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Compact Stars in *F*(*R*) Gravity Theory

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Abstract: $F(R)$ gravity is a modified gravity theory, and its applications for the compact star have attracted attention in the last decades. We review the basics of the $F(R)$ gravity theory and the modified Tolman-Oppenheimer-Volkoff (TOV) equation. Recent studies show that the model dependence of equation of state (EOS) and modification of gravity degenerate to each other, which suggests the mass-radius (M-R) relation of the compact star alone cannot completely determine the EOS of the inner matter. Moreover, the effects of a new scalar field predicted in $F(R)$ gravity on both the internal and external structure of the compact star are illustrated in the benchmark R^2 model. Finally, We discuss the future directions for testing gravitational theories by observational measurements of the compact stars.

Key words: modified gravity; compact star; TOV equation

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0 Introduction

Despite plenty of success in the standard model (SM) of particle physics based on quantum field theory and that of cosmology based on general relativity (GR), there are unsolved problems in a wide range of energy scales, from the dark side of the Universe to quantum gravity. In search for new physics, gravitational theories alternative to GR, known as modified gravity, have been intensively investigated so far. The Lovelock theorem states that GR is a unique metric theory whose equation of motion includes up to the second-order derivatives in four-dimensional spacetime[1−2]. However, various modified gravity theories, which violate assumptions in the Lovelock theorem, have been studied by modifying the Einstein-Hilbert action and including, for instance, the higher derivatives of the metric and new gravitational degrees of freedom[3−9]. Moreover, from the perspective of cosmological observations, the recent Hubble tension[10−12] and Dark Energy Spectroscopic Instrument (DESI) result[13−14] motivate us to investigate the cosmological models based on the modified gravity theories.

$F(R)$ gravity is one of the modified gravity theories[15−17], where the Einstein-Hilbert action in GR is replaced with a function of the Ricci scalar R, $F(R)$. This modification introduces an additional scalar degree of freedom (DOF), and it is known that the $F(R)$ gravity theory is equivalent to the scalar-tensor theory[18] that describes GR coupled with the scalar field. This scalar field can derive the accelerated expansion of the Universe, and $F(R)$ gravity theory has been investigated in the context of the dark energy[19−22] and the inflaton[23−24]. Moreover, from the perspective of particle physics, the new scalar field has also been examined as the dark matter candidate[25−27] and the new particle beyond SM[28−31].

Compact stars provide a multi-disciplinary research field in high-energy physics involving astrophysics, particle physics, and gravitational physics. In GR, the compact star is described by the Tolman-Oppenheimer-Volkoff (TOV) equation on the static and spherically symmetric spacetime. Solving the TOV equation, we can compute the M-R relation of the compact star for a given EOS, and inversely, the observations of the M-R relation can constrain the EOS[32−36]. Recently, the study of compact stars has been one of the main interests in modified gravity theories. Observations of phenomena around compact stars allow us to test gravitational theories in a strong and nonperturbative gravitational-field environment through the electromagnetic signals and gravitational waves[37−38].

The modified gravity predicts the modified TOV equa-

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tion and thus the different M-R relation, allowing us to distinguish the gravitational theories by the astrophysical observations. Thus, compact star physics is one of the phenomenological applications of the modified gravity theory. In $F(R)$ gravity theory, the *M*-*R* relations have been well studied for the neutron star (NS)[39−46] as well as for the quark stars and the white dwarfs[47−48]. Existing works have shown that the maximum mass of the compact star can increase as the modification of gravity becomes significant, which may explain the observed existence of massive NS with $\sim 2.0\,M_\odot$ [49−50] from the viewpoint of the gravitational theory.

In this article, we briefly introduce the basic properties of the $F(R)$ gravity in Sections 1 and 2 and the derivation of the modified TOV equation in Section 3. Then, we review the following two points based on recent results[51−52] in Sections 4~7. First, from theoretical aspects of $F(R)$ gravity, measuring the *M*-*R* relation cannot completely determine the EOS in the $F(R)$ gravity theory because the ambiguities in models of EOS and $F(R)$ degenerate with each other. Moreover, because the vacuum solution for the spherically symmetric spacetime in the $F(R)$ gravity is not unique, the boundary conditions are to be handled carefully. Second, from phenomenological aspects of the new scalar field, the scalar field influences the internal and external structure of the compact star. The existence of the scalar field can be a unique signal distinguishing $F(R)$ gravity from GR.

1 $F(R)$ gravity theory and scalaron field

The action of $F(R)$ gravity is defined as follows

$$S = \frac{1}{2\kappa^2}\int \mathrm{d}^4x\,\sqrt{-g}F(R) + S_M\,. \tag{1}$$

$g_{\mu\nu}$ is the metric, and $g = \det(g_{\mu\nu}) \cdot \kappa^2 = 8\pi G$ where G is Newton's gravitational constant, and S_M is the matter action. Performing the variation of the action (1) with respect to the metric, we obtain the field equation:

$$\kappa^2 T_{\mu\nu} = F_R(R)R_{\mu\nu} - \frac{1}{2}F(R)\,g_{\mu\nu} + (g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu)\,F_R(R)\,. \tag{2}$$

We denote the derivative with respect to R by the subscript, $F_R = \mathrm{d}F/\mathrm{d}R$. The matter energy-momentum tensor is defined by the matter action S_M as

$$T_{\mu\nu} = \frac{-2}{\sqrt{-g}}\frac{\delta S_M}{\delta g^{\mu\nu}}\,. \tag{3}$$

