supergravity seems to have all ingredients already in 10D; a runaway exponential potential for the dilaton and a dilaton coupling to the $U(1)$-field:

$$S_{10} \supset \int \sqrt{|g_{10}|}\Big(R - \tfrac{1}{2}(\partial\phi)^2 - \tfrac{1}{4}e^{3\phi/2}F_2^2 - m^2\epsilon^{5\phi/2}\Big)\,. \tag{12.2.21}$$

We could look for magnetic solutions of the form $dS_2 \times S^8$, with Q units of F_8 flux threading the S^8. However, it turns out that (12.2.19) is not satisfied, since $7 \cdot 5/2$ is not smaller than $3/2$. On top of this, the equation of motion for H_3

$$d(e^{-\phi} * H_3) = mF_8 \tag{12.2.22}$$

means we could not truncate H_3.

As explained above, when we try to run our argument in higher-dimensional fieldstrenghts we actually find perturbatively unstable de Sitter solutions. These are not too interesting anyway (in spite of the observation in the literature that mildly tachyonic dS solutions are not found in the perturbative corners of string theory, or TCC [164, 327, 489, 490]), but it is fun to observe that (12.2.19) is not satisfied in this case either. As a simple example, consider the consistent truncation of the non-SUSY $O(16) \times O(16)$ heterotic string [491] with the B_2 field as abelian $p-1$-form fieldstrength. The 10d action in the Einstein frame reads (we have normalized the dilaton so that the string coupling is $e^{-\phi}$)

$$S_E = \frac{1}{2\kappa_{10}^2}\int d^{10}x\,\sqrt{|g_E^{10}|}\left[R_E - \frac{1}{2}(\partial\phi)^2 - e^{\phi}|H_3|^2 - V_0 e^{-\frac{5}{2}\phi}\right], \tag{12.2.23}$$

where [491]

$$V_0 = 0.03(\alpha')^5. \tag{12.2.24}$$

The right hand side of (12.2.19) is 1, and the left hand side is 5, so no de Sitter solutions are possible. The cosmological constant in ten dimensions only arises

as a one-loop effect, which means that the usual $e^{-2\phi}$ factor in the string frame gets cancelled out, and we get the above result. Notice that the fact that there is no dS solution in this case lies out of the scope of the usual Maldacena-Nuñez result. While the non-supersymmetric $O(16) \times O(16)$ string can be connected to a non-supersymmetric compactification of M theory in nine dimensions, Maldacena-Nuñez refers to classical M-theory compactifications.

One generically expects runaway potentials whenever supersymmetry is broken, even in situations where Maldacena-Nuñez does not apply. Our point is that in such situation, whenever abelian p-form fields are present too, there is an easy way to potentially obtain de Sitter solutions. It would be interesting to check whether our constraint applies in more general setups, such as e.g. those discussed in [471].

We can elaborate a little bit on this case. The $O(16) \times O(16)$ example above failed to produce a (n unstable) de Sitter, but it came quite close. The vacuum energy was generated at one loop, but suppose one was somehow able to produce a model with a tree level contribution to the vacuum energy as well. The potential in (12.2.23) then gets replaced as

$$V_0 e^{-\frac{5}{2}\phi} \to V_0 e^{-\frac{5}{2}\phi} + V_{\rm tree} e^{-\frac{\phi}{2}}. \tag{12.2.25}$$

and now, for large charge,

$$\left|\frac{V'}{V}\right| = \frac{(5/2)V_0 e^{-\frac{5}{2}\phi} + (1/2)V_{\rm tree} e^{-\frac{\phi}{2}}}{V_0 e^{-\frac{5}{2}\phi} + V_{\rm tree} e^{-\frac{\phi}{2}}} > \frac{1}{2}, \tag{12.2.26}$$

so the inequality (12.2.19) is not satisfied. In fact, the violation is proportional to $e^{-2\phi}$, so it can be made arbitrarily small, but it is always positive.

A tree-level (worldsheet) de Sitter in string theory[9] would still not be a de Sitter compactification of quantum gravity, since at any finite value of the coupling there would be a runaway potential for the dilaton. Even so, one could have tried to stabilize it with our techniques to turn this tree-level de Sitter into a different, exact dS solution, via a flux compactification. This would be an example of a parametrically controlled dS saddle point in string theory. Equation (12.2.26) shows that even this cannot happen.

It therefore seems that our statement (12.2.19) is verified in string theory, even though when violations would only lead to a tachyonic dS, which doesn't seem too bad. One possibility is that even mildly tachyonic dS solutions are not allowed in string theory [164, 489, 490]. Another possible explanation is that it might be possible to further compactify the higher-dimensional solutions to

[9]Even this already faces serious obstacles in heterotic string theory [476].

produce dS_2 solutions, where the instability is absent [492]. We leave exploration of this to future work.

We also need to address the Swampland Distance Conjecture [325], since generically our dS solutions sit at a certain value of ϕ that depends on Q. For us to be able to trust the analysis for abitrarily large Q, the lagrangian descriptions (12.2.1) or (12.2.18) should be valid in the deep IR for large values of ϕ. This is the case in some examples, like the $O(16) \times O(16)$ perturbative heterotic string discussed above, where the only light fields are the dilaton, metric, NSNS field, gauge fields (which can be consistently truncated to zero) and fermions. Everything else is at least string-scale heavy. While the tower of states coming from stringy modes comes down in Planck units as $\phi \to \infty$, the solutions we look at are always within the validity of the EFT.

12.2.4 The web of conjectures is self-consistent

There is one last thing we need to check. Our claim is that (12.2.17) holds for scalars with an arbitrary potential $V(\phi)$. So what about any of the very massive string or KK modes that are ubiquitous in string compactifications? When $V = 1/2m^2\phi^2$, we will get a dS solution only if

$$\phi \geq \frac{2(p-1)}{|f'/f|}. \tag{12.2.27}$$

That is, the dS solution is at a displaced value of the scalar. If this is a stringy or KK mode, we expect coupling to additional modes to spoil the analysis anyway, but it is interesting that for f'/f of order one the value of ϕ is Planckian. This is a significant distance away and it is very unclear one should trust the effective field theory in this case.

In the above simple analysis we assumed that the vacuum energy at $\phi = 0$ vanishes. It is also interesting to see what happens in the AdS case. For that consider an action of the form (12.2.1) with a potential that has a non-tachyonic AdS_4 minimum:

$$V(\phi) = -\frac{1}{L_{AdS}^2} + m^2\phi^2 + \mathcal{O}(\phi^3)\,. \tag{12.2.28}$$

Now if f'/f is order one we can try to construct a dS_2 by stabilizing ϕ away from its AdS_4 minimum using electromagnetic fluxes in such a way that the 4d potential in that part of moduli space is positive. Then we will have a dS_2. Interestingly, for that to happen the scalar needs to travel at least

$$\Delta\phi > \frac{1}{Lm}\,. \tag{12.2.29}$$

The distance conjecture [325] however tells us that this should not be super-Planckian since otherwise the EFT is not to be trusted [493]. So we have that we get a de Sitter vacuum in 2d if

$$Lm > 1\,, \tag{12.2.30}$$

but this violates the AdS-moduli scale separation conjecture [210], which is a weaker form of the AdS-KK scale separation conjecture of [392] (see also [494]). Note that satisfying (12.2.30) was conjectured to be impossible exactly because it would lead to dS vacua in 4D after SUSY breaking, quasi-independent of the details of that breaking [210]. Here we arrive at the same condition from demanding the absence of de Sitter vacua after an S^2 compactification.

So the assumption of both the distance conjecture and the more speculative no-dS conjecture implies the AdS scale separation condition. The only thing we assumed is that f'/f is order 1, which seems true for the dilaton or volume modulus in string theory.

12.3 Phenomenological implications

In the previous Section, we have explained how one can generically construct de Sitter solutions from flux compactifications where there is a scalar with a runaway potential, and how this technique resists application to string theory. Our analysis has been completely rigorous up to this point, but we now switch to conjecture mode. Reference [163] (see also [161]) raised the possibility of no de Sitter solutions in String Theory. Coupled to our observations above, this can only be possible if in any string compactification, the converse of (12.2.19) is satisfied. In other words, whenever a string compactification leads to a low-energy effective field theory such that (12.2.18) is a consistent truncation, one must have

$$(p-1)\left|\frac{V'}{V}\right| \geq \left|\frac{f'}{f}\right|. \tag{12.3.1}$$

So this is a concrete prediction of the no dS conjecture for string compatifications, but there is more to say. If the real world is indeed described by string theory, and the conjecture is correct, it follows that the presently observed positive dark energy must be in some form of quintessence. But then, our construction in the previous Section works and gives us a two-dimensional dS solution. As explained above, this should be thought of as the near-horizon limit of a charged Nariai black hole. There are two possibilities:

- The no dS conjecture or its variants do not apply in two dimensions. Gravity in two-dimensions is qualitatively different and some Swampland constraints can be absent (e.g. there are global symmetries in the string worldsheet).
- The conjecture should also apply. After all, the kind of arguments we have nowadays to rule out dS solutions in corners of moduli space come essentially from dimensional reduction of 10d string theory, and do not treat the two-dimensional case differently in any obvious way.

Modulo subtleties about stability in 2D and the Bousso beading process, we will now explore the consequences of the second possibility. We will go back to the scenario described in Section 12.2, but take the gauge field to be the electromagnetic $U(1)$ we see in the real world. ϕ will be the quintessence field. Some references which look at two-dimensional compactifications of the Standard Model are [441, 487, 488, 495]. We just add a slowly rolling scalar to the mix. In particular, [495] describes the cousin $AdS_2 \times S^2$ compactifications describing the near-horizon limit of an extremal but large RN black hole in dS_4. To avoid a dS_2 solution, we must violate either (12.2.12) or (12.2.17). Equation (12.2.12) depends only on the current-day values of the vacuum energy and f (fine structure constant), and allow for solutions with huge values of $\mathcal{Q}^2$. So the only possiblity is to violate (12.2.17), that is, to have

$$\sqrt{\left(\frac{f'}{f}\right)^2 + \left(\frac{\theta'}{4\pi^2 f}\right)^2} \leq \left|\frac{V'}{V}\right|. \tag{12.3.2}$$

Thus, we cannot have a large dependence on the quintessence field on either the QED axion decay constant, or the gauge coupling itself. Since the quintessence field is rolling, this translates to a constraint on the time dependence of the fine structure constant as well as well as that of the would-be axion decay constant (we emphasize however that θ has not even been measured to be nonzero).

Ignoring the θ dependence, we get the following result for the time variation of the axion decay constant, where we have introduced $c \equiv |V'/V|$ and used the slow-roll estimate $\dot{\phi} \approx H$ as well as $f = 1/\alpha$:

$$\left|\frac{\dot{\alpha}}{\alpha}\right| \leq c\, H. \tag{12.3.3}$$

Using that $H \approx 10^{-10}(yrs)^{-1}$, we get[10]

$$\left|\frac{\dot{\alpha}}{\alpha}\right| \lesssim \frac{c}{10^{10}\,\text{years}}, \tag{12.3.4}$$

[10]For $1/H = 14.4 * 10^9\, yrs$, one has $H = 6.94 * 10^{-11}\,(yrs)^{-1}$.

and since experimentally we know that $c \lesssim 1$ [496], this is an upper bound for the time variation of α. Currently, the best experimental upper bound for the time variation of the fine structure constant comes from the Oklo natural nuclear reactor [483], which constraints

$$\frac{\dot{\alpha}}{\alpha} < 10^{-15}(yrs)^{-1} \tag{12.3.5}$$

Therefore, our constraint is satisfied experimentally. We would need $c \sim 10^{-5}$ to provide new meaningful bounds on $\dot{\alpha}/\alpha$.

We can also put bounds on any axion that couples to the QED axion [484]. This is a periodic scalar field χ with a standard kinetic term that couples to the electromagnetic $U(1)$ field via a coupling

$$g_\chi \frac{\chi}{8\pi^2} F \wedge F \tag{12.3.6}$$

in the Lagrangian. We will also assume The has a periodic potential, of the form

$$V(\chi) = qV_0 \left[1 - \cos\left(\frac{\chi}{f_\chi}\right)\right]. \tag{12.3.7}$$

Here, q is a dimensionless parameter, and f_χ is the axion decay constant.

We will consider a situation where there is a rolling quintessence field, such as the one we considered so far, and also an axion χ coupling to electromagnetism. In principle, both q and f_χ could depend on ϕ as well, but we will assume they do not for simplicity. As we will see, avoiding dS_2 minima leads to bounds on the axion mass and axion decay constant. Our formalism, and in particular equations (12.2.9), (12.2.10) and (12.2.11) work in the same way in situations with more than one scalar. We just need to include equations like (12.2.9) for all the scalar directions we take into consideration. So in a scenario with quintessence and an additional axion field the constraint (12.3.5) gets modified to involve the axion mass and coupling g_χ in a nontrivial way. We just need to apply (12.3.2) to the axionic direction too. Explicitly, we obtain[11] a constraint in terms of $\alpha = 1/(4\pi f)$

$$p \equiv \left|\frac{2\pi}{\alpha} f_\chi g_\chi\right| \leq \frac{\pi}{2\alpha^3}. \tag{12.3.8}$$

When the axion χ is the QCD axion, the parameter p is directly related to the electromagnetic and color anomalies of the axial current associated with the

[11] In obtaining this constraint we have used that $q \gg 1$, worked out the location of the minimum for any value of χ, ϕ, and evaluated at $\chi = \pi/2f_\chi$. We have also checked stability of the minimum with respect to ϕ and χ, and that the resulting electric and magnetic charges are real and positive. On top of this, the constraint (12.2.12) that ensures that the radius is positive must be enforced as well.

axion, E and N [484][12]

$$p = \frac{\pi}{\alpha}\left(\frac{E}{N} - 1.92\right). \tag{12.3.9}$$

Equation (12.3.8) then puts a bound on E/N, that it should be lower than ~ 9400. This is way above what any reasonable model attains [484], so it is not a very interesting constraint.

Our constraints also apply to models of axion quintessence [497], but they are trivially satisfied, since the axion decay constant must be transplanckian. Of course, this kind of models is in trouble anyway with the axion version of the WGC (see e.g. the review [317]).

12.4 Discussion

In this note we have pointed out that a simple flux compactification ansatz can lead to stable, parametrically controlled de Sitter solutions starting from a theory with a runaway scalar potential and abelian p-form gauge fields, provided the simple constraint (12.2.19) involving the slope of the potential and gauge kinetic functions is satisfied. The dS_n solutions we get are perturbatively unstable for $n > 2$, but perturbatively stable for $n = 2$ This corresponds to the interesting case of compactifications of a four-dimensional theory.

We then check in three simple stringy examples, not covered by usual Maldacena-Nuñez arguments, how this constraint fails to be satisfied. Indeed, if any version of the no-dS conjecture actually holds in string theory, then the converse of (12.2.19) must follow as well. We find that (12.2.19) is satisfied even in the higher-dimensional cases where our construction would produce an unstable de Sitter solution anyway. Perhaps, as [164, 489, 490] suggests, even mildly tachyonic de Sitter is problematic string theory, at least close to perturbative corners. It could also be that the constraint is satisfied because we might be able to further compactify our solutions to dS_2 solutions. We leave this to future work.

Applying our constraint to a quadratic potential for a canonically normalized field (such as any stabilized modulus or KK mode in a string compactification) leads to interesting consequences. If the vacuum is Minkowski, then the dS minimum is at a Planckian distance, so one would not trust it anyway. If the vacuum is AdS, then one can have a de Sitter minimum at a subplanckian

[12] To compare to [484], one must use that the normalization in that reference is that $\int F_{\rm PDG} = 2\pi\sqrt{f}n$ (see Sec. 116, review on magnetic monopoles), where n is an integer. So $F_{\rm PDG} = \frac{1}{g}$, and our θ term becomes $\frac{g_\chi \alpha \chi}{2\pi}\int F_p \wedge F_p$, so that $G_{A\gamma\gamma} = \frac{g_\chi \alpha}{\pi}$.

distance only if the lightest scalar has a mass that is parametric above the AdS scale (and the consistent truncation we work in is valid). This goes agains the AdS moduli separation and AdS-KK scale separation conjectures [210, 392], and so we also do not know of stringy examples where this works. If one finds a parametric scale-separated AdS with a light $p-1$-form gauge field, or more generally any AdS solution where our truncation (with a value of the mass above the AdS scale) is valid, and our techniques automatically provide a (perturbatively unstable, except in two dimensions) dS solution.

In the second part of our note, we have explored the phenomenological consequences of the no dS conjecture coupled to our result. We have been able to constrain both the time-variation of the fine structure constant in a quintessence scenario, as well as any axion-photon coupling (including the QCD axion). The no dS conjecture implies $\dot{\alpha}/\alpha < 10^{-10}\text{year}^{-1}c$, where c is the slope of the quintessence potential, while current experimental bounds are at $\dot{\alpha}/\alpha < 10^{-15}\text{year}^{-1}$. We consider this very interesting since it demonstrates the self-consistency of the no-dS conjecture; the problematic running of the fine-structure constant in generic quintessence models is strongly constrained by the conjecture. We have also explored constraints for the coupling of any axion to electromagnetism. In particular, for the QCD axion, we obtain weak constraints that are easily accommodated by any reasonable GUT.

The first part of our note is just a general way of constructing de Sitter solutions, and is independent of the validity of the no dS conjecture or any of its variants. But the phenomenological implications really depend on the validity of the statement [160,161,163,164,462]. If this conjecture is wrong so are all conclusions drawn from this, including the simple observation made in this note.[13]

We have also discussed the subtleties of the classical stability analysis of a dS_2 solution. Although perturbations in the homogeneous mode are stable, there are inhomogeneous perturbations that are classically unstable, leading to "Bousso beading". These inhomogeneities clump and collapse, leading to a crunch, while away from the inhomogeneities the solution is still dS_2. We interpret these as pair production of 2d black holes by the expanding dS_2 background, and thus not an actual vacuum instability, although of course we cannot be sure this is correct. Finally, we also have to address quantum mechanical instabilities. Since the $\text{dS}_2 \times S^2$ solutions can be seen as the boundary of the space of black hole

[13]Strictly speaking we demanded something more than the specific "no-dS" conjecture as stated in [160, 161, 163, 164], we extend it to the non-existence of meta-stable dS vacua in $D < 4$. Even if higher-dimensional de Sitter is somehow pathological, it may be that lower-dimensional solutions are OK. Indeed, [459] studies a version of the SYK model which is holographically dual to JT gravity in a dS_2 spacetime (although it is unclear whether the theory is Einstein). However if we consider the lack of trustworthy dS constructions from string compactifications as a reason for the conjecture as in [161, 462], then the no-dS conjecture in two dimensions is as not obviously wrong as the four-dimensional case.

solutions in 4D it should not come as a surprise that these solutions are quantum mechanically unstable with respect to Hawking radiation and Schwinger pair production. Unlike the classical instability described before, this can happen even in the homogeneous sector. An in-depth study of this was recently carried out by us in [5], where it was found that the lifetime of the two-dimensional de Sitter solution is always finite, and we argued that avoidance of a certain crunch singularity leads to the dS_2 being metastable (in the sense that it gets a lifetime that grows exponentially as the gauge coupling of the gauge theory goes to zero).

In the allowed regime, these quantum instabilities are essentially bubble nucleation instabilities of 4D meta-stable dS vacua with the role of the Brown–Teitelboim bubbles replaced by electron-positron pairs. So our instabilities in 2d are still such that we can think of the dS_2 solutions (away from the clumps of the beading process) as metastable.

The take-home message of this note is that one can often get controlled lower-dimensional de Sitter solutions from runaway potentials. One may regard this as a simple setup where one can explicitly construct de Sitter solutions in string theory, or as a way to place certain runaway potentials in the Swampland. It will be important to figure out which attitude is correct, if either.

Part V

Coda

Chapter 13

Conclusions and outlook

13.1 String vacua - quantum or classical?

After reading part III, the reader may have felt some disappointment. Did we not promise them quantum gravity at the start of the thesis? What then did we end up doing to construct a four-dimensional vacuum in our theory of quantum gravity? We used ten-dimensional supergravity, a low energy EFT, to find vacua. Is this not precisely what we rallied against in the discussion surrounding eq. (1.1.11), arguing that an EFT discussion cannot predict the cosmological constant?

Most of our work was done in a supersymmetric context. Here, one can hope that supersymmetry provides the necessary cancellations to make higher-order corrections to the classical solutions irrelevant for at least some sources of corrections. But this is clearly not a satisfactory response as we will need to dynamically break supersymmetry at low energies to obtain a phenomenologically relevant vacuum. In fact, we did mention that our compactification should be in a reliable regime. If $g_s \ll 1$, field gradients are small such that higher-derivate corrections are small, this means that higher order corrections are small compared to the classical result [161]. In the language of eq. (1.1.11), this means $\sum_{i=1}^{\infty} c_i \ll c_0$.

This makes our string compactifications strange beasts, driven into a small corner of the landscape of string vacua.

On the one hand, this may mean that these vacua are wise beyond what their classical nature lets on. If for instance particle production backreaction were

to always destroy de Sitter space, these vacua should already know about this at leading order. The difficulty of constructing de Sitter vacua in this controlled corner might then indicate what in a generic situation might only be understood from a difficult and ambiguous quantum analysis. While there exist string theory vacua that are understood at a worldsheet level, these are far from phenomenologically relevant. The string vacuum arguments for the more phenomenologically relevant swampland conjectures are inspired by this particular group of controlled string vacua with small corrections.

On the other hand, it could well be that there exist string vacua far from the regimes that we control which behave vastly different from the vacua we understand. In the end, one does for now what one can. We can only hope that as time goes on new tools are developed that allow us to cast light on parts of the landscape still in the dark.

13.2 de Sitter holography and the swampland

In part II we studied de Sitter space holographically and in part IV from a swampland perspective. Both should inform us of what a consistent theory of quantum gravity in de Sitter should look like. But do these perspectives line up? In this section, we will argue that they do.

A first concern is that using dS/CFT, we have really only been able to study higher spin theory. Meanwhile, the swampland program deals with low energy EFTs that are approximately Einstein gravity + matter. Clearly Einstein gravity is the phenomenologically more interesting case. But are these two cases deeply different, or can higher spin theory effectively act as a mimic for an EFT? We have seen evidence that higher spin theory can act as such a mimic. A great example can be found in chapter 6, where one can use a massless spinor and a background scalar field to effectively produce a massive spinor field and reproduce the occupation level expected from a massive spinor in Einstein gravity.

If higher spin theory does act as a mimic, one expects that the relevant swampland constraints should show up in some form. Indeed, the ban on transplanckian excursions of fields does manifest in dS/CFT.

Throughout part II, we argued that the correct measure for evaluating the Hartle-Hawking wavefunction involved setting the measure to zero beyond a certain field range. In Hubble units, we always found the restriction to be to a range of $\mathcal{O}(\sqrt{N})$, the rank of the $U(N)$ global symmetry on the CFT side.

Since

$$G_N = \frac{1}{N\Lambda}\,, \tag{13.2.1}$$

this means that one restricts these fields to a range of order the Planck mass M_p, $\mathcal{O}(M_p)$, very much in line with the distance conjecture, conjecture 2.

Black holes played a major role in our analysis in part IV. In chapter 11 we considered the evaporation of charged black holes in de Sitter space. We saw that there exists a class of "largest possible" black holes in de Sitter space, the charged Nariai black holes. We argued that for for charged Nariai black to be able to evaporate sensibly, particles with mass m and charge q charged under a $U(1)$ gauge theory with coupling g must obey

$$m^2 \gg qgM_P H\,, \tag{13.2.2}$$

with H the Hubble rate. We saw that empty de Sitter space has a future conformal boundary with S^3 topology. Eternal charged Nariai black holes have a future conformal boundary with topology $S^1 \times S^2$. It would be interesting to see if by extending our supersymmetric dS/CFT model to a boundary theory on $S^1 \times S^2$ we can use holography to say something about black holes in de Sitter space. If such a boundary exists, it would seem that one is studying an eternal rather than an evaporating black hole. The difference between Vasiliev theory and Einstein gravity might also pose a problem in using such a holographic analysis to say something about de Sitter black holes in Einstein gravity + matter.

