

# STUDY OF ULTRA-LOW EMITTANCE DESIGN FOR SPEAR3\*

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## Abstract

Since its 2003 construction, the SPEAR3 synchrotron light source at SLAC has continuously improved its performance by raising beam current, top-off injection, small alpha and smaller emittance. This makes SPEAR3 one of the most productive light sources in the world. Now to further enhance the operation of SPEAR3, we are looking into the possibility of converting SPEAR3 to a multi-bend achromat storage ring within its site constraint.

## INTRODUCTION

The X-ray brightness of ID beamlines is the major figure-of-merit of 4<sup>th</sup> generation light source. The brightness B of an up-right phase ellipse is:

$$B \propto \frac{N_\gamma}{(\Delta\lambda/\lambda)\Delta t \sum_x \sum_{x'} \sum_y \sum_{y'}}, \quad (1)$$

where  $N_\gamma$  is the number of photons in the central radiation cone,  $(\Delta\lambda/\lambda)$  is the radiation bandwidth;  $\Delta t$  is the time scale of interest, and  $\Sigma_x$ ,  $\Sigma_{x'}$ , etc., are the convolution of the electron beam phase space and the single electron radiation distribution given by

$$\Sigma_x = \sqrt{\sigma_x^2 + \sigma_y^2} \quad \Sigma_{x'} = \sqrt{\sigma_{x'}^2 + \sigma_{y'}^2}, \quad (2)$$

where  $\sigma_x$  and  $\sigma_{x'}$  are the transverse rms size and divergence of the electron beam for horizontal plane. A similar equation holds for the vertical plane.  $\sigma_y$  and  $\sigma_{y'}$  are the transverse beam sizes and divergences of the photon beam for a zero emittance electron beam. For an undulator of length L, it is approximately described by a radiation source at the center of the undulator given by

$$\sigma_r \approx \sqrt{\frac{\lambda}{2L}} \quad \sigma_r \approx \frac{1}{2\pi} \sqrt{2\lambda L}. \quad (3)$$

The product  $\sigma_r \sigma_r' = \lambda/(2\pi)$  sets the minimum possible emittance, which is achieved in the limit of zero electron beam emittance.

To maximize the brightness, we must minimize the effect of convolution of electron distribution. That is the electron phase space has to be well matched to the radiation. For a given radiation  $\sigma_r \sigma_r'$  the minimization is when:

$$\varepsilon_{x,y} \leq \frac{\lambda}{2\pi}, \quad \frac{\sigma_{x,y}}{\sigma_{x',y'}} = \frac{\sigma_r}{\sigma_{r'}} \approx \frac{L}{\pi}. \quad (4)$$

The equilibrium electron emittance  $\varepsilon_0$  is resulting from

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the balance between quantum excitation and radiation damping [1]

$$\varepsilon_0 \approx F(\nu, \text{lattice}) \frac{C_q \gamma^2}{J_x} \theta_B^3, \quad (5)$$

where  $\theta_B$  is the bending angle per dipole,  $C_q = 3.84 \times 10^{-13}$  m,  $J_x$  is horizontal damping partition. Here we assume it's a horizontal bend.  $J_x$  is in the range of 1~3 depending on the gradient in dipole and  $\gamma$  is the Lorentz factor. F is a function of lattice design. It involves the H integral in dipole:

$$H = \beta_x \eta_x^2 + 2\alpha_x \eta_x \eta_x' + \frac{1 + \alpha_x}{\beta_x} \eta_x^2, \quad (6)$$

where  $\beta_x$ ,  $\alpha_x$  are the amplitude functions of electron motion and  $\eta_x$  and  $\eta_x'$  are the dispersion function and its derivative. It is proved that with matched lattice functions to the dipole length and  $\theta_B$  the minimum emittance are [2]:

$$\varepsilon_{MEDBA} = \frac{C_q \gamma^2}{4\sqrt{15} J_x} \theta_B^3, \quad \varepsilon_{METME} = \frac{C_q \gamma^2}{12\sqrt{15} J_x} \theta_B^3. \quad (7)$$

Here DBA means the dipole is dispersion free at one side and two of this type of dipole can make the cell dispersion free outside the dipoles. The TME (theoretical minimum emittance) dipole has dispersion at both ends. The minimum emittance of TME dipole is one third of the DBA dipole.