Operating the covariant derivative to Eq. (2), we can show the conservation law $\nabla^\mu T_{\mu\nu} = 0$; that is,

$$\nabla^\mu\left(R_{\mu\nu}F_R - \frac{1}{2}g_{\mu\nu}F + g_{\mu\nu}\Box F_R - \nabla_\mu\nabla_\nu F_R\right) = 0\,. \tag{4}$$

Taking trace of Eq. (2), we find

$$\Box F_R(R) = \frac{1}{3}\left[2F(R) - RF_R(R) + \kappa^2 T\right], \tag{5}$$

where T represents the trace of energy-momentum tensor $T = T^\mu{}_\mu$. The trace-part Eq. (5) takes nontrivial form even for the vacuum $T = 0$, instead of $R = 0$ in GR. Thus, Birkhoff theorem[53], which states the Schwarzschild solution is a unique solution for a spherically symmetric and vacuum system in GR, is generally absent in the $F(R)$ gravity. Consequently, in-vacuum, static, and spherically symmetric solutions with nontrivial curvature distribution are allowed[54]. This fact stems from the additional scalar DOF, which we call the scalaron field.

It is well known that the $F(R)$ gravity can be rewritten in the form of the scalar-tensor theory. Defining the scalaron field $\varPhi = F_R(R)$, we find that the action of the gravity sector in Eq. (1) is written as

$$S_G = \frac{1}{2\kappa^2}\int \mathrm{d}^4x\,\sqrt{-g}\left[\varPhi R - V(\varPhi)\right], \tag{6}$$

$$V(\varPhi) = R(\varPhi)\varPhi - F\big(R(\varPhi)\big)\,. \tag{7}$$

The field equations for the metric $g_{\mu\nu}$ and scalaron $\varPhi$ are

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{\kappa^2}{\varPhi}\left(T_{\mu\nu} + T^{(\varPhi)}_{\mu\nu}\right), \tag{8}$$

$$\Box\varPhi = \frac{1}{3}\left[\varPhi R(\varPhi) - 2V(\varPhi) + \kappa^2 T\right], \tag{9}$$

where $T^{(\varPhi)}_{\mu\nu}$ is the effective energy-momentum tensor of the scalaron field:

$$T^{(\varPhi)}_{\mu\nu} = \frac{1}{\kappa^2}\left[\nabla_\mu\nabla_\nu\varPhi - g_{\mu\nu}\Box\varPhi - \frac{1}{2}g_{\mu\nu}V(\varPhi)\right]. \tag{10}$$

Therefore, the gravitational field in the $F(R)$ gravity acts as GR with a non-minimally coupled scalar field, which possesses 2+1 DOF.

We can further reformulate the action (6) by the frame transformation:

$$\tilde{g}_{\mu\nu} = \varPhi g_{\mu\nu}\,,\ \varPhi \equiv \mathrm{e}^{2\sqrt{1/6}\kappa\varphi}\,. \tag{11}$$

For the original Jordan-frame metric $g_{\mu\nu}$, we call $\tilde{g}_{\mu\nu}$ the Einstein-frame metric after the transformation. We denote the scalaron field in the Einstein frame by φ. The gravitational action in the Einstein frame is written as

$$S_G = \int \mathrm{d}^4x\,\sqrt{-\tilde{g}}\left[\frac{1}{2\kappa^2}\tilde{R} - \frac{1}{2}\tilde{g}^{\mu\nu}\tilde{\nabla}_\mu\varphi\tilde{\nabla}_\nu\varphi - U(\varphi)\right], \tag{12}$$

$$U(\varphi) = \frac{1}{2\kappa^2}\mathrm{e}^{-4\sqrt{1/6}\kappa\varphi}V(\varphi)\,, \tag{13}$$

where $\tilde{R}$ and $\tilde{\nabla}$ are the Ricci scalar and covariant derivative composed by the Einstein-frame metric $\tilde{g}_{\mu\nu}$. The field equations for the metric $\tilde{g}_{\mu\nu}$ and the scalaron φ are

$$\tilde{R}_{\mu\nu}-\frac{1}{2}\tilde{R}\tilde{g}_{\mu\nu}=\kappa^2\left(\mathrm{e}^{-2\sqrt{1/6}\kappa\varphi}T_{\mu\nu}+T^{(\varphi)}_{\mu\nu}\right), \tag{14}$$

$$\tilde{\Box}\varphi=U'(\varphi)+\frac{\kappa}{\sqrt{6}}\mathrm{e}^{-4\sqrt{1/6}\kappa\varphi}T. \tag{15}$$

$T^{(\varphi)}_{\mu\nu}$ is the energy-momentum tensor of the scalaron field in the Einstein frame

$$T^{(\varphi)}_{\mu\nu}=-\frac{1}{2}\tilde{g}_{\mu\nu}\tilde{\nabla}^{\alpha}\varphi\tilde{\nabla}_{\alpha}\varphi+\tilde{\nabla}_{\mu}\varphi\tilde{\nabla}_{\nu}\varphi-\tilde{g}_{\mu\nu}U(\varphi). \tag{16}$$

Unlike Eq. (6) in the Jordan frame, Eq. (12) describes GR with a minimally coupled scalar field in the Einstein frame. It is known that the Jordan frame and Einstein frame are equivalent to each other at the classical level[55−57].