13.3 Open issues in supersymmetric de Sitter space

In part II, we studied a supersymmetric higher spin theory in de Sitter space holographically and argued that its ground state is stable. We believe our analysis gave convincing evidence for stability, but some issues remain unaddressed. By focusing on a CFT analysis of the ground state wavefunction, the unusual fermion reality conditions of the bulk fields disappear into the background.

In fact, [498] argued that states with such reality conditions either have an imaginary energy spectrum or obey $\mathcal{H}|\Psi\rangle = 0$. The Wheeler-deWitt equation guarantees that $\mathcal{H}|\Psi\rangle = 0$. We have argued that quantum gravity in de Sitter space is deeply different from quantum field theory on a fixed de Sitter background. The bound eq. (13.2.2) which does not trivialize in the gravity decoupling limit is an example of this. However, to merely invoke the Wheeler-deWitt equation to evade this issue is perhaps a rather weak excuse.

Several ways forward to better understand this issue are open to us.

We do not known how to perform a direct bulk computation of the no-boundary wavefunction in higher-spin theory. A bulk minisuperspace computation of the wavefunction could be done in $\mathcal{N} = 1$ supergravity, with the unusual fermion reality conditions assigned to the gravitino. If done carefully, such an analysis might be partly informative for the higher-spin case.

A different approach would be to extend our boundary theory to capture a Hilbert space rather than a single state, possibly along the lines of [70]. One can then look at state with fermions excited, where the appearance of negative-norm states would be indicative of possible issues.

We have argued in section 5.4 that supersymmetric higher spin theories in de Sitter space should be related to a limit of a class of exotic string string theories. See also [499] for more recent work on such exotic string theories. Perhaps the best way to understand matters in a UV-complete theory of quantum gravity would be to fully develop the worldsheet formalism for exotic superstring theories. In the best case, one might even hope to use these exotic string theories for a stringy derivation of dS/CFT.

We intend to report on these matters in future work.

Appendix A

Quantization fermions in de Sitter space

In this appendix we provide some additional details on the quantization of a free massless spinor in four-dimensional de Sitter space and the definition of $\breve{\psi}$. None of these results are new, we merely summarize some results of [121, 135] without derivation, specialized to the case of a massless two-component spinor and add to this some comments related to higher spin theory. The interested reader can find a more complete treatment in those texts, including also the extension to a massive dirac spinor and the inclusion of dynamical metric and scalar fields. We will consider the metric for a spacetime whose spatial slices are three-spheres,

$$ds^2 = -dt^2 + e^{2\alpha(t)} d\Omega_3^2 \,, \tag{A.0.1}$$

with $d\Omega_3^2$ the metric on the unit three-sphere. We will denote the metric on a spatial slice by $h_{ij}(t)$. When $e^{\alpha(t)} \simeq l \, \cosh(t/l)$, one is approximately in a de Sitter regime, with de Sitter radius l. One could demand this behaviour for the entire spacetime or asymptotically at late times as desired. Our discussion will be valid for generic metrics of the form eq. (A.0.1).

The standard action for a two-component massless spinor ψ^A in tertad e^a_μ and bispinor for tangent vector $a \to AA'$ notation has the volume term

$$I_V = -\frac{\mathrm{i}}{2} \int d^4x \, e \, \breve{\psi}^{A'} e^\mu_{AA'} D_\mu \psi^A \,. \tag{A.0.2}$$

Here $\breve{\psi}^{A'}$ is ordinarily the Dirac adjoint $\overline{\psi}^{A'} = (\psi^A)^\dagger$ of ψ^A (but read on!). We follow [121, 135] in introducing a seperate notation to indicate that for, e.g.,

the purpose of varying the action, computing conjugate momenta, canonically quantizing in an Hamiltinioan approach, one treats ψ^A and $\breve{\psi}^{A'}$ *as if* they were independent. This is the standard textbook approach, though the notation is nonstandard.

Following this breve notation provides one further advantage in our case. In the supersymmetric de Sitter higher spin theory of [86], which forms the bulk side of the specific dS/CFT duality treated in this paper, there appears a conjugate, which we will denote with c, which acts as the hermitian conjugate $\dagger$ on bosonic quantities. On fermionic quantities, however, it has the special property that $((\psi^A)^c)^c = -\psi^A$. Conjugate spinors in the higher spin theory are defined using this c conjugate rather than the hermitian conjugate. Because we treat ψ^A and $\breve{\psi}^{A'}$ *as if* they were independent, we can treat the cases $\breve{\psi}^{A'} = (\psi^A)^\dagger$ and $\breve{\psi}^{A'} = (\psi^A)^c$ at the same time and the need to distinguish the two will not come up in this appendix or section 6.2. We can thus discuss the general framework for including fermions in dS/CFT in both ordinary gravity + matter theories and higher spin theories at the same time.

In the presence of an initial Σ_I and/or final Σ_F boundary surface, the action also requires boundary terms[1]

$$I_B = \frac{\mathrm{i}}{2}\left(\int_{\Sigma_I} + \int_{\Sigma_F}\right) d^3x\, h^{1/2}\, \breve{\psi}^{A'} n_{AA'} \psi^A \,, \tag{A.0.3}$$

with $n^{AA'}$ the unit timelike vector normal to the surface. The conjugate momentum to ψ^A is given by

$$\pi_A = -\frac{\mathrm{i}}{2} h^{1/2} n_{AA'} \breve{\psi}^{A'} \,. \tag{A.0.4}$$

Using this identity, we can and will take the fundamental dynamical variables of the theory to be ψ^A and $\breve{\psi}^A$ rather than ψ^A and π_A.

One finds the classical Dirac bracket

$$\{h^{1/4}\psi^A(x)\,, h^{1/4}\breve{\psi}^{A'}(x')\}_D = -2\mathrm{i} n^{AA'}\delta(x,x')\,, \tag{A.0.5}$$

and all other Dirac brackets zero. One quantizes by the usual procedure, turning the Dirac brackets into anticommutators with factor of $\mathrm{i}\hbar$ inserted. One can expand spinor-valued functions in the background of eq. (A.0.1) in spatial three-sphere harmonics. We denote the positive frequency three-sphere harmonics by ρ_A^{np} and $\sigma_{A'}^{np}$; the negative frequency harmonics by $\overline{\rho}_{A'}^{np}$ and $\overline{\sigma}_A^{np}$. The indices

[1] When computing the Hartle-Hawking state, there would be only a final surface but no initial one. Further, the metric would also be dynamical and allowed to take complex values. Here, we discuss the free fermion generically.

run over $n = 0, 1, 2, ...$ and $p = 1, 2, .., (n+1)(n+2)$. The expansions of ψ^A and $\breve{\psi}^{A'}$ are then given by[2]

$$\psi_A = \frac{e^{-3\alpha/2}}{2\pi} \sum_n \sum_p \left(s_{np}(t) \rho_A^{np} + \breve{t}_{np}(t) \overline{\sigma}_A^{np} \right), \tag{A.0.6a}$$

$$\breve{\psi}_{A'} = \frac{e^{-3\alpha/2}}{2\pi} \sum_n \sum_p \left(\breve{s}_{np}(t) \overline{\rho}_{A'}^{np} + t_{np}(t) \sigma_{A'}^{np} \right). \tag{A.0.6b}$$

The dynamical coeffecients $s_{np}(t)$, $\breve{t}_{np}(t)$, $\breve{s}_{np}(t)$, $t_{np}(t)$ are Grassmann-valued for anticommuting spinors, as is the case here. The prefactors in front of these expansions could equally well have been absorbed into the dynamical coefficients. This choice of prefactors is convenient as they counteract the factors of $h^{1/4}$ appearing in eq. (A.0.5). This ensures that upon quantization one obtains eq. (6.2.2b) without any further factors or metric dependence.

[2]This expansion is completely general and valid. However, in practical bulk computations it is more convenient to expand e.g. as $\alpha_n^{pq} s_{nq} \rho_{pA}^n$ instead of $s_{np}\rho^{np}$, with the α_n^{np} constant matrices auspiciously chosen to ensure there are no couplings between different terms with the same value of n when filling these expansions back into the action. These issues related to the degeneracy are not relevant in our context as we focus only on one of the two $n = 0$ modes.

Appendix B

Conventions for 10d view on KKLT

The type IIB supergravity action (in units where $2\pi\ell_s = 1$) is

$$\begin{aligned} S =& 2\pi \int \star_{10} \left(R_{10} - \phi d\tau \cdot \dot{\mathrm{d}}\bar{\tau} 2(\mathrm{Im}\,\tau)^2 - \phi G_3 \cdot \overline{G_3} 2(\mathrm{Im}\,\tau) - \phi 14 |F_5|^2 \right) \\ & - \phi\pi 2i \int \phi C_4 \wedge G_3 \wedge \overline{G_3} \mathrm{Im}\,\tau \ , \end{aligned} \tag{B.0.1}$$

where R_{10} is the ten-dimensional Ricci scalar calculated using the metric g_{MN} with determinant g_{10}. The axion-dilaton is denoted by $\tau = C_0 + i\epsilon^{-\phi}$ and the NSNS and RR 3-form field strengths have been combined into a single complex 3-form

$$G_3 = F_3 - i\epsilon^{-\phi} H = dC_2 - \tau dB_2 \ , \quad H = dB_2 \ , \quad F_3 = dC_2 - HC_0 \ . \tag{B.0.2}$$

Throughout the paper we use short-hand notation to denote form contractions, let ω_p and ψ_p denote two p-forms, then

$$\begin{aligned} \star_{10}\omega_p \wedge \psi_p &= \omega_p \cdot \psi_p \ \star_{10} 1 \, , \\ \omega_p \cdot \psi_p &= \phi 1 p! \omega_{M_1 M_2 \cdots M_p} \psi^{M_1 M_2 \cdots M_p} \, , \\ |\omega_p|^2 &= \overline{\omega}_p \cdot \omega_p \ . \end{aligned} \tag{B.0.3}$$

The complex-conjugation in the last expression is to allow for the possibility that ω_p is a complex p-form.

Appendix C

Variation of the action for 10d view on KKLT

In this appendix we derive eq. (8.4.10) step-by-step. We also fill in some of the gaps of the subsequent calculations in section 8.4. First we show that we can obtain equation (8.4.3) directly by varying the ten-dimensional type IIB supergravity action with respect to the warp factor A, as defined in (F.0.7). After establishing this, we include the D7-branes and redo the same computation.

The ten-dimensional action for the Ansatz (F.0.7, 8.4.2) can be found to be:

$$S_{\text{IIB}} = 2\pi \int d^{10}x \sqrt{-g_4}\sqrt{g_6} e^{-2A}\, \mathcal{I}\,, \tag{C.0.1}$$

where

$$\mathcal{I} = e^{-2A}R_4 + e^{2A}\left(R_6 + 2\triangle_6 A - 8|dA|_6^2\right) - e^{2A}\frac{|d\tau|_6^2}{2(\text{Im}\tau)^2} - e^{6A}\frac{|G_3|_6^2}{2\text{Im}\tau} + e^{-6A}\frac{|d\alpha|_6^2}{2}\,. \tag{C.0.2}$$

The $|\cdots|_6^2$ denote metric contractions using purely the six-dimensional metric ds_6^2 without the warp factor. We now compute a variation with respect to A. We find:

$$0 = R_4 - \triangle_6 e^{4A} + e^{-6A}|de^{4A}|^2 + e^{2A}\frac{|G_3|^2}{2\text{Im}\tau} + e^{-6A}|d\alpha|^2\,. \tag{C.0.3}$$

This is simply the trace-reversed ten-dimensional Einstein-equation traced over the four-dimensional indices. Adding the Bianchi identity, we recover equation (8.4.3):

$$\triangle_6 \Phi^- = R_4 + e^{2A}\frac{|G_3^-|^2}{\text{Im}\tau} + e^{-6A}|d\Phi^-|^2\,. \tag{C.0.4}$$

We did not include any sources, but including O7/O3 planes does not modify the final result. Now we would now like to add D7-branes to the configuration. As explained in the main text, the inclusion of the D7-brane implies that we should perform the replacement (8.4.8) and vary that with respect to the warp factor. We have two crucial remarks. The careful reader will notice that I as defined in (8.4.6) contains a warp factor dependence of e^{-4A}. This factor should *not* be varied when deriving the equivalent to (C.0.3), since it does not arise from a metric in the action. On the other hand, the delta function and the holomorphic three-form do have a warp factor dependence. The holomorphic three-form goes like e^{-3A} and the delta function like e^{2A}. We can therefore write the perfect square action as

$$S_3 = -2\pi \int d^{10}x \sqrt{-g_4}\sqrt{g_6} \frac{e^{4A}}{2\mathrm{Im}\,\tau} \left|G_3 - e^{-A}(\mathrm{Im}\,\tau)\tilde{\mathcal{P}}(\bar{I})\right|_6^2 , \tag{C.0.5}$$

where we have made the warp-factor dependence explicit by momentarily writing $\mathcal{P}(I) = e^{-A}\tilde{\mathcal{P}}(\bar{I})$. Using this we find

$$\begin{aligned} -\frac{e^{4A}}{8\pi\sqrt{-g_4}\sqrt{\tilde{g}_6}}\frac{\delta S_3}{\delta A} =& \frac{e^{2A}}{2\mathrm{Im}\,\tau}\left|G_3 - (\mathrm{Im}\,\tau)\mathcal{P}(\bar{I})\right|^2 \\ &+ \frac{e^{2A}}{4}\mathrm{Re}\left[\left(G_3 - (\mathrm{Im}\,\tau)\mathcal{P}(\bar{I})\right)\cdot\mathcal{P}(I)\right] \end{aligned} \tag{C.0.6}$$

$$= e^{2A}\left(\frac{|G_3|^2}{2\mathrm{Im}\,\tau} - \frac{3}{4}\mathrm{Re}\,(G_3\cdot\mathcal{P}(I)) + \frac{\mathrm{Im}\,\tau}{4}|\mathcal{P}(I)|^2\right). \tag{C.0.7}$$

We should replace the $|G_3|^2$-term in (C.0.3) by (C.0.7). This modifies (C.0.4) to exactly (8.4.10) which reads:

$$\triangle_6\Phi^- = R_4 + e^{-6A}|d\Phi^-|^2 + e^{2A}\left(\phi|G_3^-|^2\mathrm{Im}\,\tau - \phi 34\mathrm{Re}\left(G_3\cdot\mathcal{P}(I)\right) + \phi\mathrm{Im}\,\tau 4|\mathcal{P}(I)|^2\right) .$$

If one now writes $\mathcal{P}(I) = I^{\mathrm{sing}} + I^{\mathrm{reg}}$ as in (8.4.13) and substitutes the solution to the equation of motion $G_3 = (\mathrm{Im}\,\tau)\overline{I^{\mathrm{sing}}} + G_3^0$ (while making the approximation that $\Phi^- = 0$ and warping, C_4-potential and axion-dilaton are constant) into (C.0.6), one finds together with the other terms in the IIB action and the

Bianchi identity the following equation:

$$\begin{aligned}0 = R_4 &+ \frac{e^{2A}}{2\mathrm{Im}\,\tau}\left|G_3^0 - (\mathrm{Im}\,\tau)\overline{I^{\mathrm{reg}}}\right|^2 \\ &+ \frac{e^{2A}}{4\mathrm{Im}\,\tau}\mathrm{Re}\left[\left(G_3^0 - (\mathrm{Im}\,\tau)\overline{I^{\mathrm{reg}}}\right)\cdot(\mathrm{Im}\,\tau)\left(I^{\mathrm{sing}} + I^{\mathrm{reg}}\right)\right] \\ &+ \frac{e^{2A}}{2\mathrm{Im}\,\tau}\left(|G_3^{0,-}|^2 + 2\mathrm{Re}\left(G_3^{0,-}\cdot\overline{I^{\mathrm{sing},-}}\right) - |G_3^{0,+}|^2 - 2\mathrm{Re}\left(G_3^{0,+}\cdot\overline{I^{\mathrm{sing},+}}\right)\right)\end{aligned} \tag{C.0.8}$$

The substitution already took into account that the Bianchi identity does not contribute δ^2 singularities, which is discussed in appendix D. Recall that I^{sing} is by definition exact, so its inner product with a harmonic form (G_3^0 or I^{reg}) is therefore zero when the expression is integrated over (the (A)ISD-parts vanish in the same fashion). In this way, we arrive at

$$0 = \int \star_6 \left(R_4 + \frac{e^{2A}}{\mathrm{Im}\,\tau}|G_3^{0,-}|^2 - \frac{3e^{2A}}{4}\mathrm{Re}\left(G_3^0\cdot I^{\mathrm{reg}}\right) + \frac{e^{2A}}{4}(\mathrm{Im}\,\tau)|I^{\mathrm{reg}}|^2\right). \tag{C.0.9}$$

Appendix D

(Non)renormalisation of the Chern-Simons action

The effective bulk action of IIB string theory contains a Chern-Simons term,

$$S_{\rm CS} = -\frac{\pi}{2i}\int \frac{C_4 \wedge G_3 \wedge \overline{G_3}}{{\rm Im}\,\tau}\,. \tag{D.0.1}$$

This term is topological and does not affect the Einstein equation directly. Still one can be concerned and wonder whether we should renormalise the Chern-Simons action or not. If we should, the renormalisation could introduce metric dependence and affect our results.

Under the assumptions of constant C_4 and axion-dilaton made in the main body of the text, we should not renormalise the Chern-Simons term. Not renormalising produces no disasters and thus there is perhaps no obvious need for renormalisation. Let us check this.

At first, it may appear that filling the solution for G_3 into the Chern-Simons action will produce a badly divergent action. However, this is not the case.

Filling the on-shell value of G_3 into the Chern-Simons action yields

$$\begin{aligned} S_{\text{CS,on-shell}} = &- \frac{\pi g_s}{2i} \int \Big\{ C_4 \wedge G_3^0 \wedge \overline{G_3^0} + (\text{Im}\,\tau) C_4 \wedge G_3^0 \wedge \overline{I^{\text{sing}}} \\ &+ (\text{Im}\,\tau) C_4 \wedge I^{\text{sing}} \wedge \overline{G_3^0} \Big\} \\ &- \frac{\pi g_s}{2} \int \left(|I^{\text{sing}\,-}|^2 - |I^{\text{sing}\,+}|^2 \right) (\text{Im}\,\tau)^2 C_4 \wedge \tilde{\star}_6 1 \,, \end{aligned} \tag{D.0.2}$$

where we introduced the symbols $I^{\text{sing}\,\pm} = \frac{1}{2}(\star_6 \pm i) I^{\text{sing}}$. The last two terms in this expression seem divergent while all other terms are clearly finite upon evaluation of the integral. Interestingly the two divergent terms exactly cancel against each other and the on-shell action is well-behaved. To see this, note that near a D7-brane we can locally write the internal manifold as $\mathcal{M} = \Sigma_2 \times \Sigma_4$, with Σ_2 transverse and Σ_4 parallel to the D7-brane. We then have

$$\begin{aligned} \overline{I^{\text{sing}}} &\sim d \left(\frac{\partial G}{\partial z} \right) \wedge \bar{\Omega}_2 \\ &= \left(\frac{\partial^2 G}{\partial z \partial z} \right) dz \wedge \bar{\Omega}_2 + \left(\frac{\partial^2 G}{\partial z \partial \bar{z}} \right) d\bar{z} \wedge \bar{\Omega}_2 \end{aligned} \tag{D.0.3}$$

with G the Green's function, which varies transverse to the D7-brane, and dz and $d\bar{z}$ are defined on Σ_2. It is clear that the first of these terms is $(1,2)$ while the second is $(0,3)$. First we notice that the two terms have the same magnitude:

$$\int \star_6 \left| \left(\frac{\partial^2 G}{\partial z \partial z} \right) dz \wedge \bar{\Omega}_2 \right|^2 = \int \star_6 \left| \left(\frac{\partial^2 G}{\partial z \partial \bar{z}} \right) d\bar{z} \wedge \bar{\Omega}_2 \right|^2 , \tag{D.0.4}$$

which follows from integration by parts and the reality of G. Then we use that the first term in (D.0.3) corresponds to the IASD part of $\overline{I^{\text{sing}}}$, while the second term corresponds to the ISD part. It is immediate that the $(0,3)$ term is ISD as the unique $(0,3)$ form at our disposal is $\bar{\Omega}$ and this is ISD in our convention. That the $(1,2)$ form is IASD follows from the fact that it is primitive and primitive $(1,2)$ forms are IASD in our convention. The primitivity can be seen by decomposing J as

$$J = J_{\Sigma_2} + J_{\Sigma_4} \,, \tag{D.0.5}$$

with the two terms defined in the obvious way. Using this, evaluation yields

$$\left(\frac{\partial^2 G}{\partial z \partial z} \right) dz \wedge \bar{\Omega}_2 \wedge J = 0 \,. \tag{D.0.6}$$

We thus see that $I^{\text{sing}\,+}$ and $I^{\text{sing}\,-}$ have the same magnitude. Therefore, the two problematic terms in (D.0.2) cancel against each other and the on-shell Chern-Simons action is finite without the introduction of any renormalisation counterterms.

Appendix E

Conventions for G2 holonomy from SU(3) structure

Our form conventions are:

$$\alpha_p = \frac{1}{p!}\alpha_{m_1\dots m_p}\mathrm{d}x^{m_1\dots m_p}\,, \tag{E.0.1}$$

$$\omega_q \lrcorner \alpha_p = \frac{1}{q(p-q)!}\omega^{m_1\dots m_q}\alpha_{m_1\dots m_q\dots m_p}\mathrm{d}x^{m_{q+1}\dots m_p}\,. \tag{E.0.2}$$

Concerning SU(3)-structures we follow the notation and conventions of [176]. A summary of the most relevant equations for SU(3) structures, that fix our conventions are:

$$\frac{3i}{4}\Omega\wedge\bar{\Omega} = J^3\,, \tag{E.0.3}$$

$$\star_6\,\Omega = i\Omega\,, \tag{E.0.4}$$

$$\star_6\,J = \tfrac{1}{2}J\wedge J\,, \tag{E.0.5}$$

$$J\wedge J\wedge J = 6\mathrm{dVol}\,, \tag{E.0.6}$$

$$\star_6\,W_2 = -J\wedge W_2\,. \tag{E.0.7}$$

The convention for the Hodge-$\star$ is the one in footnote 13 of [176]. This means that, given a 2-form $w_2 = w^{(2,0)} + w^{(0,2)} + w^{(1,1)}$:

$$\star_6 w^{(2,0)} = J \wedge w^{(2,0)}\,, \tag{E.0.8}$$

$$\star_6 w^{(0,2)} = J \wedge w^{(0,2)}\,, \tag{E.0.9}$$

$$\star_6 w^{(1,1)}_{prim} = -J \wedge w^{(1,1)}_{prim}\,, \tag{E.0.10}$$

where the last identity is valid only for the primitive part (if any) of $w^{(1,1)}$, i.e. such that $J^{mn} w^{(1,1)prim}_{mn} = 0$ (or $J \wedge J \wedge w^{(1,1)}_{prim}$).

Appendix F

Localised O6 solution

The solution with localised sources is straightforward to obtain by dressing forms with the appropriate warping as explained in [176, 500]. A factor of e^{W} (e^{-W}) is added in J, Ω for every coordinate one-form e_{+}^{α} (e_{-}^{α}) parallel (transverse) to O6's (we use a tilde to refer to solutions in the smeared case):

$$e_{+}^{\alpha} = e^{W}\tilde{e}_{+}^{\alpha}\,, \qquad e_{-}^{i} = e^{-W}\tilde{e}_{-}^{i}\,. \tag{F.0.1}$$

This method only works for localising sources that are all parallel.

