On the Question of Gamma-ray Bursts in the Vicinity of Black Holes
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К вопросу о гамма-вспышках в окрестности черных дыр

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Abstract

In the present work the question connected with the most adequate mathematical approach for description probable resonant effects in black holes vicinities is considered. Metrics of Schwarzschild and Reissner-Nordstrem are analyzed.

Аннотация

В работе рассмотрен вопрос о наиболее адекватном математическом подходе к описанию возможных резонансных эффектов в окрестности черных дыр. Проанализированы случаи метрик Шварцшильда и Рейсснера-Нордстрема.

1. Introduction

The question connected with falling of a scalar field packet on horizon of a black hole (and passing across it) was discussed in many works and from various positions (see, for example, [1 - 10]). Nevertheless, the treatment as statements of the problems connected with it, and the conclusions obtained at their solution, is extremely important both from general theoretical positions, and from the point of view of a possibility of experimental check of effects of behaviour of a matter in a vicinity of a black hole, is still far from full and settling. Therefore authors continue a cycle of the works, devoted to interaction of non-point object with the metrics of the distorted space-time in the vicinity of collapsar. In the present work some qualitative aspects of possible mechanism x-ray and gamma-ray bursts, connected with "dyadosphere" [11 - 15] of ëlectromagnetic black hole" are considered. Also quasi-local processes of dynamics of a scalar (and electromagnetic) field packet in metrics of Schwarzschild and Reissner-Nordstrem are analyzed.

2. Non-monochromatic wave packet in the Schwarzschild metrics

The Schwarzschild metrics (historically it would be logically to name Hilbert's metrics) has the form :

$$ds^2 = -\,(1-2M/r)\,dt^2 + dr^2\,/\,(1-2M/r) + r^2\,d\,W^2, \qquad (1)$$

where: $d\,W^2 = d\,J^2 + sin^2\,J d\,j^2$; r, J, j are spherical coordinates; $R = 2GM$ is Schwarzschild radius, M is a mass of the black hole, G is a gravitational constant (G=c=1). Klein-Gordon equation (the wave equation for a mass field) for a scalar field f([(r)\vec], t) in the given metrics is:

$$-\frac{1}{r^2 - 2Mr}\left[r^2\frac{\P}{\P t}\right]^2 f + \frac{\P}{\P r}\left(\left[r^2 - 2Mr\right]\frac{\P}{\P r} f\right) +$$

$$+\mathrm{cosec}\,\theta\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) + \left[\mathrm{cosec}\,\theta\frac{\partial}{\partial\varphi}\right]^2\psi - m^2 r^2\psi = 0, \quad (2)$$

where m is the field parameter ("mass"). For electromagnetic and gravitational waves to the left-hand side it is added term $-j\psi/r^3$, where for an electromagnetic field (Wheeler equation [16]) there is j=1, and for a gravitational field there is j=2 (Regge-Wheeler [17] and Zerilli equations [18]). The equation (2) supposes various possibilities of separation of variables; usually as angular parts choose spherical harmonics $Y_{l[(m)]}(\theta,\varphi)$. However if we introduce in consideration more general than Schwarzschild metrics ones, here are required the certain updating. So, it is represented rational to use following decomposition: $\psi(r, t, \theta, \varphi) = R(r)\,S(\theta)\exp i([(m)]\varphi - \omega t)$, resulting for angular $S(\theta)$ and radial parts $R(r)$ to two following equations [19]:

$$\frac{\partial}{\partial\theta}\left[\sin\theta\frac{\partial S(\theta)}{\partial\theta}\right] + \left[\lambda\sin\theta - \frac{\bar{m}^2}{\sin\theta}\right]S(\theta) = 0, \quad (3)$$

$$\frac{\partial}{\partial r}\left((r^2-2Mr)\frac{\partial R(r)}{\partial r}\right) + \left(\frac{\omega^2 r^3}{r-2M} - m^2 r^2 - \lambda\right)R(r) = 0, \quad (4)$$