2 Chameleon mechanism

In both Jordan and Einstein frames, the scalaron field couples with the trace of matter energy-momentum tensor T as in Eqs. (9) and (15). This coupling introduces the environment-dependent effective mass of the scalaron field. In the Jordan frame, we define the effective potential $V_{\rm eff}(\Phi,T)$ including T:

$$\Box\Phi=\frac{\mathrm{d}V_{\rm eff}(\Phi,T)}{d\Phi}, \tag{17}$$

$$\frac{\mathrm{d}V_{\rm eff}(\Phi,T)}{\mathrm{d}\Phi}\equiv\frac{1}{3}\left[\Phi R(\Phi)-2V(\Phi)+\kappa^2T\right]. \tag{18}$$

$V_{\rm eff}$ explicitly depends on T, and we denote it by $V_{\rm eff}(\Phi,T)$. The mass is related to the second derivative or curvature of the effective potential given by

$$\begin{aligned}\frac{\mathrm{d}^2V_{\rm eff}}{\mathrm{d}\Phi^2}&\equiv\frac{1}{3}\frac{\mathrm{d}}{\mathrm{d}\Phi}\left[\Phi R(\Phi)-2V(\Phi)+\kappa^2T\right]\\&=\frac{1}{3}\left[\frac{F_R(R)}{F_{RR}(R)}-R\right].\end{aligned} \tag{19}$$

By convention, the second derivative at the minimum of the effective potential is used to define the effective mass,

$$m^2_{\Phi}\equiv\left.\frac{\mathrm{d}^2V_{\rm eff}}{\mathrm{d}\Phi^2}\right|_{\Phi=\Phi_{\rm min}}. \tag{20}$$

The minimum of the effective potential is determined by $\mathrm{d}V_{\rm eff}/\mathrm{d}\Phi=0$,

$$2F(R_{\rm min})-F_R(R_{\rm min})A+\kappa^2T=0, \tag{21}$$

and one can find $\Phi=\Phi_{\rm min}$, or equivalently $R=R_{\rm min}$, for a given T. Although the second derivative (19) does not explicitly depend on T, the effective mass (20) depends on T through Eq. (21). Note that we obtain similar results even in the Einstein frame.

The environment-dependent mass of the scalar field coupled with the external matter is known as the chameleon mechanism[58−60]. It is worth mentioning that the chameleon mechanism plays an important role in the DE models of $F(R)$ gravity theory or scalar-tensor theory. For instance, the trace of matter energy-momentum tensor is characterized by the energy density ρ, $T=-\rho$, for the nonrelativistic matter. In the low-density region or at the cosmological scale, the scalaron acts as DE. However, in the locally high-density region, such as the Solar System and Earth, the scalaron becomes heavy, and the propagation of the scalaron is suppressed. In this way, the DE models can explain the accelerated expansion of the current Universe, while the chameleon mechanism hides the scalar field from local tests of gravity, known as fifth force constraints[61−62].

We emphasize that the chameleon mechanism is a specific model for describing the effective mass in the interacting system. It is analogous to the heavy fermion materials or matter effect on neutrino oscillations. Moreover, the chameleon mechanism is one of the screening mechanisms that suppress the fifth force at local scales[63−67].

3 Modified TOV equation in $F(R)$ gravity

We consider the static and spherically symmetric star composed of the perfect fluid with a certain EOS. The geometry inside the compact star is given as

$$\mathrm{d}s^2=-\mathrm{e}^{2\nu(r)}\mathrm{d}t^2+\mathrm{e}^{2\lambda(r)}\mathrm{d}r^2+r^2(\mathrm{d}\theta^2+\sin^2\theta\,\mathrm{d}\phi^2). \tag{22}$$

The matter component is described by the EOS $p=p(\rho)$, and the energy-momentum tensor is defined as

$$T_{\mu\nu}=\left[\varepsilon(r)+p(r)\right]u_{\mu}u_{\nu}+p(r)g_{\mu\nu}, \tag{23}$$

with rest-mass density $\rho(r)$, total energy density $\varepsilon(r)$, pressure $p(r)$, and 4-velocity of static fluid u^{μ}. Note that the compact star is in a highly relativistic system, and the rest-mass density $\rho(r)$ and total energy density $\varepsilon(r)$ should be strictly distinct.

The non-vanishing components of Eq. (2) are given as follows:

$$\begin{aligned}-\kappa^2\epsilon=&-\frac{1}{2}F-\mathrm{e}^{-2\lambda}\left[\nu''+(\nu'-\lambda')\nu'+\frac{2\nu'}{r}\right]F_R+\\&\mathrm{e}^{-2\lambda}\left[F''_R+\left(-\lambda'+\frac{2}{r}\right)F'_R\right],\end{aligned} \tag{24}$$

$$\begin{aligned}-\kappa^2p=&\frac{1}{2}F+\mathrm{e}^{-2\lambda}\left[\nu''+(\nu'-\lambda')\nu'-\frac{2\lambda'}{r}\right]F_R-\\&\mathrm{e}^{-2\lambda}\left(\nu'+\frac{2}{r}\right)F'_R,\end{aligned} \tag{25}$$

$$-\kappa^2 p = \frac{1}{2}F - \frac{1}{r^2}\Big\{1+\big[-1-r(\nu'-\lambda')\big]\mathrm{e}^{-2\lambda}\Big\}F_R - \mathrm{e}^{-2\lambda}\Big[F_R'' + \Big(\nu'-\lambda'+\frac{1}{r}\Big)F_R'\Big]. \tag{26}$$

Here, we denote the derivative by the prime ($'$) with respect to the radial coordinate r. The conservation law $\nabla^\mu T_{\mu\nu}=0$ leads to

$$0 = \nabla^\mu T_{\mu r} = \nu'(\epsilon+p)+p'. \tag{27}$$

Using Eq. (27), we finds Eqs. (24)~(26) reproduces the conservation law in Eq. (4).