We can now compute F_2 from (9.2.6) and this requires Ω_I. The anti-holomorphic O6 involution σ is supposed to act on the pure spinors as [176]:

$$\sigma(J) = -J\,, \qquad \sigma(\Omega) = -\bar{\Omega}\,. \tag{F.0.2}$$

From this fact we find that J has a foot along and the other transverse to the O6's. Ω_R can have either a foot transverse and 2 feet parallel to O6's or all 3 feet transverse. On the other hand, Ω_I can have either a foot parallel and 2 transverse or all parallel. In our case, $\Omega_I = -2e^{124} + e^{135} + e^{236} + 2e^{456} = -2e^{-W}\tilde{e}^{124} + e^{-W}\tilde{e}^{135} + e^{-W}\tilde{e}^{236} + 2e^{3W}\tilde{e}^{456}$, and then

$$\mathrm{d}(e^{W}\Omega_I) = \mathrm{d}(-2\tilde{e}^{124} + \tilde{e}^{135} + \tilde{e}^{236} + 2e^{4W}\tilde{e}^{456}) \tag{F.0.3}$$

$$= [2\mathrm{d}e^{4W} \wedge \tilde{e}^{456} + 2e^{4W}(\tilde{e}^{1256} - \tilde{e}^{1346} + \tilde{e}^{2345})]\,. \tag{F.0.4}$$

In order to compute F_2 one needs to compute the Hodge-$\star_6$ with respect to the (warped) metric g:

$$\star_6 \tilde{e}^{abcd} = \frac{1}{2}\sqrt{g} g^{aa'} \cdots g^{dd'} \epsilon_{efa'\ldots d'} \tilde{e}^{ef} \tag{F.0.5}$$

$$= \frac{1}{2}\sqrt{\tilde{g}} g^{aa'} \cdots g^{dd'} \epsilon_{efa'\ldots d'} \tilde{e}^{ef} \,, \tag{F.0.6}$$

where $g^{aa'} = e^{\mp 2W}\tilde{g}^{aa'}$ if a, a are directions parallel or transverse to O6's respectively. The unwarped metric $\tilde{g}_{mn}$ can be extracted from the expression for J, since $J = \tilde{J} = i\tilde{g}_{i\bar{j}}\tilde{z}^i \wedge \bar{\tilde{z}}^j$. One finds $\tilde{g}_{i\bar{j}} = \frac{1}{2}\delta_{i\bar{j}}$, which in real coordinates means

$$\tilde{g} = \mathrm{diag}(1,1,1,4,1,1)\,. \tag{F.0.7}$$

Therefore

$$\star_6\, \tilde{e}^{1256} = 2\tilde{e}^{34}\,, \qquad \star_6 \tilde{e}^{1346} = \frac{1}{2}\tilde{e}^{25}\,, \qquad \star_6 \tilde{e}^{2345} = \frac{1}{2}\tilde{e}^{16}\,. \tag{F.0.8}$$

We can finally compute F_2:

$$g_s F_2 = -e^{-4W} \star_6 \mathrm{d}(e^W \Omega_I) \tag{F.0.9}$$

$$= 2e^{4W} \star_6 (\mathrm{d}e^{-4W} \wedge \tilde{e}^{456}) - 2(2\tilde{e}^{34} - \frac{1}{2}\tilde{e}^{25} + \frac{1}{2}\tilde{e}^{16}) \tag{F.0.10}$$

$$= e^W \star_3 \mathrm{d}e^{-4W} - 4\tilde{e}^{34} + \tilde{e}^{25} - \tilde{e}^{16}\,, \tag{F.0.11}$$

where we have used the fact that $\star_6$ decomposes as $\star_3\star_3^{\parallel}$ and $\star_3^{\parallel}\tilde{e}^{456} = \frac{e^{-3W}}{2}$. As we pointed out below (9.2.9), F_2 splits into the background component, plus a component sourced by local objects. By taking the exterior derivative:

$$g_s \mathrm{d}F_2 = \mathrm{d}(e^W \star_3 \mathrm{d}e^{-4W}) + 6\tilde{e}^{123} \tag{F.0.12}$$

$$= (\tilde{\nabla}^2_- e^{-4W} + 6)\tilde{e}^{123}\,, \tag{F.0.13}$$

and the Bianchi identity becomes an equation for the warp factor:

$$(\tilde{\nabla}^2_- e^{-4W} + 6)\tilde{e}^{123} = g_s Q \delta\,. \tag{F.0.14}$$

Appendix G

SU(3) torsion classes

The deviation of an SU(3)-structure manifold from having SU(3) holonomy is measured by the torsion (classes), which expresses the non-closure of Ω and J via

$$\mathrm{d}J = \tfrac{3}{2}\mathrm{Im}\,(\bar{W}_1\Omega) + W_4 \wedge J + W_3\,, \tag{G.0.1}$$

$$\mathrm{d}\Omega = W_1 J^2 + W_2 \wedge J + \bar{W}_5 \wedge \Omega\,. \tag{G.0.2}$$

To gain more understanding of the flux solutions in IIA supergravity we compute the torsion classes. The closure of J implies

$$W_1 = W_3 = W_4 = 0\,. \tag{G.0.3}$$

Since $\mathrm{d}\Omega = (\mathrm{d}\Omega)^{3,1} + (\mathrm{d}\Omega)^{2,2}$ (this follows from the fact that Ω is a complex $(3,0)$-form), and $\star_6 F_2 = (\star_6 F_2)^{3,1} + (\star_6 F_2)^{2,2} + (\star_6 F_2)^{1,3}$, we can rewrite (9.2.5), (9.2.6) as

$$(\mathrm{d}\Omega)^{3,1} = (\mathrm{d}W)^{0,1} \wedge \Omega = -ie^{\phi} J \wedge F^{2,0}\,, \tag{G.0.4}$$

$$(\mathrm{d}\bar{\Omega})^{1,3} = (\mathrm{d}W)^{1,0} \wedge \bar{\Omega} = ie^{\phi} J \wedge F^{0,2}\,, \tag{G.0.5}$$

$$(\mathrm{d}\Omega)^{2,2} = -(\mathrm{d}\bar{\Omega})^{2,2} = -ie^{\phi}(\star_6 F_2)^{2,2}\,. \tag{G.0.6}$$

By matching these equations with (G.0.2), which can be decomposed in

$$(\mathrm{d}\Omega)^{2,2} = W_2 \wedge J = -\star_6 W_2\,, \tag{G.0.7}$$

$$(\mathrm{d}\Omega)^{3,1} = \bar{W}_5 \wedge \Omega\,, \tag{G.0.8}$$

one can extract W_2 and W_5. From the first one and (G.0.6), one can immediately read W_2:

$$W_2 = ie^{\phi} F^{1,1} \tag{G.0.9}$$

which means that $F^{1,1}$ has to be primitive. Indeed, one can check it from $J \wedge (\mathrm{d}\Omega)^{2,2} = J^2 \wedge W_2 = 0$, which is also $J \wedge (\mathrm{d}\Omega)^{2,2} = -ie^{\phi} J \wedge \star_6 F^{(1,1)} \sim J^{mn} F^{(1,1)}_{mn}$. Hence, $J^{mn} F^{(1,1)}_{mn} = 0$.

Let us compute W_5. By contracting (G.0.2) with with $\bar{\Omega}$:

$$\begin{aligned} \bar{\Omega} \lrcorner \mathrm{d}\Omega = \bar{\Omega} \lrcorner (\mathrm{d}\Omega)^{3,1} &= \bar{\Omega} \lrcorner (\bar{W}_5 \wedge \Omega) \\ &= \frac{1}{3!} \bar{\Omega}^{mnp} \left(3 \bar{W}_{5m} \Omega_{npq} - \bar{W}_{5q} \Omega_{mnp} \right) \mathrm{d}x^q \\ &= -\frac{1}{3!} \bar{\Omega}^{ijk} \bar{W}_{5\bar{\imath}} \Omega_{ijk} \, \mathrm{d}\bar{z}^{\bar{\imath}} \\ &= -|\Omega|^2 \bar{W}_5 \, . \end{aligned} \tag{G.0.10}$$

Upon using the supersymmetry equation (G.0.4), one finds

$$\begin{aligned} \bar{\Omega} \lrcorner (\mathrm{d}\Omega)^{3,1} &= -ie^{\phi} \bar{\Omega} \lrcorner (J \wedge F^{2,0}) \\ &= -\frac{ie^{\phi}}{3!} \bar{\Omega}^{ijk} (J \wedge F^{2,0})_{ijk\bar{k}} \mathrm{d}\bar{z}^k \, , \end{aligned} \tag{G.0.11}$$

and finally, since in our conventions $|\Omega|^2 = 8$:

$$\bar{W}_5 = \frac{1}{8} ie^{\phi} \bar{\Omega} \lrcorner (J \wedge F^{2,0}) = \frac{ie^{\phi}}{3!8} \bar{\Omega}^{ijk} (J \wedge F^{2,0})_{ijk\bar{k}} \mathrm{d}\bar{z}^k \, . \tag{G.0.12}$$

For future reference, notice that (G.0.5) implies (by expressing $(\mathrm{d}\bar{\Omega})^{1,3}$ in terms of torsion structures):

$$\mathrm{d}W = W_5 + \bar{W}_5 \, . \tag{G.0.13}$$

Appendix H

(Co-)Closure of Φ

Here, we show that the G2-form

$$\Phi = e^{-\phi}\Omega_I - J \wedge (\mathrm{d}z + A) \tag{H.0.1}$$

is closed and co-closed, iff supersymmetric equations are satisfied. Closure, $\mathrm{d}\Phi = 0$, is achieved first by observing that, due to the supersymmetric condition $\mathrm{d}J = 0$,

$$\mathrm{d}\Phi = \mathrm{d}(e^{-\phi}\Omega_I) - J \wedge F_2\,, \tag{H.0.2}$$

Then, by using (9.2.6), (G.0.5),(G.0.4), (G.0.13), (9.2.3), it is possible to show that the RHS vanishes. In fact:

$$\mathrm{d}(e^{-\phi}\Omega_I) = -\frac{4}{3}e^{-\phi}\mathrm{d}\phi \wedge \Omega_I - \star_6(F^{(2,0)} + F^{(0,2)} + F^{(1,1)}_{prim}) \tag{H.0.3}$$

$$= 2ie^{-\phi}(\bar{W}_5 \wedge \Omega - W_5 \wedge \bar{\Omega}) - J \wedge (F^{(2,0)} + F^{(0,2)} - F^{(1,1)}_{prim}) \tag{H.0.4}$$

$$= 2(J \wedge F^{(2,0)} + J \wedge F^{(0,2)}) - J \wedge (F^{(2,0)} + F^{(0,2)} - F^{(1,1)}_{prim}) \tag{H.0.5}$$

$$= J \wedge F_2\,. \tag{H.0.6}$$

In order to prove co-closure, $\mathrm{d} \star_7 \Phi = 0$, we have to compute $\star_7\Phi$ first. In order to act with the Hodge-$\star_7$, we have to recast Φ in terms of 7d quantities. We will use siebenbeins. Since the metric is $\mathrm{d}s^2_{11} = e^{-\frac{2}{3}\phi}\mathrm{d}s^2_{10} + e^{\frac{4}{3}\phi}(\mathrm{d}z + A)^2$,

seibenbeins are defined as $e^A = (e^a_{(7)}, e^7) = (e^{-\phi/3}e^a, e^{2\phi/3}(\mathrm{d}z + A))$ (while sechsbeins are e^a with $a = 1...6$). We can now compute the $\star_7\Phi$ by defining $\epsilon_{abcdef7} \equiv \epsilon_{abcdef}$. The first terms yields

$$\star_7[e^{-\phi}\Omega_I] = \frac{1}{3!}e^{-\phi} \star_7 [e^{\phi}(\Omega_I)_{abc}e^{abc}_{(7)}] \tag{H.0.7}$$

$$= -\frac{1}{3!}(\Omega_I)_{abc}\frac{1}{3!}\epsilon_{def}{}^{abc}e^{-\phi/3}e^{def} \wedge (\mathrm{d}z + A) \tag{H.0.8}$$

$$= -e^{-\phi/3} \star_6 \Omega_I \wedge (\mathrm{d}z + A) \tag{H.0.9}$$

$$= -e^{-\phi/3}\Omega_R \wedge (\mathrm{d}z + A)\,, \tag{H.0.10}$$

where we used the definition of $\star_6$ and the property $\star_6\Omega_I = \Omega_R$. The second term is

$$\star_7[J \wedge (\mathrm{d}z + A)] = \star_7[J_{(7)} \wedge e^7] \tag{H.0.11}$$

$$= \frac{1}{2 \cdot 4!}J_{ab}\epsilon_{cdef}{}^{ab}e^{-4\phi/3}\tilde{e}^{cdef} \tag{H.0.12}$$

$$= e^{-4\phi/3} \star_6 J \tag{H.0.13}$$

$$= \frac{1}{2}e^{-4\phi/3}J \wedge J\,, \tag{H.0.14}$$

where we used $\star_6 J = \frac{1}{2}J^2$ in the last step. We can show now that

$$\star_7\Phi = -e^{-\phi/3}\Omega_R \wedge (\mathrm{d}z + A) - \frac{1}{2}e^{-4\phi/3}J^2 \tag{H.0.15}$$

is closed:

$$\mathrm{d} \star_7 \Phi = e^{-\phi/3}\Omega_R \wedge F_2 - \frac{1}{2}\mathrm{d}e^{-4\phi/3} \wedge J \wedge J = 0\,, \tag{H.0.16}$$

where we used the supersymmetry equations (9.2.4)-(9.2.5) in the first step and

$$-\frac{1}{2}\mathrm{d}e^{-4\phi/3} \wedge J \wedge J = \frac{2}{3}e^{-4\phi/3}\mathrm{d}\phi \wedge J \wedge J = -e^{-\phi/3}F_2 \wedge \Omega_R \tag{H.0.17}$$

in the final step. The relation (H.0.17) can be proven by using (as one may check from (G.0.13), (G.0.12) and some little algebra [1])

$$\mathrm{d}\phi \wedge J \wedge J = 3\left(W_5 + \bar{W}_5\right) \wedge J \wedge J \tag{H.0.18}$$

$$= -\frac{3}{4} e^{\phi} \left(F^{0,2} \wedge \Omega + F^{2,0} \wedge \bar{\Omega}\right) \tag{H.0.19}$$

$$= -\frac{3}{2} e^{\phi} F_2 \wedge \Omega_R \,. \tag{H.0.20}$$

[1] Explicitly, $\bar{W}_5 \wedge J^2 = i\frac{e^{\phi}}{3!8}\bar{\Omega}^{ijk}(J \wedge F^{(2,0)})_{ijk\bar{k}}\mathrm{d}\bar{z}^k \wedge J^2 = -\frac{e^{\phi}}{16}\bar{f}\epsilon_{\overline{kbc}}g^{\bar{b}j}g^{\bar{c}k}F_{jk}g_{l\bar{m}}g_{p\bar{n}}\epsilon^{\overline{kmn}}\mathrm{d}z^{\overline{123}} \wedge \mathrm{d}z^{lp} = \ldots = -\frac{e^{\phi}}{4}\bar{f}\mathrm{d}z^{\overline{123}} \wedge F_2 = -\frac{e^{\phi}}{4}\bar{\Omega} \wedge F_2$, where we used $\Omega = \frac{1}{3!}f\epsilon_{ijk}\mathrm{d}z^{ijk}$, and $J = ig_{i\bar{j}}\mathrm{d}z^{i\bar{j}}$.

Appendix I

Details about the festina lente quasistatic equations of motion

Here we provide some details about the semiclassical equations of motion (11.3.18) quoted in the main text. To first order in ϵ, the coefficients of the

Einstein tensor expansion are

$$\delta_1 = \frac{1}{2}\left(\frac{2\delta A\left(3M^2r^2 - 5MQ^2r - 3Mr^5 + Mr^3 + 2Q^4 + \left(2Q^2-1\right)r^4 + 2r^6\right)}{r^4\left(2Mr - Q^2 + r^4 - r^2\right)}\right.$$
$$+\frac{2\delta B\left(M^2r^2 + MQ^2r + 7Mr^5 - Mr^3 - Q^4 - 4Q^2r^4 + r^8 - 2r^6\right)}{\left(2Mr - Q^2 + r^4 - r^2\right)^3}$$
$$+\frac{r^2\delta B''}{-2Mr + Q^2 - r^4 + r^2} + \frac{r\left(-3Mr + 2Q^2 + r^2\right)\delta B'}{\left(-2Mr + Q^2 - r^4 + r^2\right)^2} - \frac{8Q\delta F}{-2Mr + Q^2 - r^4 + r^2}$$
$$\left.+\frac{\left(-3Mr + 2Q^2 + r^2\right)\delta A'}{r^3}\right), \tag{I.0.1}$$

$$\delta_2 = \frac{1}{2}\left(-\frac{2\delta A\left(r\left(-M^2 - Mr^3 + 3Mr + 2r^6 - r^2\right) + Q^2\left(M + 2r^3 - 2r\right)\right)}{r^3\left(2Mr - Q^2 + r^4 - r^2\right)^{-1}}\right.$$
$$+\frac{2\delta B\left(r^2\left(-5M^2 - 5Mr^3 + 3Mr + r^6\right) + Q^2r\left(5M + 4r^3 - 2r\right) - Q^4\right)}{2Mr - Q^2 + r^4 - r^2}$$
$$+r^2\left(2Mr - Q^2 + r^4 - r^2\right)\delta B'' + 8Q\delta F\left(-2Mr + Q^2 - r^4 + r^2\right)$$
$$\left.-\frac{\left(M + 2r^3 - r\right)\left(-2Mr + Q^2 - r^4 + r^2\right)^2\delta A'}{r^2} + r^2\left(-M - 2r^3 + r\right)\delta B'\right), \tag{I.0.2}$$

$$\delta_3 = \frac{4r\left(r\dot{M} - Q\dot{Q}\right)}{-2Mr + Q^2 - r^4 + r^2}, \tag{I.0.3}$$

$$\delta_4 = \frac{1}{2}\left(-\frac{2\delta A\left(r\left(-M^2 + 5Mr^3 + Mr + 5r^6 - 4r^4\right) - Q^2\left(-M + r^3 + r\right)\right)}{r^5}\right.$$
$$-\frac{2\delta B\left(r\left(M^2 + 7Mr^3 - Mr + r^6 - 2r^4\right) + Q^2\left(-M - 5r^3 + r\right)\right)}{r\left(-2Mr + Q^2 - r^4 + r^2\right)^2}$$
$$-\frac{\left(-3Mr + 2Q^2 + r^2\right)\delta B'}{r\left(2Mr - Q^2 + r^4 - r^2\right)} - \frac{\left(M + 2r^3 - r\right)\left(2Mr - Q^2 + r^4 - r^2\right)\delta A'}{r^4} - \frac{4Q\delta F}{r^2}$$
$$\left.+\delta B''\right). \tag{I.0.4}$$

To first order in ϵ, Einstein's equations are simply $\delta_i = 8\pi G\eta_i$. The δ_3 component gives (11.3.18) in the main text; here we focus on the remaining components, that give the equations of motion for δA, δB, δF. We have omitted the dependence on the slow time scale t_1, since there are no time-derivatives anywhere in these expressions. The perturbation δF also shows up in Maxwell's equations, but these are redundant, as Einstein's equations imply energy-momentum conservation $\nabla_a T^{ab} = 0$ which is in turn equivalent to Maxwell's equations (this is the case because we only have one field; in the quintessence discussion in Appendix J, one needs to introduce an additional function and impose the equation of motion for the radial perturbation by hand, as we do in the main text).

Since there are no time derivatives, the time dependence on the δ_i is fixed by the time dependence of the η_i. For the cases under consideration, where η_i comes from Hawking or Swchinger effects, these only depend on t_1, since the stress-energy for these depend on time only through M and Q.

To sum up, we have a system of three coupled second-order ODE's for the perturbations with non-singular coefficients outside the horizons. Working in first-order formalism, $\delta B''(r) = C'(r)$, the equations $\delta_i = 8\pi G\eta_i$ together with $C = \delta B'(r)$ can be written as a matrix ODE

$$\mathbf{M}_1\vec{y}' + \mathbf{M}_2\vec{y} = \vec{\eta}, \tag{I.0.5}$$

where $\vec{y} = (\delta A, \delta B, \delta F, C)$, $\vec{\eta} = 8\pi G(\eta_1, \eta_2, \eta_4, 0)$, and $\mathbf{M}_1, \mathbf{M}_2$ are coefficient matrices. Crucially,

$$\det(\mathbf{M}_1) = \frac{Q\left(r\left(-2M - r^3 + r\right) + Q^2\right)^2}{r^3} \neq 0, \tag{I.0.6}$$

so $\mathbf{M}_1$ is invertible. Then, as long as there are no singularities in the η_i between the two horizons, existence of the solution given boundary conditions is then guaranteed by the general theory of ODE's e.g. Caratheodory's theorem [501].

Since we have solved all of Einstein's equations, our solutions satisfy the Hamiltonian and momentum constraints, to first order in ϵ. Again, we can generically expect violation of the Hamiltonian constraints to take place at second order in ϵ.

Appendix J

Generalization to quintessence

In this Appendix, we describe the generalization of the quasistatic and adiabatic equations of motion (11.3.18) when one introduces a very slowly rolling scalar field φ. This will be relevant for us because the conjecture in [163, 326] suggests that exactly stable Einsteinian de Sitter solutions do not exist; hence, we need to know to what extent are our results robust against a small slow-roll.

J.1 Quasistatic

We will parametrize the rolling of φ by a time-dependent Hubble radius $\ell(t)$, on top of $M(t)$ and $Q(t)$. So in this Appendix we will restore ℓ. The scalar will couple via a Lagrangian

$$\mathcal{L}_\varphi = \frac{1}{2\epsilon}(\partial\varphi)^2 - V(\varphi), \tag{J.1.1}$$

where ϵ is a small parameter. The method of separation of scales requires an unperturbed problem and a parametric family of solutions for it. We will choose the unperturbed problem to be the $\epsilon \to 0$ limit of (J.1.1). The family of solutions are the RN-dS solutions with the three parameters $(Q, M, \ell(\varphi))$, where the Hubble radius is a function of the scalar potential given by

$$\frac{3}{\ell(\varphi)^2} = V(\varphi_0). \tag{J.1.2}$$

To fully specify the family of solutions, we also need to specify the profile for the quintessence field. Equation (J.1.2) implies that it is a constant. This is consistent with the equations of motion of the system, which are

$$G_{\mu\nu} = 8\pi G(T^{\rm EM}_{\mu\nu} + V(\varphi)g_\mu\nu), \quad \nabla_\mu\nabla^\mu\varphi = 0. \tag{J.1.3}$$

Now, we work to first order in ϵ, and introduce a small time dependence,

$$M(\epsilon t),\, Q(\epsilon t),\, \ell(\epsilon t) = \frac{3}{V(\varphi_0(\epsilon t))}. \tag{J.1.4}$$

On top of this, we introduce order ϵ but time-independent perturbations to the metric, gauge field and quintessence. The metric and gauge field components work out in the same way as above, so in order not to clutter the equations we will not re-discuss them. For quintessence, we have

$$\varphi(r,t) = \varphi_0(\epsilon t) + \epsilon\, u(r), \tag{J.1.5}$$

with an equation of motion $\nabla_\mu\nabla^\mu\varphi = \epsilon V'(\varphi)$. To first order in ϵ, we get

$$g^{tt}\ddot{\varphi} + \frac{1}{r^2}\partial_r\left(g^{rr}r^2u'(r)\right) = V'(\varphi_0(0)), \tag{J.1.6}$$

which fixes $\ddot{\varphi} = 0$ (familiar from slow roll approximation) and ODE which uniquely determines $u(r)$, given boundary conditions. Einstein's equations become as before give equations of motion for the time-independent metric perturbations, and the off-diagonal components as before give the equation of motion for M[1],

$$\frac{dM}{dt} = -\pi G\ell^2 r^4 U\mathcal{T} - \frac{4\pi\mathcal{J}}{Q} + r^2\frac{\dot{\ell}}{\ell^3}r. \tag{J.1.7}$$

The extra last term is the contribution from quintessence. The equation of motion for $\dot{Q}$ remains the same. Out of the δ_4 component of the Einsteins equations we also get

$$-\frac{6\dot{l}}{\ell^3} = V'(\varphi_0(0))\dot{\varphi}_0(0), \tag{J.1.8}$$

which is consistent with (J.1.2).