Today's sources deliver radiation from 100 eV to 100 keV. The corresponding electron emittance for diffraction-limited source is from 2 nm to 2 pm. The horizontal emittance of SPEAR3 now is 9.8 nm and vertical emittance is 10 pm with 0.1% coupling. The emittance of SPEAR3 has to be reduced in order to match the hard x-ray radiation.

## MBA DESIGN

It's clear that the effective way to reduce the emittance is to put as many small angle TME dipoles as possible in a cell. This is the concept of multi-bend achromatic (MBA) cell. Figure 1 shows the theoretical minimum emittance of a 3 GeV 18 period storage ring with horizontal damping partition of 1. The horizontal axis is number of dipoles per unit cell. The blue curve is the emittance of MBA cell with achromatic match. The emittance of minimum MBA achromatic cell is [2]:

$$\varepsilon_{MEMBA} = \frac{C_q \gamma^2}{4\sqrt{15} J_x} \theta_1^3 \text{ (outer dipole angle)}. \quad (8)$$

In order to match the H integral between TME and DBA dipole, the length of TME dipole is  $3^{1/3}$  times the length of DBA dipole in an isomagnetic cell. The  $\theta_1$  in

Table 1: Summary of Various MBA Storage Ring Light Source Designs without Intrabeam Scattering.  $M1 = \epsilon_0 C^3/E^2$  is given in units of  $\text{pm km}^3/\text{GeV}^2$ .  $M2 = \epsilon_0 C^3/E^5$  is given in units of  $\text{pm km}^3/\text{GeV}^5$  and  $M3 = \epsilon_0/E^2/\theta_1^3/1E6$  is given in units of  $\text{pm}/\text{GeV}^2$

Name	Energy (GeV)	Circumference (km)	Emittance (pm)	Structure	M1	M2	M3
MAX IV [4]	3	0.528	263	7-BA 20	4.301	0.159	0.672
Sirius [5]	3	0.518	280	5-BA 20	4.324	0.160	0.254
SPRING8-II [6]	6	1.43595	67	6-BA 48	5.510	0.026	0.389
ESRF upgrade [7]	6	0.8444	150	7-HBA 32	2.509	0.012	0.430
APS-U [8]	6	1.104	60	7-HBA 40	2.243	0.010	0.336
ALS II [9]	2	0.2	100	9-BA 12	0.200	0.025	0.308
SPEAR3* upgrade	3	0.234	750	4-BA 18	1.068	0.040	0.228

\*The emittance shown here is a double-4BA cell. A full ring matching is under-way.

Eq. 8 is the bend angle of the DBA type dipole in the MBA cell. The emittance of an MBA cell is always larger than the pure TME cell because two TME dipoles are replaced by two DBA type dipoles. A 3 GeV, 18 cells, four-bend MBA gives emittance of 300 pm. These are the theoretical minimum values. Taking into consideration realistic constraints like allowed cell length and magnetic field strength, the design value usually is a few times larger than the theoretic minimum values.

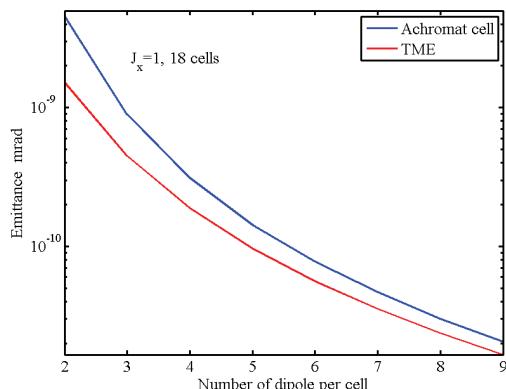


Figure 1: Theoretic minimum emittance (TME) of a 3 GeV, 18 cell ring with different number of bends per cell. The blue curve is the minimum emittance of MBA cell with achromatic match.