Moreover, using the expression of the curvature,

$$R = \mathrm{e}^{-2\lambda}\Big[-2\nu''-2(\nu'-\lambda')\nu'\Big] + \mathrm{e}^{-2\lambda}\Big[-\frac{4(\nu'-\lambda')}{r} + \frac{2\mathrm{e}^{2\lambda}-2}{r^2}\Big], \tag{28}$$

we can rewrite the field Eqs. (24)~(26) as follows:

$$\lambda' = \frac{1}{2r(2F_R + rF_{RR}R')}\times\Big\{\mathrm{e}^{2\lambda}\Big[r^2(2\kappa^2\epsilon - F) + F_R(r^2R-2)\Big] + 2r^2F_{RRR}(R')^2 + 2rF_{RR}(rR''+2R') + 2F_R\Big\}, \tag{29}$$

$$\nu' = \frac{1}{2r(2F_R + rF_{RR}R')}\times\Big\{\mathrm{e}^{2\lambda}\Big[r^2(2\kappa^2 p + F) - F_R(r^2R-2)\Big] - 2(2rF_{RR}R' + F_R)\Big\}, \tag{30}$$

$$R'' = \frac{F_R}{F_{RR}}\Big[\frac{1}{r}\Big(3\nu'-\lambda'+\frac{2}{r}\Big) + \mathrm{e}^{2\lambda}\Big(\frac{1}{2}R - \frac{2}{r^2}\Big)\Big] + \Big(\lambda'+\frac{1}{r}\Big)R' - \frac{F_{RRR}}{F_{RR}}R'^2. \tag{31}$$

The above are the modified TOV equations in general $F(R)$ gravity. Here, we treat the Ricci scalar $R(r)$ as an independent field corresponding to the scalaron field $\Phi(r)$, as seen in Eq. (5). Note that we can substitute Eq. (28) into Eq. (31) and so replace the differential equation for R with that of metric functions.

4 Degeneracy between EOS and $F(R)$ function

Combining Eqs. (24) and (25), Eqs. (24) and (26), and Eqs. (25) and (26), we obtain the following three equations, respectively:

$$\kappa^2(\epsilon+p) = \frac{2(\nu'+\lambda')}{r}\mathrm{e}^{-2\lambda}F_R - \mathrm{e}^{-2\lambda}\Big[F_R'' - (\nu'+\lambda')F_R'\Big], \tag{32}$$

$$\kappa^2(\epsilon+p) = \Big\{\frac{1}{r^2} + \mathrm{e}^{-2\lambda}\Big[\nu''+(\nu'-\lambda')\nu' + \frac{\nu'+\lambda'}{r} - \frac{1}{r^2}\Big]\Big\}F_R + \mathrm{e}^{-2\lambda}\Big(\nu'-\frac{1}{r}\Big)F_R', \tag{33}$$

$$0 = \Big\{\frac{1}{r^2} + \mathrm{e}^{-2\lambda}\Big[\nu''+(\nu'-\lambda')\nu' - \frac{\nu'+\lambda'}{r} - \frac{1}{r^2}\Big]\Big\}F_R + \mathrm{e}^{-2\lambda}\Big[F_R'' + \Big(-\lambda'-\frac{1}{r}\Big)F_R'\Big]. \tag{34}$$

Since one of the above three equations is redundant by the conservation law as in Eq. (27), the analysis below focuses on Eqs. (32) and (33).

In the following, we symbolically solve the modified TOV equations. First, We assume that the density profile $\rho=\rho(r)$ and EOS $p=p(\rho)$ are given. Because the M-R relation is determined by the integration of the density profile, assuming the density profile corresponds to using the observational data of the M-R relation. Then, the pressure p is written as a function of the radial coordinate, $p=p(r)$. Moreover, the total energy density ε and rest-mass density ρ are related through the first law of thermodynamics:

$$\mathrm{d}\Big(\frac{\epsilon}{\rho}\Big) = -p\mathrm{d}\Big(\frac{1}{\rho}\Big). \tag{35}$$

The total energy density is also given as a function of the radial coordinate, $\epsilon=\epsilon(r)$. For example, using a polytrope EOS,

$$p(\rho) = K\rho^\Gamma \tag{36}$$

with constants K and Γ, we find

$$\epsilon(\rho) = (1+a)\rho + \frac{K}{\Gamma-1}\rho^\Gamma \tag{37}$$

where a is a constant. Thus, Eq. (27) gives $\nu=\nu(r)$.