So now that we have the modified equations of motion, we just need to check what motion do they enforce on the (M,Q,ℓ) space. To check whether we leave the Nariai branch, again, we just need to compute $\dot{\Delta}$, where Δ is the discriminant of the quadratic equation $U(r) = 0$. We cannot neglect the ℓ dependence in the discriminant now,

$$\Delta = \ell^4M^2 - \ell^4Q^2 - 27\ell^2M^4 + 36\ell^2M^2Q^2 - 8\ell^2Q^4 - 16Q^6, \tag{J.1.9}$$

[1]Notice that the equations are written in terms of $\mathcal{T}$, $\mathcal{J}$ instead of $\mathscr{T}$, $\mathscr{J}$.

and becoming superextremal is equivalent to $\dot{\Delta} > 0$ on the Nariai branch. Using (J.1.7), one gets

$$\dot{\Delta} = 0, \tag{J.1.10}$$

so that again one moves along the charged Nariai branch, without changing the charge. This is precisely what [402] found for the extremal case. Their analysis is in fact general for classical effects on the charged Nariai branch, and suggests that other modifications such as e.g. introducing a coupling of φ to the gauge field cannot change the picture; we have checked explicitly that this is indeed the case.

J.2 Adiabatic

We will only analyze the Nariai branch, as in the main text. The 2d effective action is now

$$8\pi G\, S = \int d\Omega_2 d\tau dx \left\{ (e^{\dot{2\phi}})\dot{a} - e^{-\phi} a + \left(V(\varphi) - \frac{1}{2}\dot{\varphi}^2 \right) e^{\phi} a \right\} + 8\pi G \int dV_2 d\Omega_2 \sqrt{-g}\mathcal{L}, \tag{J.2.1}$$

where the quintessence potential and kinetic term replace the $+3$ cosmological constant term in (J.2.1). We will assume no coupling to the electromagnetic field. Defining again $s \equiv e^{2\phi}$ and assuming isotropic radiation stress-energy, as in the main text, the equations of motion are

$$\ddot{\varphi} = -\frac{V'(\varphi)}{\sqrt{s}} - \left[\frac{\dot{a}}{a} + \frac{\dot{s}}{s} \right] \dot{\varphi}, \tag{J.2.2}$$

$$\ddot{s} = V(\varphi)\sqrt{s} - s\dot{\varphi}^2 - \frac{1}{\sqrt{s}} \left(1 + \frac{\rho_0}{a^{4/3}} \right), \tag{J.2.3}$$

$$\frac{\ddot{a}}{a} = \frac{1}{2s^{3/2}} \left(1 - \frac{\rho_0}{a^{4/3}} + V(\varphi)\, s \right) - \dot{\varphi}^2, \tag{J.2.4}$$

where ρ_0 is again the energy density of the electromagnetic field just before decay, and we have assumed that $V(\varphi_0) = 3$, so that we can use expressions like (11.2.6).

The question is whether the addition of quintessence can alleviate or even make the crunch disappear. We will limit ourselves to a numerical analysis illustrating these qualitative features in the particular case of an exponential potential $V = 3e^{-\alpha\varphi}$. Results are presented in Figure J.1. Quintessence causes a slight delay of the crunch for negative values of α, but this effect is small unless $|\alpha| \gg 1$, in which case the model does not make sense anyway.

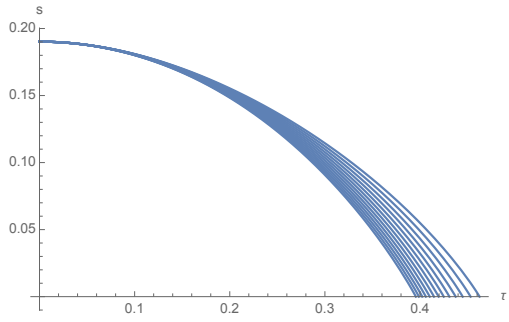


Figure J.1: Numerical solution for $s(t)$ of the system of equations (J.2.2)-(J.2.4) for an initial charge $Q = 0.99 \cdot 12^{-1/2}$ and different values of α from $-1/3$ to $+1/3$ in $1/6$ increments. Central value corresponds to $\alpha = 0$. This illustrates that reasonable quintessence potentials are not enough to avoid the Big Crunch on the charged Nariai branch.

Appendix K

Exact Schwinger pair production on the charged Nariai branch

Here we compute the exact evolution equation on the Nariai branch, using the results in [426]. The steady current for a particle of two dimensional mass m_{2d} and charge q is

$$\mathscr{J}_{dS_2,r}(m_{2d},q)=\frac{qH_{dS_2}}{\pi}\frac{\sigma}{\sinh(2\pi\sigma)}\sinh(2\pi qE/H_{dS_2}^2), \tag{K.0.1}$$

where[1]

$$\sigma=\sqrt{\frac{1}{4}-\frac{m^2}{H^2}-\frac{q^2E^2}{H^4}},\quad qE=\sqrt{\frac{1}{4\pi G}}\frac{gq}{\ell}\frac{Q\ell^2}{r_g^2}, \tag{K.0.2}$$

and

$$r_g^2=\frac{\ell^2}{6}\left(1+\sqrt{1-12Q^2}\right),\quad H_{dS_2}^2\ell\text{`}2=6\left(\frac{1}{1+\sqrt{1-12Q^2}}-1\right). \tag{K.0.3}$$

It is convenient to introduce parameters

$$\chi\equiv\frac{qE}{\tilde{m}^2}=\frac{qE}{m^2-H^2/4} \tag{K.0.4}$$

[1]Under dimensional reduction we have that $1/g_{2d}^2=\frac{4\pi r^2}{g_{4d}^2}$ and $E_{4d}=\frac{g_{4d}^2Q_r}{4\pi r^2}=g_{2d}^2Q_rq=E_{2d}$, so both the product qE and m can be computed in 4d and then one can use the 2d formula directly. The normalization factor in the scalar accounts for integration over the sphere.

and

$$y \equiv \tilde{m}^2/H^2. \tag{K.0.5}$$

In terms of these, equation (K.0.1) becomes

$$\mathscr{J}_{dS_2,r}(m_{2d},q) = \frac{q}{\pi} H_{dS_2} \frac{\sqrt{y^2\chi^2+y}}{\sinh(2\pi\sqrt{y^2\chi^2+y})} \sinh(2\pi\chi y). \tag{K.0.6}$$

Notice that

$$\chi y = \frac{qE}{H_{dS_2}^2} > 0. \tag{K.0.7}$$

If one takes $y > 0$, which amounts to having a field more massive than $H_{dS_2}^2/4$, it is possible to write down a simple lower bound on the current. For any $a > 0, b > 0$ we have

$$\frac{\sinh(a)}{\sinh(\sqrt{a^2+b})} \geq \frac{\tanh(a)}{e^c}, \tag{K.0.8}$$

where $c \equiv \sqrt{a^2+b} - a \leq \frac{b}{2a}$. Due to this, one can write

$$\frac{\sinh(a)}{\sinh(\sqrt{a^2+b})} \geq \tanh(a) e^{-\frac{b}{2a}}. \tag{K.0.9}$$

Now taking $a = 2\pi\chi y$, $b = 4\pi^2 y$, one can bound

$$\begin{aligned} \mathscr{J}_{dS_2}(m_{2d},q) &\gtrsim \frac{q}{\pi} H_{dS_2} \frac{\sqrt{y^2\chi^2+y}}{\sinh(2\pi\sqrt{y^2\chi^2+y})} \sinh(2\pi\chi y) \\ &\geq \frac{q^2 E}{\pi H_{dS_2}} \tanh\left(2\pi \frac{qE}{H_{dS_2}^2}\right) e^{-\frac{\pi}{\chi}}. \end{aligned} \tag{K.0.10}$$

The bound becomes tighter and tighter the higher the hierarchy between m^2 and qE. The factor $e^{-\frac{\pi}{\chi}}$ is precisely the Schwinger space prefactor (11.3.1); the bound becomes tight when $y \gg 1$, which means that the curvature effects become unimportant and the pair-production happens essentially in flat space.

As in the main text, a 4d field of mass m and charge q decomposes into a tower of 2d fields with masses

$$m_{2d}(s)^2 = m^2 + \frac{s(s+1)}{r_g^2}. \tag{K.0.11}$$

Here, $s = 0, 1, 2, ...$ and there is a multiplicity $2s+1$ for the sth mass. Note that depending on the spectrum of the four-dimensional theory, it may well be

possible that $m_{4d}^2 \gg \frac{s(s+1)}{r_c^2}$ and the KK modes are densely packed compared to the lowest mass. It is then important to sum over the KK modes when computing the Schwinger current. This is reflecting that the physics of the problem is intrinsically four-dimensional, but simple enough that it can be analyzed in 2d terms.

Then the current equation of motion in (11.3.51) becomes a sum over KK modes, which we can evaluate explicitly using (K.0.10), since only the last factor depends on m. In this way, (K.0.10) becomes

$$\begin{aligned} \mathscr{J}_r &\gtrsim \frac{q^2 r_c^2}{\pi^2} |E| H_{dS_2} \frac{\sqrt{y^2\chi^2 + y}}{\sinh(2\pi\sqrt{y^2\chi^2 + y})} \sinh(2\pi\chi y) \\ &\geq \frac{q^2 r_c^2 E|E|}{\pi^2 H_{dS_2}} \tanh\left(2\pi \frac{qE}{H_{dS_2}^2}\right) e^{-\frac{\pi}{\chi}}. \end{aligned} \tag{K.0.12}$$

The electric field on the Nariai branch is given by (K.0.2). Using this, equation (11.3.51) then becomes[2]

$$\mathscr{J} = \sqrt{\frac{g^2 G}{4\pi\ell^2}}\, \mathscr{J}_r \gtrsim \sqrt{\frac{g^2 G}{4\pi\ell^2}}\, \frac{q^2 r_c^2 E|E|}{\pi^2 H_{dS_2}} \tanh\left(2\pi \frac{qE}{H_{dS_2}^2}\right) e^{-\frac{\pi}{\chi}} \tag{K.0.13}$$

This is equation (11.3.38) of the main text.

[2]The prefactor accounts for the definition of Q (11.2.7) in terms of the actual integer-valued current. The factor of $4\pi r_g^2$ in (11.3.22) is already taken into account in the expression for $\mathscr{J}_{dS_2}$, since this is a two-dimensional calculation.

Appendix L

Big Crunch and singularity theorems

In this Appendix we show that the Big Crunch in the adiabatic approximation discussed in the main text is actually independent of the details of the model and is in fact guaranteed by a singularity theorem. The starting point of our discussion will be the dimensionally reduced Lagrangian to two dimensions of the Einstein-Maxwell system, (11.3.45) of the main text. However, we will now have a general two-dimensional metric g_2, and in the two-dimensional Lagrangian $\mathcal{L}$ we will include a sum over all the KK modes of metric components and any matter fields we include. By working high up the Nariai branch, we can have a parametric hierarchy between the size of the two-spheres on which we reduce the theory and the non-compact space, so we mostly are thinking of the regime where a honest KK reduction is possible, but our discussion is general.

We will apply one of the original singularity theorems [502]. In the main text, we picked a particular spacelike hypersurface in the Nariai geometry, declared that this was the "starting point" of the evolution, and then let the electric field be screened out. We want to prove a crunch exists too for an arbitrary initial data hypersurface $\mathcal{C}$, even one that crosses the cosmological or black hole horizons. We also want to work (slightly) outside of the Nariai limit. Thus, we need to consider the maximal analytic extension of the RN-dS black hole, a spacetime described in [405]. Removing singularities, it has topology $S^2 \times S^1 \times \mathbb{R}$. In particular, any global Cauchy slice $\mathcal{C}$ is compact. The singularity theorems have a loophole precisely in this case [503]. One can lift this problem however by working in the universal cover of the RN-dS black hole, which describes an infinite sequence of "Bousso beads". This is the spacetime we will work in.

For a particular Cauchy surface $\mathcal{C}$, the singularity theorem guarantees that every geodesic is not future-extendible provided that

1. The expansion of a congruence of null geodesics at any point on $\mathcal{C}$ is negative, and
2. $R_{ab}k^a k^b \geq 0$ for any null vector k^a.

The second condition is equivalent, by Einstein's equations, to the null energy condition, which we take to hold (notice that unlike e.g. the strong energy condition, the null energy condition is marginally satisfied by a positive cosmological constant).

The first point can be established by noting that locally in a small neighbourhood of any point in $\mathcal{C}$, the situation is the one described in the main text: The electric field decays locally to radiation, and immediately afterwards we have a FRWL metric. Using the ansatz (11.3.44) (allowing for angular dependence as well), a null radially affinely parametrized congruence of ingoing/outgoing null geodesics is given by (see [504])

$$K^a_{\pm} = \frac{r}{a(\tau)}\left(1, \pm a(\tau), 0, 0\right). \tag{L.0.1}$$

The corresponding expansions $\theta_{\pm} \equiv \nabla_a K^a_{\pm}$ are

$$\theta_{\pm} = \frac{1}{ra}\left((\dot{r^2}) \pm \partial_x(r^2 a)\right). \tag{L.0.2}$$

Exactly on the Nariai branch, the second term vanishes, and the first does as well if there are no charged particles. In the adiabatic approximation, however, $(\dot{r^2})$ becomes negative at any time > 0, leading to contraction for both congruences as long as the gradients $\partial_x(r^2 a)$ are small enough (i.e. close enough to the Nariai branch).

Since these arguments work for any point of the initial surface, every null geodesic must be incomplete – no one is escaping to a hypothetical de Sitter region. The crunch is unavoidable.

It is also interesting to consider why the singularity theorem does not lead to the same conclusion in the quasistatic case. The null energy condition is famously not satisfied by Hawking radiation [505], and perhaps this is the case for the Schwinger radiation as well (we have not checked). It is also not straightforward to argue that any null congruence of geodesics has negative expansion in the same way as above; up in the Nariai branch, the stress-energy of the produced pairs is localized on the angular S^2.

Appendix M

A dS obstruction equations of motion

Here we derive the equations of motion (12.2.9)-(12.2.11) in the main text. Using the ansatz (12.2.2), and only allowing time dependence of the fields, the action becomes (after integration by parts)

$$S = \int dt \Big(2h - R^2 \frac{d}{dt}\Big(\frac{\dot{h}}{h}\Big) + 2\dot{R}^2 + \frac{hR^2}{2}\dot{\phi}^2 - \frac{fR^2}{2}\left(\frac{E^2}{h} - B^2 h\right) - VhR^2 + \frac{\theta}{4\pi^2} EBR^2 \Big), \tag{M.0.1}$$

where E is the electric field, and B is the magnetic field. It has the value $E = \frac{Q_e h}{R^2 f}$. The magnetic field is quantized, $B = Q_m/R^2$. These two conditions are enforced off-shell in two dimensions since the gauge field has no dynamical degrees of freedom. The resulting action is

$$S = \int dt \left(2h - R^2 \frac{d}{dt}\Big(\frac{\dot{h}}{h}\Big) + 2\dot{R}^2 + \frac{hR^2}{2}\dot{\phi}^2 - \frac{h}{2R^2}\mathcal{Q}^2 - VhR^2 \right), \tag{M.0.2}$$

Setting $\dot{R} = 0$ and $\dot{\phi} = 0$ since we are looking for static solutions, the equation of motion for R^2 is just

$$\mathcal{R}_2 = \frac{1}{h}\frac{d}{dt}\Big(\frac{\dot{h}}{h}\Big) = V - \frac{\mathcal{Q}^2}{2R^4}. \tag{M.0.3}$$

In the above, the left hand side is precisely the curvature of the two-dimensional manifold, $\mathcal{R}_2$, so we recover the main text. The equation of motion for h, which one obtains by first integrating by parts the second term in (M.0.2), is

$$\frac{2}{R^2} - \frac{\mathcal{Q}^2}{2R^4} - V = 0, \tag{M.0.4}$$

or, multiplying by $-R^4$,

$$VR^4 - 2R^2 + \frac{\mathcal{Q}^2}{2} = 0, \tag{M.0.5}$$

which is the equation in the main text. Finally, the equation of motion for ϕ is

$$0 = V' + \frac{(\mathcal{Q}^2)'}{2R^4}. \tag{M.0.6}$$

In case we deal with a truncation to several scalars, as we do in the axion case in the main text, (M.0.6) becomes a vector equation where the prime denotes derivative with respect to every scalar. We do not expect to find a solution in situations with more than two scalars.

As for the stability analysis, we can just refer to Appendix C.3 of [488]. We just need to ensure that the second derivative of the scalar potential with respect to the quintessence field is positive. One obtains

$$V'' - \frac{V'\left(ff'' + (f')^2\right)}{ff'} > 0 \tag{M.0.7}$$

which for exponential dependence $f = e^{\delta\phi}$, $V = V_0 e^{-\gamma\phi}$ become

$$\gamma R^2 V_0(\gamma + 2\delta)e^{-\gamma\phi} > 0 \tag{M.0.8}$$

and so are satisfied for any positive values of γ and δ.

Appendix N

Higher dimensions

The main idea in this note is that, when compactifying to two dimensions a higher-dimensional solution with a runaway potential and a nontrivial gauge kinetic function satisfying certain constraints, de Sitter solutions are unavoidable. Here we generalize the picture to higher dimensions. One can consider a general D-dimensional solution with an abelian p-form fieldstrength and a scalar with an action of the form

$$S = \frac{1}{2\kappa_{10}^2}\int d^{10}x\,\sqrt{|g_E^{10}|}\left[R_E - \frac{1}{2}(\partial\phi)^2 - f(\phi)|F_p|^2 - V(\phi)\right], \qquad \text{(N.0.1)}$$

where without loss of generality we can take $\gamma > 0$ and $\delta > 0$ (if δ is negative for a certain F_p, it will be positive for its electromagnetic dual).

Following [506], we can now reduce on a product manifold $X \times Y$, of dimensions d_x, d_y, $d_x + d_y = D$ and $d_X > 2$, and with metric

$$ds_D^2 = e^{2\alpha\rho}ds_X^2 + e^{2\beta\rho}ds_Y^2, \qquad \text{(N.0.2)}$$

where

$$\alpha^2 \equiv \frac{d_y}{2(d_x-2)(d_x+d_y-2)}, \quad \beta = -\frac{d_x-2}{d_y}\alpha. \qquad \text{(N.0.3)}$$

This ansatz ensures that the scalar ρ has a canonical kinetic term after dimensional reduction, and that the lower-dimensional theory is in the Einstein frame. The kinetic term of both scalars is

$$\mathcal{L}^{\text{kin.}}_{\text{scalars}} = \frac{1}{2}\left[(\partial\rho)^2 + e^{2\alpha\rho}(\partial\phi)^2\right]. \qquad \text{(N.0.4)}$$

We will consider the particular case where $Y = S^p$ is a sphere threaded by $p-$form flux, and only look at a magnetic solution; the electric one can be

obtained by electric-magnetic duality. The resulting d_x-dimensional effective action has two scalars ϕ and ρ, and a potential

$$V_X = -d_y(d_y-1)e^{2(\alpha-\beta)\rho} + \frac{Q^2}{2}e^{2(d_x-1)\alpha\rho}f(\phi) + V(\phi)e^{2\alpha\rho}. \tag{N.0.5}$$

There is only one critical point, where V takes the value:

$$V_X^{\text{min.}} = \frac{(d_x-2)e^{\frac{d_y\rho}{(d_x-2)(d_x+d_y-2)}}\left((d_y-1)fV' + Vf'\right)}{(d_x+d_y-2)f'}. \tag{N.0.6}$$

Demanding that this is positive leads to (12.2.19) in the main text. The charge is determined by

$$Q^2 = -\frac{2e^{-2\alpha d_x - 1\rho}V'}{f'}. \tag{N.0.7}$$

In this case there is no obstruction akin to (12.2.12); in other words, there is no bound on the charge the solutions can attain. In the $d_x = 2$ case, the upper bound (12.2.12) can be interpreted as the condition for an "ultracold black hole" [418], and it is related to the maximum charge that can one can fit on a static patch. For $d_x > 2$ there is no such interpretation since the would-be brane whose near horizon geometry we are computing has to stretch out of the static patch anyway.

Just as in the two-dimensional case, we need to study stability of the solution. As before, we will compute one diagonal element and the determinant of the Hessian $\mathcal{H}$. We work in the specific case of $V = V_0e^{-\gamma\phi}$ and $f = e^{\delta\phi}$.

$$\begin{aligned}\partial_\phi^2 V &= \frac{1}{2}\delta Q^2(\gamma+\delta)e^{\delta\phi+(d_x-1)\rho},\\ \det(\mathcal{H}) &= -\frac{\delta(d_x-2)Q^4(\delta+\gamma(d_x-1))(\gamma(1-d_y)+\delta)e^{2\delta\phi+2(d_x-1)\rho}}{4\gamma d_y}.\end{aligned} \tag{N.0.8}$$

Taking into account (12.2.19), we see that the determinant is always negative. Thus, the solution is always unstable. Presumably, we also get similar instabilities for more general f and V. Note that, as discussed earlier, this conclusion does not apply to the dS_2 case, as here the sphere radius is fixed by the Hamiltonian condition.

We can compute explicitly the mass of the tachyonic direction, to then compare to the refined no dS conjecture [164, 489, 490]. The general expression is not illuminating, so we will not write it down here, but at the threshold (12.2.19) it is

$$\frac{|m_{\mathcal{T}}^2|}{V_X^{\text{min.}}} = \frac{\delta^2(d_x+d_y-2)^2}{(d_x-2)^2(d_y-1)^2+\delta^2 d_y^2}. \tag{N.0.9}$$

This is an $\mathcal{O}(1)$ quantity, for δ of order 1. So whether or not this satisfies [164] depends on the precise value of the parameter c' that one takes.

Bibliography

[1] T. Hertog, G. Tartaglino-Mazzucchelli, T. Van Riet, and G. Venken, "Supersymmetric dS/CFT," *JHEP* **02** (2018) 024, arXiv:1709.06024 [hep-th].

[2] T. Hertog, G. Tartaglino-Mazzucchelli, and G. Venken, "Spinors in Supersymmetric dS/CFT," *JHEP* **10** (2019) 117, arXiv:1905.01322 [hep-th].

[3] F. F. Gautason, V. Van Hemelryck, T. Van Riet, and G. Venken, "A 10d view on the KKLT AdS vacuum and uplifting," arXiv:1902.01415 [hep-th].

[4] S. Andriolo, G. Shiu, H. Triendl, T. Van Riet, G. Venken, and G. Zoccarato, "Compact G2 holonomy spaces from SU(3) structures," *JHEP* **03** (2019) 059, arXiv:1811.00063 [hep-th].

[5] M. Montero, T. Van Riet, and G. Venken, "Festina Lente: EFT Constraints from Charged Black Hole Evaporation in de Sitter," *JHEP* **01** (2020) 039, arXiv:1910.01648 [hep-th].

[6] M. Montero, T. Van Riet, and G. Venken, "A dS obstruction and its phenomenological consequences," arXiv:2001.11023 [hep-th].

[7] S. Weinberg, *Cosmology*. 9, 2008.

[8] **WMAP** Collaboration, D. N. Spergel *et al.*, "Wilkinson Microwave Anisotropy Probe (WMAP) three year results: implications for cosmology," *Astrophys. J. Suppl.* **170** (2007) 377, arXiv:astro-ph/0603449 [astro-ph].

[9] A. G. Riess, A. V. Filippenko, P. Challis, A. Clocchiatti, A. Diercks, P. M. Garnavich, R. L. Gilliland, C. J. Hogan, S. Jha, R. P. Kirshner, and et al., "Observational evidence from supernovae for an accelerating

universe and a cosmological constant," *The Astronomical Journal* **116** no. 3, (Sep, 1998) 1009–1038. http://dx.doi.org/10.1086/300499.

[10] **Supernova Cosmology Project** Collaboration, S. Perlmutter *et al.*, "Measurements of Ω and Λ from 42 high redshift supernovae," *Astrophys. J.* **517** (1999) 565–586, arXiv:astro-ph/9812133 [astro-ph].