where λ is a constant of separation (eigenvalue). The general solution $S^{l}_{[(m)],0}(\theta)$ the equations for an angular part (3) is spheroidal angular function with spin weight s=0 [4]. If we change of independent ($r \to x$) and dependent variables ($R(r) \to F(x)$) in the equation (8)

$$r=2M(1-x), \quad R(r) = R_0(r)\,F(x),$$

$$R_0(r) \propto (r/M-2)^{2i\omega M}\exp\left(iM\sqrt{\omega^2 - m^2}\,(r/M-1)\right),$$

then (8) may be re-writing as:

$$\frac{d^2F(x)}{dx^2} + A(x)\frac{dF(x)}{dx} + B(x)\,F(x) = 0 \quad (5)$$

$$A(x) = \left(\beta + \frac{\gamma}{x} + \frac{\delta}{x-1}\right), \quad B(x) = \frac{\alpha\beta x - h}{x(x-1)}, \quad (6)$$

$$\alpha = -(1+2i\omega M) + \frac{i(2\omega^2 M - m^2 M)}{\sqrt{\omega^2 - m^2}}, \quad (7)$$

$$\beta = 4iM\sqrt{\omega^2 - m^2}, \quad \gamma = 1+4i\omega M, \quad \delta = 1, \quad (8)$$

$$h = \lambda - 2iM\sqrt{\omega^2 - m^2} - 2i\omega M + 8\omega M^2\sqrt{\omega^2 - m^2} - 4M(2\omega^2 M - m^2 M). \quad (9)$$

The equation (5) belongs to a class *confluent Heun equations* (CHE) [20]. It is necessary to note that the spheroidal equation (3) is special case CHE (if the term corresponding β in (5) is equal to zero)).

The point x=0 corresponds to event horizon r=2M, and the point x=1 is singularity r=0. The standard solution of the equation (5) in a vicinity x=0 (at $|x| < 1$, $\omega \neq i/4M, 2i/4M$...) (so-called "Frobenius solution") has the form:

$$^{(1)}F(x) \in Hc^{(a)}(\mathrm{b, a, g, d}, h; x) = \overset{\Gamma}{\underset{k=0}{e}} c^{\{a\}}{}_{k}\, x^{k},$$

$$Hc^{(a)}(\mathrm{b, a, g, d}, h; x=0) = 1, \qquad (10)$$

where factors $c^{\{a\}}{}_{k}$ are defined from a recurrent formula

$$-(k+1)(k + \mathrm{g})\, c^{\{a\}}{}_{k+1} + (k((k + \mathrm{g}\text{-}1 + \mathrm{d}\text{-} \mathrm{b}) + h)\, c^{\{a\}}{}_{k}-$$

$$+ \mathrm{b}(k\text{-}1 + \mathrm{a})\, c^{\{a\}}{}_{k\text{-}1} = 0, \quad c^{\{a\}}{}_{0} = 1, \quad c^{\{a\}}{}_{\text{-}1} = 0. \qquad (11)$$

The second linearly independent solution of the equation (5) in the vicinity x=0 is the local solution in a vicinity of infinity ($x = \Gamma$), so called "Thome solution" [21] :

$$^{(2)}F(x) \in Hc^{(r)}(\mathrm{b, a, g, d}, h; x) = \overset{\Gamma}{\underset{k=0}{e}} c^{\{r\}}{}_{k}\, x^{\mathrm{a}\text{-}\,k},$$

$$\lim_{x \,®\, \text{-}\Gamma} Hc^{(r)}(\mathrm{b, a, g, d}, h; x) = 1, \qquad (12)$$

$$-\,\mathrm{b}(k+1)\, c^{\{r\}}{}_{k+1} + ((k + \mathrm{a})(k - \mathrm{b}\text{-}\mathrm{d}\text{-}\, \mathrm{g}+1)\text{-}h)\, c^{\{r\}}{}_{k}-$$

$$-(k + \mathrm{a}\text{-}\, \mathrm{g})(k\text{-}1 + \mathrm{a})\, c^{\{r\}}{}_{k\text{-}1} = 0, \quad c^{\{r\}}{}_{0}=1, \quad c^{\{r\}}{}_{\text{-}1} = 0. \qquad (13)$$

Legitimacy of such choice is caused by that second Frobenius solution is not linearly independent with first one ($Hc^{(a)}$) for all w: if w = 0 then characteristic exponent 1-g = 0.