Next, we solve the remaining two equations. Defining a new variable N as

$$N \equiv \mathrm{e}^{-2\nu-2\lambda}, \tag{38}$$

we rewrite Eqs. (32) and (33):

$$\mathrm{e}^{-2\nu}\kappa^2(\epsilon+p) = -N'\Big(\frac{F_R}{r} + \frac{1}{2}F_R'\Big) - NF_R'', \tag{39}$$

$$\mathrm{e}^{-2\nu}\Big[\kappa^2(\varepsilon+p) - \frac{1}{r^2}F_R\Big] = \Big[N\Big(\nu''+2\nu'^2-\frac{1}{r^2}\Big) + N'\Big(\frac{\nu'}{2} - \frac{1}{2r}\Big)\Big]F_R + N\Big(\nu'-\frac{1}{r}\Big)F_R'. \tag{40}$$

Because LHS of Eq. (39) is written in known functions, $\{\nu(r), \varepsilon(r), p(r)\}$, we can solve Eq. (39) with respect to $N = N(F_R'', F_R', F_R, r)$. Substituting N into Eq. (40), we can obtain the differential equation symbolically denoted by

$$E\left(F_R'', F_R', F_R, \nu''(r), \nu'(r), \nu(r), \epsilon(r), p(r), r\right) = 0. \qquad (41)$$

Solving the above differential equation of F_R with respect to r, we obtain $F_R = F_R(r)$, and thus $N = N(r)$ and $\lambda = \lambda(r)$ from Eq. (38). Then, the Ricci scalar as in Eq. (28)

$$R = R\left(\nu''(r), \nu'(r), \lambda'(r), \lambda(r), r\right) \qquad (42)$$

gives $r = r(R)$. Finally, we obtain $F_R(r) = F_R(R)$ and thus $F = F(R)$.

The above calculation shows that for a given M-R relation (or equivalently, the density profile) and EOS, one can obtain the $F(R)$ function as a solution of the modified TOV equation. Therefore, we conclude that for the given EOS and $p(r)$, one can reconstruct the model of $F(R)$ gravity that reproduces an arbitrary M-R relation. In other words, functional DOF in $F(R)$ degenerates to that in EOS $p(\rho)$ [51], and measuring the M-R relation cannot determine the EOS in the $F(R)$ gravity theory.

5 Boundary condition and non-integer power model

The degeneracy issue is inevitable due to the model-dependence of the EOS and $F(R)$ function, and thus, one can reconstruct the $F(R)$ function to lead to any M-R relation. However, it does not mean any functional form of $F(R)$ is allowed. Because field equations include the second-order derivative of the Ricci scalar R, the first derivative of the Ricci scalar R' must be continuous at the surface of the compact star $r = r_s$ even if the values of the energy density and pressure jump at the surface.

By demanding the exterior of the compact star to be Schwarzschild geometry, thus $R = 0$, near the surface,

$$R \sim R_0 \left(1 - \frac{r}{r_s}\right)^a, \qquad (43)$$

where $a \geqslant 2$ so that R'' converges and R' is continuous at the surface $r = r_s$, and R_0 is a dimensional constant. Because R includes $\{\nu'', \nu', \lambda', \lambda\}$, the continuity of R' suggests the continuities of $\{\nu''', \nu'', \nu', \lambda'', \lambda', \lambda\}$. We assume that the $F(R)$ function describes the correction to GR as

$$F_R \sim F_0 + F_1 \left(\frac{R}{R_0}\right)^b \sim F_0 + F_1 \left(1 - \frac{r}{r_s}\right)^{ab}, \qquad (44)$$

where b is a constant. We also assume a mass-polytrope EOS in the following form:

$$\epsilon = \rho + np, \; p = K\rho^{1+\frac{1}{n}}, \qquad (45)$$

where $n > 0$ and K is a constant as in Eq. (36). Demanding continuities at the surface $r = r_s$, we find

$$F(R) \sim \Lambda + F_0 R + \frac{F_1 R_0}{b+1}\left(\frac{R}{R_0}\right)^{b+1} + \cdots. \qquad (46)$$

The index b of the Ricci scalar, which characterizes the correction to GR, is thus determined:

$$b = \frac{n+2}{a}. \qquad (47)$$

Here, Λ is an integration constant, which can be cast as the cosmological constant.

Moreover, continuities of the pressure p at the surface $r = r_s$ indicate that a needs to be an integer (Ref. [51] for detailed calculation). Because n corresponds to the value $0.5 \leqslant n \leqslant 1$ for neutron stars, b in Eq. (46) cannot be an integer, and thus the non-integer power of R can generally show up in the functional form of $F(R)$. Note that the constant Λ can spoil our assumption that the $F(R)$ gravity possesses the Schwarzschild solution as a vacuum solution, indicating $\Lambda = 0$. However, Λ is negligible if it takes the value of observed dark energy compared with a typical compact star scale.

To understand the origin of the non-integer power correction to GR, we reconsider the scalaron field. In the $F(R)$ gravity with the non-integer power of R, the chameleon mechanism works, and the deviation from GR outside the star is expected to be suppressed. To see how the chameleon mechanism works in this model, we consider the effective mass of the scalaron field. Applying the mass formula (19) to Eq. (46), one finds the effective mass approximately evaluated as follows:

$$m_\Phi^2 \approx \frac{1}{3}\frac{R_0}{bF_1}\left[F_0\left(\frac{R}{R_0}\right)^{1-b} + (1-b)F_1\frac{R}{R_0}\right]. \qquad (48)$$

Equation (48) shows the effective mass depends on the curvature (or roughly, the energy density), and the first term is dominant around the surface of the star for small F_1 in the case that $0 < 1 - b < 1$.

