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On emergent gravity, ungravity and Λ

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ABSTRACT

In this work we study the *"ungravity"* modifications to the Friedmann equations. The *"ungravity"* contributions are encoded in a modified entropy-area relation. To derive the modified Friedmann equations we use the first law of thermodynamics and the new entropy-area relationship. From the modified Friedmann equations (in the late time regime) we find an effective cosmological constant. Therefore, this simple model can provide an *"ungravity"* origin to the cosmological constant Λ.

1. Introduction

The discovery of the late time acceleration of the Universe has deeply impacted our understanding of fundamental physics. If we propose a solution in the context of general relativity (GR) we either assume the existence of a primordial energy density or a new type of matter (not contained in the SM) with the property of having a negative pressure. In the standard model of cosmology (ΛCDM), the late time acceleration is attributed to a cosmological constant Λ. Although the ΛCDM model is compatible with observations there are some serious problems. Recently there is a growing tension between the Planck observations of the cosmic microwave background anisotropies and the local measurement of the Hubble constant [1–5]. Moreover, there are inconsistencies with traditional quantum field theory [6,7] (these problems have been addressed by different approaches [8]). Also, there is no known mechanism that guarantees a zero or nearly zero value for Λ in a stable vacuum, no explanation of the similar value between the associated energy density of Λ and the energy density of present day matter. Finally, one of the biggest issues, is the 120 orders of magnitude discrepancy between the predicted vacuum energy density derived from QFT and the energy density associated to Λ. Of course, if we entertain the possibility that the problems related to Λ and the current acceleration are consequence of the poor understanding of gravity and therefore a reformulation of gravity is warranted.

A recent approach for understanding the incompatibility of gravity with quantum mechanics, is to consider the gravitational interaction as an emergent phenomenon. Starting with an entropy proportional to the area, in [9] Einstein's equations are derived, verifying that one can consider GR as an entropic force. Interest on the entropic origin of gravity was rekindled by Verlinde [10]. He explored an entropic origin to Newtonian gravity and also proposed that gravity is an entropic forces.[1] Considering that in this formulation Newtonian gravity has an entropic origin, by introducing changes to the Bekenstein-Hawking entropy-area relation one can induce modifications to Newtonian gravity (see [11] and references therein). In a more recent paper [12], Verlinde claims that the dark matter and dark energy problems can be simultaneously solved in the emergent formulation of gravity. The dark matter predictions for this theory have been put to test in several works [13,14]. Even if Verlinde's proposal is not a rigorous formulation, it has some intriguing ideas that warrant a more detailed exploration. One interesting idea is the connection of entropic gravity to the dark matter and dark energy problems, giving a new approach to study dark sector of the Universe.

In the last decade, the presence of *"unparticle"* degrees of freedom in low energy physics was extensively explored in the context of particle physics. These *"unparticles"* are originated from a UV scale invariant sector that couples to the SM. Scale invariant QFT's represent massless particles, if we extend the notion of scale invariance to include a new kind of *"particle"*, it will have a contin-

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[1] In this context, gravity is similar to the emergent forces that are present in polymers.

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uous mass spectrum and therefore no definite mass, for this reason this extension is known as *"unparticle"* sector [15]. The effects of the *"unparticle"* sector are easily calculated and can be probed in current accelerators, unfortunately the search for these effects has been negative. Nonetheless, one can explore this new sector in the context of gravity. There have been attempts to study *"unparticle"* effects in connection to Newtonian gravity. The analysis is based on scalar modifications [16] to the gravitational potential. Also in [17], the authors study the effects of a scalar *"unparticle"* in the cosmological scenario. This approach introduces an *"unparticle"* scalar field. Because they do not consider an *"unparticle"* to the graviton field, they are not extensions to GR (we do have to clarify that in [15] they are working with extension to Newtonian gravity so the modifications can be considered extensions to gravity). If we want to consider *"unparticle"* extension to GR we need to assume the existence of *"ungravitons"*. In [18], the authors find an effective action that allows the study of gravitational effects beyond the weak field approximation. Moreover, in [19] using an effective action that incorporates *"unparticle"* effects to GR, the authors study the ungravity effects to Schwarzschild black hole and derive its temperature and entropy. They find that the entropy area relationship [19], is given by

$$S=\frac{k_Bc^3}{4\hbar G}\frac{(2\sqrt{\pi}R)^{2-2d_U}}{d_U\Gamma_U}A^{d_U}. \tag{1}$$

Inspired in these ideas, we explore the late time behavior of the Friedmann-Robertson-Walker (FRW) universe in the context of entropic gravity, by considering the effects of the *"ungravity"* sector. For this, we will calculate the modification to Friedmann equations and analyze it late time behavior. In order to calculate the modifications to the Friedmann equations, we will use the first law of thermodynamics [20,21] in conjunction with a modified entropy-area relationship. The modified entropy-area relationship, was derived from the *"ungravity"* effective action for the Schwarzschild black hole. Finally, analyzing the late time behavior we find an effective cosmological constant in terms of to the *"ungravity"* parameters.