Further, it is necessary to note, that $^{(2)}F(x)$ is the recessive solution only if $Re(\text{-b}) > 0$, i.e. if $|\mathrm{w}| < m$. In a vicinity of w = m both solutions possess the structure with singularities of different type. The first solution $^{(1)}F(x)$ has singularity due to factors $c^{\{a\}}{}_{k}$ (at $|\mathrm{w}| > m$), and the second one $^{(2)}F(x)$ has singularity due to the term x^{a} (at $|\mathrm{w}| < m$). Thus, if we consider the central two-point connection problem with given asymptotics of solutions (its correspond to "falling" and "reflected" waves in terms quantum mechanics) near to regular (event horizon) and irregular points (in the infinity), then general solution will represent parametric resonant function.

It is necessary to note, that interpretation of this fact can be based on the introduction of än abnormal dispersion" analog concept for the metrics considered as the medium of a scalar field dynamics. Amplitude of a field

$$F(x) \sim Hc^{(r)}(\mathrm{b(w,w_0), a(w,w_0), g(w)}, 1, h(\mathrm{w,w_0}); x), \quad \mathrm{w}_0 \in m,$$

in this case is an analog a role of displacement of an electric charge in the standard theory of dispersive media. It is possible to enter also a parameter of refraction and permettivity of the metrics for a scalar field. As it has above mentioned, the equation (2) for an electromagnetic field (at j=1 , Wheeler case) is led to CHE so the approach is quite universal.

Continuation of the solution äcross the horizon" (in a vicinity of a singular point x=1) demands the application corresponding s -homotopic transformation [22] of CHE leading to following pair of basic

Heun equation solutions:

$$^{(1)}\widetilde{F}(x) = (x\text{-}1)^{1\text{ - d}}\, Hc^{(a)}\,(\text{b, - a+ d-1, g, 2 - d, } h \text{ - g(1 - d); } x) =$$

$$= Hc^{(a)}\,(\text{b, - a, g, 1, } h\text{; } x), \qquad (14)$$

$$^{(2)}\widetilde{F}(x) = (x\text{-}1)^{1\text{ - d}}\, Hc^{(r)}\,(\text{b, -a+ d-1, g, 2 - d, } h \text{ - g(1 - d); } x) =$$

$$= Hc^{(r)}\,(\text{b, - a, g, 1, } h\text{; } x). \qquad (15)$$

Thus, the general solution of the equation (5) äbove" and ünder horizon" ($0 < x < 1$) represents a composition of confluent Heun functions $Hc^{(a)}$ (ängular") and $Hc^{(r)}$ ("radial"):

$$R\text{-}region\text{:}\quad C_1\, Hc^{(a)}\,(\text{b, a, g,1, } h\text{; } x) + C_2\, Hc^{(r)}\,(\text{b, a, g, 1, } h\text{; } x), \qquad (16)$$

$$F\text{-}region\text{:}\quad \widetilde{C}_1\, Hc^{(a)}\,(\text{b,- a, g, 1, } h\text{; } x) + \widetilde{C}_2\, Hc^{(r)}\,(\text{b, -a, g, 1, } h\text{; } x). \qquad (17)$$