In this model, the scalar field becomes massive inside the compact star, although we expect it is almost massless outside the star when Λ is cast as the dark energy. Thus, the chameleon mechanism can screen the scalaron field around the compact star in the non-integer power model. Our assumption of the Schwarzschild geometry outside the compact star seems relevant, although the Birkhoff theorem is absent in $F(R)$ gravity. Note that we only need the behavior near the surface in the above argument. Therefore, we do not need the boundary conditions on the core of the compact star and spatial infinity.

6 R^2 model and massive scalaron field

To explore the relations among the model of $F(R)$ gravity, scalaron-field distribution, and boundary condition in terms of the chameleon mechanism, we investigate the

R^2 model of $F(R)$ gravity:

$$F(R) = R + \alpha R^2 . \tag{49}$$

R^2 model has been utilized in the existing works[39−46] as a benchmark model of $F(R)$ gravity theory. In the R^2 gravity, Eqs. (2) and (5) are given as

$$\kappa^2 T_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \alpha R\left(2R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}\right) + 2\alpha\,(g_{\mu\nu}\Box - \nabla_\mu \nabla_\nu) R , \tag{50}$$

and

$$2\alpha \Box R = \frac{1}{3}(R + \kappa^2 T) . \tag{51}$$

The $\alpha \to 0$ limit recovers the ordinary Einstein equation in GR.

For the R^2 model (49), the scalaron field Φ is defined as the linear form of the curvature:

$$\Phi \equiv F_R(R) = 1 + 2\alpha R . \tag{52}$$

Thus, we can use the curvature as the independent field for the scalar DOF instead of using only metric $g_{\mu\nu}$ for both tensorial and scalar DOF. The equation for the scalaron field is given as

$$\Box \Phi = \frac{1}{6\alpha}\left[\Phi - 1 + 2\alpha\kappa^2 T\right] , \tag{53}$$

and $V(\Phi)$ is

$$V(\Phi) = \frac{1}{4\alpha}(\Phi - 1)^2 . \tag{54}$$

Note that Eq. (53) is equivalent to Eq. (51), and thus we use Eq. (51) as the field equation for curvature R.

The effective potential $V_{\rm eff}(\Phi, T)$ in the R^2 gravity is given as

$$V_{\rm eff}(\Phi, T) = \frac{1}{12\alpha}(\Phi - \Phi_{\rm min})^2 - \frac{\Phi_{\rm min}^2}{12\alpha} , \tag{55}$$

$$\Phi_{\rm min} = 1 - 2\kappa^2 \alpha T , \tag{56}$$

up to a constant term. The stationary condition $\partial V_{\rm eff} / \partial \Phi = 0$ reduces to

$$\frac{\partial V_{\rm eff}}{\partial \Phi}(\Phi_{\rm min}, T) = \frac{1}{3}\left[R(\Phi_{\rm min}) + \kappa^2 T\right] = 0 . \tag{57}$$

$R = -\kappa^2 T$, which holds automatically in GR, gives the condition for the potential minimum. In other words, there is no difference between GR and the R^2 gravity if the scalaron field always stays at the potential minimum. The excitation from the minimum causes nontrivial profiles of the spacetime rather than GR ones.

The effective mass in the R^2 gravity is read as the coefficient of Φ^2,

$$m_\Phi^2 = \frac{1}{6\alpha} . \tag{58}$$

The effective mass is constant in the R^2 model, and the scalaron field merely behaves as a massive particle with the mass $m_\Phi \sim \alpha^{-1/2}$ for positive α. Therefore, the screening is absent in this model. In the following analysis, the T-dependent behavior of the scalaron field, not the ordinary sense of the screening mechanism, plays an important role. Note that for negative α, the mass m_Φ becomes pure imaginary, and the scalaron field behaves as a tachyon[68−69].

7 Scalaron inside and outside compact stars

We assume the external region of the compact star $r > r_s$ is a vacuum, and the cosmological constant vanishes. The Birkhoff theorem is generally absent in the $F(R)$ gravity, and the outer solution is not necessarily the Schwarzschild one, $R = 0$. Because the curvature is related to the scalaron field, the R^2 gravity may show the nonvanishing scalaron field surrounding the compact star, which could be called the scalarized solution.

We analyze the asymptotic behaviors of the scalaron or the curvature. The scalaron field Φ behaves as a free massive particle in the R^2 gravity:

$$\Box \Phi = m_\Phi^2 (\Phi - \Phi_{\rm min}) . \tag{59}$$

Assuming the asymptotically flat spacetime at the external region and $T = 0$, the asymptotic solution of Eq. (59) would be

$$\Phi(r) - 1 \sim \frac{c_1}{r} e^{m_\Phi r} + \frac{c_2}{r} e^{-m_\Phi r} , \tag{60}$$

where c_1 and c_2 are constants. Eq. (60) shows the decaying and growing modes in space, reflecting the static spacetime. For the assumption of the asymptotic flat spacetime, it is relevant to consider the decaying mode, and thus $\Phi \to 1$. Moreover, Eq. (52) suggests $\Phi \to 1$ corresponds to the limit to GR and $R \to 0$. Therefore, the deviation from the GR itself is expected to be exponentially decreasing:

$$\Phi(r) - 1 \propto \frac{1}{r} e^{-m_\Phi r} . \tag{61}$$

In other words, compact stars in the R^2 gravity have exponentially decaying scalar hair, reflecting the absence of Birkhoff theorem[70−72]. Moreover, the scalar hair is not screened due to the absence of the chameleon mechanism[44−46] but simply decaying due to the finite mass. From the equivalence between the curvature and the scalaron field, the curvature decreases exponentially with some typical length, which is of the same order as the Compton length of the scalaron field $m_\Phi^{-1} = \sqrt{6\alpha}$. Because the deviation from GR decreases in asymptotic flat spacetime, we can conclude that the Birkhoff theorem in GR is restored in the region $r \gg \sqrt{\alpha}$ and that the spacetime solution is asymptotically the Schwarzschild one.