The paper is organized as follows. In section 2, we briefly review the ungravity modification to gravity and the new entropy area relationship and derive the modified Friedmann equations. Section 3, is devoted for discussion and final remarks.

2. The ungravity Friedmann equations

As is well known the Friedmann equations together with the continuity equation, is all that is needed to study the dynamics of the universe. Using the Clausius equation and a linear relationship between the entropy and the area, one can derive Friedmann equations [20]. To study modifications to the Friedmann equations, we can start from a modified entropy-area relationship and derived the modified equations [21]. In this approach the *"new physics"* are encoded in the entropy-area expression. The ungravity effective action is composed by the actions for classical matter, classical gravity and "quantum" ungravity. In [19], the authors consider this model as a perturbation of Einstein gravity. The effective action for the ungravitons extends Einstein-Hilbert action to include ungraviton dynamics. Nonetheless, when solving the field equations, we can leave the l.h.s. to have the usual form for Einsteins equations and put the ungravity terms together with the usual matter. Therefore they can solve the model as usual gravity coupled to exotic matter.

To explore the effects of the *"ungravity* sector to cosmology, we use an entropy-area relationship that includes the *"ungravity"* degrees of freedom [18]. From the effective action for the Schwarzschild black hole, the *"ungravity* contributions to the temperature and entropy of the black-hole are calculated [19]. The temperature for the Schwarzschild black hole after incorporating the *"ungravity* effects is

$$T_U=\frac{\hbar c}{4\pi k_B\tilde{r}_h}\left[1+\frac{2(2d_U-1)\Gamma_U}{1+\Gamma_U\left(\frac{R}{\tilde{r}_h}\right)^{2d_U-2}}\left(\frac{R}{\tilde{r}_h}\right)^{2d_U-2}\right], \tag{2}$$

where Γ_U is defined as

$$\Gamma_U=\frac{2}{\pi^{2d_U-1}}\frac{\Gamma(d_U-1/2)\Gamma(d_U+1/2)}{\Gamma(2d_U)}, \tag{3}$$

and R is

$$R=\frac{1}{\lambda_U}\left(\frac{M_{\rm Pl}}{M_U}\right)^{1/(d_U-1)}. \tag{4}$$

The constant, M_U is the *"ungravity"* coupling constant and is related to the interaction between the *"ungraviton"* and the usual matter. Also, λ_U is the critical energy scale at which the scale invariant properties of *"ungraviton"* emerge. Finally, d_U is the scaling parameter[2] that labels the continuous mass spectrum of the *"ungraviton"* and can take values $1<d_U<2$. The thermodynamic energy of the system as a function of the horizon is

$$U(\tilde{r}_h)=\tilde{r}_h\frac{c^4}{2G}\left(\frac{1}{1+\Gamma_U\left(\frac{R}{\tilde{r}_h}\right)^{2d_u-2}}\right), \tag{5}$$

from which we obtain

$$dS_U=d(A)\frac{k_Bc^3}{4\hbar G}\left(\frac{1}{1+\Gamma_U\left(\frac{R}{\tilde{r}_h}\right)^{2d_U-2}}\right). \tag{6}$$

Finally, integrating the previous equation we get Eq. (1), the *"ungravity"* entropy. This new entropy-area relationship was used to obtain the *"ungravity"* modifications to Newtonian gravity [22]. The correction was derived using the entropic approach and is consistent with the result obtained by only considering the *"unparticle"* contribution [23]. This gives confidence that the contributions derived from an entropic formulation are encoded in the modified entropy-area relationship. Going back to Eq. (1), this expression includes the *"ungravity"* effects. Although for $d_U=1$ the entropy and temperature have the same functional form as in GR,[3] $d_U=1$ is not valid value for this theory (for example, Eq. (4) is not defined). Therefore, $d_U=1$ is a not regular limit for the theory.

To derive the *"ungravity"* Friedmann equations we will follow the approach in [20], where the Friedmann equations are calculated using the Clausius equation and the entropy-area relationship evaluated at the apparent horizon. This approach allows us to introduce modifications to the Friedmann equations from the entropy-area relationship.