[11] A. G. Riess *et al.*, "A 2.4% Determination of the Local Value of the Hubble Constant," *Astrophys. J.* **826** no. 1, (2016) 56, arXiv:1604.01424 [astro-ph.CO].

[12] A. G. Riess, "The Expansion of the Universe is Faster than Expected," *Nature Rev. Phys.* **2** no. 1, (2019) 10–12, arXiv:2001.03624 [astro-ph.CO].

[13] K. C. Wong *et al.*, "H0LiCOW XIII. A 2.4% measurement of H_0 from lensed quasars: 5.3σ tension between early and late-Universe probes," arXiv:1907.04869 [astro-ph.CO].

[14] **Planck** Collaboration, N. Aghanim *et al.*, "Planck 2018 results. VI. Cosmological parameters," arXiv:1807.06209 [astro-ph.CO].

[15] W. de Sitter, "On Einstein's Theory of Gravitation and its Astronomical Consequences. Third Paper.," *Monthly Notices of the Royal Astronomical Society* **78** no. 1, (11, 1917) 3–28. https://doi.org/10.1093/mnras/78.1.3.

[16] S. Weinberg, "The Cosmological Constant Problem," *Rev. Mod. Phys.* **61** (1989) 1–23.

[17] M. Spradlin, A. Strominger, and A. Volovich, "Les Houches lectures on de Sitter space," in *Unity from duality: Gravity, gauge theory and strings. Proceedings, NATO Advanced Study Institute, Euro Summer School, 76th session, Les Houches, France, July 30-August 31, 2001*, pp. 423–453. 2001. arXiv:hep-th/0110007 [hep-th].

[18] E. Witten, "Quantum gravity in de Sitter space," in *Strings 2001: International Conference Mumbai, India, January 5-10, 2001*. 2001. arXiv:hep-th/0106109 [hep-th].

[19] D. Anninos, "De Sitter Musings," *Int. J. Mod. Phys.* **A27** (2012) 1230013, arXiv:1205.3855 [hep-th].

[20] G. W. Gibbons and S. W. Hawking, "Cosmological Event Horizons, Thermodynamics, and Particle Creation," *Phys. Rev.* **D15** (1977) 2738–2751.

[21] T. Banks, "Cosmological breaking of supersymmetry?," *Int. J. Mod. Phys.* **A16** (2001) 910–921, arXiv:hep-th/0007146 [hep-th]. [,270(2000)].

[22] E. Mottola, "Particle Creation in de Sitter Space," *Phys. Rev.* **D31** (1985) 754.

[23] A. M. Polyakov, "De Sitter space and eternity," *Nucl. Phys.* **B797** (2008) 199–217, arXiv:0709.2899 [hep-th].

[24] U. H. Danielsson, "The quantum swampland," arXiv:1809.04512 [hep-th].

[25] J. A. Isaacson and B. Rogers, "The Stability of de Sitter space with a scalar quantum field. 1. The Linearized semiclassical Einstein equations," *Nucl. Phys.* **B364** (1991) 381–400.

[26] B. Rogers and J. A. Isaacson, "The Stability of De Sitter space with a scalar quantum field. 2. The Linear analysis," *Nucl. Phys.* **B368** (1992) 415–443.

[27] C. Busch, "The Semiclassical stability of de Sitter space-time," arXiv:0803.3204 [gr-qc].

[28] D. Marolf and I. A. Morrison, "The IR stability of de Sitter QFT: results at all orders," *Phys. Rev.* **D84** (2011) 044040, arXiv:1010.5327 [gr-qc].

[29] L. Aalsma, M. Parikh, and J. P. Van Der Schaar, "Back(reaction) to the Future in the Unruh-de Sitter State," *JHEP* **11** (2019) 136, arXiv:1905.02714 [hep-th].

[30] V. Gorbenko and L. Senatore, "$\lambda\phi^4$ in dS," arXiv:1911.00022 [hep-th].

[31] B. S. DeWitt, "Quantum Theory of Gravity. 1. The Canonical Theory," *Phys. Rev.* **160** (1967) 1113–1148.

[32] J. B. Hartle and S. W. Hawking, "Wave Function of the Universe," *Phys. Rev.* **D28** (1983) 2960–2975.

[33] S. W. Hawking, "The Quantum State of the Universe," *Nucl. Phys.* **B239** (1984) 257.

[34] M. B. Green, J. H. Schwarz, and E. Witten, *SUPERSTRING THEORY. VOL. 1: INTRODUCTION*. Cambridge Monographs on Mathematical Physics. 1988. http://www.cambridge.org/us/academic/subjects/

physics/theoretical-physics-and-mathematical-physics/superstring-theory-volume-1.

[35] M. B. Green, J. H. Schwarz, and E. Witten, *SUPERSTRING THEORY. VOL. 2: LOOP AMPLITUDES, ANOMALIES AND PHENOMENOLOGY*. 1988. http://www.cambridge.org/us/academic/subjects/physics/theoretical-physics-and-mathematical-physics/superstring-theory-volume-2.

[36] J. Polchinski, *String theory. Vol. 1: An introduction to the bosonic string*. Cambridge Monographs on Mathematical Physics. Cambridge University Press, 2007.

[37] J. Polchinski, *String theory. Vol. 2: Superstring theory and beyond*. Cambridge Monographs on Mathematical Physics. Cambridge University Press, 2007.

[38] C. V. Johnson, *D-branes*. Cambridge Monographs on Mathematical Physics. Cambridge University Press, 2005. http://books.cambridge.org/0521809126.htm.

[39] H. Nastase, *Introduction to the ADS/CFT Correspondence*. Cambridge University Press, 2015. http://www.cambridge.org/academic/subjects/physics/theoretical-physics-and-mathematical-physics/introduction-adscft-correspondence?format=HB&isbn=9781107085855.

[40] B. Truijen, *Dynamics of Branes and Fluxes*. PhD thesis, Leuven U., 2018-09. https://fys.kuleuven.be/itf/groups/hep/files/phd/BrechtTruijen.pdf.

[41] nLab authors, "worldline formalism." http://ncatlab.org/nlab/show/worldline%20formalism, Mar., 2020. Revision 20.

[42] J. M. Maldacena, "The Large N limit of superconformal field theories and supergravity," *Int. J. Theor. Phys.* **38** (1999) 1113–1133, arXiv:hep-th/9711200 [hep-th]. [Adv. Theor. Math. Phys.2,231(1998)].

[43] H. Ooguri and C. Vafa, "Non-supersymmetric AdS and the Swampland," arXiv:1610.01533 [hep-th].

[44] A. Strominger, "The dS / CFT correspondence," *JHEP* **10** (2001) 034, arXiv:hep-th/0106113 [hep-th].

[45] J. M. Maldacena, "Non-Gaussian features of primordial fluctuations in single field inflationary models," *JHEP* **05** (2003) 013, arXiv:astro-ph/0210603 [astro-ph].

[46] D. Harlow and D. Stanford, "Operator Dictionaries and Wave Functions in AdS/CFT and dS/CFT," arXiv:1104.2621 [hep-th].

[47] E. Schrodinger, *Expanding universes.* 1956.

[48] M. K. Parikh, I. Savonije, and E. P. Verlinde, "Elliptic de Sitter space: dS/Z(2)," *Phys. Rev.* **D67** (2003) 064005, arXiv:hep-th/0209120 [hep-th].

[49] L. Hackl and Y. Neiman, "Horizon complementarity in elliptic de Sitter space," *Phys. Rev. D* **91** no. 4, (2015) 044016, arXiv:1409.6753 [hep-th].

[50] M. A. Vasiliev, "Consistent equation for interacting gauge fields of all spins in (3+1)-dimensions," *Phys. Lett.* **B243** (1990) 378–382.

[51] V. E. Didenko and E. D. Skvortsov, "Elements of Vasiliev theory," arXiv:1401.2975 [hep-th].

[52] X. Bekaert, N. Boulanger, and P. Sundell, "How higher-spin gravity surpasses the spin two barrier: no-go theorems versus yes-go examples," *Rev. Mod. Phys.* **84** (2012) 987–1009, arXiv:1007.0435 [hep-th].

[53] G. Venken, "A supersymmetric ds/cft correspondence?," 2016.

[54] C. Fronsdal, "Massless Fields with Integer Spin," *Phys. Rev. D* **18** (1978) 3624.

[55] S. R. Coleman and J. Mandula, "All Possible Symmetries of the S Matrix," *Phys. Rev.* **159** (1967) 1251–1256.

[56] I. R. Klebanov and E. Witten, "AdS / CFT correspondence and symmetry breaking," *Nucl. Phys. B* **556** (1999) 89–114, arXiv:hep-th/9905104.

[57] S. Giombi and X. Yin, "The Higher Spin/Vector Model Duality," *J. Phys.* **A46** (2013) 214003, arXiv:1208.4036 [hep-th].

[58] I. R. Klebanov and A. M. Polyakov, "AdS dual of the critical O(N) vector model," *Phys. Lett.* **B550** (2002) 213–219, arXiv:hep-th/0210114 [hep-th].

[59] E. Sezgin and P. Sundell, "Massless higher spins and holography," *Nucl. Phys.* **B644** (2002) 303–370, arXiv:hep-th/0205131 [hep-th]. [Erratum: Nucl. Phys.B660,403(2003)].

[60] M. Eastwood and T. Leistner, "Higher symmetries of the square of the laplacian," 2006.

[61] J. Maldacena and A. Zhiboedov, "Constraining Conformal Field Theories with A Higher Spin Symmetry," *J. Phys.* **A46** (2013) 214011, arXiv:1112.1016 [hep-th].

[62] S. Giombi and X. Yin, "Higher Spin Gauge Theory and Holography: The Three-Point Functions," *JHEP* **09** (2010) 115, arXiv:0912.3462 [hep-th].

[63] C. Sleight and M. Taronna, "Higher Spin Interactions from Conformal Field Theory: The Complete Cubic Couplings," *Phys. Rev. Lett.* **116** no. 18, (2016) 181602, arXiv:1603.00022 [hep-th].

[64] E. Sezgin and P. Sundell, "Holography in 4D (super) higher spin theories and a test via cubic scalar couplings," *JHEP* **07** (2005) 044, arXiv:hep-th/0305040 [hep-th].

[65] S. Giombi, S. Minwalla, S. Prakash, S. P. Trivedi, S. R. Wadia, and X. Yin, "Chern-Simons Theory with Vector Fermion Matter," *Eur. Phys. J.* **C72** (2012) 2112, arXiv:1110.4386 [hep-th].

[66] P. Cvitanovic and A. D. Kennedy, "Spinors in Negative Dimensions," *Phys. Scripta* **26** (1982) 5.

[67] P. Cvitanovic, *Group theory: Birdtracks, Lie's and exceptional groups.* 2008. http://press.princeton.edu/titles/8839.html.

[68] D. Anninos, T. Hartman, and A. Strominger, "Higher Spin Realization of the dS/CFT Correspondence," *Class. Quant. Grav.* **34** no. 1, (2017) 015009, arXiv:1108.5735 [hep-th].

[69] W. Pauli, "The Connection Between Spin and Statistics," *Phys. Rev.* **58** (1940) 716–722.

[70] D. Anninos, F. Denef, R. Monten, and Z. Sun, "Higher Spin de Sitter Hilbert Space," arXiv:1711.10037 [hep-th].

[71] R. Monten, *De Sitter Space and Holography.* PhD thesis, Leuven U., 2017-09-01. https://lirias.kuleuven.be/handle/123456789/587058.

[72] D. Anninos, F. Denef, and D. Harlow, "Wave function of Vasiliev's universe: A few slices thereof," *Phys. Rev.* **D88** no. 8, (2013) 084049, arXiv:1207.5517 [hep-th].

[73] S. W. Hawking and T. Hertog, "A Smooth Exit from Eternal Inflation?," *JHEP* **04** (2018) 147, arXiv:1707.07702 [hep-th].

[74] D. Anninos, F. Denef, G. Konstantinidis, and E. Shaghoulian, "Higher Spin de Sitter Holography from Functional Determinants," *JHEP* **02** (2014) 007, arXiv:1305.6321 [hep-th].

[75] J. B. Hartle, S. W. Hawking, and T. Hertog, "Vector Fields in Holographic Cosmology," *JHEP* **11** (2013) 201, arXiv:1305.7190 [hep-th].

[76] S. Banerjee, A. Belin, S. Hellerman, A. Lepage-Jutier, A. Maloney, D. Radicevic, and S. Shenker, "Topology of Future Infinity in dS/CFT," *JHEP* **11** (2013) 026, arXiv:1306.6629 [hep-th].

[77] C.-M. Chang, A. Pathak, and A. Strominger, "Non-Minimal Higher-Spin DS4/CFT3," arXiv:1309.7413 [hep-th].

[78] D. Anninos, R. Mahajan, o. Radičević, and E. Shaghoulian, "Chern-Simons-Ghost Theories and de Sitter Space," *JHEP* **01** (2015) 074, arXiv:1405.1424 [hep-th].

[79] G. Conti and T. Hertog, "Two wave functions and dS/CFT on S^1 S^2," *JHEP* **06** (2015) 101, arXiv:1412.3728 [hep-th].

[80] N. Bobev, P. Bueno, and Y. Vreys, "Comments on Squashed-sphere Partition Functions," *JHEP* **07** (2017) 093, arXiv:1705.00292 [hep-th].

[81] N. Bobev, T. Hertog, and Y. Vreys, "The NUTs and Bolts of Squashed Holography," *JHEP* **11** (2016) 140, arXiv:1610.01497 [hep-th].

[82] G. Conti, T. Hertog, and Y. Vreys, "Squashed Holography with Scalar Condensates," *JHEP* **09** (2018) 068, arXiv:1707.09663 [hep-th].

[83] C.-M. Chang, S. Minwalla, T. Sharma, and X. Yin, "ABJ Triality: from Higher Spin Fields to Strings," *J. Phys.* **A46** (2013) 214009, arXiv:1207.4485 [hep-th].

[84] K. Pilch, P. van Nieuwenhuizen, and M. F. Sohnius, "De Sitter Superalgebras and Supergravity," *Commun. Math. Phys.* **98** (1985) 105.

[85] J. Lukierski and A. Nowicki, "All Possible De Sitter Superalgebras and the Presence of Ghosts," *Phys. Lett.* **151B** (1985) 382–386.

[86] E. Sezgin and P. Sundell, "Supersymmetric Higher Spin Theories," *J. Phys.* **A46** (2013) 214022, arXiv:1208.6019 [hep-th].

[87] C. M. Hull, "Timelike T duality, de Sitter space, large N gauge theories and topological field theory," *JHEP* **07** (1998) 021, arXiv:hep-th/9806146 [hep-th].

[88] V. Balasubramanian, J. de Boer, and D. Minic, "Mass, entropy and holography in asymptotically de Sitter spaces," *Phys.Rev.* **D65** (2002) 123508, arXiv:hep-th/0110108 [hep-th].

[89] D. Harlow and D. Stanford, "Operator Dictionaries and Wave Functions in AdS/CFT and dS/CFT," arXiv:1104.2621 [hep-th].

[90] J. Maldacena, "Einstein Gravity from Conformal Gravity," arXiv:1105.5632 [hep-th].

[91] T. Hertog and J. Hartle, "Holographic No-Boundary Measure," *JHEP* **1205** (2012) 095, arXiv:1111.6090 [hep-th].

[92] R. Dijkgraaf, B. Heidenreich, P. Jefferson, and C. Vafa, "Negative Branes, Supergroups and the Signature of Spacetime," *JHEP* **02** (2018) 050, arXiv:1603.05665 [hep-th].

[93] K. Skenderis, P. K. Townsend, and A. Van Proeyen, "Domain-wall/cosmology correspondence in adS/dS supergravity," *JHEP* **08** (2007) 036, arXiv:0704.3918 [hep-th].

[94] E. A. Bergshoeff, J. Hartong, A. Ploegh, J. Rosseel, and D. Van den Bleeken, "Pseudo-supersymmetry and a tale of alternate realities," *JHEP* **07** (2007) 067, arXiv:0704.3559 [hep-th].

[95] J. B. Hartle, S. Hawking, and T. Hertog, "Quantum Probabilities for Inflation from Holography," *JCAP* **1401** no. 01, (2014) 015, arXiv:1207.6653 [hep-th].

[96] S. Giombi, "Higher Spin — CFT Duality," in *Proceedings, Theoretical Advanced Study Institute in Elementary Particle Physics: New Frontiers in Fields and Strings (TASI 2015): Boulder, CO, USA, June 1-26, 2015*, pp. 137–214. 2017. arXiv:1607.02967 [hep-th].

[97] U. Danielsson and G. Dibitetto, "Fate of stringy AdS vacua and the weak gravity conjecture," *Phys. Rev.* **D96** no. 2, (2017) 026020, arXiv:1611.01395 [hep-th].

[98] B. Freivogel and M. Kleban, "Vacua Morghulis," arXiv:1610.04564 [hep-th].

[99] T. Anous, D. Z. Freedman, and A. Maloney, "de Sitter Supersymmetry Revisited," *JHEP* **07** (2014) 119, arXiv:1403.5038 [hep-th].

[100] R. G. Leigh and A. C. Petkou, "Holography of the N=1 higher spin theory on AdS(4)," *JHEP* **06** (2003) 011, arXiv:hep-th/0304217 [hep-th].

[101] Y. Hikida and T. Wada, "Marginal deformations of 3d supersymmetric U(N) model and broken higher spin symmetry," *JHEP* **03** (2017) 047, arXiv:1701.03563 [hep-th].

[102] M. Honda, Y. Pang, and Y. Zhu, "ABJ Quadrality," arXiv:1708.08472 [hep-th].

[103] C. Closset, T. T. Dumitrescu, G. Festuccia, and Z. Komargodski, "Supersymmetric Field Theories on Three-Manifolds," *JHEP* **05** (2013) 017, arXiv:1212.3388 [hep-th].

[104] B. Willett, "Localization on three-dimensional manifolds," *J. Phys.* **A50** no. 44, (2017) 443006, arXiv:1608.02958 [hep-th].

[105] S. M. Kuzenko and D. Sorokin, "Superconformal structures on the three-sphere," *JHEP* **10** (2014) 80, arXiv:1406.7090 [hep-th].

[106] A. A. Nizami, T. Sharma, and V. Umesh, "Superspace formulation and correlation functions of 3d superconformal field theories," *JHEP* **07** (2014) 022, arXiv:1308.4778 [hep-th].

[107] S. M. Kuzenko and D. X. Ogburn, "Off-shell higher spin N=2 supermultiplets in three dimensions," *Phys. Rev.* **D94** no. 10, (2016) 106010, arXiv:1603.04668 [hep-th].

[108] S. M. Kuzenko, R. Manvelyan, and S. Theisen, "Off-shell superconformal higher spin multiplets in four dimensions," *JHEP* **07** (2017) 034, arXiv:1701.00682 [hep-th].

[109] T. T. Dumitrescu and N. Seiberg, "Supercurrents and Brane Currents in Diverse Dimensions," *JHEP* **07** (2011) 095, arXiv:1106.0031 [hep-th].

[110] S. M. Kuzenko and G. Tartaglino-Mazzucchelli, "Three-dimensional N=2 (AdS) supergravity and associated supercurrents," *JHEP* **12** (2011) 052, arXiv:1109.0496 [hep-th].

[111] S. M. Kuzenko, "Prepotentials for N=2 conformal supergravity in three dimensions," *JHEP* **12** (2012) 021, arXiv:1209.3894 [hep-th].

[112] P. S. Howe, J. M. Izquierdo, G. Papadopoulos, and P. K. Townsend, "New supergravities with central charges and Killing spinors in (2+1)-dimensions," *Nucl. Phys.* **B467** (1996) 183–214, arXiv:hep-th/9505032 [hep-th].

[113] S. M. Kuzenko, U. Lindström, and G. Tartaglino-Mazzucchelli, "Off-shell supergravity-matter couplings in three dimensions," *JHEP* **03** (2011) 120, arXiv:1101.4013 [hep-th].

[114] M. G. Eastwood, "Higher symmetries of the Laplacian," *Annals Math.* **161** (2005) 1645–1665, arXiv:hep-th/0206233 [hep-th].

[115] P. S. Howe and U. Lindström, "Super-Laplacians and their symmetries," *JHEP* **05** (2017) 119, arXiv:1612.06787 [hep-th].

[116] X. Bekaert, E. Joung, and J. Mourad, "On higher spin interactions with matter," *JHEP* **05** (2009) 126, arXiv:0903.3338 [hep-th].

[117] X. Bekaert, E. Joung, and J. Mourad, "Effective action in a higher-spin background," *JHEP* **02** (2011) 048, arXiv:1012.2103 [hep-th].

[118] A. A. Tseytlin, "On limits of superstring in AdS(5) x S**5," *Theor. Math. Phys.* **133** (2002) 1376–1389, arXiv:hep-th/0201112 [hep-th]. [Teor. Mat. Fiz.133,69(2002)].

[119] A. Y. Segal, "Conformal higher spin theory," *Nucl. Phys.* **B664** (2003) 59–130, arXiv:hep-th/0207212 [hep-th].

[120] I. L. Buchbinder, S. J. Gates, and K. Koutrolikos, "Higher Spin Superfield interactions with the Chiral Supermultiplet: Conserved Supercurrents and Cubic Vertices," arXiv:1708.06262 [hep-th].

[121] P. D. D'Eath and J. J. Halliwell, "Fermions in Quantum Cosmology," *Phys. Rev.* **D35** (1987) 1100.

[122] P. D. D'Eath and D. I. Hughes, "SUPERSYMMETRIC MINISUPERSPACE," *Phys. Lett.* **B214** (1988) 498–502.

[123] P. D. D'Eath and D. I. Hughes, "Minisuperspace with local supersymmetry," *Nucl. Phys.* **B378** (1992) 381–409.

[124] T. Hertog and G. T. Horowitz, "Designer gravity and field theory effective potentials," *Phys. Rev. Lett.* **94** (2005) 221301, arXiv:hep-th/0412169 [hep-th].

[125] C. Closset, T. T. Dumitrescu, G. Festuccia, Z. Komargodski, and N. Seiberg, "Comments on Chern-Simons Contact Terms in Three Dimensions," *JHEP* **09** (2012) 091, arXiv:1206.5218 [hep-th].

[126] D. Anninos, T. Anous, D. Z. Freedman, and G. Konstantinidis, "Late-time Structure of the Bunch-Davies De Sitter Wavefunction," arXiv:1406.5490 [hep-th].

[127] T. Hertog, R. Monten, and Y. Vreys, "Lorentzian Condition in Holographic Cosmology," *JHEP* **01** (2017) 060, arXiv:1607.07471 [hep-th].

[128] I. B. Samsonov and D. Sorokin, "Superfield theories on S^3 and their localization," *JHEP* **04** (2014) 102, arXiv:1401.7952 [hep-th].

[129] Y. Imamura and D. Yokoyama, "N=2 supersymmetric theories on squashed three-sphere," *Phys. Rev.* **D85** (2012) 025015, arXiv:1109.4734 [hep-th].

[130] D. Martelli and A. Passias, "The gravity dual of supersymmetric gauge theories on a two-parameter deformed three-sphere," *Nucl. Phys.* **B877** (2013) 51–72, arXiv:1306.3893 [hep-th].

[131] T. Nishioka and K. Yonekura, "On RG Flow of τ_{RR} for Supersymmetric Field Theories in Three-Dimensions," *JHEP* **05** (2013) 165, arXiv:1303.1522 [hep-th].

[132] J. Hartle and T. Hertog, "Arrows of Time in the Bouncing Universes of the No-boundary Quantum State," *Phys. Rev.* **D85** (2012) 103524, arXiv:1104.1733 [hep-th].

[133] L. Anguelova and P. Langfelder, "Massive gravitino propagator in maximally symmetric spaces and fermions in dS / CFT," *JHEP* **03** (2003) 057, arXiv:hep-th/0302087 [hep-th].

[134] M. Henningson and K. Sfetsos, "Spinors and the AdS / CFT correspondence," *Phys. Lett.* **B431** (1998) 63–68, arXiv:hep-th/9803251 [hep-th].