Considering the solution in F-region (as analytical continuation of the solution of the equation in R-region), it is possible to show (following [23]) that reflection of horizon R is not equal to zero so interpretation of the general solution as superpositions of the falling and reflected waves is lawful. There is no influence the decomposition method of the solution of the Klein-Gordon equation over resonance existence. Decompose of the equation of Klein-Gordon (2) with use of a method of "phase shift" [24]:

$$\text{f}(r, t, \text{J, j}) = \underset{l[\Gamma(m)]}{\text{e}} \; \overset{\Gamma}{\underset{-\Gamma}{\text{y x}}} \; r^{-1} f_{\text{w},l}(r)\, Y_{l[\Gamma(m)]}(\text{J, j}) \exp(-i\,\text{w}t)\, d\,\text{w}, \qquad (18)$$

$$f_{\text{w},\,l}(r) = \text{v}\exp(i\,\text{w}r_* + i\,g\,(\text{r})), \quad r_* = r + 2M \ln|r/2M\text{-}1\,|, \quad \text{r} = 1\text{-}2M/r,$$

$$g\,(\text{r}) = \left(-\frac{(2\,\text{w}M - 2M\text{Ц}\overline{\text{w}^2 - m^2})^2}{2\text{Ц}\overline{\text{w}^2 - m^2}}\right) \ln(1 - \text{r}) + \overset{\Gamma}{\underset{n=-1}{\text{e}}} \; (a_n + b_n\,\text{r})\,(4\,\text{r}\text{-}4\,\text{r}^2)^{\,n},$$

where a_n, b_n are some factors, which obvious kind is defined by substitution of the solution in the initial equation (with corresponding recurrent formulae). The phase g(r) is defined by the same way; so as there exists next condition:

$$b_1 = \frac{1}{2\text{Ц}\overline{\text{w}^2 - m^2}}\,\text{ж}\; 2\text{Ц}\overline{\text{w}^2 - m^2}\, a_1 +$$

$$\frac{\frac{1}{4}\left(2\,\text{w}M - 2M\text{Ц}\overline{\text{w}^2 - m^2}\right)^{2\,)\,2}}{\overline{\text{w}^2 - m^2}} - \frac{1+i\left(a_1 + \frac{1}{2}\,\text{-}\,(2\,\text{w}M - 2M\text{Ц}\overline{\text{w}^2 - m^2})^2\right)}{4 \cdot 2\,\overline{\text{w}^2 - m^2}}\,\text{ц ш} - 2a_1, \qquad (19)$$

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then the resonance effect at $\omega \cong m$ will be realize in the same way, as well as at decomposition with the Heun functions. In a complex conjugated functions to the augmented waves $f_{\omega l}(r)$ together with the last ones form pair linear independent basic solutions, and analytical continuation under horizon gives effect of reflection directed by [23]. Nevertheless, obtaining the solution is in such a way possible only for additional assumptions and don't extend on other metrics and consequently can serve only in the didactic purposes. According to results of work [25], if we accept as basic functions the spherical harmonics and pseudo-flat waves:

$$\varphi(r, t, \vartheta, \varphi) = \sum_{l[\sim(m)]} \int_{-\infty}^{\infty} \varphi(r, \omega, l, \bar{m})\, Y_{l[\sim(m)]}(\vartheta, \varphi) \exp(-i\,\omega t)\, d\,\omega, \qquad (18a)$$

then the solution of the Klein-Gordon equation for Schwarzschild metric is the combination of pair linearly independent solutions with the preassigned asymptotics:

$$\varphi(r, \omega, l) = c_l(-\omega)\, \psi_l(r, \omega) + c_l(\omega)\, \psi_l(r, -\omega), \qquad (20)$$

$$\lim_{r \to 2M} \psi_l(r, \omega) \sim (|r-2M|/2M)^{-2i\,\omega M},$$

$$\lim_{r \to \infty} \varphi(r, \omega, l) \sim$$

$$\sim \exp\left(i\, r\sqrt{\omega^2 - m^2} + i\, M \ln r\, (2\,\omega^2 - m^2)/\sqrt{\omega^2 - m^2}\right) / \left(\sqrt{\omega^2 - m^2}\, r\, i^{\,l+1}\right).$$