We start with the FRW metric,

$$ds^2=-dt^2+a^2(t)\left(\frac{dr^2}{1-\kappa r^2}+r^2d\Omega^2\right), \tag{7}$$

comparing with $ds^2=h_{ab}dx^adx^b+\tilde{r}^2d\Omega^2$, we can identify h_{ab}. Following the procedure in [20], we introduce the work density W and the energy-supply vector Ψ

[2] Although we are restricting to $1<d_U<2$, higher values can't be dismissed.

[3] For $d_U=1$, the temperature is $T=\frac{\hbar c}{2\pi k_B\tilde{r}_H}$ and the entropy is $S=\frac{k_Bc^3}{4\hbar G}A$. Although these expressions have the correct functional form, we must remember that $d_U=1$ is not valid.

$$W = -\frac{1}{2}T^{ab}h_{ab}, \qquad \Psi_a = T_a^b \partial_b \tilde{r} + W \partial_a \tilde{r}, \tag{8}$$

where $\tilde{r} = a(t)r$ and T_{ab} is the projection of $T_{\mu\nu}$ in the normal direction of the 2-sphere. Using the energy-momentum tensor for a perfect fluid, the work density and the energy supply vector are

$$W = \frac{1}{2}(\rho - P), \tag{9}$$
$$\Psi_a = \left(-\frac{1}{2}(\rho + P)H\tilde{r}, \frac{1}{2}(\rho + P)a\right).$$

The amount of energy δQ crossing the apparent horizon during the time interval dt is given by

$$\delta Q = -A\Psi = A(\rho + p)H\tilde{r}_h dt, \tag{10}$$

where $A = 4\pi\tilde{r}_h^2$ is the area of the apparent horizon and $\tilde{r}_h$ is the radius of the apparent horizon

$$\tilde{r}_h = \frac{1}{\sqrt{H^2 + \kappa/a^2}}, \tag{11}$$

the apparent horizon is obtained from $h^{ab}\partial_a\tilde{r}\partial_b\tilde{r} = 0$.

From this point forward we will use the area-entropy relationship defined at the apparent horizon, this area-entropy relationship is analogous to one we get at the black hole horizon. Before deriving the Friedman equations that include the ungravity contribution, we will derive the Friedmann equations from the Hawking-Bekenstein entropy.

From the Clausius relation $\delta Q = T_U dS$ and the continuity equation for a perfect fluid $\dot{\rho} = -3H(\rho + p)$, we get

$$\frac{8\pi G}{3c^4}\frac{\partial\rho}{\partial t} = \frac{d(4\pi/A)}{dt}. \tag{12}$$

After integrating both sides of the equation and substituting the radius of the apparent horizon Eq. (11), we recover the Friedman equation.

$$H^2 + \frac{\kappa}{a^2} = \frac{8\pi G}{3c^4}\rho. \tag{13}$$

This is the usual Friedmann equation one derives from GR, moreover, to have a cosmological constant it has to be introduced by hand. To study the effects of the ungravity contribution, we follow the same procedure for arbitrary d_U (as already mentioned we will restrict for $1 < d_U < 2$). As before, using the Clausius relation and the entropy area relationship, we now get

$$\begin{aligned} \frac{8\pi G}{3}\frac{\partial\rho}{\partial t} = \\ \frac{c^4}{2}\left[1 + \frac{2(2d_U - 1)}{1 + \Gamma_U(R^2(4\pi/A))^{d_U - 1}}\Gamma_U\left(R^2\frac{4\pi}{A}\right)^{d_U - 1}\right] \\ \times\left[\frac{1}{1 + \Gamma_U(R^2(4\pi/A))^{d_U - 1}}\right]\frac{d(4\pi/A)}{dt}. \end{aligned} \tag{14}$$

Integrating both sides of the equation we get

$$\begin{aligned} \frac{8\pi G}{3}\rho = \frac{c^4 v}{2\Gamma_u^\alpha R^2}F_1^{(2)}\left(1, \alpha; 1 + \alpha; -v^{1/\alpha}\right) \\ - \frac{c^4(2d_U - 1)}{\Gamma_u^\alpha R^2}\frac{\alpha v}{(1 + v^{1/\alpha})} \\ + \frac{\alpha c^4(2d_U - 1)v}{\Gamma_u^\alpha R^2}F_1^{(2)}\left(1, \alpha; 1 + \alpha; -v^{1/\alpha}\right), \end{aligned} \tag{15}$$

where $\alpha = 1/(d_U - 1)$ and $v = 4\pi\Gamma_U^\alpha R^2 A^{-1}$.