[135] P. D. D'Eath, *Supersymmetric quantum cosmology*. Cambridge Monographs on Mathematical Physics. Cambridge University Press, 2011.

[136] M. F. Atiyah, V. K. Patodi, and I. M. Singer, "Spectral asymmetry and Riemannian Geometry 1," *Math. Proc. Cambridge Phil. Soc.* **77** (1975) 43.

[137] F. A. Berezin, "The method of second quantization," *Pure Appl. Phys.* **24** (1966) 1–228.

[138] L. D. Faddeev and A. A. Slavnov, "GAUGE FIELDS. INTRODUCTION TO QUANTUM THEORY," *Front. Phys.* **50** (1980) 1–232. [Front. Phys.,1(1991)].

[139] J. S. Dowker, "Effective actions on the squashed three sphere," *Class. Quant. Grav.* **16** (1999) 1937–1953, arXiv:hep-th/9812202 [hep-th].

[140] G. W. Gibbons, "Spectral Asymmetry and Quantum Field Theory in Curved Space-time," *Annals Phys.* **125** (1980) 98. [,471(1979)].

[141] N. Hitchin, "Harmonic spinors," *Advances in Mathematics* **14** no. 1, (1974) 1 – 55. http://www.sciencedirect.com/science/article/pii/0001870874900218.

[142] D. L. Jafferis, "The Exact Superconformal R-Symmetry Extremizes Z," *JHEP* **05** (2012) 159, arXiv:1012.3210 [hep-th].

[143] D. L. Jafferis, I. R. Klebanov, S. S. Pufu, and B. R. Safdi, "Towards the F-Theorem: N=2 Field Theories on the Three-Sphere," *JHEP* **06** (2011) 102, arXiv:1103.1181 [hep-th].

[144] I. R. Klebanov, S. S. Pufu, and B. R. Safdi, "F-Theorem without Supersymmetry," *JHEP* **1110** (2011) 038, arXiv:1105.4598 [hep-th].

[145] H. Casini and M. Huerta, "On the RG running of the entanglement entropy of a circle," *Phys. Rev.* **D85** (2012) 125016, arXiv:1202.5650 [hep-th].

[146] S. Corley, "The Massless gravitino and the AdS / CFT correspondence," *Phys. Rev.* **D59** (1999) 086003, arXiv:hep-th/9808184 [hep-th].

[147] A. Volovich, "Rarita-Schwinger field in the AdS / CFT correspondence," *JHEP* **09** (1998) 022, arXiv:hep-th/9809009 [hep-th].

[148] P. V. Moniz, "Supersymmetric quantum cosmology: Shaken not stirred," *Int. J. Mod. Phys.* **A11** (1996) 4321–4382, arXiv:gr-qc/9604025 [gr-qc].

[149] E. I. Buchbinder, J. Hutomo, and S. M. Kuzenko, "Higher spin supercurrents in anti-de Sitter space," *JHEP* **09** (2018) 027, arXiv:1805.08055 [hep-th].

[150] J. Hutomo, S. M. Kuzenko, and D. Ogburn, "$\mathcal{N} = 2$ supersymmetric higher spin gauge theories and current multiplets in three dimensions," *Phys. Rev.* **D98** no. 12, (2018) 125004, arXiv:1807.09098 [hep-th].

[151] J. Hutomo and S. M. Kuzenko, "Higher spin supermultiplets in three dimensions: (2,0) AdS supersymmetry," *Phys. Lett.* **B787** (2018) 175–181, arXiv:1809.00802 [hep-th].

[152] D. Anninos, V. De Luca, G. Franciolini, A. Kehagias, and A. Riotto, "Cosmological Shapes of Higher-Spin Gravity," *JCAP* **2019** no. 04, (2019) 045, arXiv:1902.01251 [hep-th].

[153] Y. Neiman, "Holographic quantization of linearized higher-spin gravity in the de Sitter causal patch," *JHEP* **11** (2018) 033, arXiv:1809.07270 [hep-th].

[154] R. Aros, C. Iazeolla, J. Noreña, E. Sezgin, P. Sundell, and Y. Yin, "FRW and domain walls in higher spin gravity," *JHEP* **03** (2018) 153, arXiv:1712.02401 [hep-th].

[155] B. McInnes, "Exploring the similarities of the dS / CFT and AdS / CFT correspondences," *Nucl. Phys.* **B627** (2002) 311–329, arXiv:hep-th/0110062 [hep-th].

[156] J. B. Hartle, S. W. Hawking, and T. Hertog, "The Classical Universes of the No-Boundary Quantum State," *Phys. Rev.* **D77** (2008) 123537, arXiv:0803.1663 [hep-th].

[157] E. Kiritsis and A. Tsouros, "de Sitter versus Anti de Sitter flows and the (super)gravity landscape," arXiv:1901.04546 [hep-th].

[158] T. Van Riet. Private communication, postings on Facebook and Twitter.

[159] J. B. Hartle, S. Hawking, and T. Hertog, "No-Boundary Measure of the Universe," *Phys.Rev.Lett.* **100** (2008) 201301, arXiv:0711.4630 [hep-th].

[160] T. D. Brennan, F. Carta, and C. Vafa, "The String Landscape, the Swampland, and the Missing Corner," arXiv:1711.00864 [hep-th].

[161] U. H. Danielsson and T. Van Riet, "What if string theory has no de Sitter vacua?," arXiv:1804.01120 [hep-th].

[162] S. K. Garg and C. Krishnan, "Bounds on Slow Roll and the de Sitter Swampland," arXiv:1807.05193 [hep-th].

[163] G. Obied, H. Ooguri, L. Spodyneiko, and C. Vafa, "De Sitter Space and the Swampland," arXiv:1806.08362 [hep-th].

[164] H. Ooguri, E. Palti, G. Shiu, and C. Vafa, "Distance and de Sitter Conjectures on the Swampland," *Phys. Lett.* **B788** (2019) 180–184, arXiv:1810.05506 [hep-th].

[165] D. Andriot, "New constraints on classical de Sitter: flirting with the swampland," *Fortsch. Phys.* **67** no. 1-2, (2019) 1800103, arXiv:1807.09698 [hep-th].

[166] J. Diaz Dorronsoro, "Antibranes in the early and late universe," 2018. https://lirias.kuleuven.be/handle/123456789/625437.

[167] J. Moritz, *On accelerated expansion in string theory.* PhD thesis, Hamburg U., 2019.

[168] J. Overduin and P. Wesson, "Kaluza-Klein gravity," *Phys. Rept.* **283** (1997) 303–380, arXiv:gr-qc/9805018.

[169] P. G. O. Freund and M. A. Rubin, "Dynamics of Dimensional Reduction," *Phys. Lett.* **97B** (1980) 233–235. [,80(1980)].

[170] S. Randjbar-Daemi, A. Salam, and J. Strathdee, "Spontaneous Compactification in Six-Dimensional Einstein-Maxwell Theory," *Nucl. Phys. B* **214** (1983) 491–512.

[171] F. Denef, M. R. Douglas, and S. Kachru, "Physics of String Flux Compactifications," *Ann. Rev. Nucl. Part. Sci.* **57** (2007) 119–144, arXiv:hep-th/0701050 [hep-th].

[172] P. Koerber, "Lectures on Generalized Complex Geometry for Physicists," *Fortsch. Phys.* **59** (2011) 169–242, arXiv:1006.1536 [hep-th].

[173] M. Nakahara, *Geometry, topology and physics.* 2003.

[174] M. Grana, "Flux compactifications in string theory: A Comprehensive review," *Phys.Rept.* **423** (2006) 91–158, arXiv:hep-th/0509003 [hep-th].

[175] K. Becker, M. Becker, and J. Schwarz, *String theory and M-theory: A modern introduction.* Cambridge University Press, 12, 2006.

[176] M. Grana, R. Minasian, M. Petrini, and A. Tomasiello, "A Scan for new N=1 vacua on twisted tori," *JHEP* **0705** (2007) 031, arXiv:hep-th/0609124 [hep-th].

[177] D. Joyce, *Compact manifolds with special holonomy.* Oxford Mathematical Monographs. Oxford University Press, 2000.

[178] S. Grigorian, "Moduli spaces of G(2) manifolds," *Rev. Math. Phys.* **22** (2010) 1061–1097, arXiv:0911.2185 [math.DG].

[179] S.-T. Yau, "Calabi's conjecture and some new results in algebraic geometry," *Proceedings of the National Academy of Sciences* **74** no. 5, (1977) 1798–1799, https://www.pnas.org/content/74/5/1798.full.pdf. https://www.pnas.org/content/74/5/1798.

[180] S.-T. Yau, "On the ricci curvature of a compact khler manifold and the complex monge-ampre equation, i," *Communications on Pure and Applied Mathematics* **31** no. 3, (1978) 339–411, https://onlinelibrary.wiley.com/doi/pdf/10.1002/cpa.3160310304. https://onlinelibrary.wiley.com/doi/abs/10.1002/cpa.3160310304.

[181] B. S. Acharya, "M theory, G(2)-manifolds and four-dimensional physics," *ICTP Lect. Notes Ser.* **13** (2002) 1–56.

[182] L. Randall and R. Sundrum, "A Large mass hierarchy from a small extra dimension," *Phys. Rev. Lett.* **83** (1999) 3370–3373, arXiv:hep-ph/9905221.

[183] K. Dasgupta, G. Rajesh, and S. Sethi, "M theory, orientifolds and G - flux," *JHEP* **08** (1999) 023, arXiv:hep-th/9908088 [hep-th].

[184] S. B. Giddings, S. Kachru, and J. Polchinski, "Hierarchies from fluxes in string compactifications," *Phys. Rev.* **D66** (2002) 106006, arXiv:hep-th/0105097 [hep-th].

[185] J. M. Maldacena and C. Nunez, "Supergravity description of field theories on curved manifolds and a no go theorem," *Int. J. Mod. Phys.* **A16** (2001) 822–855, arXiv:hep-th/0007018 [hep-th]. [,182(2000)].

[186] I. R. Klebanov and M. J. Strassler, "Supergravity and a confining gauge theory: Duality cascades and chi SB resolution of naked singularities," *JHEP* **08** (2000) 052, arXiv:hep-th/0007191 [hep-th].

[187] S. Gukov, C. Vafa, and E. Witten, "CFT's from Calabi-Yau four folds," *Nucl. Phys.* **B584** (2000) 69–108, arXiv:hep-th/9906070 [hep-th]. [Erratum: Nucl. Phys.B608,477(2001)].

[188] J. P. Conlon, "Moduli Stabilisation and Applications in IIB String Theory," other thesis, 2007.

[189] S. Kachru, R. Kallosh, A. D. Linde, and S. P. Trivedi, "De Sitter vacua in string theory," *Phys. Rev.* **D68** (2003) 046005, arXiv:hep-th/0301240 [hep-th].

[190] I. Affleck, M. Dine, and N. Seiberg, "Dynamical Supersymmetry Breaking in Supersymmetric QCD," *Nucl. Phys. B* **241** (1984) 493–534.

[191] D. Baumann and L. McAllister, *Inflation and String Theory*. Cambridge University Press, 2015. arXiv:1404.2601 [hep-th]. http://inspirehep.net/record/1289899/files/arXiv:1404.2601.pdf.

[192] S. Kachru, R. Kallosh, A. D. Linde, J. M. Maldacena, L. P. McAllister, and S. P. Trivedi, "Towards inflation in string theory," *JCAP* **0310** (2003) 013, arXiv:hep-th/0308055 [hep-th].

[193] S. Kachru, J. Pearson, and H. L. Verlinde, "Brane / flux annihilation and the string dual of a nonsupersymmetric field theory," *JHEP* **06** (2002) 021, arXiv:hep-th/0112197 [hep-th].

[194] S. Sethi, "Supersymmetry Breaking by Fluxes," arXiv:1709.03554 [hep-th].

[195] D. Cohen-Maldonado, J. Diaz, T. Van Riet, and B. Vercnocke, "From black holes to flux throats: Polarization can resolve the singularity," *Fortsch. Phys.* **64** (2016) 317–321, arXiv:1511.07453 [hep-th].

[196] D. Cohen-Maldonado, J. Diaz, T. Van Riet, and B. Vercnocke, "Observations on fluxes near anti-branes," *JHEP* **01** (2016) 126, arXiv:1507.01022 [hep-th].

[197] D. Cohen-Maldonado, J. Diaz, and F. F. Gautason, "Polarised antibranes from Smarr relations," *JHEP* **05** (2016) 175, arXiv:1603.05678 [hep-th].

[198] F. F. Gautason, B. Truijen, and T. Van Riet, "Smeared antibranes polarise in AdS," *JHEP* **07** (2015) 165, arXiv:1502.00927 [hep-th].

[199] I. Bena, G. Giecold, M. Grana, N. Halmagyi, and S. Massai, "The backreaction of anti-D3 branes on the Klebanov-Strassler geometry," *JHEP* **06** (2013) 060, arXiv:1106.6165 [hep-th].

[200] J. Blaback, F. Gautason, A. Ruipérez, and T. Van Riet, "Anti-brane singularities as red herrings," *JHEP* **12** (2019) 125, arXiv:1907.05295 [hep-th].

[201] I. Bena, J. Blaback, U. Danielsson, and T. Van Riet, "Antibranes cannot become black," *Phys. Rev. D* **87** no. 10, (2013) 104023, arXiv:1301.7071 [hep-th].

[202] I. Bena, M. Graña, S. Kuperstein, and S. Massai, "Polchinski-Strassler does not uplift Klebanov-Strassler," *JHEP* **09** (2013) 142, arXiv:1212.4828 [hep-th].

[203] J. Moritz, A. Retolaza, and A. Westphal, "Toward de Sitter space from ten dimensions," *Phys. Rev.* **D97** no. 4, (2018) 046010, arXiv:1707.08678 [hep-th].

[204] L. Randall, "The Boundaries of KKLT," arXiv:1912.06693 [hep-th].

[205] E. Dudas and S. Lüst, "An update on moduli stabilization with antibrane uplift," arXiv:1912.09948 [hep-th].

[206] I. Bena, E. Dudas, M. Graña, and S. Lust, "Uplifting Runaways," *Fortsch. Phys.* **2018** (2018) 1800100, arXiv:1809.06861 [hep-th].

[207] I. Bena, A. Buchel, and S. Lüst, "Throat destabilization (for profit and for fun)," arXiv:1910.08094 [hep-th].

[208] R. Blumenhagen, D. Kläwer, and L. Schlechter, "Swampland Variations on a Theme by KKLT," *JHEP* **05** (2019) 152, arXiv:1902.07724 [hep-th].

[209] S. Kachru, M. Kim, L. McAllister, and M. Zimet, "de Sitter Vacua from Ten Dimensions," arXiv:1908.04788 [hep-th].

[210] F. F. Gautason, V. Van Hemelryck, and T. Van Riet, "The tension between 10D supergravity and dS uplifts," arXiv:1810.08518 [hep-th].

[211] L. Martucci, J. Rosseel, D. Van den Bleeken, and A. Van Proeyen, "Dirac actions for D-branes on backgrounds with fluxes," *Class. Quant. Grav.* **22** (2005) 2745–2764, arXiv:hep-th/0504041 [hep-th].

[212] Y. Hamada, A. Hebecker, G. Shiu, and P. Soler, "Brane gaugino condensate in 10d," arXiv:1812.06097 [hep-th].

[213] V. Balasubramanian, P. Berglund, J. P. Conlon, and F. Quevedo, "Systematics of moduli stabilisation in Calabi-Yau flux compactifications," *JHEP* **03** (2005) 007, arXiv:hep-th/0502058 [hep-th].

[214] M. Graña and J. Polchinski, "Supersymmetric three form flux perturbations on AdS(5)," *Phys. Rev.* **D63** (2001) 026001, arXiv:hep-th/0009211 [hep-th].

[215] R. Kallosh and T. Wrase, "dS Supergravity from 10d," *Fortsch. Phys.* **2018** (2018) 1800071, arXiv:1808.09427 [hep-th].

[216] S. Banerjee, U. Danielsson, G. Dibitetto, S. Giri, and M. Schillo, "Emergent de Sitter Cosmology from Decaying Anti de Sitter Space," *Phys. Rev. Lett.* **121** no. 26, (2018) 261301, arXiv:1807.01570 [hep-th].

[217] J. Blaback, U. Danielsson, and G. Dibitetto, "A new light on the darkest corner of the landscape," arXiv:1810.11365 [hep-th].

[218] C. Cordova, G. B. De Luca, and A. Tomasiello, "Classical de Sitter Solutions of Ten-Dimensional Supergravity," arXiv:1812.04147 [hep-th].

[219] J. J. Heckman, C. Lawrie, L. Lin, J. Sakstein, and G. Zoccarato, "Pixelated Dark Energy," arXiv:1901.10489 [hep-th].

[220] M. Weissenbacher, "α'-Corrections and de Sitter Vacua - a Mirage?," arXiv:1901.09626 [hep-th].

[221] J. Moritz and T. Van Riet, "Racing through the swampland: de Sitter uplift vs weak gravity," arXiv:1805.00944 [hep-th].

[222] I. Bena, M. Graña, and N. Halmagyi, "On the Existence of Meta-stable Vacua in Klebanov-Strassler," *JHEP* **09** (2010) 087, arXiv:0912.3519 [hep-th].

[223] J. Blaback, U. H. Danielsson, and T. Van Riet, "Resolving anti-brane singularities through time-dependence," *JHEP* **02** (2013) 061, arXiv:1202.1132 [hep-th].

[224] F. F. Gautason, D. Junghans, and M. Zagermann, "Cosmological Constant, Near Brane Behavior and Singularities," *JHEP* **09** (2013) 123, arXiv:1301.5647 [hep-th].

[225] B. Michel, E. Mintun, J. Polchinski, A. Puhm, and P. Saad, "Remarks on brane and antibrane dynamics," *JHEP* **09** (2015) 021, arXiv:1412.5702 [hep-th].

[226] J. Armas, N. Nguyen, V. Niarchos, N. A. Obers, and T. Van Riet, "Meta-stable non-extremal anti-branes," arXiv:1812.01067 [hep-th].

[227] I. Bena, M. Graa, S. Kuperstein, and S. Massai, "Giant Tachyons in the Landscape," *JHEP* **02** (2015) 146, arXiv:1410.7776 [hep-th].

[228] I. Bena, J. Blaback, and D. Turton, "Loop corrections to the antibrane potential," *JHEP* **07** (2016) 132, arXiv:1602.05959 [hep-th].

[229] K. Dasgupta, R. Gwyn, E. McDonough, M. Mia, and R. Tatar, "de Sitter Vacua in Type IIB String Theory: Classical Solutions and Quantum Corrections," *JHEP* **07** (2014) 054, arXiv:1402.5112 [hep-th].

[230] R. Kallosh, "Gaugino Condensation and Geometry of the Perfect Square," arXiv:1901.02023 [hep-th].

[231] F. Carta, J. Moritz, and A. Westphal, "Gaugino condensation and small uplifts in KKLT," arXiv:1902.01412 [hep-th].

[232] Y. Hamada, A. Hebecker, G. Shiu, and P. Soler, "Understanding KKLT from a 10d perspective," arXiv:1902.01410 [hep-th].

[233] D. Baumann, A. Dymarsky, S. Kachru, I. R. Klebanov, and L. McAllister, "D3-brane Potentials from Fluxes in AdS/CFT," *JHEP* **06** (2010) 072, arXiv:1001.5028 [hep-th].

[234] D. Baumann, A. Dymarsky, I. R. Klebanov, and L. McAllister, "Towards an Explicit Model of D-brane Inflation," *JCAP* **0801** (2008) 024, arXiv:0706.0360 [hep-th].

[235] A. Dymarsky and L. Martucci, "D-brane non-perturbative effects and geometric deformations," *JHEP* **04** (2011) 061, arXiv:1012.4018 [hep-th].

[236] B. Heidenreich, L. McAllister, and G. Torroba, "Dynamic SU(2) Structure from Seven-branes," *JHEP* **05** (2011) 110, arXiv:1011.3510 [hep-th].

[237] P. Koerber and L. Martucci, "From ten to four and back again: How to generalize the geometry," *JHEP* **08** (2007) 059, arXiv:0707.1038 [hep-th].

[238] P. G. Camara, L. E. Ibanez, and A. M. Uranga, "Flux-induced SUSY-breaking soft terms on D7-D3 brane systems," *Nucl. Phys.* **B708** (2005) 268–316, arXiv:hep-th/0408036 [hep-th].

[239] P. Horava and E. Witten, "Eleven-dimensional supergravity on a manifold with boundary," *Nucl. Phys.* **B475** (1996) 94–114, arXiv:hep-th/9603142 [hep-th].

[240] P. Horava and E. Witten, "Heterotic and type I string dynamics from eleven-dimensions," *Nucl. Phys.* **B460** (1996) 506–524, arXiv:hep-th/9510209 [hep-th]. [,397(1995)].

[241] P. Horava, "Gluino condensation in strongly coupled heterotic string theory," *Phys. Rev.* **D54** (1996) 7561–7569, arXiv:hep-th/9608019 [hep-th].

[242] M. Dine, R. Rohm, N. Seiberg, and E. Witten, "Gluino Condensation in Superstring Models," *Phys. Lett.* **156B** (1985) 55–60.

[243] A. Sen, "F theory and orientifolds," *Nucl. Phys.* **B475** (1996) 562–578, arXiv:hep-th/9605150 [hep-th].

[244] V. Kaplunovsky and J. Louis, "Field dependent gauge couplings in locally supersymmetric effective quantum field theories," *Nucl. Phys.* **B422** (1994) 57–124, arXiv:hep-th/9402005 [hep-th].

[245] A. Hebecker and T. Wrase, "The asymptotic dS Swampland Conjecture - a simplified derivation and a potential loophole," *Fortsch. Phys.* **2018** (2018) 1800097, arXiv:1810.08182 [hep-th].

[246] R. Kallosh and A. D. Linde, "Landscape, the scale of SUSY breaking, and inflation," *JHEP* **12** (2004) 004, arXiv:hep-th/0411011 [hep-th].

[247] B. de Carlos, A. Guarino, and J. M. Moreno, "Flux moduli stabilisation, Supergravity algebras and no-go theorems," *JHEP* **01** (2010) 012, arXiv:0907.5580 [hep-th].

[248] E. Palti, "Low Energy Supersymmetry from Non-Geometry," *JHEP* **10** (2007) 011, arXiv:0707.1595 [hep-th].

[249] J. J. Blanco-Pillado, M. A. Urkiola, and J. M. Wachter, "Racetrack Potentials and the de Sitter Swampland Conjectures," arXiv:1811.05463 [hep-th].

[250] R. Kallosh, A. Linde, E. McDonough, and M. Scalisi, "dS Vacua and the Swampland," arXiv:1901.02022 [hep-th].

[251] D. Joyce, "Compact Riemannian 7-manifolds with holonomy G2. I, II," *J. Differential Geom.* **43** (1996) 291–328, 329–375.

[252] A. Kovalev, "Twisted connected sums and special Riemannian holonomy," arXiv:math/0012189 [math].

[253] J. Halverson and D. R. Morrison, "The landscape of M-theory compactifications on seven-manifolds with G_2 holonomy," *JHEP* **04** (2015) 047, arXiv:1412.4123 [hep-th].

[254] A. P. Braun and S. Schäfer-Nameki, "Compact, Singular G_2-Holonomy Manifolds and M/Heterotic/F-Theory Duality," *JHEP* **04** (2018) 126, arXiv:1708.07215 [hep-th].

[255] A. Kennon, "G_2-Manifolds and M-Theory Compactifications," arXiv:1810.12659 [hep-th].

[256] S. Kachru and J. McGreevy, "M theory on manifolds of G(2) holonomy and type IIA orientifolds," *JHEP* **06** (2001) 027, arXiv:hep-th/0103223 [hep-th].

[257] M. Cvetic, G. Shiu, and A. M. Uranga, "Three family supersymmetric standard - like models from intersecting brane worlds," *Phys. Rev. Lett.* **87** (2001) 201801, arXiv:hep-th/0107143 [hep-th].