MBA is the trend of design of synchrotron storage rings under constructions or for upgrade proposals. Table 1 summarizes the parameters of a few storage ring designs using MBA cells. The column  $M1 (= \epsilon_0 C^3/E^2)$  in Table 1 is one way to evaluate the effect of linear optics to emittance. It is approximately constant for machines of various energies and circumferences with similar optics [3]. The smaller value of  $M1$  means the optics design is toward the optimized emittance. It's noticeable that ALS II has significantly smaller  $M1$  than the rest of the MBA designs in Table 1. The reason is when calculating  $M1$ , the circumference is assumed linearly proportional to the number of dipole  $N_B (=2\pi/\theta_B)$ . It does not take into consideration of energy dependence of bending radius. If the dipole field is fixed,  $N_B$  scales according to  $C/E$ . The column  $M2 (= \epsilon_0 C^3/E^5)$  is a modification of the

assessment. A more straight forward way is to calculate the effect of linear optics according to Eq. 8 with  $M3 (= \epsilon_0/E^2/\theta_1^3)$ . For an upgrade machine  $M3$  is more realistic evaluation of the optics design because it reflects the constraint of cell length and super period of the existing storage to make small emittance.

Table 2: Summary of Dipole Quadrupole [11]

	B at beam [T]	Gradient [T/m]	Aperture [mm]
MAX IV	0.52	8.6	28
Sirius	0.584	7.8	28
Diamond DDBA	0.8	14.4	30
ESRF2 DQ1	0.54	37	38
ESRF2 DQ2	0.42	48	38
APS	0.5	38	26
ALS II	0.78	50	24

## GRADIENT DIPOLE

Most of the MBA designs take advantage of the dipole with a defocussing gradient. This has several advantages [4]: the number of magnets per achromat is reduced; the partition number  $J_x$  is larger than 1 which reduces the emittance at the price of smaller longitudinal partition number; the length of the cell is small, therefore

reducing the total circumference. Despite the above advantages these MBAs require strong dipole gradient which is beyond the capability of conventional dipole with sloped pole face. A shifted quadrupole as dipole which provides 0.78 T dipole field and 50 T/m gradient is proposed by ALS II team [10]. Table 2 summarizes the parameters of gradient dipoles used in different projects [11]. It falls into two categories: the first three MAX IV, Sirius, and Diamond use combined function dipoles providing low to moderate gradient in dipoles; the rest use shifted strong quadrupoles providing strong gradient.

## QUADRUPOLE AND SEXTUPOLE

In general, small emittance is made by small  $\beta_x$  and  $\eta_x$  through the dipoles. It requires strong focussing. Another reason for strong gradient of quadrupole is to make the

length of quadrupole short to make the MBA compact in order to fit in the existing cell length. Fortunately the aperture for the quadrupole is reduced too due to the smaller beam size. Taking the ALS II project for example the quadrupole aperture is reduced from ALS 35 mm to ALS II 12 mm. The maximum gradient can reach 100 T/m [10]. Because of the strong focussing the natural chromaticities are large. The small dispersion makes the strength of sextupoles for linear chromaticity correction even stronger. This tends to reduce the dynamic aperture and momentum aperture. The maximum sextupole gradient proposed by ESRF is 4900 T/m<sup>2</sup> (nominal 3200 T/m<sup>2</sup>) [11]. These quadrupole and sextupole strength limitations will be used in SPEAR3 MBA design study.