$F_1^{(2)}(a, b; c; z)$ is the hypergeometric function, for $|z| < 1$ is defined by the series expansion

$$F_1^{(2)}(a, b; c; z) = \sum_{n=0}^{\infty}\frac{\Gamma(a + n)\Gamma(b + n)\Gamma(c)}{\Gamma(a)\Gamma(b)\Gamma(c + n)\Gamma(n + 1)}z^n. \tag{16}$$

Using the area of apparent horizon is $A = 4\pi/(H^2 + k/a^2)$ we finally arrive at

$$\begin{aligned} \frac{8\pi G}{3}\rho = -\frac{c^4(2d_U - 1)}{d_U - 1}\frac{H^2 + \frac{\kappa}{a^2}}{1 + \Gamma_U\left[R^2\left(H^2 + \frac{\kappa}{a^2}\right)\right]^{(d_U - 1)}} \\ + \frac{c^4(5d_U - 3)}{2(d_U - 1)}\sum_{n=0}^{\infty}\frac{(-1)^n\Gamma_U^n R^{2n(d_U - 1)}}{(d_U - 1)(n + \frac{1}{d_U - 1})}\left(H^2 + \frac{\kappa}{a^2}\right)^{n(d_U - 1) + 1}. \end{aligned} \tag{17}$$

To study the dynamics of the Universe we need to introduce the appropriate matter content, (i.e., radiation, dust, scalar field, etc.) and solve the resulting equations. As we are interested in a more generic result that does not depend on the particular type for the matter density and pressure we don't give a particular type of matter. Because Eq. (17) is very complicated, we will make some assumption and approximations.

We will take $d_U = 3/2$, as for this case the entropy will scale with the volume. Entropy terms that scale on the volume are related to non gravitational degrees of freedom and are not present in GR (it is worth mentioning that volumetric corrections to the entropy of black holes have been derived in loop quantum gravity [24]). Moreover, in [12] it is argued that volumetric contributions to the entropy term can be related to dark energy. After taking $d_U = 3/2$ we have

$$\begin{aligned} \frac{8\pi G}{3}\rho = \frac{9c^4}{2}\sum_{n=0}^{\infty}\frac{2(-1)^n\pi^{-2n}R^n}{n + 2}\left(H^2 + \frac{\kappa}{a^2}\right)^{\frac{n}{2} + 1} \\ - 4c^4\frac{H^2 + \frac{\kappa}{a^2}}{1 + \pi^{-2}\left[R^2\left(H^2 + \frac{\kappa}{a^2}\right)\right]^{1/2}}. \end{aligned} \tag{18}$$

This modified Friedmann equation was originally derived in terms of the hypergeometric function, therefore the convergence of r.h.s. of Eq. (15) is guaranteed for $|v^{d_U - 1}| < 1$, therefore $\Gamma_U R^{2(d_U - 1)}(H^2 + k/a^2)^{d_U - 1} < 1$.

For $d_U = 3/2$, the constraint takes the form $\pi^{-2}R(H^2 + k/a^2)^{1/2} < 1$, from here on the units we are employing we have $c = 1$. In this approximation, to leading order we get

$$\frac{8\pi G}{3}\rho = \frac{9}{2}\left(\left(H^2 + \frac{\kappa}{a^2}\right) - \frac{2R\left(H^2 + \frac{\kappa}{a^2}\right)^{3/2}}{3\pi^2}\right) - 4\left(H^2 + \frac{\kappa}{a^2}\right). \tag{19}$$

Now we solve for $\left(H^2 + \frac{\kappa}{a^2}\right)$ and get

$$H^2 + \frac{\kappa}{a^2} = \frac{\pi^2(\pi^2 + \sqrt[3]{M_2 + M_3})}{108R^2} - \frac{M_1}{8748\pi^2R^2\sqrt[3]{M_2 + M_3}}, \tag{20}$$

where M_1, M_2, M_3 are defined as follows

$$\begin{aligned} M_1 = 93312\pi^5 g\rho R^2 - 81\pi^8, \\ M_2 = 497664g^2\rho^2R^4 - 1728\pi^3 g\rho R^2 + \pi^6 \\ M_3 = 13824\sqrt{1296g^4\rho^4R^8 - \pi^3g^3\rho^3R^6}. \end{aligned} \tag{21}$$

As we are interested in the late time evolution,[4] we analyze the limit $t \to \infty$, this is equivalent to the limit $\rho \to 0$,

$$H^2 + \frac{\kappa}{a^2} = \frac{\pi^4}{36R^2}, \tag{22}$$

from this equation in this limit, we can define an effective cosmological constant.