[258] M. Cvetic, G. Shiu, and A. M. Uranga, "Chiral four-dimensional N=1 supersymmetric type 2A orientifolds from intersecting D6 branes," *Nucl. Phys.* **B615** (2001) 3–32, arXiv:hep-th/0107166 [hep-th].

[259] M. Cvetic, G. Shiu, and A. M. Uranga, "Chiral type II orientifold constructions as M theory on G(2) holonomy spaces," in *Supersymmetry and unification of fundamental interactions. Proceedings, 9th International Conference, SUSY'01, Dubna, Russia, June 11-17, 2001*, pp. 317–326. 2001. arXiv:hep-th/0111179 [hep-th].

[260] S. Kachru, M. B. Schulz, P. K. Tripathy, and S. P. Trivedi, "New supersymmetric string compactifications," *JHEP* **0303** (2003) 061, arXiv:hep-th/0211182 [hep-th].

[261] P. Kaste, R. Minasian, M. Petrini, and A. Tomasiello, "Kaluza-Klein bundles and manifolds of exceptional holonomy," *JHEP* **0209** (2002) 033, arXiv:hep-th/0206213 [hep-th].

[262] P. Kaste, R. Minasian, M. Petrini, and A. Tomasiello, "Nontrivial RR two form field strength and SU(3) structure," *Fortsch.Phys.* **51** (2003) 764–768, arXiv:hep-th/0301063 [hep-th].

[263] J. Gaillard and J. Schmude, "The Lift of type IIA supergravity with D6 sources: M-theory with torsion," *JHEP* **1002** (2010) 032, arXiv:0908.0305 [hep-th].

[264] A. Fayyazuddin and T. Z. Husain, "G(2) holonomy metrics and wrapped D6-branes," *Phys.Rev.* **D75** (2007) 065017, arXiv:hep-th/0608163 [hep-th].

[265] F. Marchesano and G. Shiu, "MSSM vacua from flux compactifications," *Phys. Rev.* **D71** (2005) 011701, arXiv:hep-th/0408059 [hep-th].

[266] F. Marchesano and G. Shiu, "Building MSSM flux vacua," *JHEP* **11** (2004) 041, arXiv:hep-th/0409132 [hep-th].

[267] S. B. Giddings and A. Maharana, "Dynamics of warped compactifications and the shape of the warped landscape," *Phys. Rev.* **D73** (2006) 126003, arXiv:hep-th/0507158 [hep-th].

[268] M. R. Douglas and G. Torroba, "Kinetic terms in warped compactifications," *JHEP* **05** (2009) 013, arXiv:0805.3700 [hep-th].

[269] G. Shiu, G. Torroba, B. Underwood, and M. R. Douglas, "Dynamics of Warped Flux Compactifications," *JHEP* **06** (2008) 024, arXiv:0803.3068 [hep-th].

[270] M. B. Schulz and E. F. Tammaro, "M-theory/type IIA duality and K3 in the Gibbons-Hawking approximation," arXiv:1206.1070 [hep-th].

[271] P. Koerber and D. Tsimpis, "Supersymmetric sources, integrability and generalized-structure compactifications," *JHEP* **08** (2007) 082, arXiv:0706.1244 [hep-th].

[272] A. Hatcher, *Algebraic topology*. Cambridge Univ. Press, Cambridge, 2000. https://cds.cern.ch/record/478079.

[273] F. Marchesano, "D6-branes and torsion," *JHEP* **05** (2006) 019, arXiv:hep-th/0603210 [hep-th].

[274] J. M. Maldacena, G. W. Moore, and N. Seiberg, "D-brane instantons and K theory charges," *JHEP* **11** (2001) 062, arXiv:hep-th/0108100 [hep-th].

[275] J. F. G. Cascales and A. M. Uranga, "Chiral 4d string vacua with D branes and NSNS and RR fluxes," *JHEP* **05** (2003) 011, arXiv:hep-th/0303024 [hep-th].

[276] D. J. Gross and M. J. Perry, "Magnetic Monopoles in Kaluza-Klein Theories," *Nucl. Phys.* **B226** (1983) 29–48.

[277] M. F. Atiyah and N. J. Hitchin, "Low-Energy Scattering of Nonabelian Monopoles," *Phys. Lett.* **A107** (1985) 21–25.

[278] G. W. Gibbons and N. S. Manton, "Classical and Quantum Dynamics of BPS Monopoles," *Nucl. Phys.* **B274** (1986) 183–224.

[279] N. Seiberg and E. Witten, "Gauge dynamics and compactification to three-dimensions," arXiv:hep-th/9607163 [hep-th].

[280] N. Seiberg, "IR dynamics on branes and space-time geometry," *Phys.Lett.* **B384** (1996) 81–85, arXiv:hep-th/9606017 [hep-th].

[281] L. Martucci, "Warping the Kähler potential of F-theory/IIB flux compactifications," *JHEP* **03** (2015) 067, arXiv:1411.2623 [hep-th].

[282] C. Beasley and E. Witten, "A Note on fluxes and superpotentials in M theory compactifications on manifolds of G(2) holonomy," *JHEP* **0207** (2002) 046, arXiv:hep-th/0203061 [hep-th].

[283] T. W. Grimm and J. Louis, "The Effective action of type IIA Calabi-Yau orientifolds," *Nucl. Phys.* **B718** (2005) 153–202, arXiv:hep-th/0412277 [hep-th].

[284] R. C. Mclean, "Deformations of calibrated submanifolds," *Commun. Analy. Geom* **6** (1996) 705–747.

[285] F. Marchesano, D. Regalado, and G. Zoccarato, "On D-brane moduli stabilisation," *JHEP* **11** (2014) 097, arXiv:1410.0209 [hep-th].

[286] P. Koerber, "Stable D-branes, calibrations and generalized Calabi-Yau geometry," *JHEP* **08** (2005) 099, arXiv:hep-th/0506154 [hep-th].

[287] L. Martucci and P. Smyth, "Supersymmetric D-branes and calibrations on general N=1 backgrounds," *JHEP* **11** (2005) 048, arXiv:hep-th/0507099 [hep-th].

[288] L. Martucci, "D-branes on general N=1 backgrounds: Superpotentials and D-terms," *JHEP* **06** (2006) 033, arXiv:hep-th/0602129 [hep-th].

[289] P. Koerber and L. Martucci, "Deformations of calibrated D-branes in flux generalized complex manifolds," *JHEP* **12** (2006) 062, arXiv:hep-th/0610044 [hep-th].

[290] A. Klemm and M. Marino, "Counting BPS states on the enriques Calabi-Yau," *Commun. Math. Phys.* **280** (2008) 27–76, arXiv:hep-th/0512227 [hep-th].

[291] D. Joyce, "Compact Riemannian 7-manifolds with holonomy G2," *Journal of Differential Geometry, 43* **I** (1996) 291–328.

[292] U. H. Danielsson, S. S. Haque, P. Koerber, G. Shiu, T. Van Riet, and T. Wrase, "De Sitter hunting in a classical landscape," *Fortsch. Phys.* **59** (2011) 897–933, arXiv:1103.4858 [hep-th].

[293] J. Blaback, B. Janssen, T. Van Riet, and B. Vercnocke, "BPS domain walls from backreacted orientifolds," *JHEP* **05** (2014) 040, arXiv:1312.6125 [hep-th].

[294] G. Dall'Agata and N. Prezas, "Scherk-Schwarz reduction of M-theory on G2-manifolds with fluxes," *JHEP* **10** (2005) 103, arXiv:hep-th/0509052 [hep-th].

[295] U. Danielsson, G. Dibitetto, and A. Guarino, "KK-monopoles and G-structures in M-theory/type IIA reductions," *JHEP* **02** (2015) 096, arXiv:1411.0575 [hep-th].

[296] J. Scherk and J. H. Schwarz, "Spontaneous Breaking of Supersymmetry Through Dimensional Reduction," *Phys. Lett.* **82B** (1979) 60–64.

[297] J. Scherk and J. H. Schwarz, "How to Get Masses from Extra Dimensions," *Nucl. Phys.* **B153** (1979) 61–88. [,79(1979)].

[298] M. Graña, R. Minasian, H. Triendl, and T. Van Riet, "Quantization problem in Scherk-Schwarz compactifications," *Phys. Rev.* **D88** no. 8, (2013) 085018, arXiv:1305.0785 [hep-th].

[299] E. Palti, G. Tasinato, and J. Ward, "WEAKLY-coupled IIA Flux Compactifications," *JHEP* **06** (2008) 084, arXiv:0804.1248 [hep-th].

[300] M. Larfors, D. Lust, and D. Tsimpis, "Flux compactification on smooth, compact three-dimensional toric varieties," *JHEP* **07** (2010) 073, arXiv:1005.2194 [hep-th].

[301] M. Larfors, A. Lukas, and F. Ruehle, "Calabi-Yau Manifolds and SU(3) Structure," arXiv:1805.08499 [hep-th].

[302] S. Kachru and S. P. Trivedi, "A comment on effective field theories of flux vacua," arXiv:1808.08971 [hep-th].

[303] M. Cicoli, S. De Alwis, A. Maharana, F. Muia, and F. Quevedo, "De Sitter vs Quintessence in String Theory," arXiv:1808.08967 [hep-th].

[304] W. Lerche, D. Lust, and A. Schellekens, "Chiral Four-Dimensional Heterotic Strings from Selfdual Lattices," *Nucl. Phys. B* **287** (1987) 477.

[305] F. Quevedo, "Lectures on superstring phenomenology," *AIP Conf. Proc.* **359** (1996) 202–242, arXiv:hep-th/9603074 [hep-th].

[306] R. Bousso and J. Polchinski, "Quantization of four form fluxes and dynamical neutralization of the cosmological constant," *JHEP* **06** (2000) 006, arXiv:hep-th/0004134.

[307] A. N. Schellekens, "The Landscape 'avant la lettre'," arXiv:physics/0604134 [physics].

[308] L. Susskind, "The Anthropic landscape of string theory," arXiv:hep-th/0302219 [hep-th].

[309] J. Brown and C. Teitelboim, "Dynamical Neutralization of the Cosmological Constant," *Phys. Lett. B* **195** (1987) 177–182.

[310] J. Brown and C. Teitelboim, "Neutralization of the Cosmological Constant by Membrane Creation," *Nucl. Phys. B* **297** (1988) 787–836.

[311] S. Ashok and M. R. Douglas, "Counting flux vacua," *JHEP* **01** (2004) 060, arXiv:hep-th/0307049 [hep-th].

[312] M. R. Douglas, "Basic results in vacuum statistics," *Comptes Rendus Physique* **5** (2004) 965–977, arXiv:hep-th/0409207 [hep-th].

[313] F. Denef and M. R. Douglas, "Distributions of flux vacua," *JHEP* **05** (2004) 072, arXiv:hep-th/0404116.

[314] F. Denef and M. R. Douglas, "Computational complexity of the landscape. I.," *Annals Phys.* **322** (2007) 1096–1142, arXiv:hep-th/0602072.

[315] J. Polchinski, "The Cosmological Constant and the String Landscape," in *The Quantum Structure of Space and Time: Proceedings of the 23rd Solvay Conference on Physics. Brussels, Belgium. 1 - 3 December 2005*, pp. 216–236. 2006. arXiv:hep-th/0603249 [hep-th].

[316] M. R. Douglas and S. Kachru, "Flux compactification," *Rev. Mod. Phys.* **79** (2007) 733–796, arXiv:hep-th/0610102 [hep-th].

[317] E. Palti, "The Swampland: Introduction and Review," *Fortsch. Phys.* **67** no. 6, (2019) 1900037, arXiv:1903.06239 [hep-th].

[318] D. Harlow and H. Ooguri, "Constraints on symmetry from holography," arXiv:1810.05337 [hep-th].

[319] D. Harlow and H. Ooguri, "Symmetries in quantum field theory and quantum gravity," arXiv:1810.05338 [hep-th].

[320] N. Arkani-Hamed, L. Motl, A. Nicolis, and C. Vafa, "The String landscape, black holes and gravity as the weakest force," *JHEP* **06** (2007) 060, arXiv:hep-th/0601001 [hep-th].

[321] B. Heidenreich, M. Reece, and T. Rudelius, "Sharpening the Weak Gravity Conjecture with Dimensional Reduction," *JHEP* **02** (2016) 140, arXiv:1509.06374 [hep-th].

[322] J. S. Schwinger, "On gauge invariance and vacuum polarization," *Phys. Rev.* **82** (1951) 664–679. [,116(1951)].

[323] G. W. Gibbons, "Vacuum Polarization and the Spontaneous Loss of Charge by Black Holes," *Commun. Math. Phys.* **44** (1975) 245–264.

[324] B. Pioline and J. Troost, "Schwinger pair production in AdS(2)," *JHEP* **03** (2005) 043, arXiv:hep-th/0501169 [hep-th].

[325] H. Ooguri and C. Vafa, "On the Geometry of the String Landscape and the Swampland," *Nucl. Phys.* **B766** (2007) 21–33, arXiv:hep-th/0605264 [hep-th].

[326] P. Agrawal, G. Obied, P. J. Steinhardt, and C. Vafa, "On the Cosmological Implications of the String Swampland," *Phys. Lett.* **B784** (2018) 271–276, arXiv:1806.09718 [hep-th].

[327] A. Bedroya and C. Vafa, "Trans-Planckian Censorship and the Swampland," arXiv:1909.11063 [hep-th].

[328] A. Bedroya, R. Brandenberger, M. Loverde, and C. Vafa, "Trans-Planckian Censorship and Inflationary Cosmology," arXiv:1909.11106 [hep-th].

[329] J. McNamara and C. Vafa, "Cobordism Classes and the Swampland," arXiv:1909.10355 [hep-th].

[330] U. Schreiber, "Swampland in nlab." https://ncatlab.org/nlab/show/swampland. Accessed: 2019-10-01.

[331] T. Banks and L. J. Dixon, "Constraints on String Vacua with Space-Time Supersymmetry," *Nucl. Phys.* **B307** (1988) 93–108.

[332] L. F. Abbott and M. B. Wise, "Wormholes and Global Symmetries," *Nucl. Phys.* **B325** (1989) 687–704.

[333] S. R. Coleman and K.-M. Lee, "Wormholes made without massless matter fields," *Nucl. Phys.* **B329** (1990) 387–409.

[334] R. Kallosh, A. D. Linde, D. A. Linde, and L. Susskind, "Gravity and global symmetries," *Phys. Rev.* **D52** (1995) 912–935, arXiv:hep-th/9502069 [hep-th].

[335] T. Banks and N. Seiberg, "Symmetries and Strings in Field Theory and Gravity," *Phys. Rev.* **D83** (2011) 084019, arXiv:1011.5120 [hep-th].

[336] L. Susskind, "Trouble for remnants," arXiv:hep-th/9501106 [hep-th].

[337] C. Beem, M. Lemos, P. Liendo, L. Rastelli, and B. C. van Rees, "The $\mathcal{N} = 2$ superconformal bootstrap," *JHEP* **03** (2016) 183, arXiv:1412.7541 [hep-th].

[338] S. Fichet and P. Saraswat, "Approximate Symmetries and Gravity," arXiv:1909.02002 [hep-th].

[339] C. Cheung and G. N. Remmen, "Naturalness and the Weak Gravity Conjecture," *Phys. Rev. Lett.* **113** (2014) 051601, arXiv:1402.2287 [hep-ph].

[340] B. Heidenreich, M. Reece, and T. Rudelius, "Evidence for a Lattice Weak Gravity Conjecture," arXiv:1606.08437 [hep-th].

[341] S. Andriolo, D. Junghans, T. Noumi, and G. Shiu, "A Tower Weak Gravity Conjecture from Infrared Consistency," *Fortsch. Phys.* **66** no. 5, (2018) 1800020, arXiv:1802.04287 [hep-th].

[342] D. Klaewer and E. Palti, "Super-Planckian Spatial Field Variations and Quantum Gravity," *JHEP* **01** (2017) 088, arXiv:1610.00010 [hep-th].

[343] E. Palti, "The Weak Gravity Conjecture and Scalar Fields," *JHEP* **08** (2017) 034, arXiv:1705.04328 [hep-th].

[344] D. Lust and E. Palti, "Scalar Fields, Hierarchical UV/IR Mixing and The Weak Gravity Conjecture," *JHEP* **02** (2018) 040, arXiv:1709.01790 [hep-th].

[345] M. Montero, G. Shiu, and P. Soler, "The Weak Gravity Conjecture in three dimensions," *JHEP* **10** (2016) 159, arXiv:1606.08438 [hep-th].

[346] M. Reece, "Photon Masses in the Landscape and the Swampland," arXiv:1808.09966 [hep-th].

[347] S.-J. Lee, W. Lerche, and T. Weigand, "Tensionless Strings and the Weak Gravity Conjecture," arXiv:1808.05958 [hep-th].

[348] S.-J. Lee, W. Lerche, and T. Weigand, "A Stringy Test of the Scalar Weak Gravity Conjecture," *Nucl. Phys.* **B938** (2019) 321–350, arXiv:1810.05169 [hep-th].

[349] Q. Bonnefoy, E. Dudas, and S. Lüst, "On the weak gravity conjecture in string theory with broken supersymmetry," arXiv:1811.11199 [hep-th].

[350] D. Klaewer, D. Lüst, and E. Palti, "A Spin-2 Conjecture on the Swampland," *Fortsch. Phys.* **67** no. 1-2, (2019) 1800102, arXiv:1811.07908 [hep-th].

[351] Y. Hamada, T. Noumi, and G. Shiu, "Weak Gravity Conjecture from Unitarity and Causality," *Phys. Rev. Lett.* **123** no. 5, (2019) 051601, arXiv:1810.03637 [hep-th].

[352] C. De Rham, L. Heisenberg, and A. J. Tolley, "Spin-2 and the Weak Gravity Conjecture," arXiv:1812.01012 [hep-th].

[353] A. Hebecker, S. Leonhardt, J. Moritz, and A. Westphal, "Thraxions: Ultralight Throat Axions," *JHEP* **04** (2019) 158, arXiv:1812.03999 [hep-th].

[354] N. Craig, I. Garcia Garcia, and S. Koren, "Discrete Gauge Symmetries and the Weak Gravity Conjecture," *JHEP* **05** (2019) 140, arXiv:1812.08181 [hep-th].

[355] B. Bellazzini, M. Lewandowski, and J. Serra, "Amplitudes' Positivity, Weak Gravity Conjecture, and Modified Gravity," arXiv:1902.03250 [hep-th].

[356] A. Pal and Q. Shafi, "Supersymmetric $SU(5) \times U(1)_\chi$ and the weak gravity conjecture," *Phys. Rev.* **D100** no. 4, (2019) 043526, arXiv:1903.05703 [hep-ph].

[357] G. J. Loges, T. Noumi, and G. Shiu, "Thermodynamics of 4D Dilatonic Black Holes and the Weak Gravity Conjecture," arXiv:1909.01352 [hep-th].

[358] C. Cheung, J. Liu, and G. N. Remmen, "Entropy Bounds on Effective Field Theory from Rotating Dyonic Black Holes," *Phys. Rev.* **D100** no. 4, (2019) 046003, arXiv:1903.09156 [hep-th].

[359] S. Brahma and M. W. Hossain, "Relating the scalar weak gravity conjecture and the swampland distance conjecture for an accelerating universe," arXiv:1904.05810 [hep-th].

[360] N. Craig, I. Garcia Garcia, and S. Koren, "The Weak Scale from Weak Gravity," arXiv:1904.08426 [hep-ph].

[361] J. Kaplan and S. Kundu, "A Species or Weak-Gravity Bound for Large N Gauge Theories Coupled to Gravity," arXiv:1904.09294 [hep-th].

[362] S. Shirai and M. Yamazaki, "Is Gravity the Weakest Force?," arXiv:1904.10577 [hep-th].

[363] M. Mirbabayi, "A Weak Gravity Theorem," arXiv:1905.02736 [hep-th].

[364] P. Nath and M. Piskunov, "Enhancement of the Axion Decay Constant in Inflation and the Weak Gravity Conjecture," arXiv:1906.02764 [hep-ph].

[365] A. M. Charles, "The Weak Gravity Conjecture, RG Flows, and Supersymmetry," arXiv:1906.07734 [hep-th].

[366] E. Gonzalo and L. E. Ibáñez, "A Strong Scalar Weak Gravity Conjecture and Some Implications," *JHEP* **08** (2019) 118, arXiv:1903.08878 [hep-th].

[367] S.-J. Lee, W. Lerche, and T. Weigand, "Modular Fluxes, Elliptic Genera, and Weak Gravity Conjectures in Four Dimensions," *JHEP* **08** (2019) 104, arXiv:1901.08065 [hep-th].

[368] S.-J. Lee, W. Lerche, and T. Weigand, "Emergent Strings, Duality and Weak Coupling Limits for Two-Form Fields," arXiv:1904.06344 [hep-th].

[369] L. Aalsma, A. Cole, and G. Shiu, "Weak Gravity Conjecture, Black Hole Entropy, and Modular Invariance," arXiv:1905.06956 [hep-th]. [JHEP08,022(2019)].

[370] B. Heidenreich, M. Reece, and T. Rudelius, "Repulsive Forces and the Weak Gravity Conjecture," arXiv:1906.02206 [hep-th].

[371] Y. C. Ong and Y. Yao, "Charged Particle Production Rate from Cosmic Censorship in Dilaton Black Hole Spacetimes," arXiv:1907.07490 [gr-qc].

[372] S. de Alwis, A. Eichhorn, A. Held, J. M. Pawlowski, M. Schiffer, and F. Versteegen, "Asymptotic safety, string theory and the weak gravity conjecture," arXiv:1907.07894 [hep-th].

[373] G. N. Remmen and N. L. Rodd, "Consistency of the Standard Model Effective Field Theory," arXiv:1908.09845 [hep-ph].

[374] C. R. T. Jones and B. McPeak, "The Black Hole Weak Gravity Conjecture with Multiple Charges," arXiv:1908.10452 [hep-th].

[375] G. Goon and R. Penco, "A Universal Relation Between Corrections to Entropy and Extremality," arXiv:1909.05254 [hep-th].

[376] P. A. Cano, T. Ortin, and P. F. Ramirez, "On the extremality bound of stringy black holes," arXiv:1909.08530 [hep-th].

[377] Y. C. Ong, "The Charge of Electron, Weak Gravity Conjecture and Black Hole Evolution," arXiv:1909.09977 [gr-qc].

[378] B. Heidenreich, M. Reece, and T. Rudelius, "The Weak Gravity Conjecture and Emergence from an Ultraviolet Cutoff," *Eur. Phys. J.* **C78** no. 4, (2018) 337, arXiv:1712.01868 [hep-th].

[379] T. W. Grimm, E. Palti, and I. Valenzuela, "Infinite Distances in Field Space and Massless Towers of States," *JHEP* **08** (2018) 143, arXiv:1802.08264 [hep-th].

[380] B. Heidenreich, M. Reece, and T. Rudelius, "Emergence of Weak Coupling at Large Distance in Quantum Gravity," *Phys. Rev. Lett.* **121** no. 5, (2018) 051601, arXiv:1802.08698 [hep-th].

[381] T. W. Grimm, C. Li, and E. Palti, "Infinite Distance Networks in Field Space and Charge Orbits," arXiv:1811.02571 [hep-th].

[382] W.-C. Lin and W. H. Kinney, "Consistency of Tachyacoustic Cosmology with de Sitter Swampland Conjectures," arXiv:1812.04447 [astro-ph.CO].

[383] G. Buratti, J. Calderón, and A. M. Uranga, "Transplanckian axion monodromy!?," *JHEP* **05** (2019) 176, arXiv:1812.05016 [hep-th].

[384] A. Hebecker, D. Junghans, and A. Schachner, "Large Field Ranges from Aligned and Misaligned Winding," *JHEP* **03** (2019) 192, arXiv:1812.05626 [hep-th].

[385] E. Gonzalo, L. E. Ibanez, and A. M. Uranga, "Modular Symmetries and the Swampland Conjectures," *JHEP* **05** (2019) 105, arXiv:1812.06520 [hep-th].