Accordingly, for Green's function $G(r, r', \omega)$, being the solution of the Klein-Gordon equation with the right hand $\delta^4(X-X')$ ($X = (\mathbf{r}, t)$):

$$(-\partial_\mu\, g^{\mu\nu}\sqrt{-g}\,\partial_\nu + m^2\sqrt{-g})\, G(X, X') = \delta^4(X-X')$$

we obtain representation in the form of:

$$G(X, X') = \sum_{l[\sim(m)]} \int_{-\infty}^{\infty} G_l(r, r', \omega)\, Y_{l[\sim(m)]}(\vartheta, \varphi)\, Y^*_{l[\sim(m)]}(\vartheta', \varphi') \exp(-i\,\omega(t-t'))\, d\,\omega, \qquad (21)$$

where $G_l(r, r', \omega)$ is the kernel satisfying Klein-Gordon equation with the right hand $\delta(r-r')$. Research of the obtained Green function by means of its representation with the use of basic solutions (20) has allowed to establish property of stability of vacuum in a vicinity of Schwarzschild horizon: there is no flux of particles at infinity in conditions of stability of a black hole (öut"-conditions on horizon of the absolute past, ïn"-conditions on horizon of the absolute future). However it contradicts to Hawking result [26,2] about existence of thermal radiation of a black hole with temperature $kT=1/8\,\pi M$.

Apparently, this result is caused by acceptance of aprioristic assumptions of asymptotics of solutions of the Klein-Gordon equation on the event horizon. It is represented reasonable to use as basic functions above-stated confluent Heun functions: with its application an event horizon may be the generator of particles and property of stability of vacuum in its vicinity because of structure of the common solutions (16) and (17) is according to requirements of Hawking theory. Thus, naturally there is a question on an opportunity of generation of particles not only in the vicinity of event horizon, but also in some "macroscopical" its vicinity caused by resonances of solutions $\omega \cong m$.

3. Reissner-Nordstrem metrics and physical processes in the dyadosphere

Studying of gamma-bursts and attempts of an explanation of their mechanisms have led to creation of several new theories on a joint of astrophysics, the theory of elementary particles and cosmology. Major candidates for roles of the theories explaining all set of the phenomena observable during gamma-bursts are the mechanisms considering occurrence with shock-wave processes in a barion matter in vicinity of black holes (of external and internal genesis), Compton processes in magnetized plasma and generation of electron-positron pairs in "dyadosphere" of an ëlectromagnetic black hole" (EMBH) with the electric charge Q [12-14]. From the point of view of developed above the approach based on use of basic functions with "natural" asymptotics, it is appropriately consider the problem on its generalization on the case of Reissner-Nordtream metrics, describing space-time in a vicinity of an electromagnetic black hole. Besides, this approach is adequate applied in enough big region of "classical" dyadosphere, and can give its additional description. According to the dyadosphere theory, there is region above EMBH horizon

$$r_{EMBH} \in 1.47 \cdot 10^5 \, \mathrm{m}(1 + \sqrt{1 - \mathrm{x}^2}) < r < r_{ds} \in 1.12 \cdot 10^8 \sqrt{\mathrm{mx}} \text{ cm},$$

$$\mathrm{m} = M/M_{\mathrm{Sun}} > 3.2, \quad \mathrm{x} = Q/Q_{max} \, \mathrm{J} \, 1,$$

which it is possible to present in the form of set of concentric condensers with thickness $\mathrm{d}_0 << MG/c^2$ and charge surface density $\mathrm{s}(r) = Q/4\,\mathrm{p}r^2$. If to assume $\mathrm{d}_0 = (^{\mathrm{h}}/_{2\mathrm{p}})/m_e\, c$, rate of generation of pairs e^+-e^- during polarization of Schwinger-Heisenberg vacuum [28,29] represented in the form:

$$\frac{dN}{dt} = \frac{1}{4\,\mathrm{p}c}\left(\frac{4e\,\mathrm{s}}{(^{\mathrm{h}}/_{2\mathrm{p}})}\right)^2 \exp(-\mathrm{ps}_c / \mathrm{s})\, 4\,\mathrm{p}r^2 \left(\frac{(^{\mathrm{h}}/_{2\mathrm{p}})}{m_e\, c}\right), \quad \mathrm{s}_c = E_c/4\,\mathrm{p} = m^2{}_e\, c^3/4\,\mathrm{p}(^{\mathrm{h}}/_{2\mathrm{p}})\, e.$$