Finally, the cosmological constant (for the case $d_U = 3/2$) in terms on the *"ungravity"* parameters is

$$\Lambda_{eff} \sim \lambda_U^2 \left(\frac{M_U}{M_{pl}}\right)^{\frac{2}{d_u - 1}}. \tag{23}$$

3. Discussion and final remarks

In this work, we consider the *"unparticle"* effects in the cosmology. This is done in order to try to answer the question, is *"unparticle"* physics relevant at the cosmological scale?. The result of the previous section point to a positive answer. In particular it gives insight on a possible origin to the cosmological constant from the *"ungravity"* sector. So let us take seriously this possibility and consider that the cosmological constant is originated from the *"ungravity"* sector.

We can impose the current value of Λ and find a relationship between M_U and λ_U. In Fig. 1, the lines represent the values of λ_U and M_U that give the correct value for the cosmological constant Λ. For the values of d_U considered in Fig. 1 we can rewrite the effective cosmological constant as $\Lambda_{eff} \sim \frac{1}{R^2}$. Then for the case $d_U = \frac{3}{2}$ we have can conclude that the degrees of freedom give an effective cosmological constant.

Now we can ask, is Eq. (23) is valid for other values of d_U? If the answer is positive, then this result is more or less general for the *"ungravity"* sector, if the answer is negative, at least it is consistent with the fact the volumetric contributions to the entropy are related to the dark energy sector.

For $d_U = \frac{4}{3}$ Eq. (23) holds, but for arbitrary values of d_U an analytical solution for Λ_{eff} in terms of M_U and λ_U can not be found. Nonetheless, if we consider that for $d_U = \frac{3}{2} + \epsilon$ with ϵ a small number the functional behavior of Λ_{eff} as a function of R is the same (we know it holds for $d_U = \frac{4}{3}$), therefore we can argue that the cosmological constant from the *"ungravity"* sector is

$$\Lambda_{eff} \sim \frac{1}{R^2}. \tag{24}$$

Of course there are couple of caveats to consider. First, M_U and λ_U are not necessarily the same that appear in the *"unparticle"* extensions of the standard model or the *"unparticle"* scalar extensions to Newtonian gravity and we have to be careful as we are lacking a fully consistent formulation of *"unparticle"* physics and the problems are inherited by the *"ungravity"* theory.

Therefore, if we want to attribute the dark energy sector of the Universe to the gravitational interaction, we need gravitational degrees of freedom whose entropy scales as $S \sim A^m$ with $m > 1$. Unfortunately this necessary implies a violation of the holographic principle, this is rather reasonable principle and systems dominated by gravity are expected to fulfill it [25]. Therefore one way out of this conundrum is to consider the *"ungravity"* sector. This sector can violate the holographic principle and has an entropy that scales[5] as $S \sim A^{d_U}$ with $1 < d_U < 2$. With this in mind we can argue that in the context of an *"ungravity"* theory, we can obtain an effective cosmological constant that depends on the *"ungravity"* coupling constant M_U and the critical energy scale λ_U that is the scale were invariant properties of *"ungraviton"* emerge.

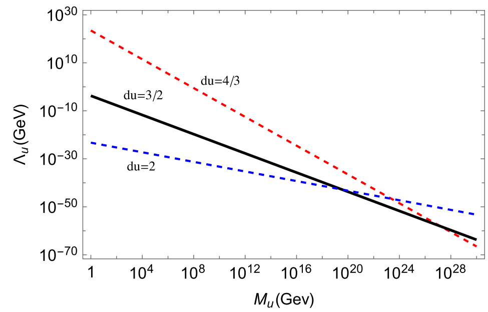

Fig. 1. Valid region of parameters λ_U and M_U for different values of d_U in order to have the values for the cosmological constant.

In summary, we have found an effective cosmological constant in the late time limit. As we are interpreting the effects from the ungravity sector as "dark energy". This "dark energy" (or ungravity effects) are only constant in the asymptotic limit, but in general is a dynamical dark energy. Consequently, it will have effects on the H0 tension. Can this model solve the H0 tension? To answer this question, we can follow a similar approach as in [1]. We will need to analyze the perturbations starting from the full effective theory because it is not clear how derive them from the formalism we have used. This is an interesting line of research and will be reported elsewhere.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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