We now solve the modified TOV equations and discuss the scalaron-field distribution and its consequences. We numerically solve Eqs. (29)~(31) and Eq. (27) with SLy EOS in the R^2 gravity. For the numerical integration, we define dimensionless variables as follows:

$$M_g = M_\odot \simeq 1.99 \cdot 10^{33}\,\mathrm{g},$$
$$r_g = \frac{GM_\odot}{c^2} \simeq 1.48 \cdot 10^5\,\mathrm{cm},$$
$$\rho_g = \frac{M_g}{r_g^3} \simeq 6.18 \cdot 10^{17}\,\mathrm{g\,cm^{-3}},$$
$$p_g = \frac{M_g c^2}{r_g^3} \simeq 5.55 \cdot 10^{38}\,\mathrm{g\,cm^{-1}\,s^{-2}}. \tag{62}$$

We show the results of numerical integration as follows (see Ref. [52] for details on implementation and setup).

Figure 1 shows the M-R relation with SLy EOS in R^2 gravity. The mass is defined as the Schwarzschild mass M, and the radius is the surface radius r_s;

$$M = m(r_s),\ m(r) \equiv \frac{r}{2}\left(1 - e^{-2\lambda(r)}\right),\ p(r_s) = 0. \tag{63}$$

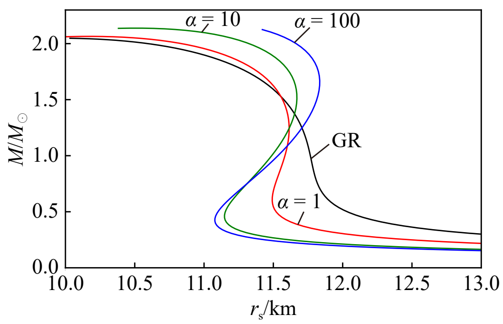

Fig. 1 (color online) The mass-radius relation with SLy for several values of α in R^2 model. The black, red, green, and blue curves correspond to $\alpha = 0$ (GR), 1, 10, and 100.

The *M-R* curves seem to rotate clockwise in response to α, and the maximum mass increases as α increases[44]. Figures 2 and 3 show the *M-R* relation in the high-mass and low-mass regions. For instance, in the case $\alpha = 10$, the mass is different from GR one by roughly 10% in the high-mass region and by two times in the low-mass region.

Figure 4 shows the scalaron-field distribution in spacetime. The dashed vertical line is the surface radius for GR as a reference value. Nontrivial scalaron concentration becomes more pronounced as α increases. Figure 5 shows the deviations from GR in terms of $\Phi - 1$, that is, the scalar hair outside the compact star. The horizontal solid line

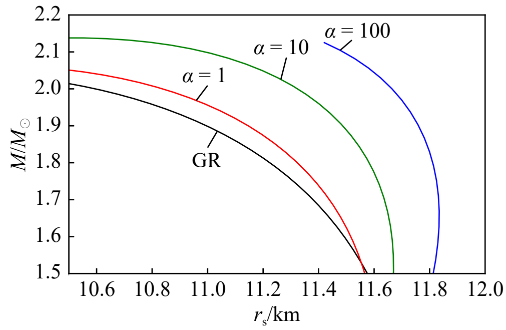

Fig. 2 (color online) *M-R* relation in the high-mass region.

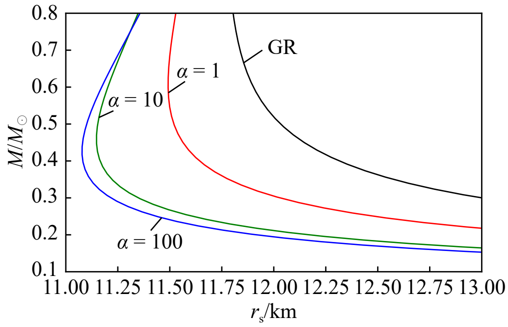

Fig. 3 (color online) *M-R* relation in the low-mass region.

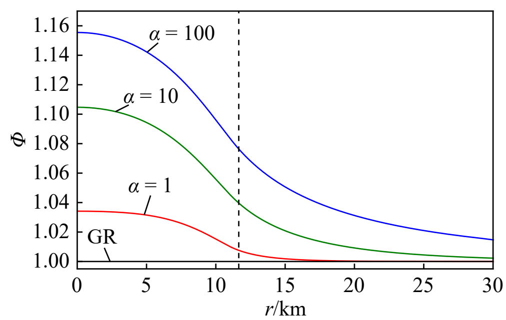

Fig. 4 (color online) The profiles of scalaron field $\varPhi$ for $\alpha = 1$, 10, and 100. We have chosen the central density $\rho_c = 1.00 \cdot 10^{15}\,\mathrm{g/cm^3}$ with SLy. $\varPhi = 1$ corresponds to GR case ($\alpha = 0$).