[386] M. Scalisi and I. Valenzuela, "Swampland distance conjecture, inflation and α-attractors," *JHEP* **08** (2019) 160, arXiv:1812.07558 [hep-th].

[387] F. Marchesano and M. Wiesner, "Instantons and infinite distances," *JHEP* **08** (2019) 088, arXiv:1904.04848 [hep-th].

[388] A. Kehagias, D. Lust, and S. Lüst, "Swampland, Gradient Flow and Infinite Distance," arXiv:1910.00453 [hep-th].

[389] P. Corvilain, T. W. Grimm, and I. Valenzuela, "The Swampland Distance Conjecture for Kähler moduli," *JHEP* **08** (2019) 075, arXiv:1812.07548 [hep-th].

[390] A. Font, A. Herraez, and L. E. Ibanez, "The Swampland Distance Conjecture and Towers of Tensionless Branes," *JHEP* **08** (2019) 044, arXiv:1904.05379 [hep-th].

[391] T. W. Grimm and D. Van De Heisteeg, "Infinite Distances and the Axion Weak Gravity Conjecture," arXiv:1905.00901 [hep-th].

[392] D. Luest, E. Palti, and C. Vafa, "AdS and the Swampland," *Phys. Lett.* **B797** (2019) 134867, arXiv:1906.05225 [hep-th].

[393] W. H. Kinney, "Eternal Inflation and the Refined Swampland Conjecture," *Phys. Rev. Lett.* **122** no. 8, (2019) 081302, arXiv:1811.11698 [astro-ph.CO].

[394] G. Goswami, "Enhancement of axion decay constants in type IIA theory?," *Phys. Rev.* **D100** no. 6, (2019) 066009, arXiv:1812.11909 [hep-th].

[395] N. Kaloper, "Dark Energy, H_0 and Weak Gravity Conjecture," arXiv:1903.11676 [hep-th].

[396] C. Gómez, "Gravity as universal UV completion: Towards a unified view of Swampland conjectures," arXiv:1907.13386 [hep-th].

[397] M. Montero, "A Holographic Derivation of the Weak Gravity Conjecture," *JHEP* **03** (2019) 157, arXiv:1812.03978 [hep-th].

[398] T. Banks, "The Top 10^{500} Reasons Not to Believe in the Landscape," arXiv:1208.5715 [hep-th].

[399] T. Crisford and J. E. Santos, "Violating the Weak Cosmic Censorship Conjecture in Four-Dimensional Anti–de Sitter Space," *Phys. Rev. Lett.* **118** no. 18, (2017) 181101, arXiv:1702.05490 [hep-th].

[400] T. Crisford, G. T. Horowitz, and J. E. Santos, "Testing the Weak Gravity - Cosmic Censorship Connection," *Phys. Rev.* **D97** no. 6, (2018) 066005, arXiv:1709.07880 [hep-th].

[401] G. T. Horowitz and J. E. Santos, "Further evidence for the weak gravity — cosmic censorship connection," *JHEP* **06** (2019) 122, arXiv:1901.11096 [hep-th].

[402] R. Bousso and S. W. Hawking, "Pair creation of black holes during inflation," *Phys. Rev.* **D54** (1996) 6312–6322, arXiv:gr-qc/9606052 [gr-qc].

[403] R. Bousso, "Charged Nariai black holes with a dilaton," *Phys. Rev.* **D55** (1997) 3614–3621, arXiv:gr-qc/9608053 [gr-qc].

[404] R. Bousso and S. W. Hawking, "(Anti)evaporation of Schwarzschild-de Sitter black holes," *Phys. Rev.* **D57** (1998) 2436–2442, arXiv:hep-th/9709224 [hep-th].

[405] R. Bousso, "Proliferation of de Sitter space," *Phys. Rev.* **D58** (1998) 083511, arXiv:hep-th/9805081 [hep-th].

[406] R. Bousso, "Quantum global structure of de Sitter space," *Phys. Rev.* **D60** (1999) 063503, arXiv:hep-th/9902183 [hep-th].

[407] R. Bousso, "Adventures in de Sitter space," in *The future of theoretical physics and cosmology: Celebrating Stephen Hawking's 60th birthday. Proceedings, Workshop and Symposium, Cambridge, UK, January 7-10, 2002*, pp. 539–569. 2002. arXiv:hep-th/0205177 [hep-th].

[408] F. Belgiorno and S. L. Cacciatori, "Massive Dirac particles on the background of charged de-Sitter black hole manifolds," *Phys. Rev.* **D79** (2009) 124024, arXiv:0810.1642 [gr-qc].

[409] F. Belgiorno, S. L. Cacciatori, and F. Dalla Piazza, "Pair-production of charged Dirac particles on charged Nariai and ultracold black hole manifolds," *JHEP* **08** (2009) 028, arXiv:0906.1520 [gr-qc].

[410] F. Belgiorno, S. L. Cacciatori, and F. Dalla Piazza, "Quantum instability for charged scalar particles on charged Nariai and ultracold black hole manifolds," *Class. Quant. Grav.* **27** (2010) 055011, arXiv:0909.1454 [gr-qc].

[411] S. P. Kim, "Schwinger Effect in (A)dS and Charged Black Hole," in *Proceedings, 12th International Conference on Gravitation, Astrophysics and Cosmology (ICGAC-12): Moscow, Russia, June 28-July 5, 2015*, pp. 79–85. 2016. arXiv:1509.05532 [hep-th].

[412] F. Belgiorno, S. L. Cacciatori, and F. Dalla Piazza, "Tunneling method for Hawking radiation in the Nariai case," *Gen. Rel. Grav.* **49** no. 8, (2017) 109, arXiv:1602.09047 [gr-qc].

[413] W. A. Hiscock and L. D. Weems, "Evolution of Charged Evaporating Black Holes," *Phys. Rev.* **D41** (1990) 1142.

[414] O. J. C. Dias, H. S. Reall, and J. E. Santos, "Strong cosmic censorship: taking the rough with the smooth," *JHEP* **10** (2018) 001, arXiv:1808.02895 [gr-qc].

[415] J. Dinsmore, P. Draper, D. Kastor, Y. Qiu, and J. Traschen, "Schottky Anomaly of deSitter Black Holes," arXiv:1907.00248 [hep-th].

[416] C. V. Johnson, "de Sitter Black Holes, Schottky Peaks, and Continuous Heat Engines," arXiv:1907.05883 [hep-th].

[417] H. Nariai, "On some static solutions of einstein's gravitational field equations in a spherically symmetric case," *General Relativity and Gravitation* **31** (1999) 951–961.

[418] L. J. Romans, "Supersymmetric, cold and lukewarm black holes in cosmological Einstein-Maxwell theory," *Nucl. Phys.* **B383** (1992) 395–415, arXiv:hep-th/9203018 [hep-th].

[419] O. J. C. Dias, H. S. Reall, and J. E. Santos, "Strong cosmic censorship for charged de Sitter black holes with a charged scalar field," *Class. Quant. Grav.* **36** no. 4, (2019) 045005, arXiv:1808.04832 [gr-qc].

[420] J. Kevorkian and J. Cole, *Multiple Scale and Singular Perturbation Methods*. Applied mathematical sciences. Springer, 1996. https://books.google.com/books?id=ehRtPwAACAAJ.

[421] T. D. Cohen and D. A. McGady, "The Schwinger mechanism revisited," *Phys. Rev.* **D78** (2008) 036008, arXiv:0807.1117 [hep-ph].

[422] A. I. Nikishov, "Barrier scattering in field theory removal of klein paradox," *Nucl. Phys.* **B21** (1970) 346–358.

[423] C. Itzykson and J. Zuber, *Quantum Field Theory*. Dover Books on Physics. Dover Publications, 2012. https://books.google.com/books?id=CxYCMNrUnTEC.

[424] J. A. Harvey, "TASI 2003 lectures on anomalies," 2005. arXiv:hep-th/0509097 [hep-th].

[425] R. Bousso and S. W. Hawking, "Primordial black holes: Pair creation, Lorentzian condition, and evaporation," *Int. J. Theor. Phys.* **38** (1999) 1227–1252.

[426] M. B. Fröb, J. Garriga, S. Kanno, M. Sasaki, J. Soda, T. Tanaka, and A. Vilenkin, "Schwinger effect in de Sitter space," *JCAP* **1404** (2014) 009, arXiv:1401.4137 [hep-th].

[427] D. Anninos and T. Anous, "A de Sitter Hoedown," *JHEP* **08** (2010) 131, arXiv:1002.1717 [hep-th].

[428] T. Banks, "de sitter space and the swampland." Navigating the Swampland, IFT Madrid, 2019, year.

[429] T. Banks, "To appear." 2019.

[430] J. C. Niemeyer and R. Bousso, "The Nonlinear evolution of de Sitter space instabilities," *Phys. Rev.* **D62** (2000) 023503, arXiv:gr-qc/0004004 [gr-qc].

[431] T. Banks, "Some thoughts on the quantum theory of de sitter space," in *Cosmic inflation. Proceedings, Meeting, Davis, USA, March 22-25, 2003*. 2003. arXiv:astro-ph/0305037 [astro-ph].

[432] T. Banks, "Some thoughts on the quantum theory of stable de Sitter space," arXiv:hep-th/0503066 [hep-th].

[433] L. Ibáñez and A. Uranga, *String Theory and Particle Physics: An Introduction to String Phenomenology*. Cambridge University Press, 2012. https://books.google.com/books?id=MIkhAwAAQBAJ.

[434] S. S. Gubser, "Curvature singularities: The Good, the bad, and the naked," *Adv. Theor. Math. Phys.* **4** (2000) 679–745, arXiv:hep-th/0002160 [hep-th].

[435] R. M. Wald, "Gravitational collapse and cosmic censorship," in *Black Holes, Gravitational Radiation and the Universe: Essays in Honor of C.V. Vishveshwara*, pp. 69–85. 1997. arXiv:gr-qc/9710068 [gr-qc].

[436] P. H. Ginsparg and M. J. Perry, "Semiclassical Perdurance of de Sitter Space," *Nucl. Phys.* **B222** (1983) 245–268.

[437] O. J. C. Dias and J. P. S. Lemos, "Pair creation of higher dimensional black holes on a de Sitter background," *Phys. Rev.* **D70** (2004) 124023, arXiv:hep-th/0410279 [hep-th].

[438] G. F. Giudice, "Naturally Speaking: The Naturalness Criterion and Physics at the LHC," arXiv:0801.2562 [hep-ph].

[439] C. Csáki and P. Tanedo, "Beyond the Standard Model," in *Proceedings, 2013 European School of High-Energy Physics (ESHEP 2013): Paradfurdo, Hungary, June 5-18, 2013*, pp. 169–268. 2015. arXiv:1602.04228 [hep-ph].

[440] G. F. Giudice, "The Dawn of the Post-Naturalness Era," in *From My Vast Repertoire ...: Guido Altarelli's Legacy*, A. Levy, S. Forte, and G. Ridolfi, eds., pp. 267–292. World Scientific, 2019. arXiv:1710.07663 [physics.hist-ph].

[441] L. E. Ibanez, V. Martin-Lozano, and I. Valenzuela, "Constraining the EW Hierarchy from the Weak Gravity Conjecture," arXiv:1707.05811 [hep-th].

[442] S. Davidson, S. Hannestad, and G. Raffelt, "Updated bounds on millicharged particles," *JHEP* **05** (2000) 003, arXiv:hep-ph/0001179 [hep-ph].

[443] J. Preskill, "Cosmological Production of Superheavy Magnetic Monopoles," *Phys. Rev. Lett.* **43** (1979) 1365.

[444] Ya. B. Zeldovich and M. Yu. Khlopov, "On the Concentration of Relic Magnetic Monopoles in the Universe," *Phys. Lett.* **79B** (1978) 239–241.

[445] X. Chen, Y. Wang, and Z.-Z. Xianyu, "Loop Corrections to Standard Model Fields in Inflation," *JHEP* **08** (2016) 051, arXiv:1604.07841 [hep-th].

[446] D. Baumann, "Inflation," in *Physics of the large and the small, TASI 09, proceedings of the Theoretical Advanced Study Institute in Elementary Particle Physics, Boulder, Colorado, USA, 1-26 June 2009*, pp. 523–686. 2011. arXiv:0907.5424 [hep-th].

[447] A. Maleknejad, M. M. Sheikh-Jabbari, and J. Soda, "Gauge Fields and Inflation," *Phys. Rept.* **528** (2013) 161–261, arXiv:1212.2921 [hep-th].

[448] R. K. Jain and M. S. Sloth, "On the non-Gaussian correlation of the primordial curvature perturbation with vector fields," *JCAP* **1302** (2013) 003, arXiv:1210.3461 [astro-ph.CO].

[449] N. Barnaby, R. Namba, and M. Peloso, "Observable non-gaussianity from gauge field production in slow roll inflation, and a challenging connection with magnetogenesis," *Phys. Rev.* **D85** (2012) 123523, arXiv:1202.1469 [astro-ph.CO].

[450] J. Rubio, "Higgs inflation," *Front. Astron. Space Sci.* **5** (2019) 50, arXiv:1807.02376 [hep-ph].

[451] P. Draper and S. Farkas, "Gravitational Instabilities and Censorship of Large Scalar Field Excursions," *JHEP* **05** (2019) 158, arXiv:1901.00515 [hep-th].

[452] M. Ragsdale and D. Singleton, "Schwinger effect for non-Abelian gauge bosons," *J. Phys. Conf. Ser.* **883** no. 1, (2017) 012014, arXiv:1708.09753 [hep-th].

[453] F. Denef and S. A. Hartnoll, "Landscape of superconducting membranes," *Phys. Rev.* **D79** (2009) 126008, arXiv:0901.1160 [hep-th].

[454] M. Montero, "Are tiny gauge couplings out of the Swampland?," *JHEP* **10** (2017) 208, arXiv:1708.02249 [hep-th].

[455] C. Gogolin and J. Eisert, "Equilibration, thermalisation, and the emergence of statistical mechanics in closed quantum systems," *Rept. Prog. Phys.* **79** no. 5, (2016) 056001, arXiv:1503.07538 [quant-ph].

[456] K. Jensen, "Chaos in AdS_2 Holography," *Phys. Rev. Lett.* **117** no. 11, (2016) 111601, arXiv:1605.06098 [hep-th].

[457] J. Maldacena, D. Stanford, and Z. Yang, "Conformal symmetry and its breaking in two dimensional Nearly Anti-de-Sitter space," *PTEP* **2016** no. 12, (2016) 12C104, arXiv:1606.01857 [hep-th].

[458] J. Cotler, K. Jensen, and A. Maloney, "Low-dimensional de Sitter quantum gravity," arXiv:1905.03780 [hep-th].

[459] J. Maldacena, G. J. Turiaci, and Z. Yang, "Two dimensional Nearly de Sitter gravity," arXiv:1904.01911 [hep-th].

[460] D. Anninos, D. M. Hofman, and J. Kruthoff, "Charged Quantum Fields in AdS_2," arXiv:1906.00924 [hep-th].

[461] P. Saad, S. H. Shenker, and D. Stanford, "JT gravity as a matrix integral," arXiv:1903.11115 [hep-th].

[462] T. Van Riet, "On classical de Sitter solutions in higher dimensions," *Class. Quant. Grav.* **29** (2012) 055001, arXiv:1111.3154 [hep-th].

[463] D. Andriot, "Open problems on classical de Sitter solutions," *Fortsch. Phys.* **67** no. 7, (2019) 1900026, arXiv:1902.10093 [hep-th].

[464] D. Junghans, "Weakly Coupled de Sitter Vacua with Fluxes and the Swampland," *JHEP* **03** (2019) 150, arXiv:1811.06990 [hep-th].

[465] C. Caviezel, P. Koerber, S. Kors, D. Lust, T. Wrase, and M. Zagermann, "On the Cosmology of Type IIA Compactifications on SU(3)-structure Manifolds," *JHEP* **04** (2009) 010, arXiv:0812.3551 [hep-th].

[466] C. Roupec and T. Wrase, "de Sitter extrema and the swampland," arXiv:1807.09538 [hep-th].

[467] U. H. Danielsson, S. S. Haque, G. Shiu, and T. Van Riet, "Towards Classical de Sitter Solutions in String Theory," *JHEP* **09** (2009) 114, arXiv:0907.2041 [hep-th].

[468] T. Wrase and M. Zagermann, "On Classical de Sitter Vacua in String Theory," *Fortsch. Phys.* **58** (2010) 906–910, arXiv:1003.0029 [hep-th].

[469] E. Silverstein, "Simple de Sitter Solutions," *Phys. Rev.* **D77** (2008) 106006, arXiv:0712.1196 [hep-th].

[470] M. P. Hertzberg, S. Kachru, W. Taylor, and M. Tegmark, "Inflationary Constraints on Type IIA String Theory," *JHEP* **12** (2007) 095, arXiv:0711.2512 [hep-th].

[471] T. W. Grimm, C. Li, and I. Valenzuela, "Asymptotic Flux Compactifications and the Swampland," arXiv:1910.09549 [hep-th].

[472] C. Córdova, G. B. De Luca, and A. Tomasiello, "New de Sitter Solutions in Ten Dimensions and Orientifold Singularities," arXiv:1911.04498 [hep-th].

[473] N. Cribiori and D. Junghans, "No classical (anti-)de Sitter solutions with O8-planes," *Phys. Lett.* **B793** (2019) 54–58, arXiv:1902.08209 [hep-th].

[474] S. R. Green, E. J. Martinec, C. Quigley, and S. Sethi, "Constraints on String Cosmology," *Class. Quant. Grav.* **29** (2012) 075006, arXiv:1110.0545 [hep-th].

[475] F. F. Gautason, D. Junghans, and M. Zagermann, "On Cosmological Constants from alpha'-Corrections," *JHEP* **06** (2012) 029, arXiv:1204.0807 [hep-th].

[476] D. Kutasov, T. Maxfield, I. Melnikov, and S. Sethi, "Constraining de Sitter Space in String Theory," *Phys. Rev. Lett.* **115** no. 7, (2015) 071305, arXiv:1504.00056 [hep-th].

[477] C. Quigley, "Gaugino Condensation and the Cosmological Constant," *JHEP* **06** (2015) 104, arXiv:1504.00652 [hep-th].

[478] R. R. Caldwell, R. Dave, and P. J. Steinhardt, "Cosmological imprint of an energy component with general equation of state," *Phys. Rev. Lett.* **80** (1998) 1582–1585, arXiv:astro-ph/9708069 [astro-ph].

[479] P. J. E. Peebles and B. Ratra, "The Cosmological constant and dark energy," *Rev. Mod. Phys.* **75** (2003) 559–606, arXiv:astro-ph/0207347 [astro-ph]. [,592(2002)].

[480] E. J. Copeland, M. Sami, and S. Tsujikawa, "Dynamics of dark energy," *Int. J. Mod. Phys.* **D15** (2006) 1753–1936, arXiv:hep-th/0603057 [hep-th].

[481] A. Hebecker, T. Skrzypek, and M. Wittner, "The F-term Problem and other Challenges of Stringy Quintessence," arXiv:1909.08625 [hep-th].

[482] Y. Olguin-Trejo, S. L. Parameswaran, G. Tasinato, and I. Zavala, "Runaway Quintessence, Out of the Swampland," *JCAP* **1901** no. 01, (2019) 031, arXiv:1810.08634 [hep-th].

[483] T. Damour and F. Dyson, "The Oklo bound on the time variation of the fine structure constant revisited," *Nucl. Phys.* **B480** (1996) 37–54, arXiv:hep-ph/9606486 [hep-ph].

[484] **Particle Data Group** Collaboration, M. Tanabashi *et al.*, "Review of Particle Physics," *Phys. Rev.* **D98** no. 3, (2018) 030001.

[485] S. K. Garg, C. Krishnan, and M. Zaid Zaz, "Bounds on Slow Roll at the Boundary of the Landscape," *JHEP* **03** (2019) 029, arXiv:1810.09406 [hep-th].

[486] F. Denef, A. Hebecker, and T. Wrase, "The dS swampland conjecture and the Higgs potential," arXiv:1807.06581 [hep-th].

[487] N. Arkani-Hamed, S. Dubovsky, A. Nicolis, and G. Villadoro, "Quantum Horizons of the Standard Model Landscape," *JHEP* **06** (2007) 078, arXiv:hep-th/0703067 [HEP-TH].

[488] Y. Hamada and G. Shiu, "Weak Gravity Conjecture, Multiple Point Principle and the Standard Model Landscape," *JHEP* **11** (2017) 043, arXiv:1707.06326 [hep-th].

[489] D. Andriot, "On the de Sitter swampland criterion," arXiv:1806.10999 [hep-th].

[490] D. Andriot and C. Roupec, "Further refining the de Sitter swampland conjecture," *Fortsch. Phys.* **67** no. 1-2, (2019) 1800105, arXiv:1811.08889 [hep-th].

[491] L. Alvarez-Gaume, P. H. Ginsparg, G. W. Moore, and C. Vafa, "An O(16) x O(16) Heterotic String," *Phys. Lett.* **B171** (1986) 155–162.

[492] A. R. Brown and A. Dahlen, "Spectrum and stability of compactifications on product manifolds," *Phys. Rev.* **D90** no. 4, (2014) 044047, arXiv:1310.6360 [hep-th].

[493] R. Blumenhagen, "Large Field Inflation/Quintessence and the Refined Swampland Distance Conjecture," *PoS* **CORFU2017** (2018) 175, arXiv:1804.10504 [hep-th].

[494] F. F. Gautason, M. Schillo, T. Van Riet, and M. Williams, "Remarks on scale separation in flux vacua," *JHEP* **03** (2016) 061, arXiv:1512.00457 [hep-th].

[495] J. M. Arnold, B. Fornal, and M. B. Wise, "Standard Model Vacua for Two-dimensional Compactifications," *JHEP* **12** (2010) 083, arXiv:1010.4302 [hep-th].

[496] **Planck** Collaboration, P. A. R. Ade *et al.*, "Planck 2015 results. XIV. Dark energy and modified gravity," *Astron. Astrophys.* **594** (2016) A14, arXiv:1502.01590 [astro-ph.CO].

[497] S. M. Carroll, "Quintessence and the rest of the world," *Phys. Rev. Lett.* **81** (1998) 3067–3070, arXiv:astro-ph/9806099 [astro-ph].

[498] M. A. Vasiliev, "De Sitter Supergravity With Positive Cosmological Constant and Generalized Lie Superalgebras," *Class. Quant. Grav.* **2** (1985) 645–659.

[499] R. Blumenhagen, M. Brinkmann, A. Makridou, L. Schlechter, and M. Traube, "dS Spaces and Brane Worlds in Exotic String Theories," arXiv:2002.11746 [hep-th].

[500] J. Blaback, U. H. Danielsson, D. Junghans, T. Van Riet, T. Wrase, and M. Zagermann, "Smeared versus localised sources in flux compactifications," *JHEP* **12** (2010) 043, arXiv:1009.1877 [hep-th].

[501] W. Rudin, *Real and complex analysis*. Mathematics series. McGraw-Hill, 1987. https://books.google.com/books?id=Z_fuAAAAMAAJ.

[502] S. W. Hawking and G. F. R. Ellis, *The Large Scale Structure of Space-Time*. Cambridge Monographs on Mathematical Physics. Cambridge University Press, 2011.

[503] J. M. M. Senovilla and D. Garfinkle, "The 1965 Penrose singularity theorem," *Class. Quant. Grav.* **32** no. 12, (2015) 124008, arXiv:1410.5226 [gr-qc].

[504] G. F. R. Ellis, "Closed trapped surfaces in cosmology," *Gen. Rel. Grav.* **35** (2003) 1309–1319, arXiv:gr-qc/0304039 [gr-qc].

[505] P. Candelas, "Vacuum Polarization in Schwarzschild Space-Time," *Phys. Rev.* **D21** (1980) 2185–2202.

[506] M. S. Bremer, M. J. Duff, H. Lu, C. N. Pope, and K. S. Stelle, "Instanton cosmology and domain walls from M theory and string theory," *Nucl. Phys.* **B543** (1999) 321–364, arXiv:hep-th/9807051 [hep-th].



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