### SPEAR3 MBA CELL

SPEAR3 is a two-fold symmetry storage ring of circumference 234.14 m. It has 14 normal DBA cells with cell length 11.69 m and arc length 6.4 m (the dipole length included) and four matching DBA cells two per half ring. The matching cell has bending angle three quarter of the normal cell with cell length 18.76 m and arc length 6.79 m. SPEAR3 has the shortest arc length among all the designs listed in Table 1. The upgrade requires keeping the same number of ID beamline and keeping the ID beamline at same location. These make it a challenge to put large number bend MBA inside the DBA arc. The highest dipole field of 0.8 T from Table 2 is chosen. This reduces the total dipole length as much as possible. The gradient in dipole can be high, using ALS II 50 T/m shift quadrupole, or medium using Diamond 14.4 T/m combined function dipole. Different bend MBA from three to five bends had been studied without magnet field constraints. With the above magnet field constraints a four bend MBA is the best that can be used to fit into the 6.4 m arc length. Further study shows no good location to put orthogonal sextupoles inside the achromat. This makes the sextupole strength needed for chromaticity correction extremely high. In order to solve this problem two QBA cells are combined with a dispersive middle straight to add a pair of chromatic sextupoles SD/SF for chromaticity correction. Fig. 2 shows the linear optics of the Double QBA (DQBA). The tunes per QDBA cell are  $\Delta\nu_x/\Delta\nu_y$  3.50/1.24. The  $\beta_x/\beta_y$  at two ID straights are: 2.3 m/2.5 m, and 3.1 m/9.6 m. The dispersion at the middle straight is 0.06 m. The gradients of all the dipoles are smaller than 15T/m. The Diamond type combined function dipole can be used. All the gradients of independent quadrupole are smaller than 100T/m. The length of sextupole is 20 cm. It is put inside the quadrupole similar to MAX IV. The strength of SD/SF for linear chromaticity correction is -5400/6300 T/m<sup>2</sup>. This strength exceeds the maximum strength that ESRF proposed.

### SUMMARY

A preliminary DQBA design with sextupole scheme is presented. The emittance of the DQBA cell is 750 pm at 3

GeV. Under the constraints of the cell length and the number of super periods it already reaches the level of ultra-low emittance design compared to other MBA designs listed in Table 1. The ID beta functions at dispersion free straight are also close to the optimal values. A pair of sextupoles is used to correct the linear chromaticity. The strength of sextupole is highly correlated with the dispersion and the emittance. In order to keep the low emittance the sextupole strength exceeds the target value. Adding one more pair of SD/SF and dynamic aperture optimization are under investigation. A similar design to replace the matching DBA cell is also undergoing. The whole ring will be composed of 2 matching DQBA, 6 normal DQBA and two QBA extracted from the normal DQBA. The genetic optimization method will be used to help searching the best combinations of cell tunes, ID beta, sextupole scheme, dynamic aperture and beam lifetime of the ring.

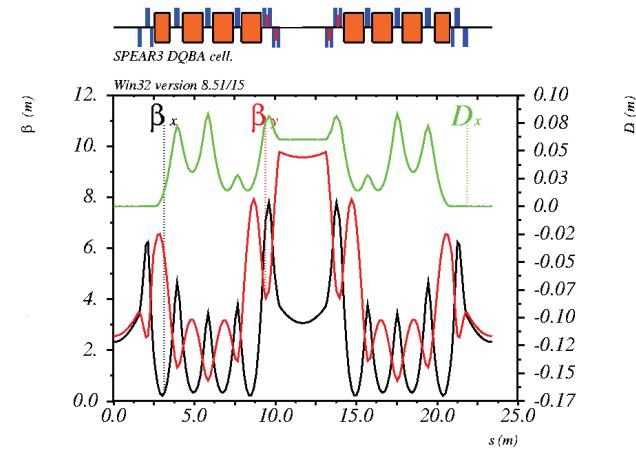


Figure 2: Optics function of SPEAR3 DQBA cell.

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