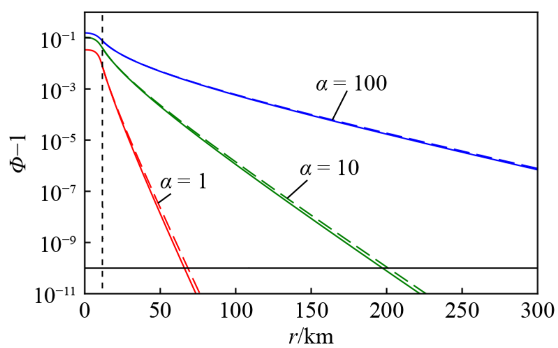

Fig. 5 (color online) The profiles of deviation from GR, $\varPhi - 1$, for $\alpha = 1$, 10, and 100 with SLy. Solid curves show the numerical solutions, and dashed lines are fitting results based on Eq. (61)

represents $\varPhi - 1 = 10^{-10}$. We can find that the decay of the scalaron becomes mild, and the scalarization radius $r_\varPhi$ defined as $\varPhi(r_\varPhi) - 1 = 10^{-10}$ is prolonged as α increases. Fig. 5 also shows the fitting results with Eq. (61), and we can observe the Yukawa-potential-like distribution of the massive scalaron field.

Figure 6 shows the energy condition of the scalaron field for a time-like vector, where we use the four-velocity of fluid $u^\mu = (e^{-\nu}, 0, 0, 0)$. Inside the star, the energy conditions can be partially broken, $T^{(\Phi)}_{\mu\nu} u^\mu u^\nu < 0$, and the scalaron behaves as a quintessence field. These features become significant for the lower value of ρ_c. The quintessential scalaron field effectively gives negative energy densities to the compact star.

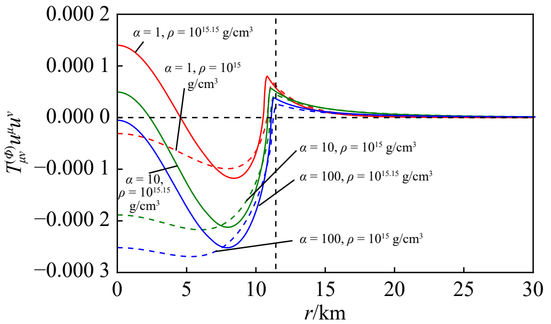

Fig. 6　(color online) The energy condition of the scalaron field $T^{(\Phi)}_{\mu\nu} u^\mu u^\nu$ for $\alpha = 1$, 10, and 100 with SLy. We have chosen the central density $\rho_c = 1.00 \cdot 10^{15}\ \mathrm{g/cm^3}$ and $1.00 \cdot 10^{15.15}\ \mathrm{g/cm^3}$ for each value of α.

8　Conclusion and discussion

We have reviewed the recent progress in the study of compact stars in the $F(R)$ gravity and several unique features that stem from the scalar DOF. Under the assumption of static and spherically symmetric spacetime and perfect fluid, we found that the *M*-*R* relation alone cannot constrain models of $F(R)$ gravity and EOS of internal matter separately. This consequence is analogous to the well-known reconstruction technique for cosmology[73−76], where an arbitrary time-evolution of scale factor can be reproduced by choosing the $F(R)$ function. It has also shown that consistency with the exterior solution of the compact star can give us constraints on the $F(R)$ function. It is nontrivial if the exact Schwarzschild spacetime is a proper boundary condition for the compact star in arbitrary $F(R)$ gravity, and the existence of the chameleon mechanism is tightly correlated with the boundary condition.

We must consider more measurements than the *M*-*R* relation to solve the degeneracy between modified gravity and EOS. It should be noted that the *M*-*R* relation is the background solution of the (modified) TOV equation. Therefore, the following measurements would be helpful. The tidal deformation includes the information of the perturbed TOV equation, and the scalar mode of perturbation can generate the scalar mode of gravitational wave[77−80]. Related to the perturbation, the stellar oscillations (asteroseismology) and related Quasi Period Oscillation can separately constrain the gravitational theory and EOS[81] together with the M-R relation. Note that the limit on sound speed $c_s^2 < 1$ by the causality can constrain EOS, which allows us, at least, to distinguish the gravitational theory from GR.

It is also intriguing to investigate the phenomenon induced by the scalaron field in the $F(R)$ gravity. The particle picture of the scalaron field suggests the new scalar particle coupling to the internal matters of the compact stars. Considering the thermal evolution, the scalaron particle may affect the cooling curves of neutron stars, similar to the axion cooling[82−85] and the dark matter capture in the neutron stars[86−88]. Moreover, the scalar hair surrounding the compact star can produce observable electromagnetic signals by the interaction between the scalaron and photon[25−27].

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$F(R)$ 引力理论中的致密星

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摘要: $F(R)$ 引力作为一种修改引力理论，近年来在致密星研究中的应用备受关注。本文回顾了 $F(R)$ 引力理论的基本原理及其修改后的 Tolman–Oppenheimer–Volkoff(TOV) 方程。研究表明，状态方程 (EOS) 的模型依赖性与引力修正效应存在简并性，说明仅通过致密星的质量-半径 (M-R) 关系不足以完全确定其内部物质的状态方程 (EOS)。此外，在基准 R^2 模型中，展示了 $F(R)$ 引力所预测的新标量场对致密星内外结构的影响。最后，本文探讨了通过致密星观测来检验引力理论的未来发展方向。

关键词: 修改引力理论；致密星；TOV 方程

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