After the rise of "double layers" of electron-positron plasma the evolution of dyadosphere and PEM (pair-elecromagnetic pulse) origin is described by the various theoretical models based on relativistic hydrodynamics [15,30]. Application of the mathematical approach based on the theory of confluent Heun equation, in case of EMBH is caused by essential similarity of Schwarzschild and Reissner-Nordstrem metrics and corresponding Klein-Gordon equations solutions.

In the vicinity of EMBH horizon it is possible to write the equation of type (2) with an additional member $-\mathrm{f}/r^3$ (generalization of the Wheeler equation) for electromagnetic waves; after of some its transformations it is possible to lead to CHE form. Dispersion of electromagnetic waves on the metrics in a dyadosphere is very complicated phenomenon; there are a possibility, when dependence of a parameter of refraction of the metrics will correspond to a case of existence of a zone of öptical less dense medium", so for short-wave radiation (x-rays and the gamma-rays) arises full reflection.

If we consider dynamics of a scalar field of electromagnetic oscillators with which it is possible to simulate double layers in the dyadosphere, then after separation of variables in the Klein-Gordon equation for a radial part gets solution (5), but parameters in it, naturally, already others:

$$\mathrm{a} = -\,(1 + i\,(2\,\mathrm{w}M - qQ)) + i\,(2\,\mathrm{w}M - qQ)\,/\sqrt{\mathrm{w}^2 - m^2},$$

$$\mathrm{b} = 4i\,(M - Q^2/2M)\sqrt{\mathrm{w}^2 - m^2}, \quad \mathrm{g} = 1 + 2i\,(2\,\mathrm{w}M - qQ), \quad \mathrm{d} = 1,$$

$$h = 1 - \mathrm{b}/2 - i\,(2\,\mathrm{w}M - qQ) + 4\,(M - Q^2/2M)\sqrt{\mathrm{w}^2 - m^2}(2\mathrm{w}M - qQ)\; -$$

$$-4\ ((2\ \mathrm{w}M\text{-}qQ)\ \mathrm{w}\text{-}m^2M)\ (M\text{-}Q^2/2M.)$$

Nevertheless, an origin of a parametrical resonance probably and in this case. How it is possible to interpret the resonant phenomena in dyadosphere and corresponding disassembled earlier in a small vicinity of horizon asymptotic effects of behaviour of common CHE solution? For the exact quantitative analysis introduction in consideration of effects of quantum electrodynamics and precision details of the theory of polarization of vacuum is necessary. However already now it is possible to assume, that the behaviour electron-positron plasma will be defined by essentially nonlinear effects connected with dynamic structure of a scalar electromagnetic field, connected with generation in "condenser layers" of dyadosphere e^+ - e^--pairs. Consideration and the comparative analysis of effects in the dyadosphere demand the use of bifurcation theory and development of special codes for numerical modeling processes at the gamma-bursts, using it.

We shall note, that the theory of gamma-bursts is extremely roughly developing section of the astrophysics essentially influencing already settled concepts. Rather interesting in this plan its communication with the theory of black holes is represented. For example, in work [31] the hypothesis of compact objects of new type (ßtars of dark energy"), most advantageously explaining many effects observable at gamma-bursts is put forward. It is interesting, that here there is a certain coordination with results of work [32]. It is necessary to note, that conclusions rather (in particular) the structures of the horizon made by means of considered above mathematical device somewhat also demand the certain updatings of concept of horizon and influence of its topology on external processes.

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